A PROCEDURE FOR COMPUTING THE MEAN INTENSITY AND THE FLUX

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We take the transfer equation in the form

$$\frac{\mu^2}{\chi}\frac{d}{dx}\left(\frac{1}{\chi}\frac{d}{dx}J\right) = J - S \tag{1}$$

with $J(\mu) = \dfrac{I(\mu) + I(-\mu)}{2}$.

x is an arbitrary depth variable, and χ the associated absorption coefficient where

$$\frac{dI}{I} = \chi\, dx$$

A division of the x axis is given: $\{x_i\}_{i=0,N+1}$

$$x_0 = 0$$

Suppose that we know the values of $\chi_i = \chi(x_i)$, $S_i = S(x_i)$

Equation (1) is replaced by the approximation:

$$\frac{\mu^2}{\chi_i}\;\frac{\left(\frac{1}{\chi_{i+1}} + \frac{1}{\chi_i}\right)\frac{J_{i+1} - J_i}{x_{i+1} - x_i} - \left(\frac{1}{\chi_i} + \frac{1}{\chi_{i-1}}\right)\frac{J_i - J_{i-1}}{x_i - x_{i-1}}}{x_{i+1} - x_{i-1}} = J_i - S_i , \tag{2}$$

which we may write as

$$a_i J_{i-1} + b_i J_i + c_i J_{i+1} = S_i . \tag{3}$$

The boundary conditions are

$$\frac{\mu}{\chi}\left(\frac{dJ}{dx}\right)_0 = J_0 ,$$

which we replace by the approximate form:

$$\frac{\mu}{\chi_0}\,\frac{J_1 - J_0}{x_1 - x_0} = J_0 , \tag{4}$$

which we write as:

$b_0 J_0 + c_0 J_1 = 0$, and, at great depths, $J_{N+1} = S_{N+1}$.

This allows us to write the last of equations (3) in the form

$$a_N J_{N-1} + b_N J_N = S_N - C_N S_{N+1} \quad . \tag{5}$$

Thus we have a tridiagonal linear system:

$$b_0 J_0 + C_0 J_1 = 0 \tag{3}$$

$$a_i J_{i-1} + b_i J_i + C_i J_{i+1} = S_i \qquad i = 1, N-1(1) \tag{4}$$

$$a_N J_{N-1} + b_N J_N \qquad = S_N - C_N S_{N+1} \quad . \tag{5}$$

We resolve this system with the formulas:

$$d_0 = c_0 / b_0 \ , \ g_0 = 0$$

$$d_i = c_i / (b_i - a_i \cdot d_{i-1}) \ , \ g_i = (S_i - a_i S_{i-1}) / (b_i - a_i d_{i-1})$$
$$i = 1 (1) N$$

$$J_N = g_N$$
$$J_i = g_i - d_i \cdot J_{i+1} \text{ for } i = N (-1) 0 \ .$$

We repeat the process for values of μ forming a Gaussian division of the interval $(0, 1)$.

The mean intensity is given by

$$\bar{I}_i = \sum_{k=1}^{m} h_k J_i (\mu_k) \ .$$

To compute the flux we must compute:

$$F_i = F_0 + \int_0^{x_i} \chi(x) \left(\bar{I}(x) - S(x) \right) dx ,$$

where $F_0 = 4 \sum h_k \mu_k J_0(\mu_k)$,

since at $x = 0$ $\begin{cases} J(\mu) = \frac{1}{2} I(\mu) \\ I(-\mu) = 0 \end{cases}$.

When we require the values of the mean intensities at a large number of points, this method is more rapid than those consisting of the evaluation of the integrals

$$\frac{1}{2} \int_0^{\infty} E_1 (|\tau - t|) \, S(\tau) d\tau .$$

If we want N points, and use M values of μ, the classical method takes a time $\propto N^2$, whereas the method given here takes a time $\propto NM$. In general, we may use low values of M, of the order of 3 or 4. Furthermore, this method avoids the long calculation of the function E_1 .

A program for the calculation of the mean intensity following this method has been established at Meudon. It remains yet to study the precision of the results, as well as the optimum choice of the division points $\{ x_i \}$.