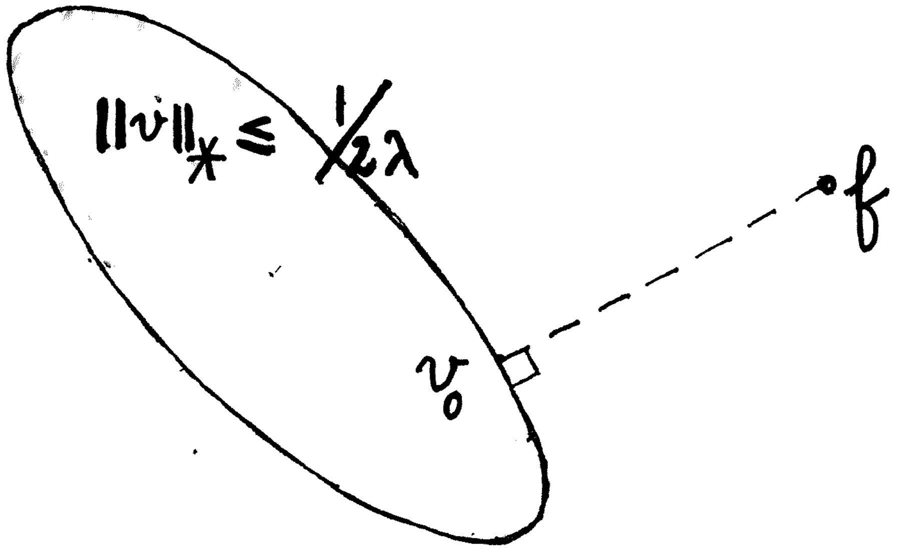
Theorem. [Antonin Chambolle]

If $f \in L^2(\mathbb{R}^2)$, then the optimal Osher-Rudin decomposition $f = u + v$ which minimizes $\|u\|_{BV} + \lambda \|v\|_2^2$ is given by $f = u_0 + v_0$ where

$$\boxed{v_0 = \operatorname{Arg\,inf} \{ \|f - v\|_2 \; ; \; \|v\|_* \leq 1/2\lambda \}}$$



Then $\|f\|_* \leq 1/2\lambda \Rightarrow u_0 = 0, \; v_0 = f$

$\|f\|_* > 1/2\lambda \Rightarrow f = u_0 + v_0$ where

$\|v_0\|_* = \dfrac{1}{2\lambda}$ and $\int u_0 v_0 \, dx = \|u_0\|_{BV} \|v_0\|_*$

$BV \subset L^2 \subset G = (BV)^*$

$\|\cdot\|_*$ is the norm in G = weak norm

$\lambda > 0$ is fixed

$f \in L^2(\mathbb{R}^2)$ and $u \in BV$ minimizes

$\|u\|_{BV} + \lambda \|f - u\|_2^2$.

<u>Theorem</u>. For $f_1 \in L^2$, $f_2 \in L^2$ we have

$$\boxed{\|u_2 - u_1\|_2 \leq 13\sqrt{\lambda}\, \|f_2 - f_1\|_*^{1/2} \left(\|f_1\|_2 + \|f_2\|_2\right).}$$

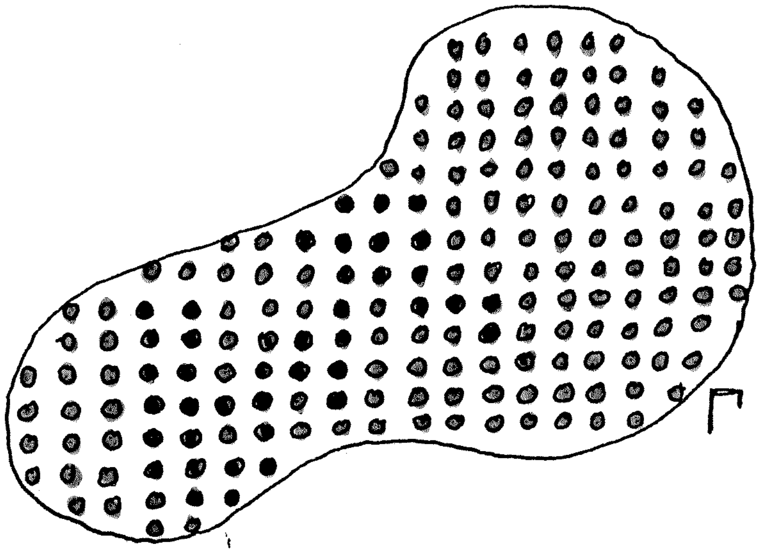
This is only interesting if

$\|f_2 - f_1\|_* \leq 1/\lambda$.

This does not hold for $\|v_2 - v_1\|_2$.

[Ali Haddad, Ph.D.]

Textures delimited by contours.



$\Gamma = \partial\Omega$

$f(x) = a$ (inside the small discs),
$f(x) = 0$ outside Ω
$f(x) = b$ if $x \in \Omega$, $x \notin$ small discs

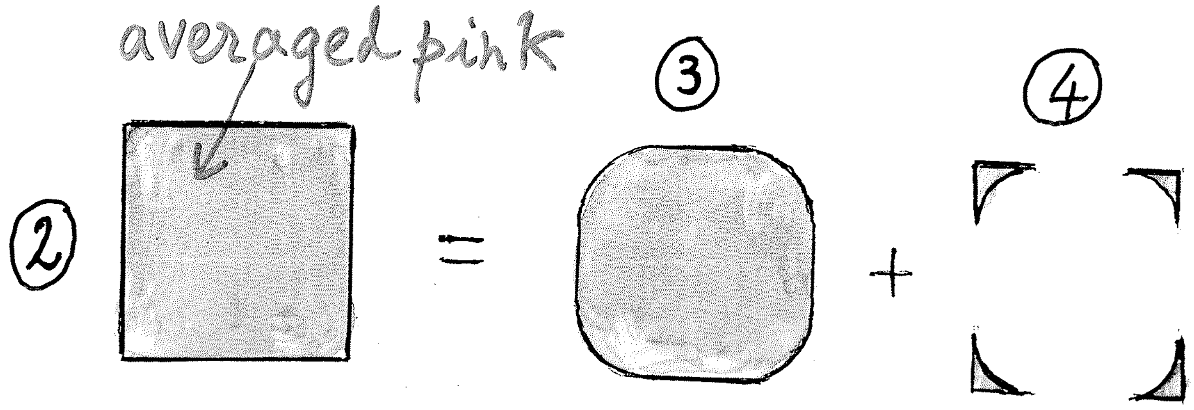
$$c = \frac{1}{|\Omega|}\int_\Omega f(x)\,dx.$$

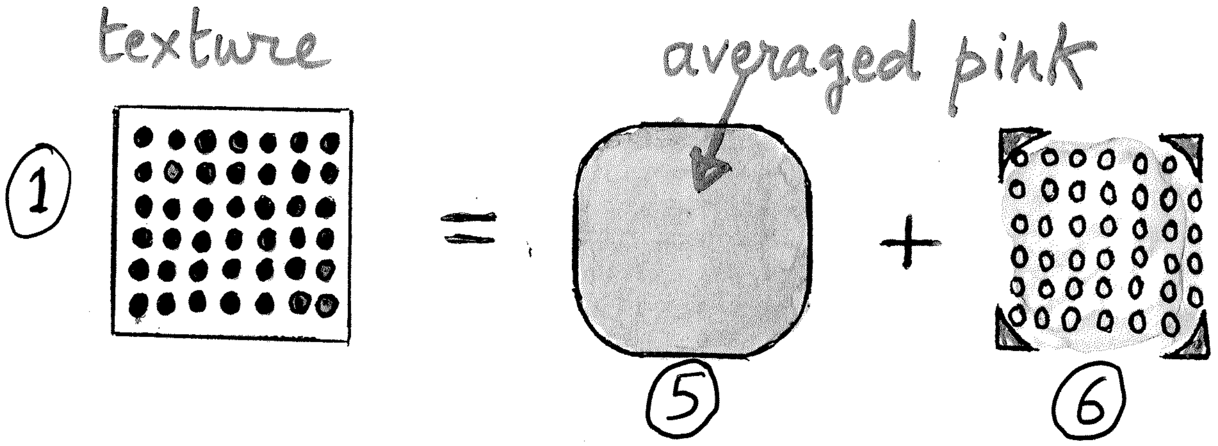
$\theta(x)$ = indicator function of Ω

$$\boxed{f(x) = c\,\theta(x) + w(x)}$$

where $\|w\|_G = O(\varepsilon)$, ε = radius of the small discs.

Is it the Osher-Rudin decomposition of f?

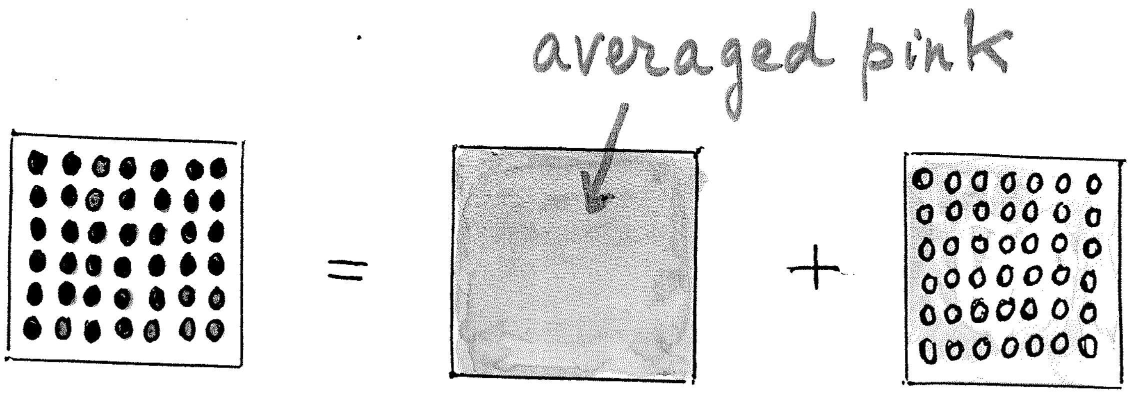




$$f = u + v$$

given by the Osher-Rudin algorithm.

This sharply contrasts with the expected decomposition:



zero mean texture

$$f = u + v$$