A METHOD FOR IMPROVING THE T(x) LAW

P. Feautrier

We take the equation of transfer in the form

$$\mu^2 \frac{d}{dx}\left(\frac{1}{\chi_\nu}\frac{d}{dx}J_\nu\right) = \chi_\nu J_\nu - \kappa_\nu B_\nu - \sigma \bar{I}_\nu ,$$
$$\chi_\nu = \kappa_\nu + \sigma . \tag{1}$$

Supposing that we know an approximate temperature law, we may linearize the equation, taking into account the radiative equilibrium condition

$$\int_0^\infty \kappa_\nu B_\nu \, d\nu = \int_0^\infty \kappa_\nu \bar{I}_\nu \, d\nu ,$$

$$\mu^2 \frac{d}{dx}\left(\frac{1}{\chi_\nu}\frac{dJ_\nu}{dx}\right) = \chi_\nu J_\nu - \frac{\kappa_\nu B_\nu}{\int_0^\infty \kappa_\nu B_\nu \, d\nu}\int_0^\infty \kappa_\nu \bar{I}_\nu \, d\nu - \sigma \bar{I}_\nu \tag{2}$$

Take a series of abscissae $\left\{ x_i \right\}_{i=0,\, N+1}$ with

$$x_0 = 0 ;$$

a Gaussian division of the interval (0,1): $\left\{ \mu_k \right\}_{k=1,\, M}$, and the dividing points for an integration formula for $\int .. \, d\nu \, \left\{ \nu_\ell \right\}_{\ell=1,\, L}$. Then in (2) we replace the differential operator by

$$\mu_k^2 \frac{\left(\frac{1}{\chi_{\ell i+1}} + \frac{1}{\chi_{\ell i}}\right)\frac{J_{k\ell i+1} - J_{k\ell i}}{x_{i+1} - x_i} - \left(\frac{1}{\chi_{\ell i}} + \frac{1}{\chi_{\ell i-1}}\right)\frac{J_{k\ell i} - J_{k\ell i-1}}{x_i - x_{i-1}}}{x_{i+1} - x_{i-1}}$$

and the integrals by the appropriate sums.

Now let J_i be the ML dimensional vector with components $J_{k\ell i}$. Then the equation of transfer becomes

$$A_i J_{i-1} + B_i J_i + C_i J_{i+1} = 0 , \tag{3}$$

A, B, and C being square matrices (ML × ML), B_i in particular being the matrix

$$\left\{B_{ik\ell k'\ell'}\right\} = \left\{ \mu_k^2 \frac{\left(\frac{1}{\chi_{\ell i+1}} + \frac{1}{\chi_{\ell i}}\right)\frac{1}{x_{i+1}-x_i} + \left(\frac{1}{\chi_{\ell i}} + \frac{1}{\chi_{\ell i-1}}\right)\frac{1}{x_i - x_{i-1}}}{x_{i+1} - x_{i-1}} + \chi_\ell \right\} \delta_{k\ell k'\ell'}$$

$$- \varphi_i(\nu_\ell)\, \kappa_{i\ell'} W_{\ell'} h_{k'} - \sigma_i h_{k'} \delta_{\ell\ell'} \quad ,$$

where the w_ℓ are the integration weights in ν ; h_k are the Gaussian weights in μ ; and φ is the function

$$\varphi(\nu) = \frac{\kappa_\nu B_\nu}{\int_0^\infty \kappa(\nu')B(\nu')d\nu'} \quad , \quad \int_0^\infty \varphi(\nu')d\nu' = 1 .$$

The boundary condition at x = 0 is

$$\frac{\mu}{\chi_\nu}\frac{dJ_\nu}{dx} = J_\nu \quad ,$$

which we approximate by

$$\frac{\mu_k}{\chi_{\ell_0}} \quad \frac{J_{k\ell_1} - J_{k\ell_0}}{x_1 - x_0} = J_{k\ell_0} \quad ,$$

which we put in the form

$$B_0 J_0 + C_0 J_1 = 0 .$$

As a second boundary condition we put

$$J_{\nu N+1} = B_\nu(T_{N+1})$$

at a great depth x_{N+1}. The real Milne problem in a semi-infinite atmosphere is obtained when x_{N+1} is made to go to infinity. T_{N+1} is a fixed temperature which serves as a temperature parameter for the star, and which will be related to the effective or the surface temperature. Putting T_{N+1} into the last of equations (3) we obtain

$$A_N J_{N-1} + B_N J_N = - C_N B(T_{N+1}) . \qquad (5)$$

The ensemble (3), (4) and (5) constitutes a linear system (tridiagonal by block)

$$
\begin{aligned}
B_0 J_0 &+ C_0 J_1 &&= 0 \\
A_i J_{i-1} + B_i J_i &+ C_i J_{i+1} &&= 0 \\
A_N J_{N-1} + B_N J_N & &&= -C_N B(T_{N+1}),
\end{aligned}
$$

which we resolve by the formulas

$$D_0 = B_0^{-1} C_0 \; ; \qquad D_i = \left(B_i - A_i D_{i-1}\right)^{-1}, \quad i = 1\,(1)\,N$$

$$J_N = \left(B_N - A_N D_{N-1}\right)^{-1} C_N J_{N+1}$$

$$J_i = -D_i J_{i+1}, \quad i = N-1\,(-1)\,0 .$$

Then we obtain an improved temperature distribution by solving the equation

$$\int_0^\infty \kappa_\nu B_\nu(T)\, d\nu = \int_0^\infty \kappa_\nu \bar{I}_\nu \, d\nu .$$

We compute $\bar{I}_\nu$ from the values of $J_{ik\ell}$ calculated above, using the equation

$$\bar{I}_{\nu_\ell}(x_i) = \sum_1^M h_k J_{ik\ell} .$$

The method is being programmed and will be applied to the standard typical model atmosphere: pure hydrogen, T_e=10000°K, g= 10^4cgs.

The problems yet to be studied are the possibility of shortening the numerical work by the introduction of

$$\mu_0^2 = \int_0^1 \mu^2 J(\mu)\, d\mu \Big/ \bar{I} ,$$

whose value will first be taken as 1/3 and will then be improved by iteration, and the optimum choice of the x_i.