TEXTURES

The model which is now discussed was proposed in my book "Oscillating patterns in image processing..." AMS (2001) Univ. Lecture Series, Vol. 22.

This model has been improved by

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$\mathcal{BV}$ = closure of compactly supported C^1-functions in BV

$\mathcal{BV} = \dot{W}^{1,1} = \{f \, ; \, \nabla f \in L^1\}$

G = dual space of $\mathcal{BV}$

$f \in G \iff f = \operatorname{div} F$

$F = (f_1, f_2)$

$\|F\|_\infty = \left\| \sqrt{|f_1|^2 + |f_2|^2} \right\|_\infty$

$\|f\|_G$ (also written $\|f\|_*$) is the infimum of $\|F\|_\infty$ where $f = \operatorname{div} F$.

Lemma 1. $|f| \in G \Rightarrow f \in G$ (but the converse is not true).

Lemma 2. If $u \in BV$, $v \in L^2$, then

$$\left| \int u(x) v(x) \, dx \right| \leq \|u\|_{BV} \|v\|_* .$$

Guy David measures: Let μ a signed Borel measure on $\mathbb{R}^2$. Then μ is a Guy David measure iff there exists a constant C s.t. for every disc $D(x, r)$ centered at $x \in \mathbb{R}^2$ with radius $r > 0$, one has

$$\boxed{|\mu|(D(x,r)) \leq C r.}$$

Theorem. Let μ be a non-negative Borel measure on $\mathbb{R}^2$. Then the following properties are equivalent ones

(a) μ is a Guy David measure

(b) μ is a (continuous) linear functional on BV.

Textures have small norms in G

G = dual of BV

Theorem. If $\theta(x) \in L^\infty(\mathbb{R}^2)$ and if $\nabla\theta = \mu$ is a Guy David measure, then there exists a constant $C = C(\theta)$ such that, for every $\omega \in \mathbb{R}^2$,

$$\boxed{\| \exp(i\omega \cdot x)\, \theta(x) \|_G \leq \frac{C(\theta)}{|\omega|}.}$$

When $|\omega|$ is large, $\exp(i\omega \cdot x)\,\theta(x)$ is a texture.

Example: $\theta(x) = 1$. We then have

$$\| \exp(i\omega \cdot x) \|_G = \frac{C_0}{|\omega|}$$

Example: $\cos^2(\omega \cdot x)\,\theta(x)$ is not small in G when $|\omega| \to +\infty$:

$$\cos^2(\omega \cdot x) = \frac{1 + \cos(2\omega \cdot x)}{2}$$

and $\| \cos^2(\omega \cdot x)\, \theta(x) \|_G \to \frac{1}{2} \| \theta \|_G$.