TIME-FREQUENCY SURROGATES

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[1] partial joint work with Pierre Borgnat, Cédric Richard and Jun Xiao

Outline

1. Surrogates?
2. Three variations:
    - Time-Frequency Distributions via signal spectrum
      $\longrightarrow$ testing for stationarity
    - Time-Frequency Distributions via ambiguity function
      $\longrightarrow$ detecting transients
    - Empirical Mode Decompositions via Intrinsic Mode Functions
      $\longrightarrow$ denoising data
3. Concluding remarks

Rationale

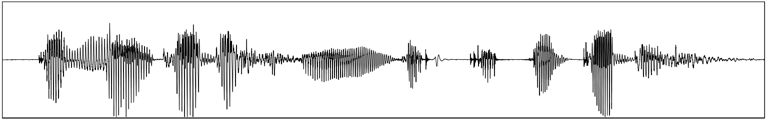
1. In many analysis/processing tasks, need for a “null hypothesis” reference:
   - stationary vs. nonstationary (test)
   - noise vs. signal (detection, denoising)
2. Elaborate the reference from the data
3. Use such surrogate data in some statistical way

Background

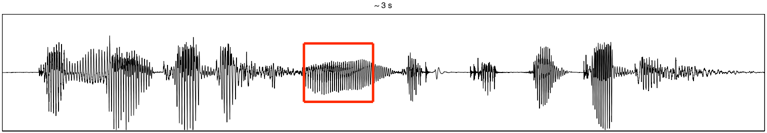
1. Surrogate data analysis previously introduced and used in the context of nonlinear dynamics (Theiler *et al.*, '92)
2. Partial overlap with bootstrap techniques and other resampling plans (Efron, '81)
3. First attempts in nonstationary signal processing:
   - Xiao, Borgnat & F., IEEE SSP Workshop '07
   - Xiao, Borgnat & F., EUSIPCO '07

nonstationary

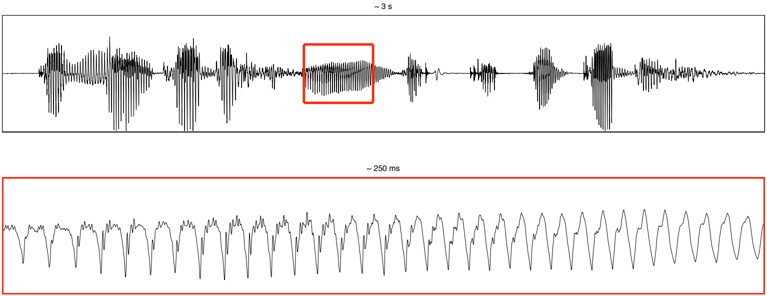
~ 3 s



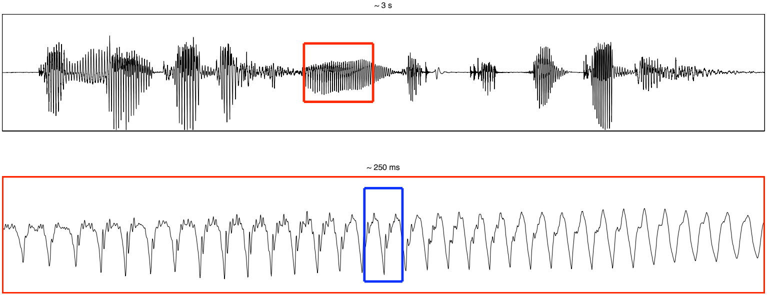
?


~3 s

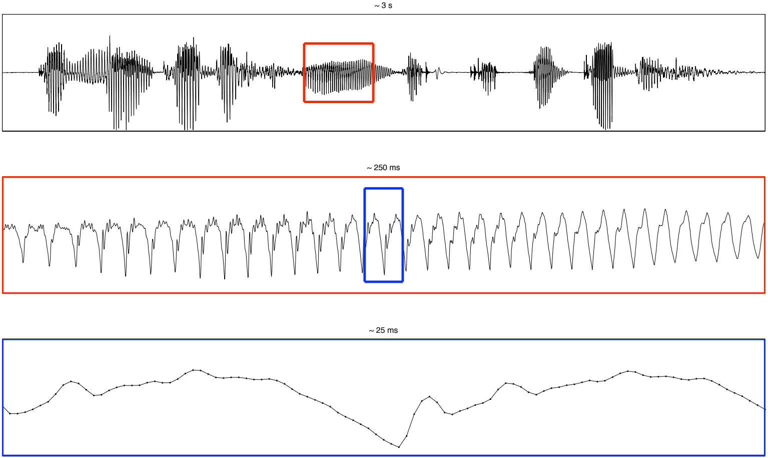
stationary



?


~ 3 s
~ 250 ms

nonstationary!



Surrogate Stationarization

1. “Stationarity” is a relative property
2. Given an observation scale, “nonstationarity” $\Rightarrow$ “local” $\neq$ “global” $\Rightarrow$ time-frequency (TF) analysis
3. Tests call for a stationary reference $\Rightarrow$ surrogate data:
   - nonstationarity encoded in time evolution or, equivalently, in spectrum phase
   - stationarization via spectrum phase randomization
4. Basic algorithm:

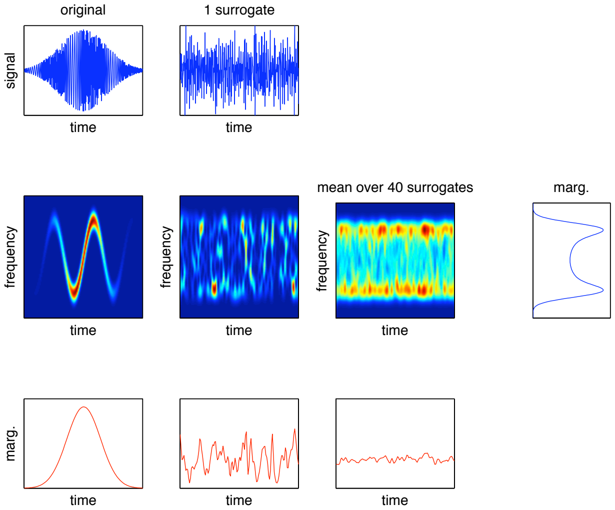
$\hat{x} = \mathrm{FFT}(x)$ % x = original data

draw WGN $\epsilon(t)$ and compute $\hat{\epsilon} = \mathrm{FFT}(\epsilon)$

$\hat{x} \leftarrow |\hat{x}| \exp\{i \arg \hat{\epsilon}\}$

$y = \mathrm{IFFT}(\hat{x})$ % y = surrogate data

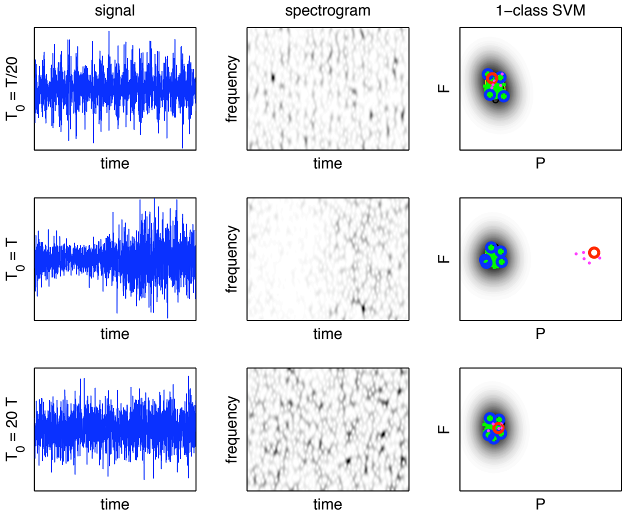
Illustration



Stationarity Test

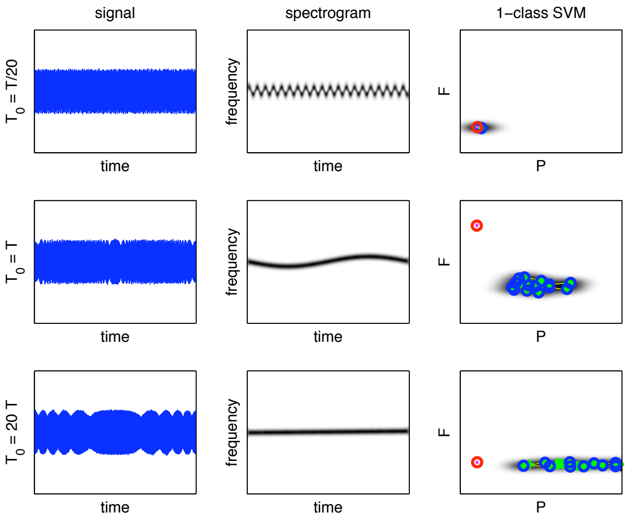
1. Compute, from the data, a set of J surrogates (typically, $J \sim 50$)
2. Attach to both data and surrogates a series of features aimed at comparing local vs. global behaviors, e.g., time fluctuations of
   - instantaneous power (P)
   - mean frequency (F)
3. Construct the test on a distance measure or a 1-class SVM classifier with surrogates as learning set

AM example



signal
spectrogram
1−class SVM
T_0 = T/20
T_0 = T
T_0 = 20 T
time
frequency
F
P

FM example


signal
spectrogram
1−class SVM
T0 = T/20
T0 = T
T0 = 20 T
time
frequency
F
P

Principle of Transient Detection

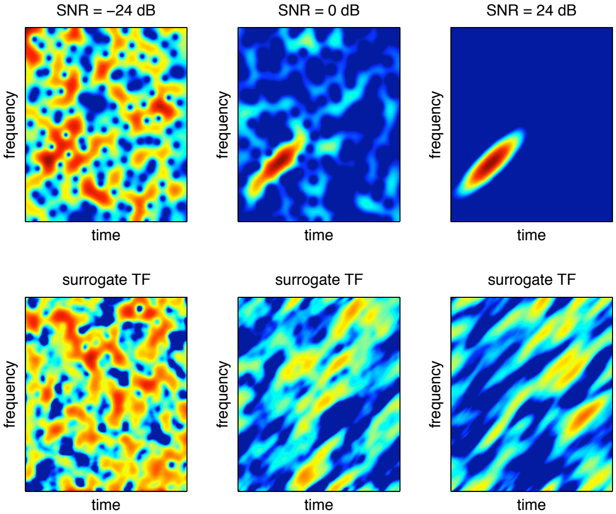
1. TF model = localized events in smoothly spread noise
2. In practice, only one observation
   $\Rightarrow$ statistical fluctuations in the estimated noise background
   $\Rightarrow$ false transients
3. Way out = compare data to a TF stationarized reference
   $\Rightarrow$ surrogates from a 2D phase randomization with a positivity constraint (spectrogram)
4. Detection via an entropy measure

Algorithm

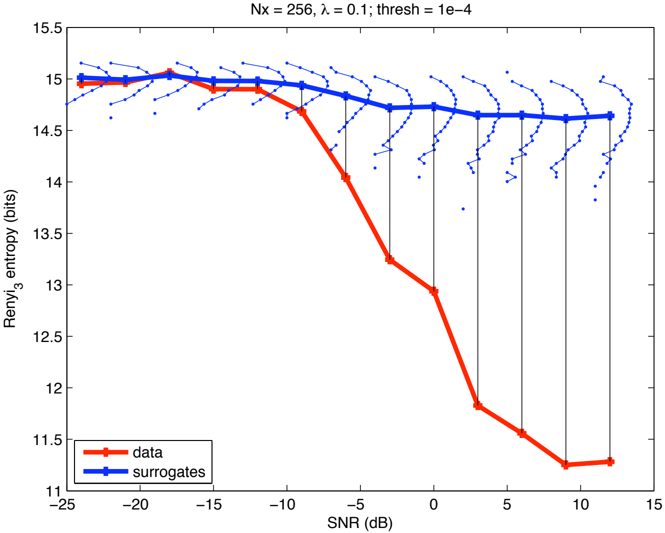
$A_x = \text{2D-FFT}(S_x)$ % S_x = spectrogram
draw WGN $\epsilon(t)$ and compute $A_\epsilon = \text{2D-FFT}(S_\epsilon)$
$A_x \leftarrow |A_x| \exp\{i \arg A_\epsilon\}$

test = $\text{test}_0 > \text{thresh}$
$r = 0$
while *test* $\geq$ *thresh* **do**
    $r \leftarrow r + 1$
    draw WGN $\epsilon(t)$ and compute $A_\epsilon = \text{2D-FFT}(S_\epsilon)$
    $A_x = \text{2D-FFT}([\text{2D-IFFT}(A_x)]_+)$
    $A_x \leftarrow |A_x| \exp\{i(\arg A_x + \lambda^r \arg A_\epsilon)\}$
    test $\leftarrow \text{vol}(S_x < 0)/\text{vol}(S_x)$

Example


SNR = −24 dB
SNR = 0 dB
SNR = 24 dB
surrogate TF
surrogate TF
surrogate TF
frequency
time

Performance



EMD-based Denoising

Low-frequency signal embedded in broadband noise:

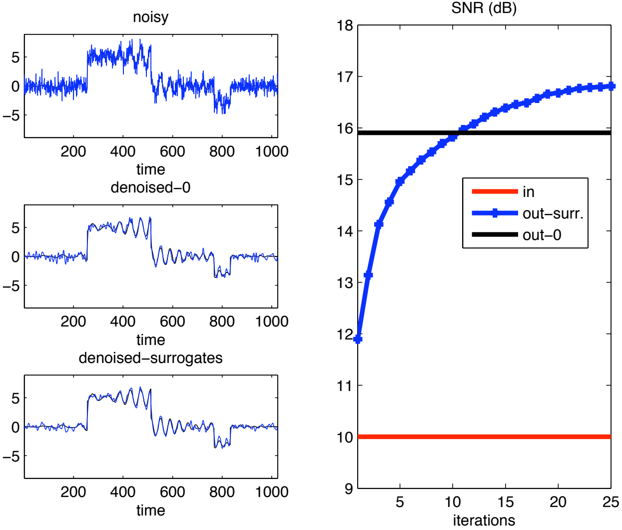
1. EMD (Empirical Mode Decomposition) $\Rightarrow$ most noise in 1st IMF (Intrinsic Mode Function)
2. Rather than removing 1st IMF, combine it with surrogates
3. Can be viewed as a matched adaptive implementation of "Ensemble EMD" (Huang *et al.*, '05)

Algorithm

for $r = 1 : R$ **do**

- $imf_{1:B} := emd(x)$ % x = signal
  - $\hat{imf}_1 = \text{FFT}(imf_1)$
  - draw WGN $\epsilon(t)$ and compute $\hat{\epsilon} = \text{FFT}(\epsilon)$
  - $\hat{imf}_1 \leftarrow |\hat{imf}_1| \exp\{i \arg \hat{\epsilon}\}$
  - $jmf_1 = \text{IFFT}(\hat{imf}_1)$ % surrogate IMF
- $imf_1 \leftarrow (imf_1 + \lambda^r jmf_1)/(1 + \lambda^r)$ % average
- $x = \sum_{k=1}^{B} imf_k$ % reconstruction

Example and performance



Concluding remarks

- Surrogates technique as a data-driven resampling plan
- Efficiency illustrated on 3 different problems
  1. testing stationarity
  2. detecting transients
  3. denoising LF signals
- Possible variations with extra constraints (e.g., pdf)
- Needs for more detailed analysis (calibration, performance evaluation, etc.)