Scaling in Heartbeat Rate Variability

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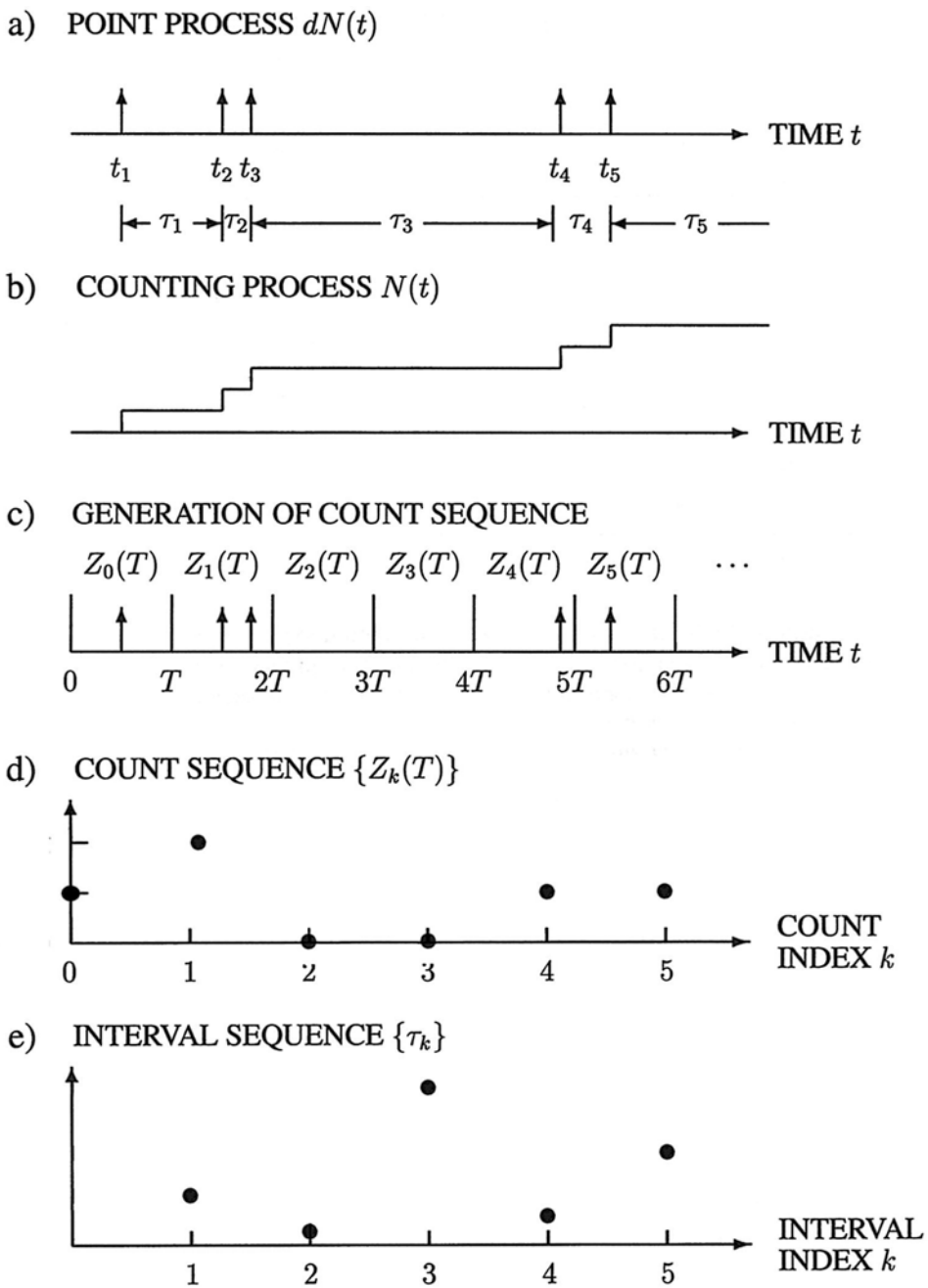
Colleagues:

- **Conor Heneghan, University College Dublin**
- **Steven Lowen, Harvard Medical School**
- **Robert Turcott, Stanford Medical School**
- **Markus Feurstein, Wirtschaftsuniversität Wien**
- **Stefan Thurner, Allgemeines Krankenhaus Wien**

Wavelet and Multifractal Analysis Summer School 2004

POINT PROCESSES

M. C. Teich 2004



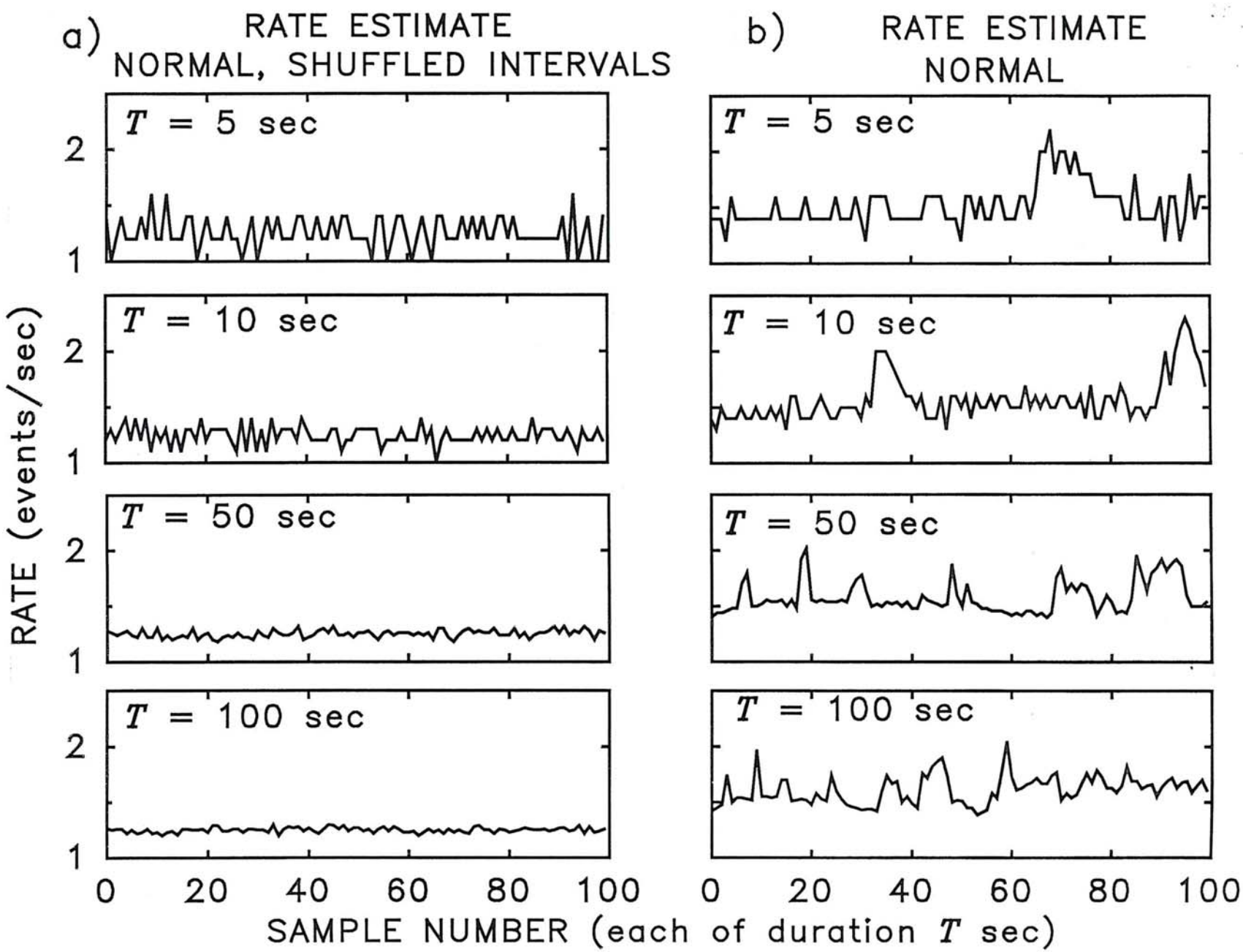
After Lowen & Teich,
Fractal-Based Point Processes,
(Wiley Series in Probability and Statistics, Hoboken, NJ, 2005)

After M. C. Teich 2004

FRACTAL-BASED POINT PROCESSES

- Fractal point processes
- Fractal-rate point processes

S. B. Lowen and M. C. Teich, *Fractal-Based Point Processes* (Wiley Series in Probability and Statistics, Hoboken, NJ, 2005)
ALMOST DONE!



After Turcott & Teich, *Ann. Biomed. Eng.* **24**, 269-293 (1996).

M. C. Teich 2004

INTERVAL-BASED MEASURES

M. C. Teich 2004

CONGESTIVE HEART FAILURE

INABILITY OF HEART TO INCREASE CARDIAC OUTPUT IN PROPORTION TO METABOLIC DEMANDS

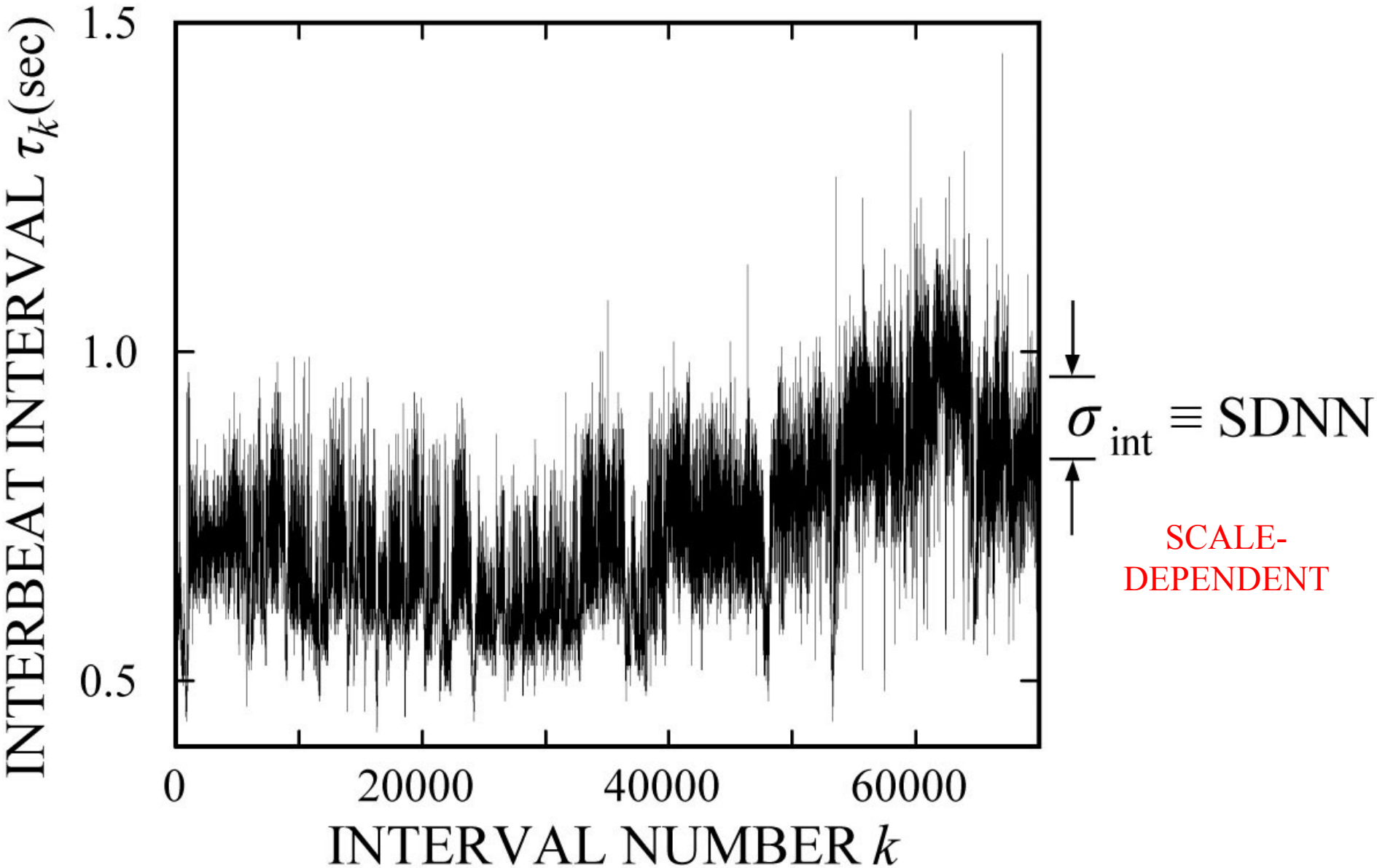
Symptom complex:
Many different presentations and etiologies

Typical symptoms:

- Shortness of breath
- Swelling in legs
- General fatigue and weakness

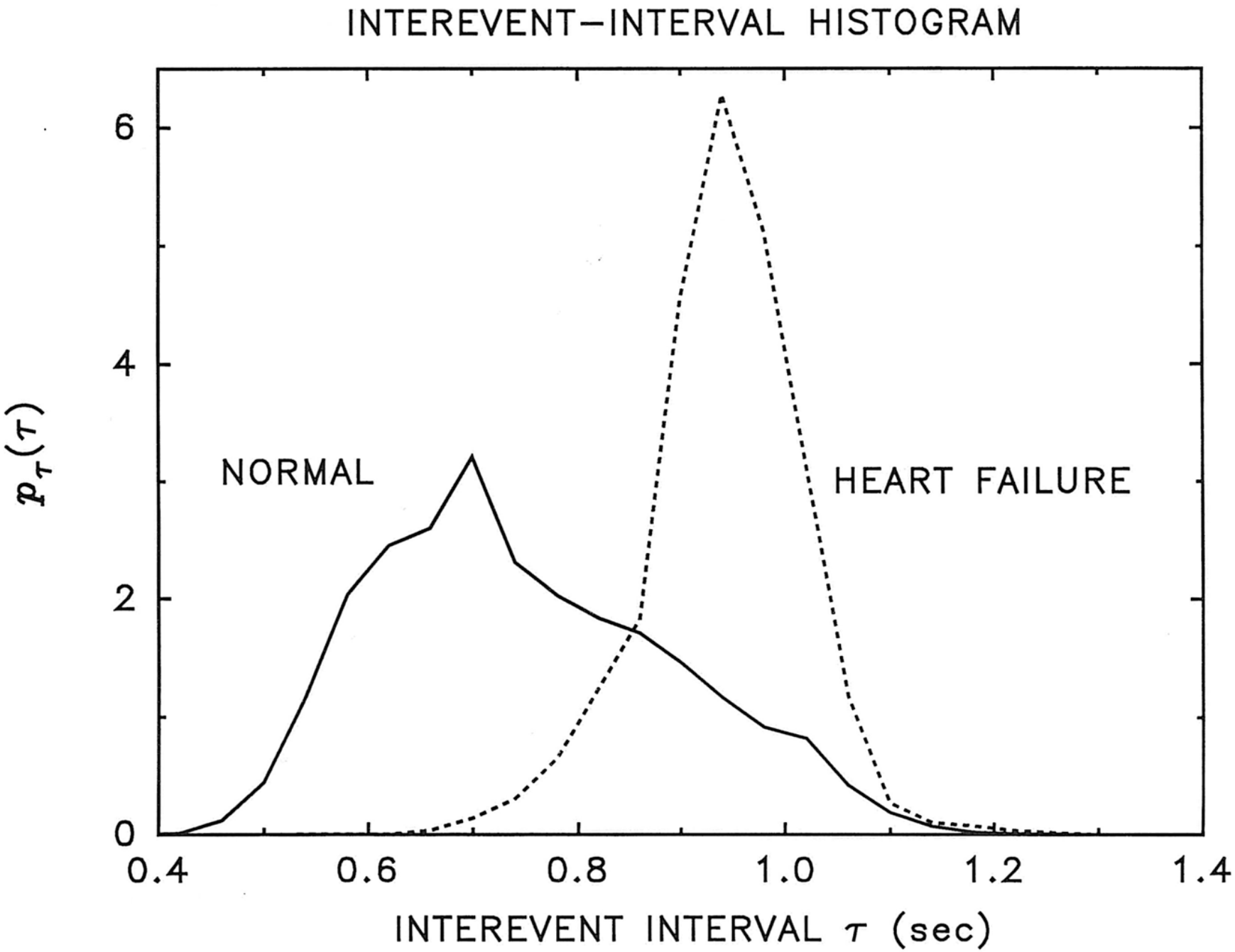
Clinical diagnostics:

- Ascultate heart
- Carotid pulse
- Electrocardiogram
- Chest radiograph

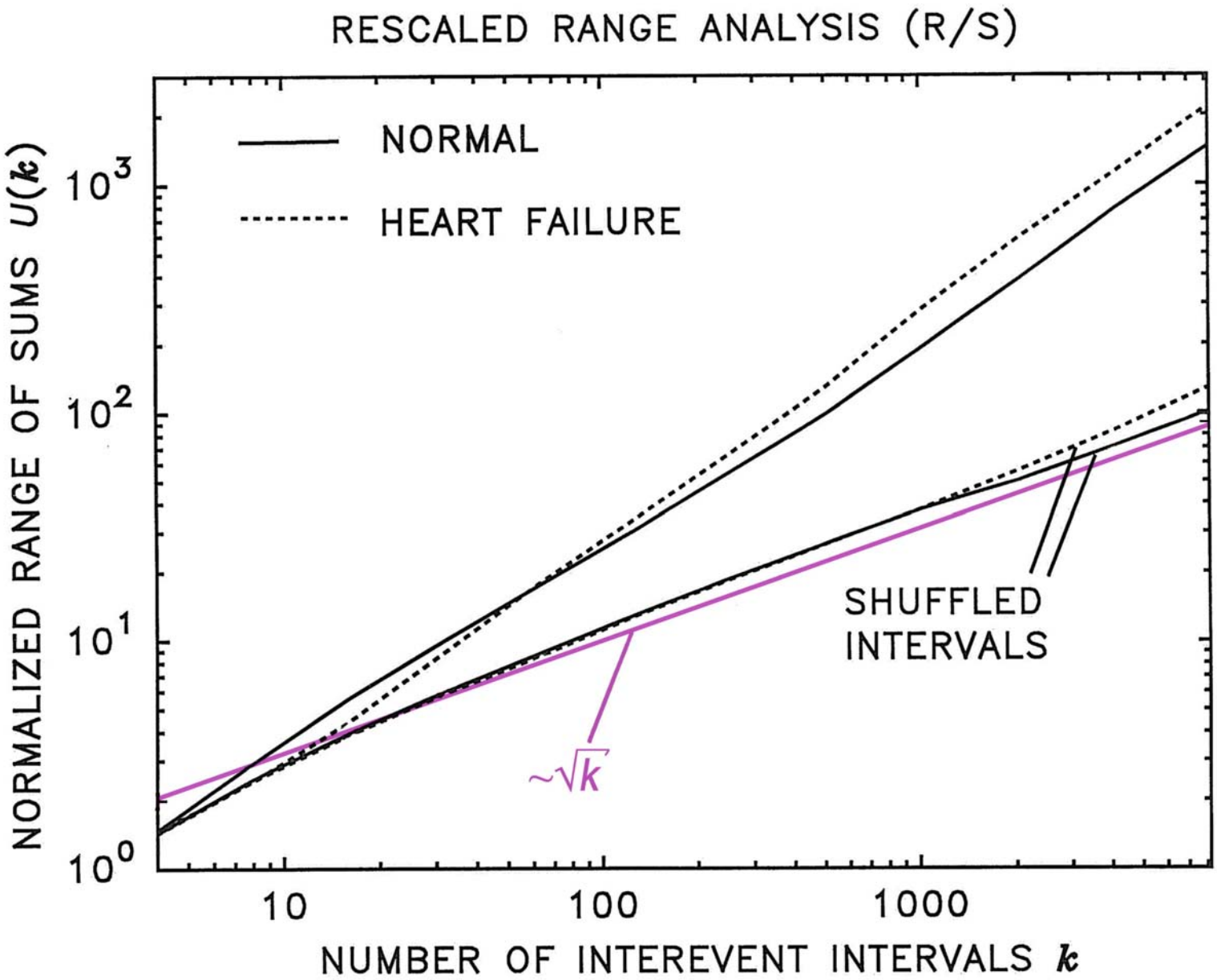


After Thurner, Feurstein & Teich, *Phys. Rev. Letters* **80**, 1544-1547 (1998).

M. C. Teich 2004

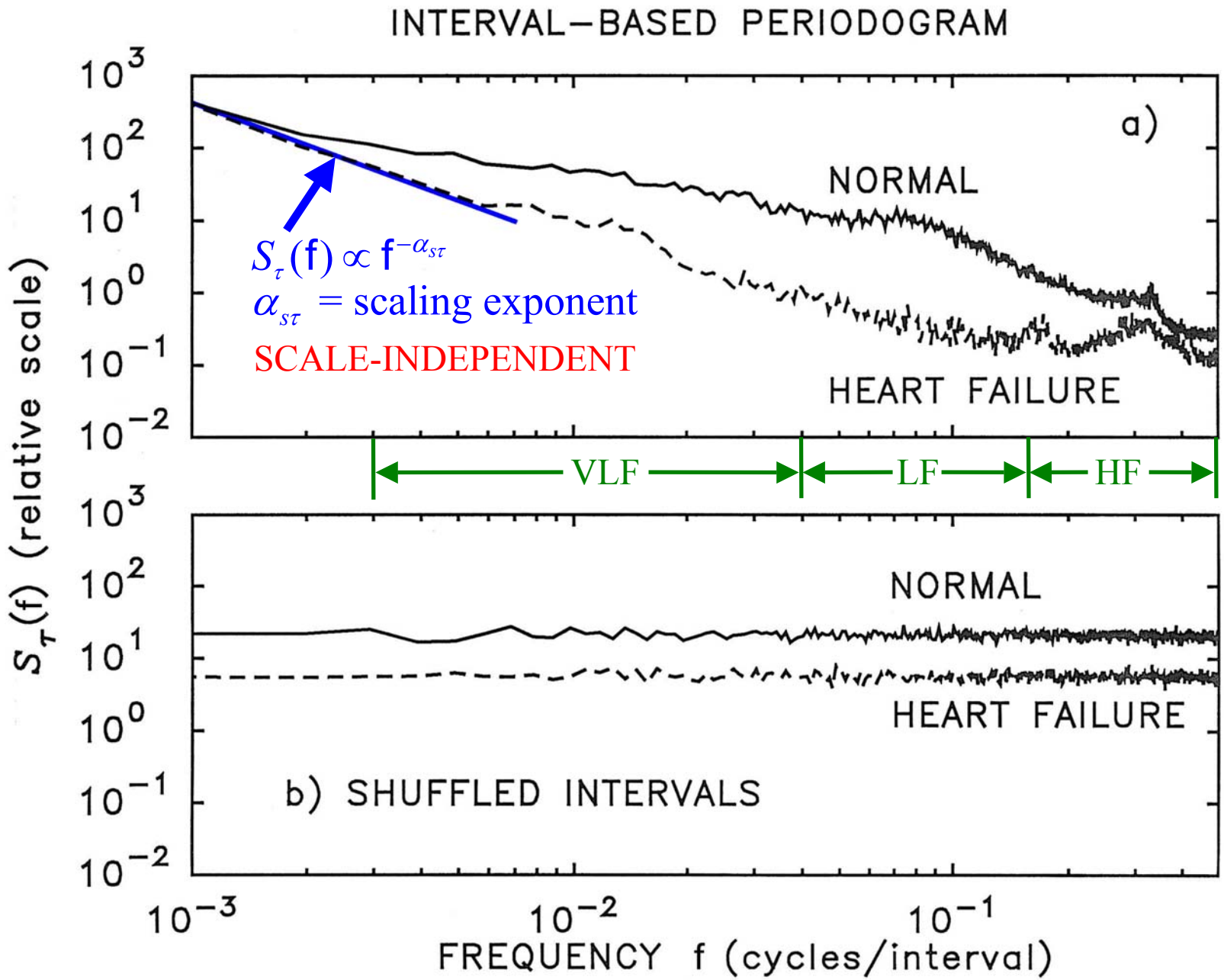


After Turcott & Teich, *Ann. Biomed. Eng.* **24**, 269-293 (1996).



After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).

SPECTRAL ANALYSIS



After Turcott & Teich, *Ann. Biomed. Eng.* **24**, 269-293 (1996).

TIME-SCALE ANALYSIS

DISCRETE WAVELET TRANSFORM

EXAMINES ALL SCALES

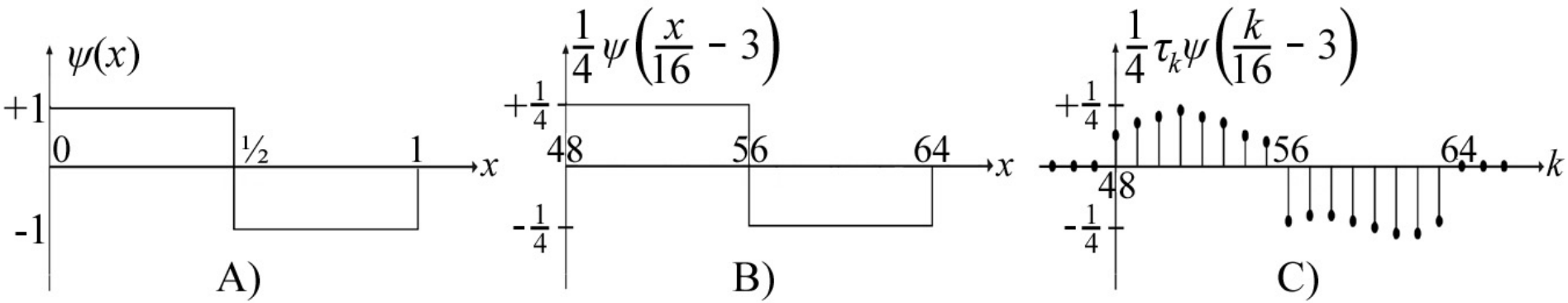
MITIGATES AGAINST NONSTATIONARITIES

m = scale index; 2^m = scale

$$W_{\psi,\tau}^{\text{wav}}(m,i) = \sum_k 2^{-m/2}\,\psi(2^{-m}k - i)\,\tau_k$$

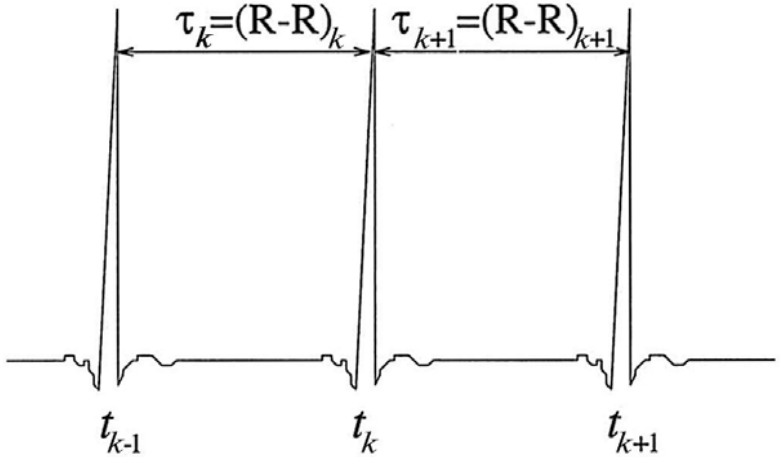
$$\sigma_{\text{wav}}^2 \equiv \text{Var}\left[W_{\psi,\tau}^{\text{wav}}(m,i)\right] = 2^{-m}\sum_k\sum_l \psi(2^{-m}k - i)\,\psi(2^{-m}l - i)\,R_\tau(l-k)$$

$$A_\tau(k) \equiv \text{Var}\left[W_{\psi,\tau}^{\text{wav}}(m,i)\right] \Big/ \text{E}^2[\tau]$$

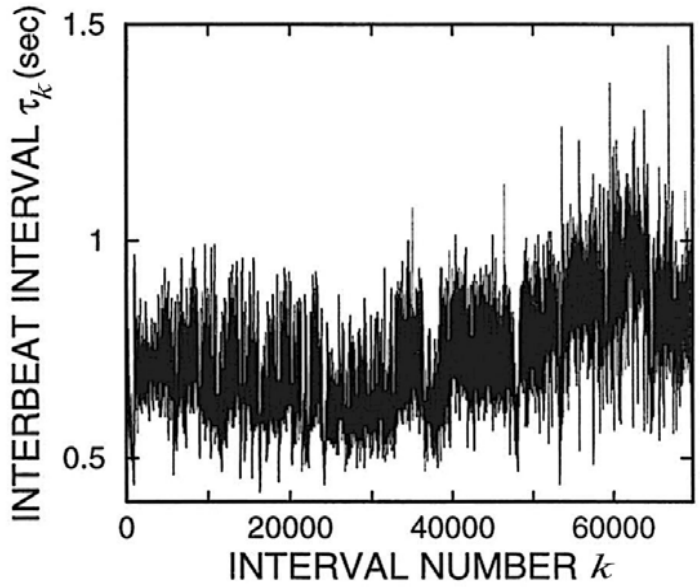


After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan,
in *Nonlinear Biomedical Signal Processing*, vol **II**, M. Akay, Ed.
(IEEE Press, NY, 2001), pp. 159-213.

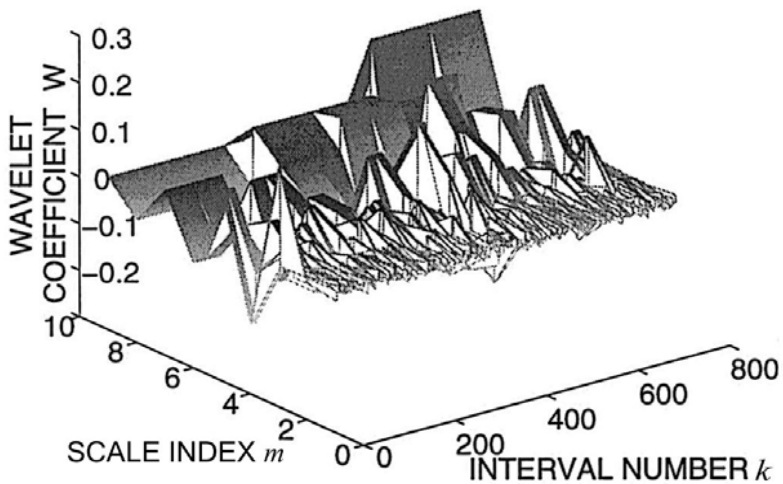
NORMAL



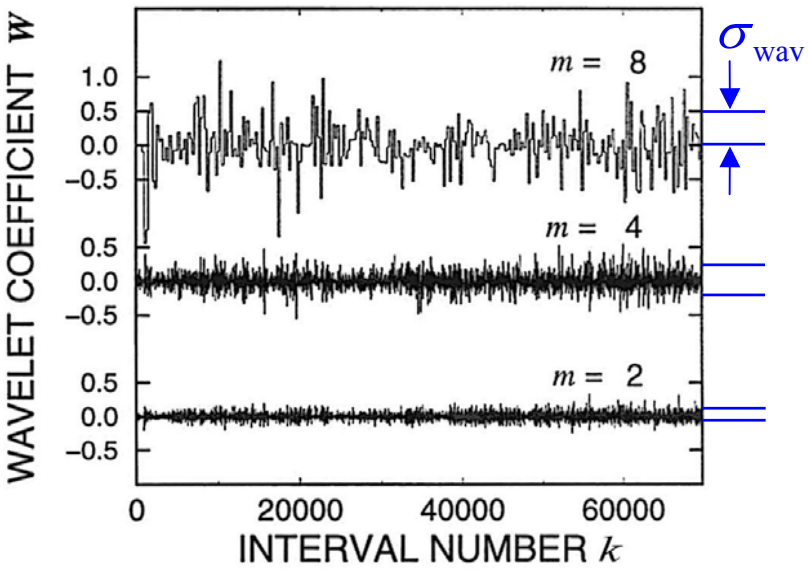
a)



b)

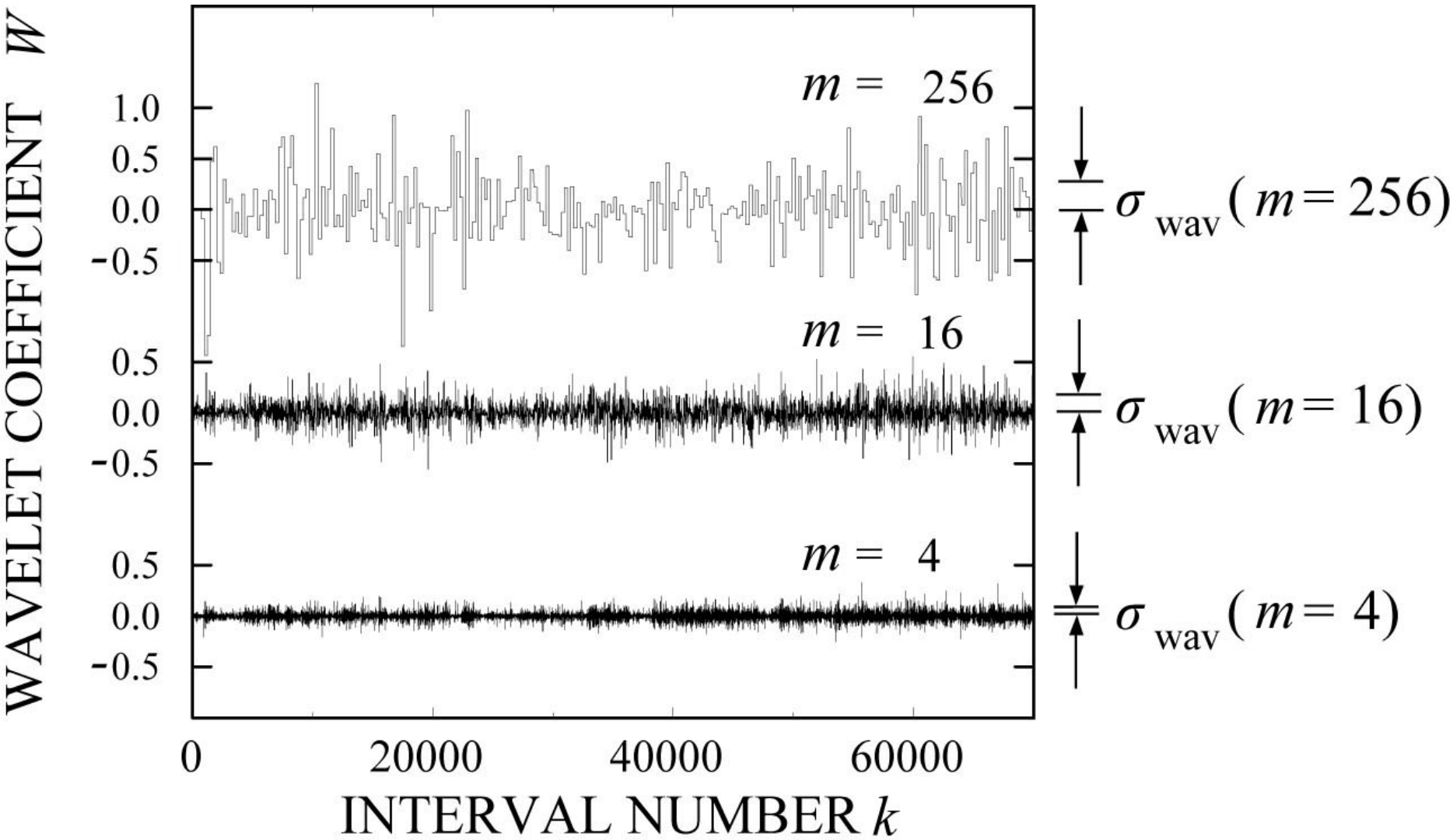


c)



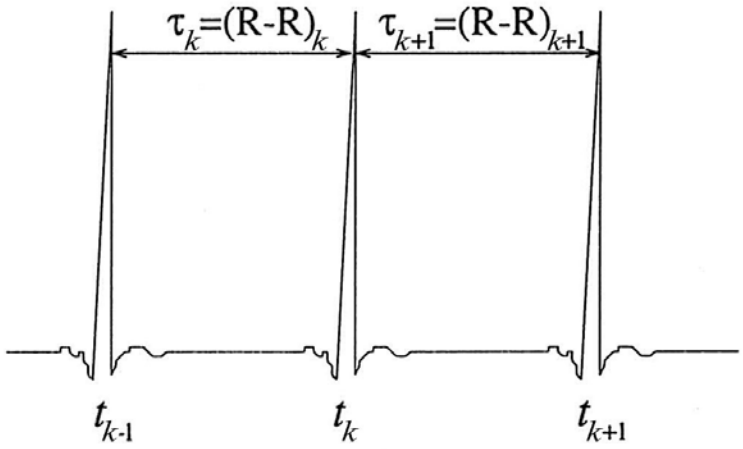
d)

After Thurner, Feurstein & Teich, *Phys. Rev. Letters* **80**, 1544-1547 (1998).

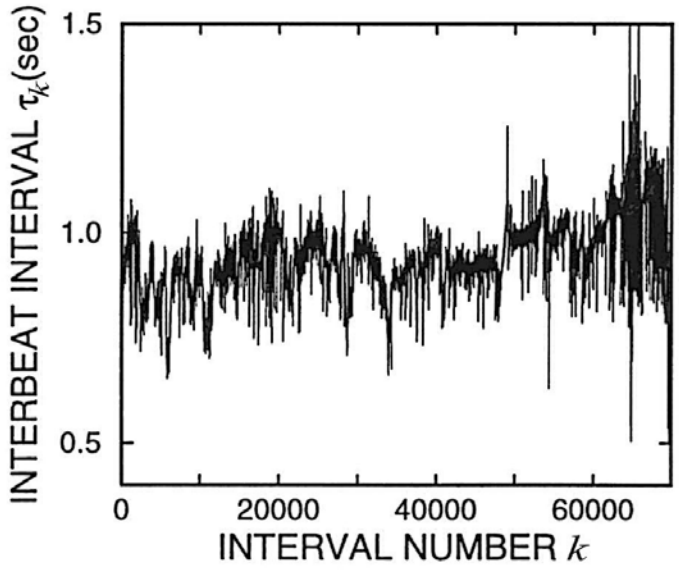


After Thurner, Feurstein & Teich, *Phys. Rev. Letters* **80**, 1544-1547 (1998).

HEART-FAILURE



a)



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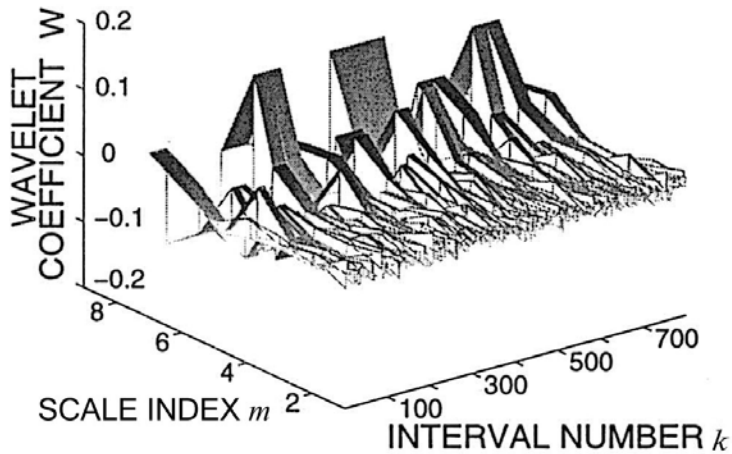
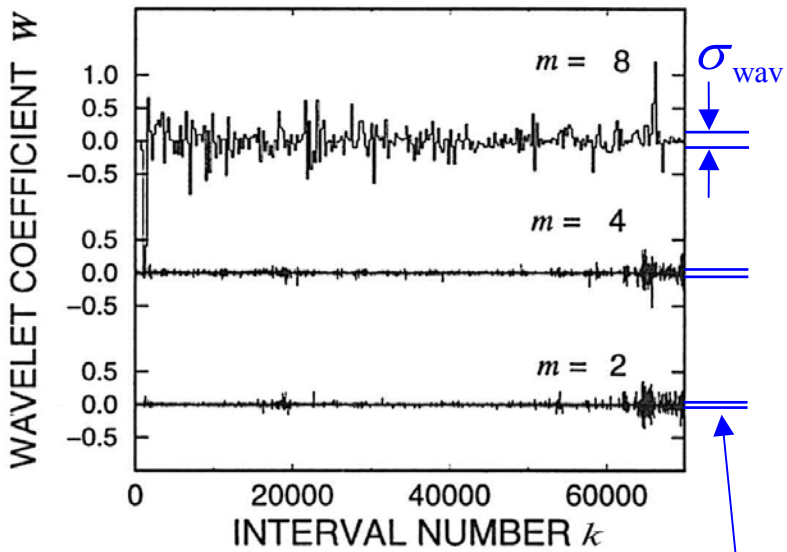


c)



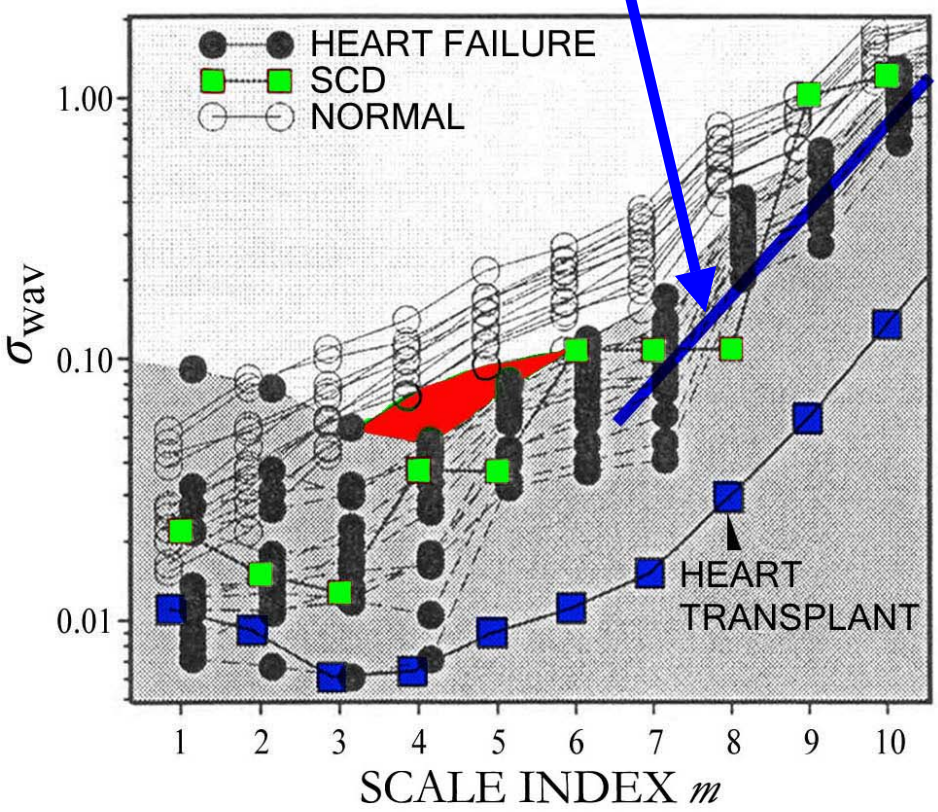
d) SMALLER VALUES OF σ_{wav} THAN FOR NORMAL SUBJECTS

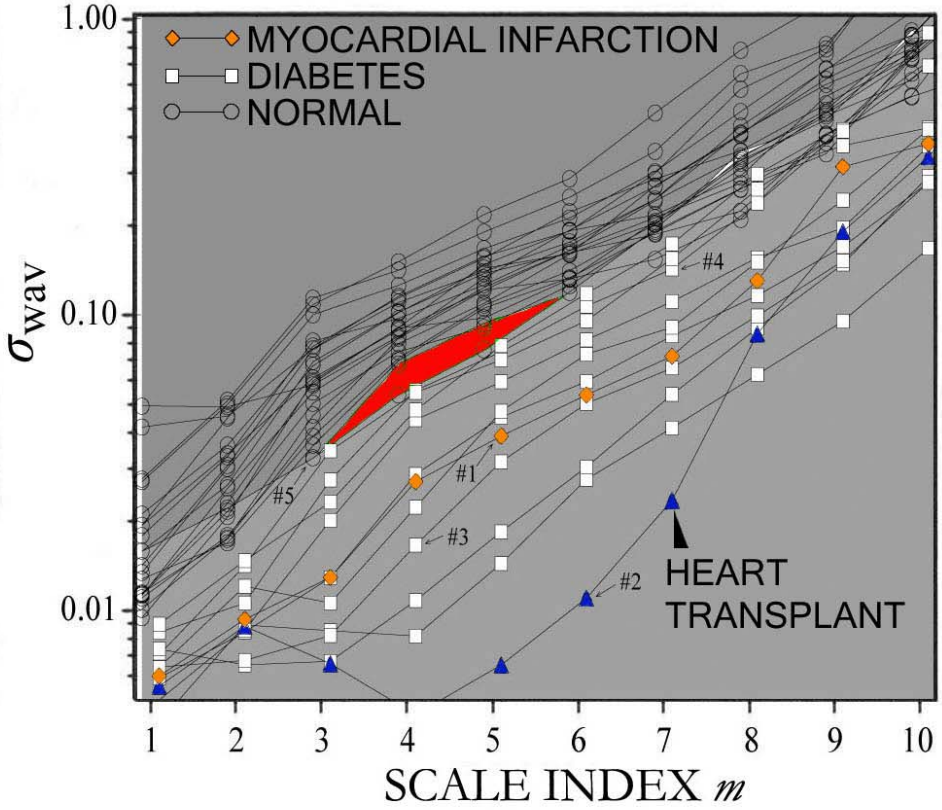
After M. C. Teich

$$\sigma^2_{\text{wav}}(T) \propto T^{\alpha_{A\tau}}$$

$\alpha_{A\tau}$ = scaling exponent

SCALE-INDEPENDENT

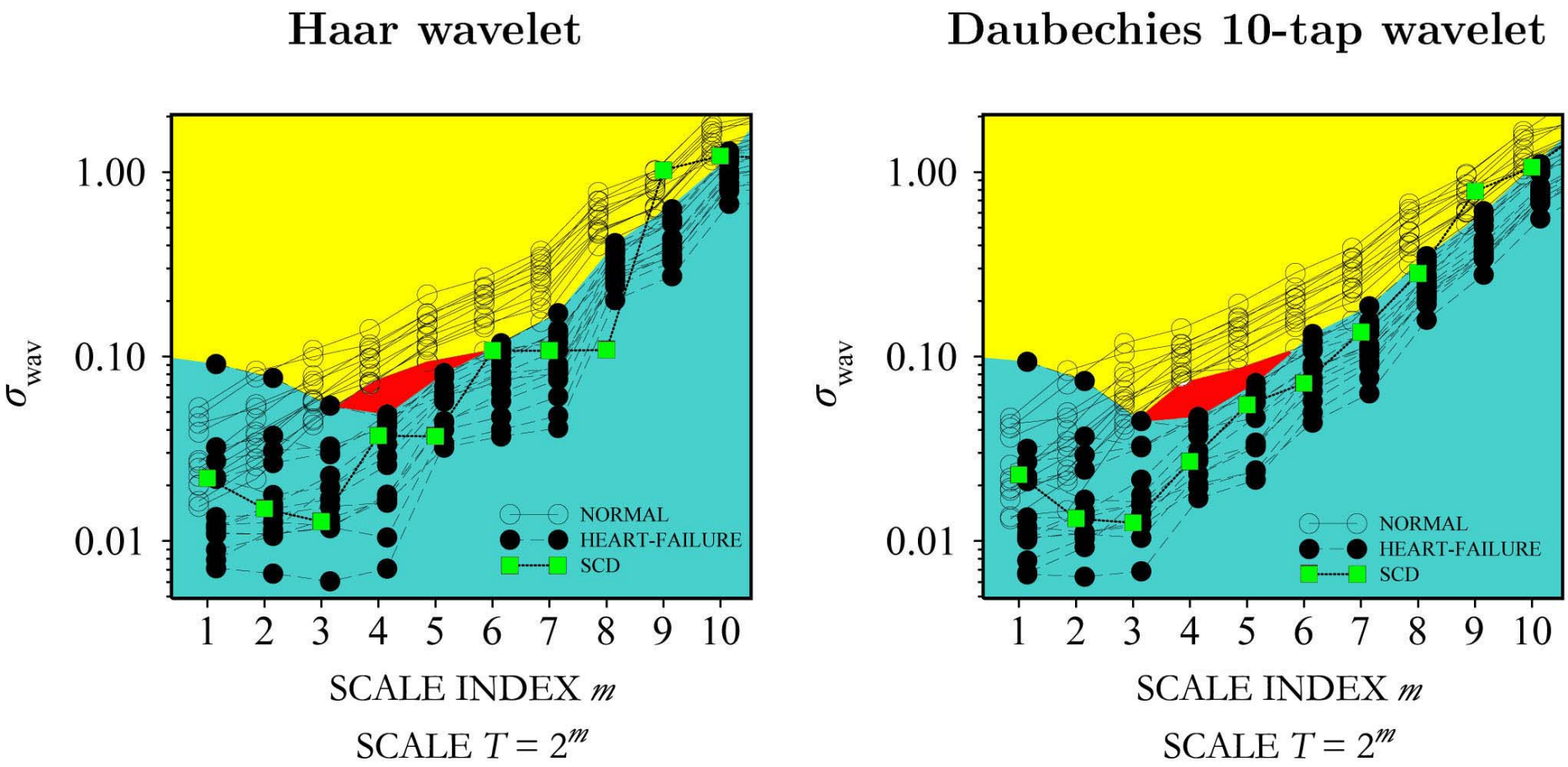




After Teich, *Proc. Int. Conf. IEEE Eng. Med. Biol. Soc.* **20**, 1136-1141 (1998).

After Ashkenazy *et al.*, *Fractals* **6**, 197-203 (1998).

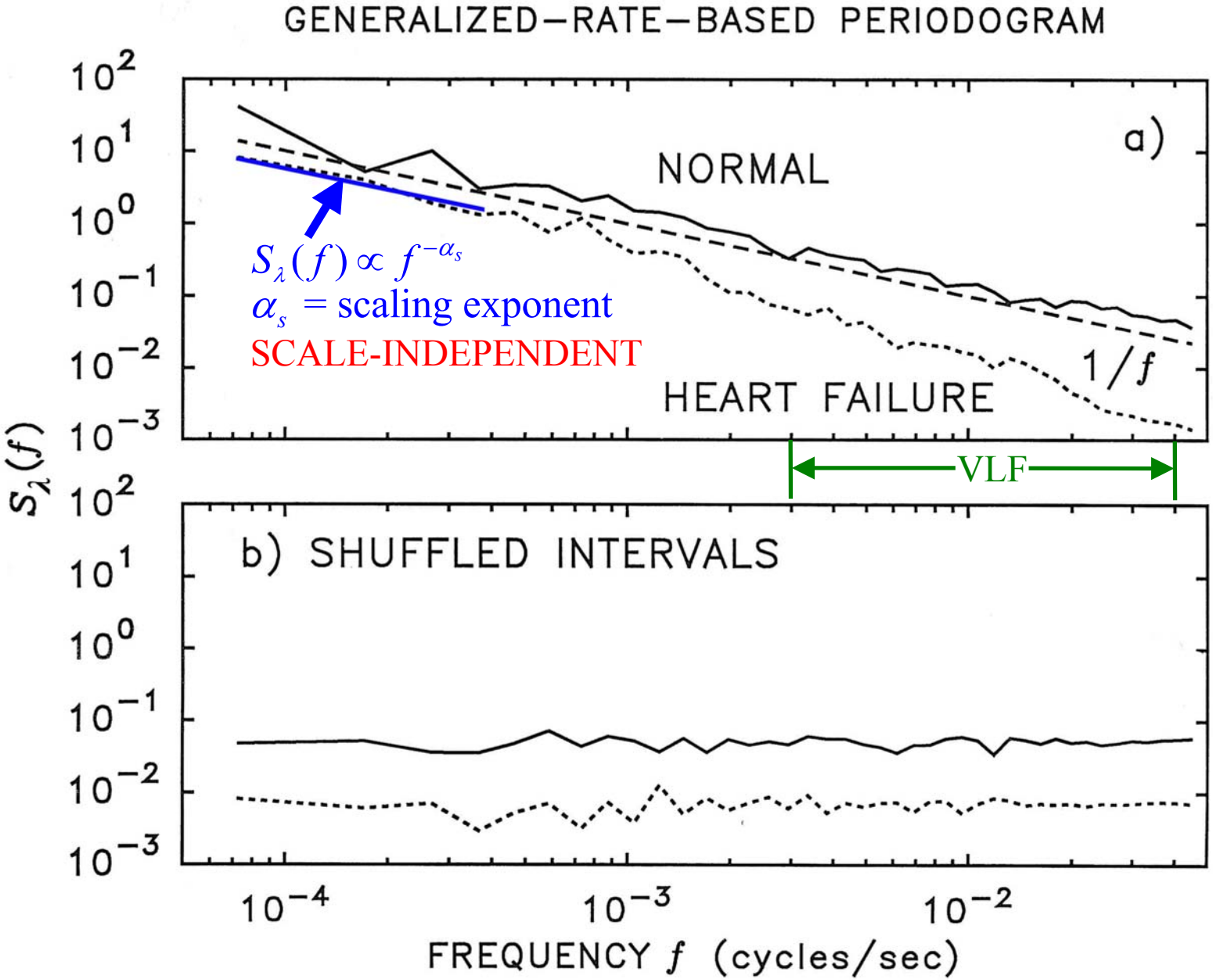
ROBUSTNESS WITH WAVELET FORM



After Thurner, Feurstein & Teich, *Phys. Rev. Letters* **80**, 1544-1547 (1998).

COUNT-BASED MEASURES

SPECTRAL ANALYSIS



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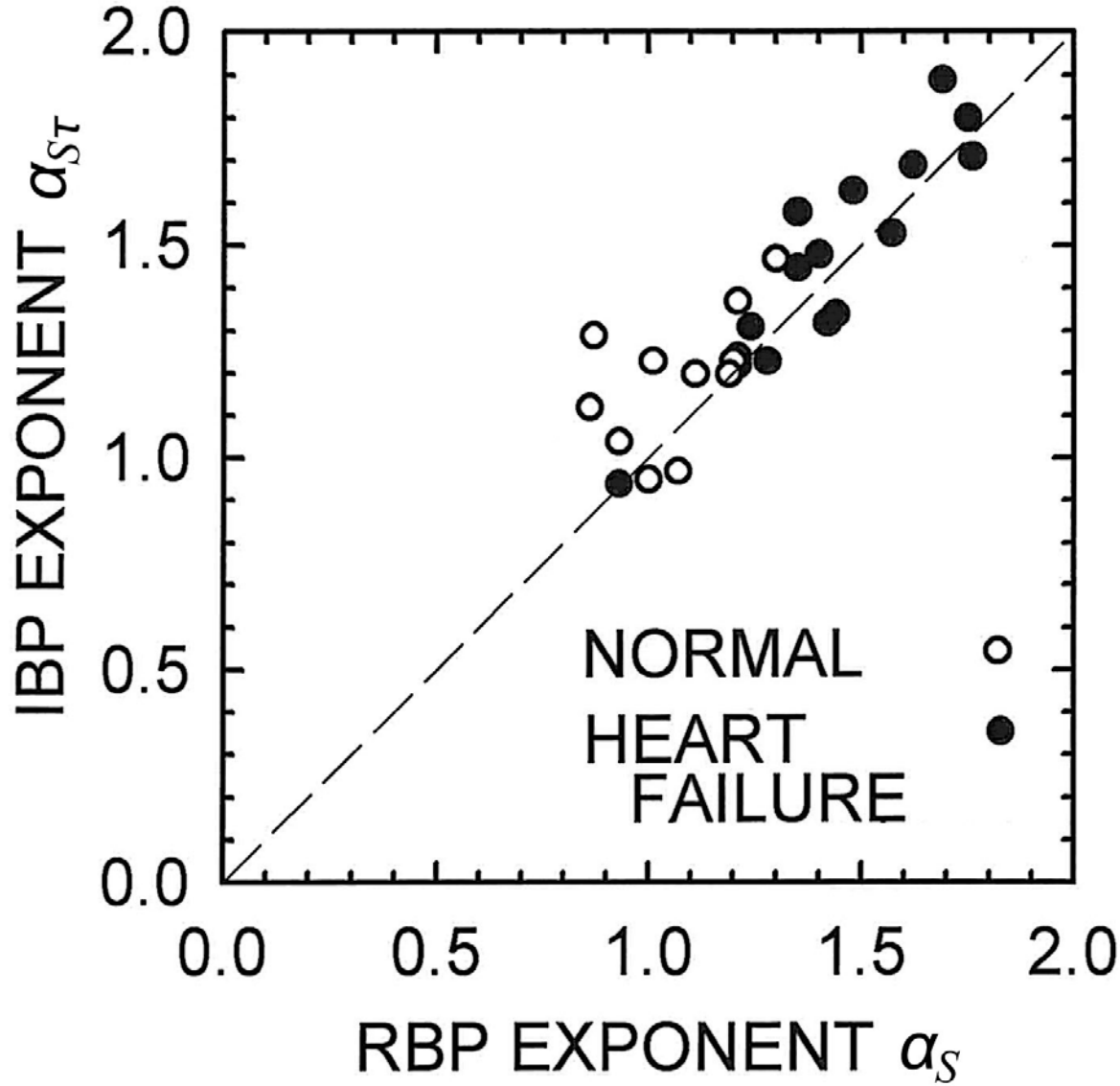
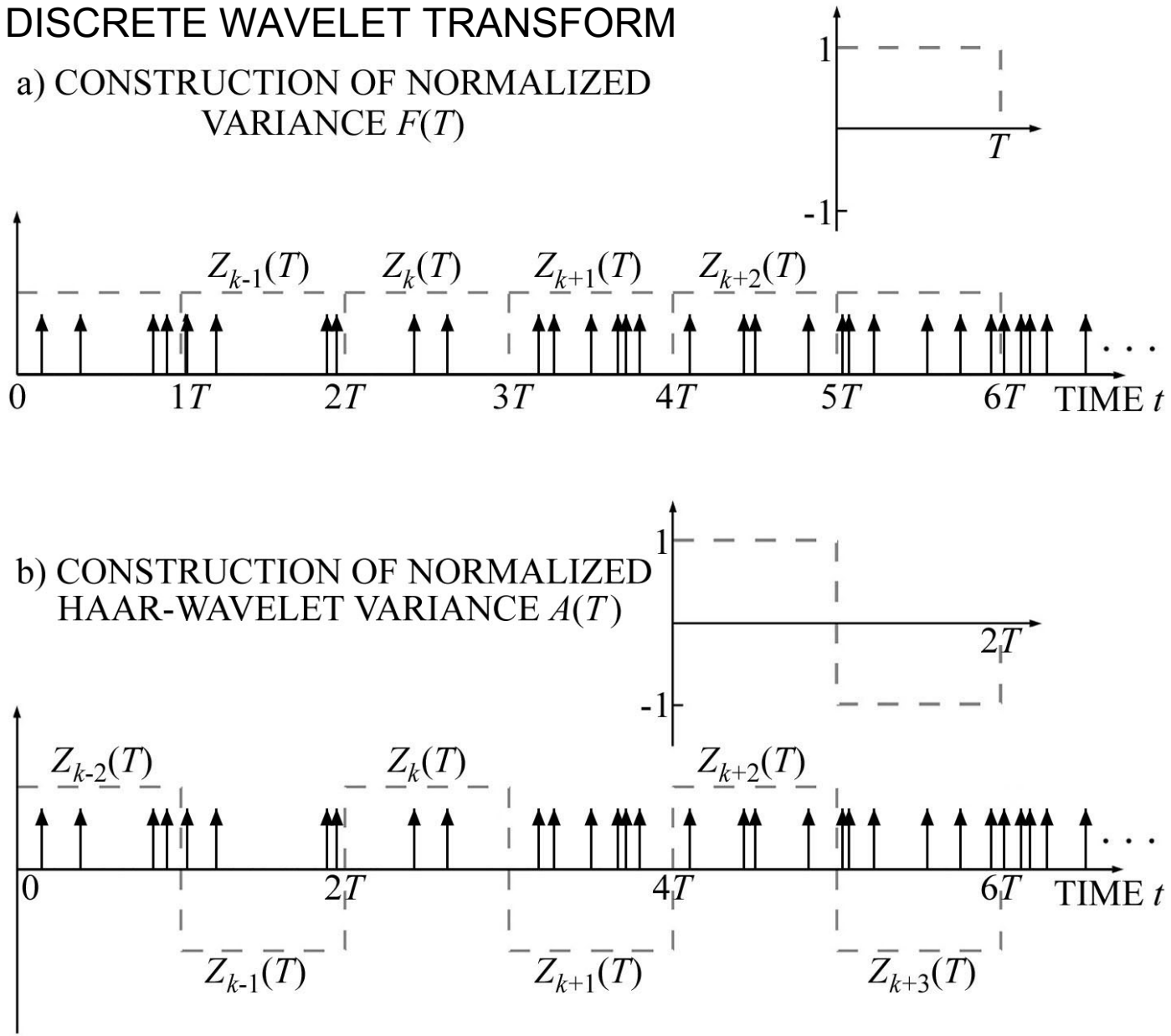


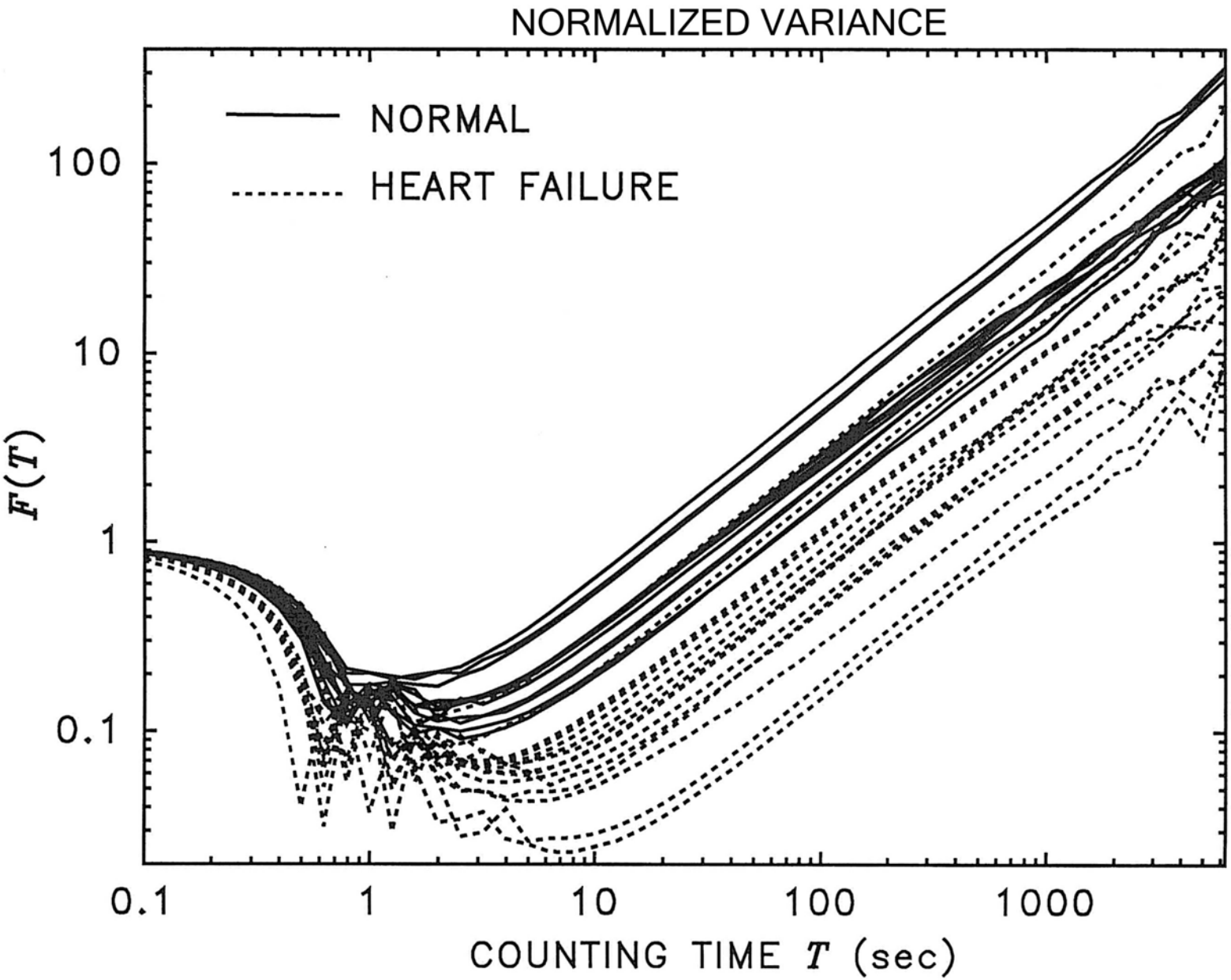
After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).

TIME-SCALE ANALYSIS

DISCRETE WAVELET TRANSFORM

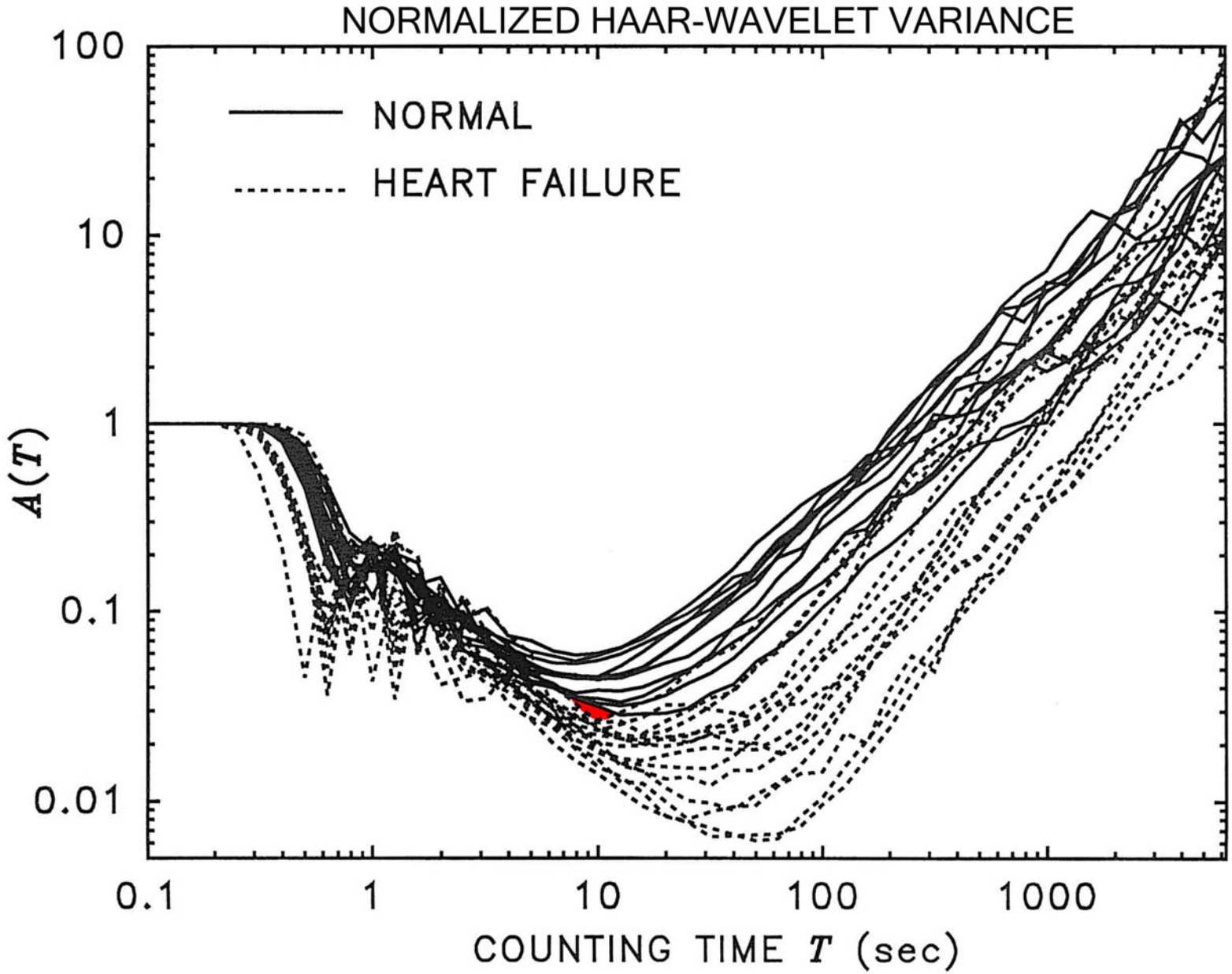


After Lowen & Teich, *Fractal-Based Point Processes*,
(Wiley Series in Probability and Statistics, Hoboken, NJ, 2005).

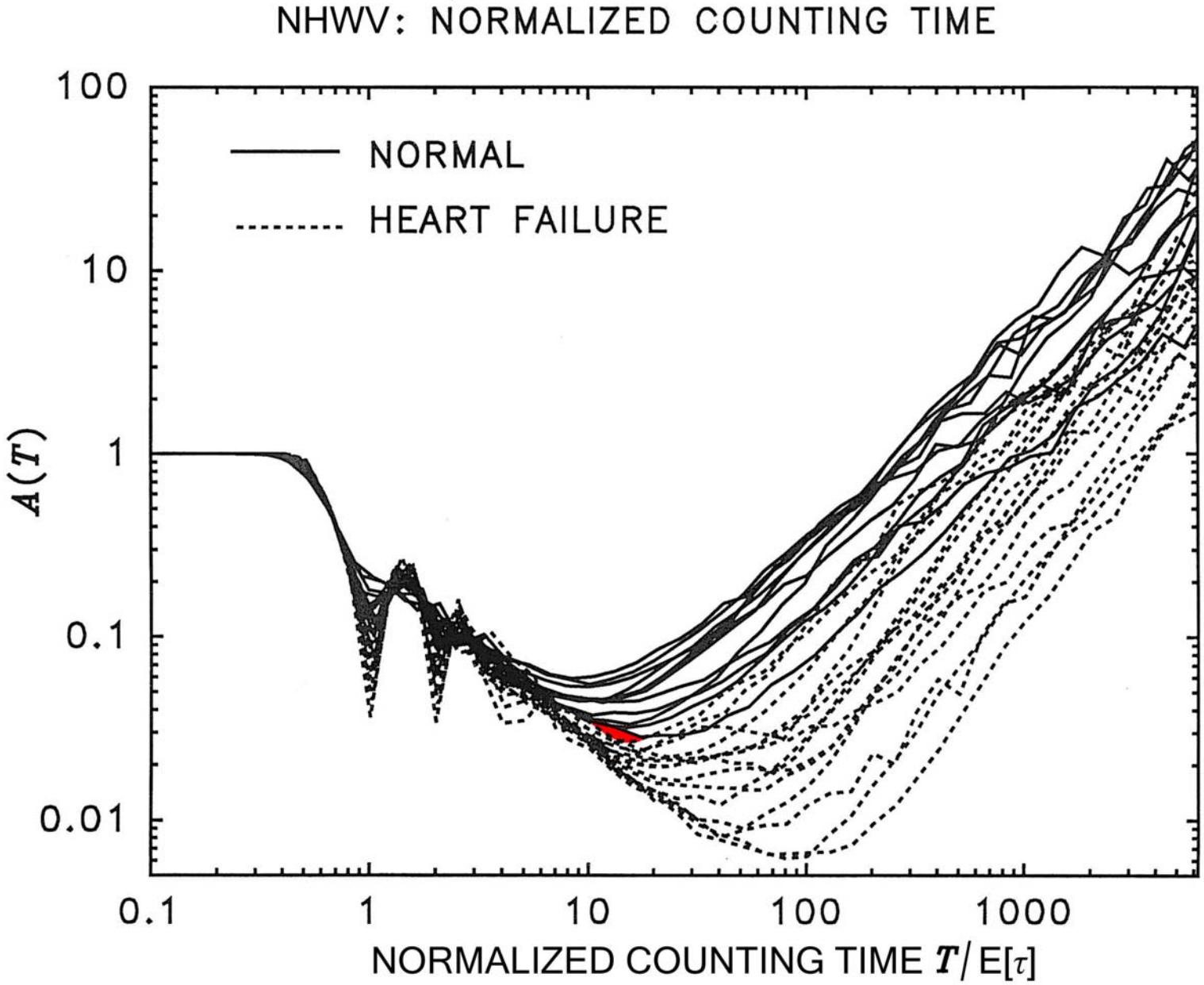


After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).

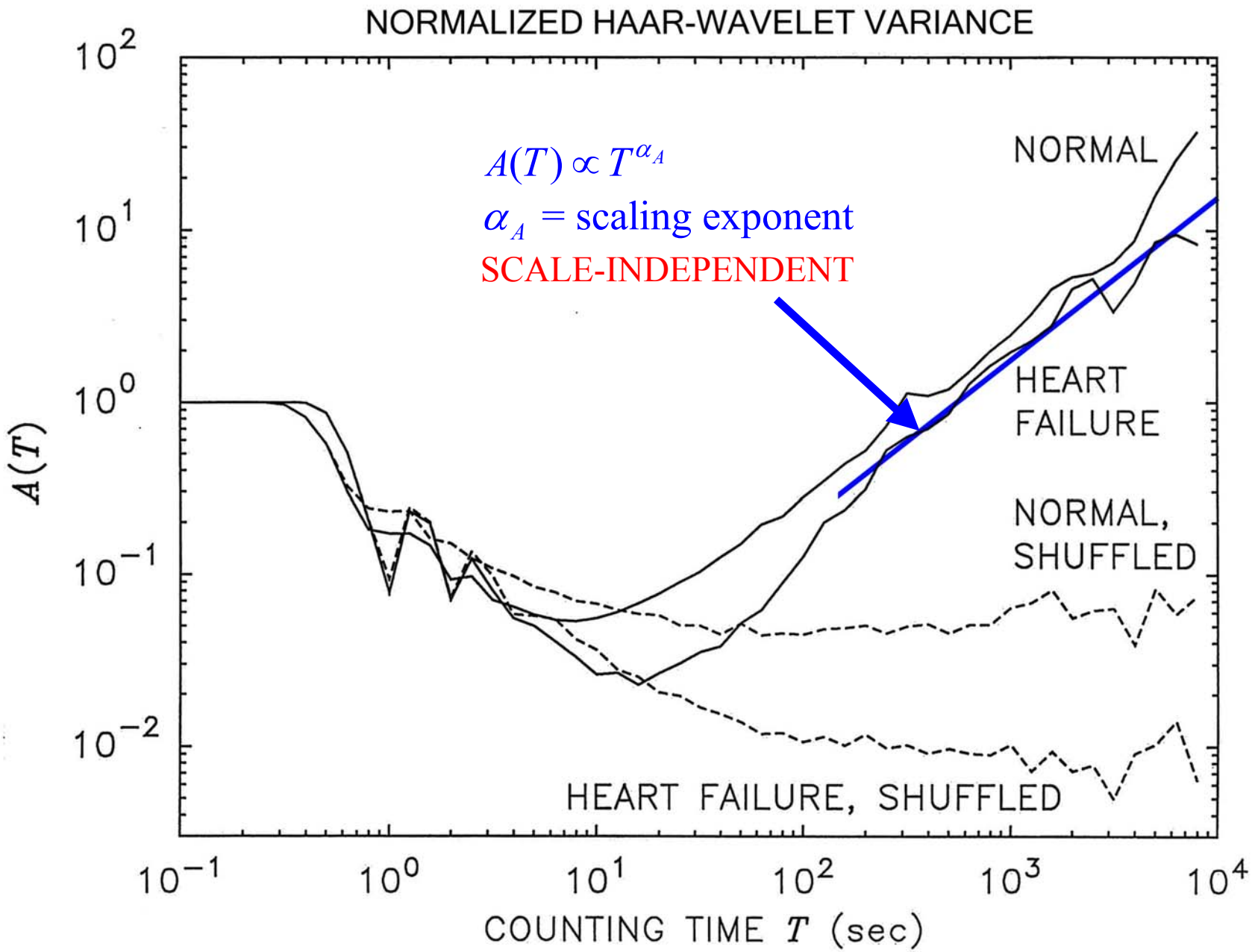
M. C. Teich 2004



After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).

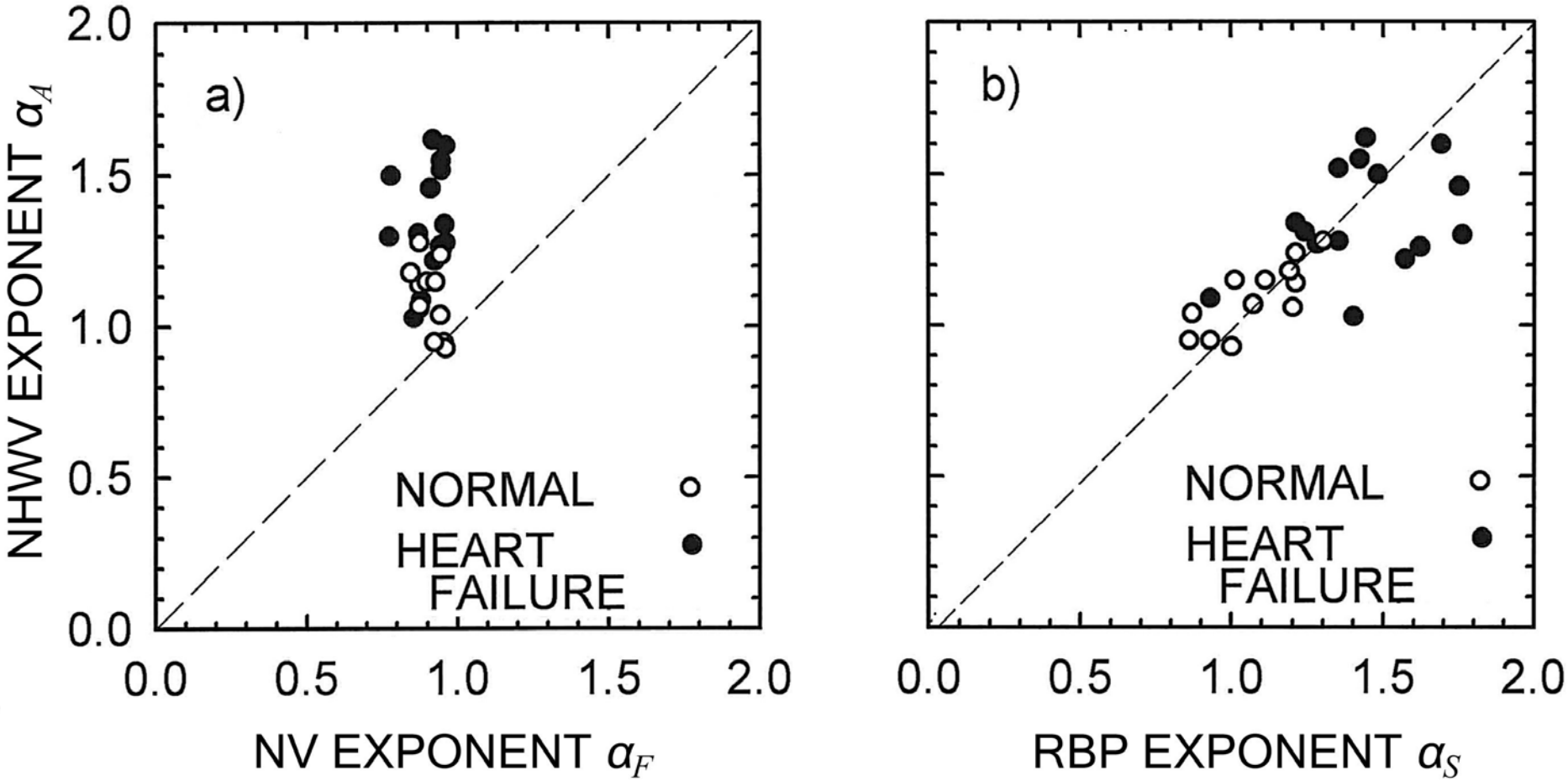


After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).



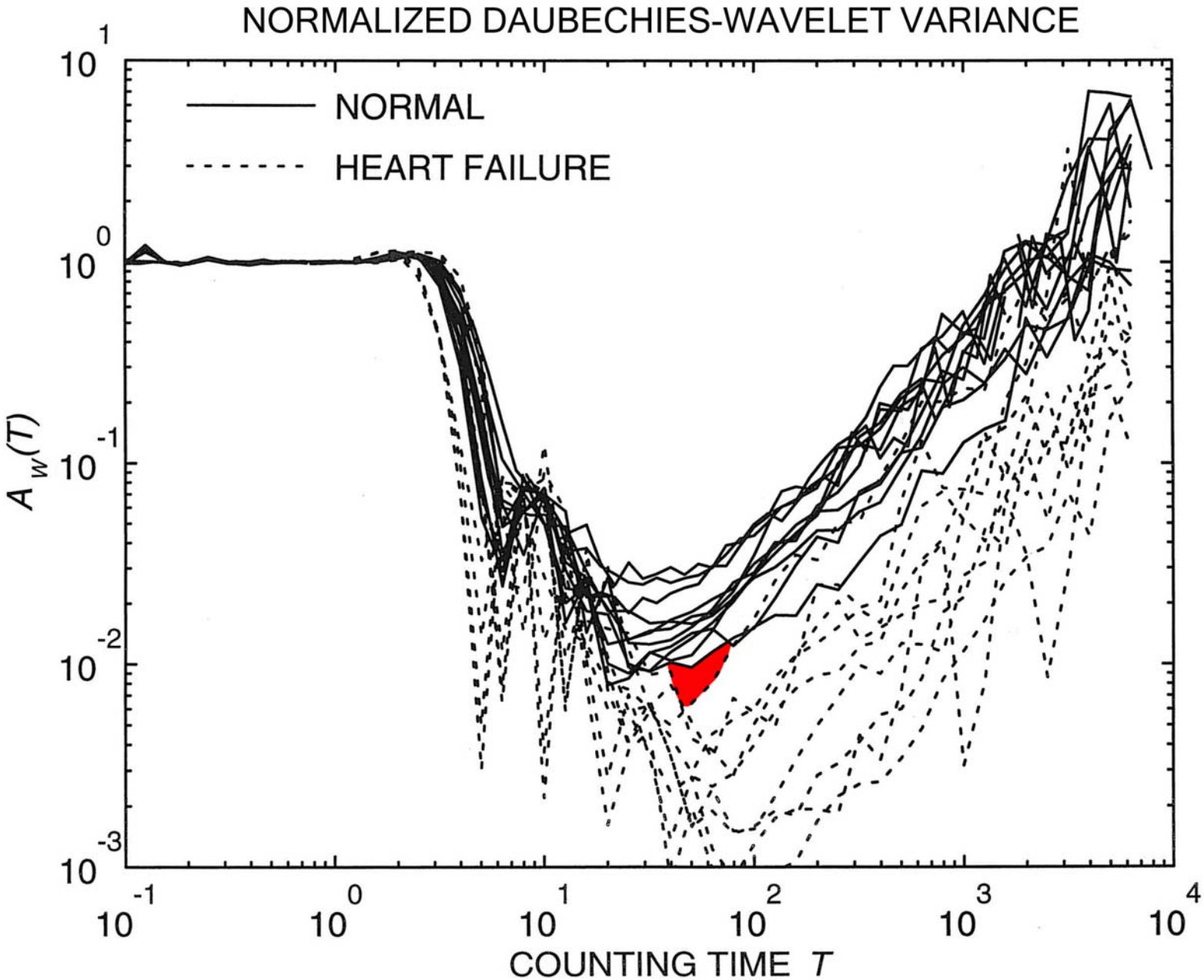
After Turcott & Teich, *Ann. Biomed. Eng.* **24**, 269-293 (1996).

FRACTAL-EXPONENT ESTIMATORS



After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).

M. C. Teich 2004

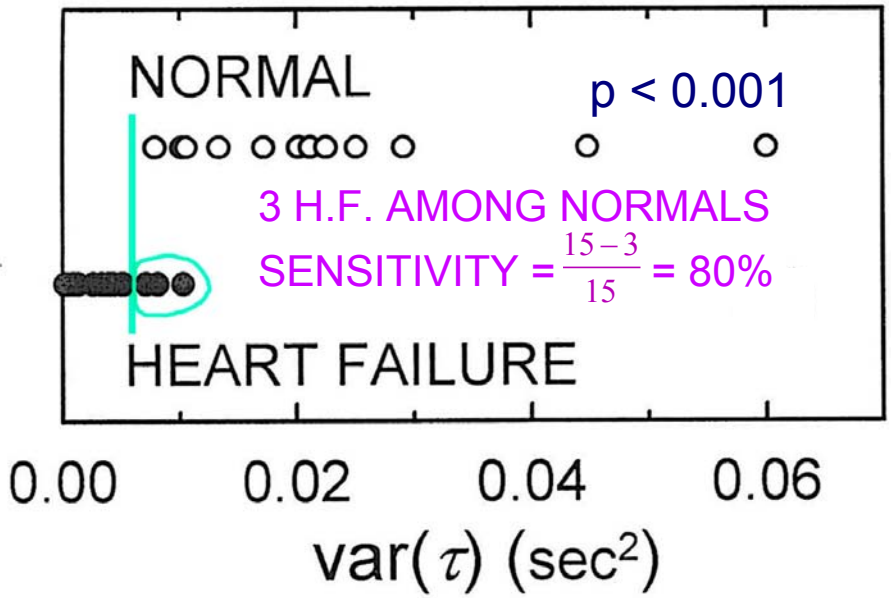


After Teich, *Proc. 18th Intern. Conf. IEEE Eng. Med. Biol. Soc.* **18**, 1128-1129 (1996).

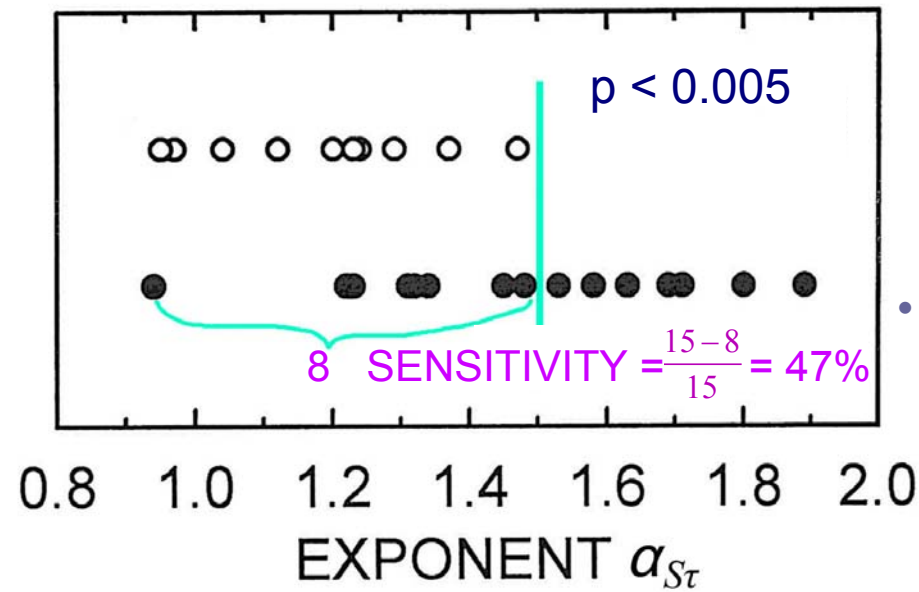
M. C. Teich 2004

IDENTIFYING PATIENTS WITH CARDIAC DYSFUNCTION

M. C. Teich 2004



SCALE-DEPENDENT



SCALE-INDEPENDENT

MEASURES OF STATISTICAL SIGNIFICANCE

- p VALUE, d', AND VARIANTS
  (rely on Gaussian assumption)

- SENSITIVITY/SPECIFICITY
  MEASURES OF CLINICAL SIGNIFICANCE
  (distribution free)

  SENSITIVITY ≡ proportion of heart-failure patients that are properly identified

  e.g., Hypothesis that all normal patients are so identified ≡ 100% SPECIFICITY

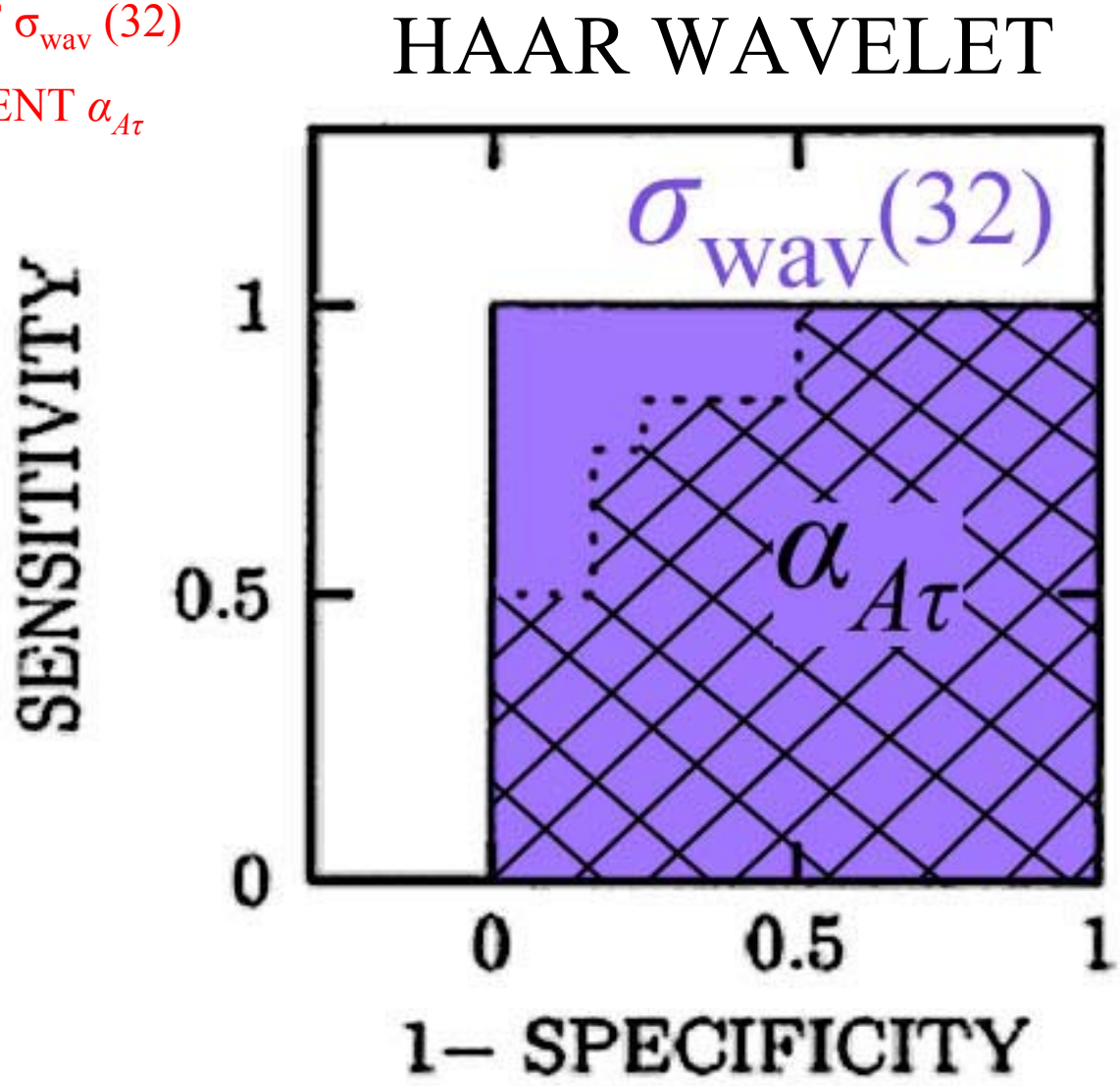
- ROC CURVES & AREA UNDER ROC

After Turcott & Teich,
Ann. Biomed. Eng. **24**, 269-293 (1996)

ROC CURVES & AREA UNDER ROC

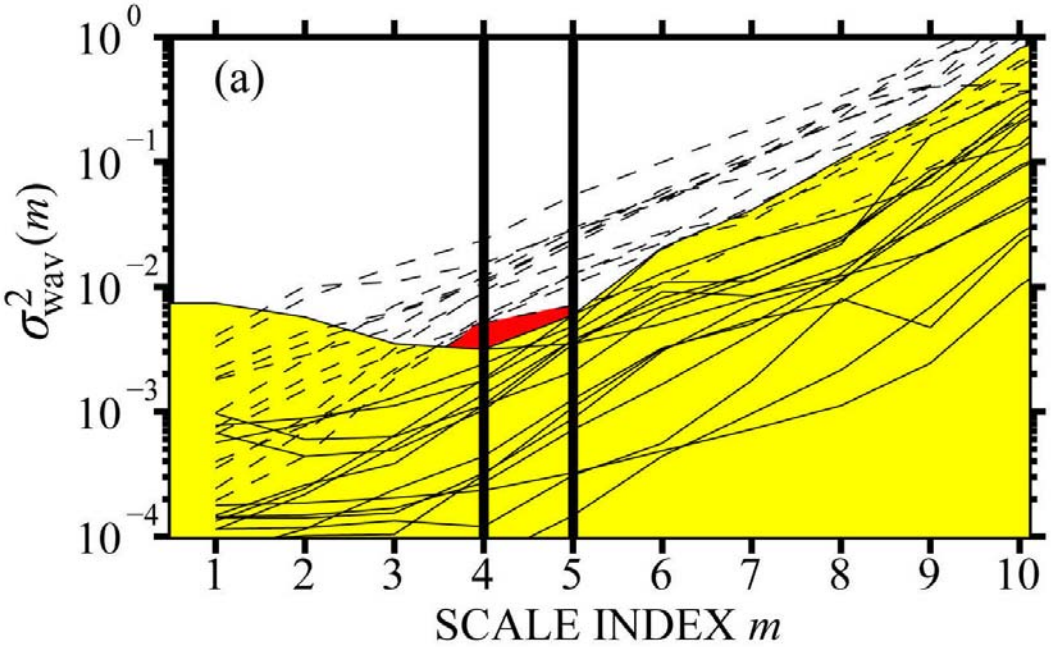
SCALE-DEPENDENT σ_{wav} (32)

SCALE-INDEPENDENT $\alpha_{A\tau}$

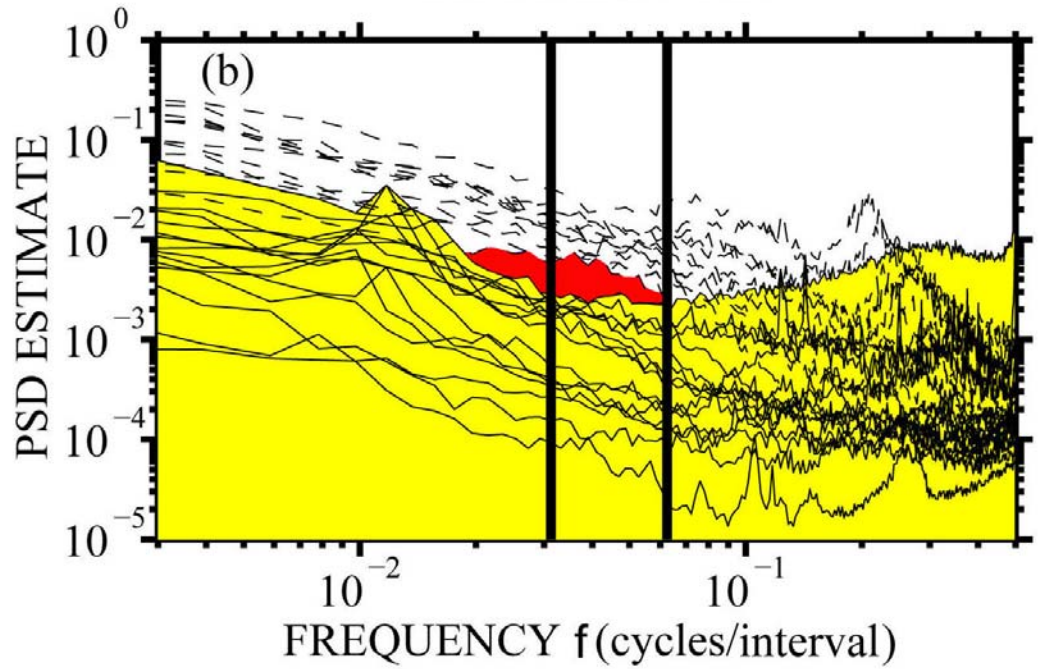


After Thurner, Feurstein, Lowen & Teich,
Phys. Rev. Letters **81**, 5688-5691 (1998).

M. C. Teich 2004

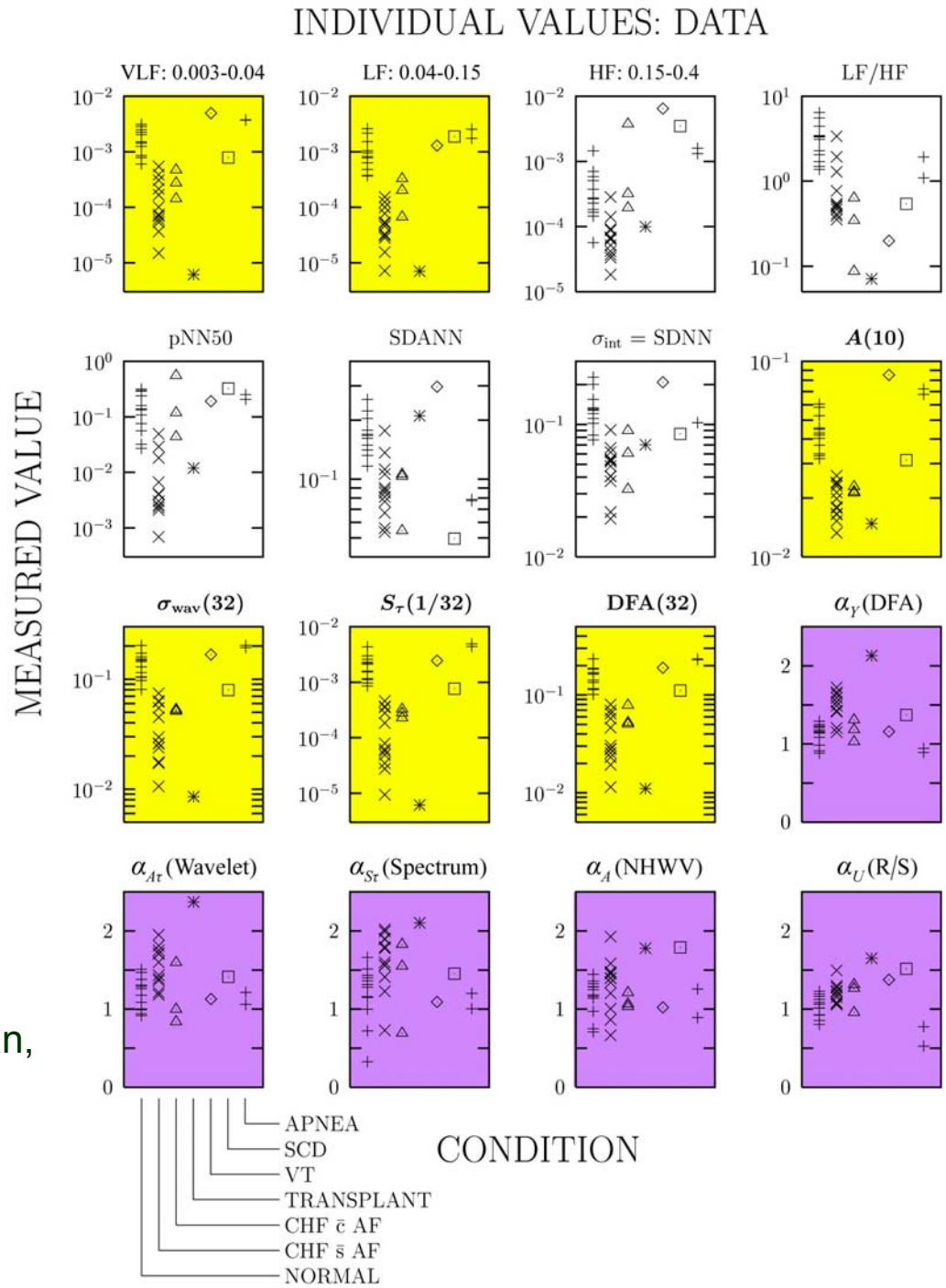


$$16 \le 2^m \ \text{(scale)} \le 32$$



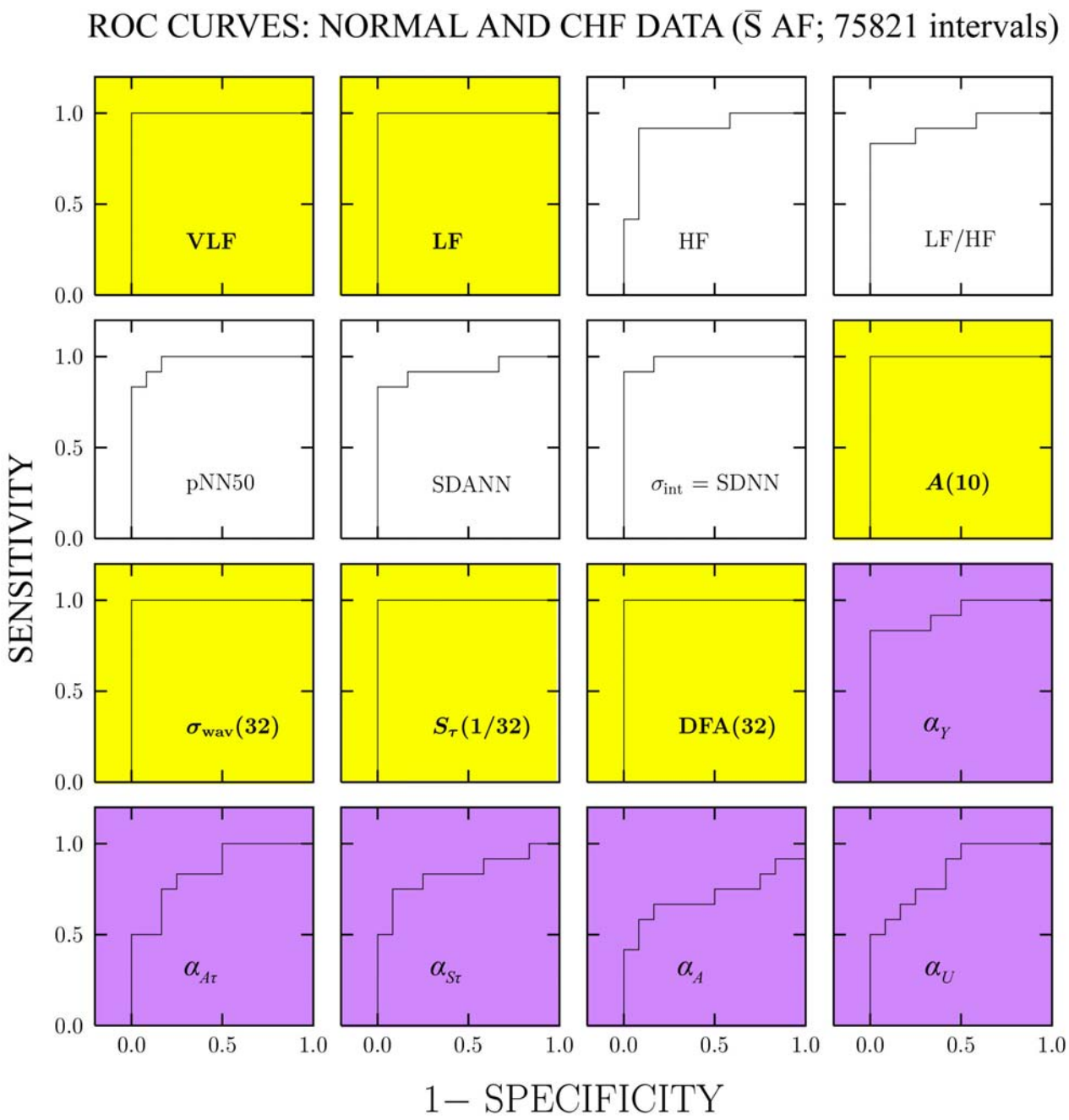
$$\frac{1}{32} < \text{f (cycles/interval)} < \frac{1}{16}$$

After Heneghan, Lowen, & Teich
Proc. 1999 ICASSP (Phoenix, AZ)
paper SPTM-8.2.



After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan, in *Nonlinear Biomedical Signal Processing*, vol **II**, M. Akay, Ed. (IEEE Press, NY, 2001), pp. 159-213.

M. C. Teich 2004



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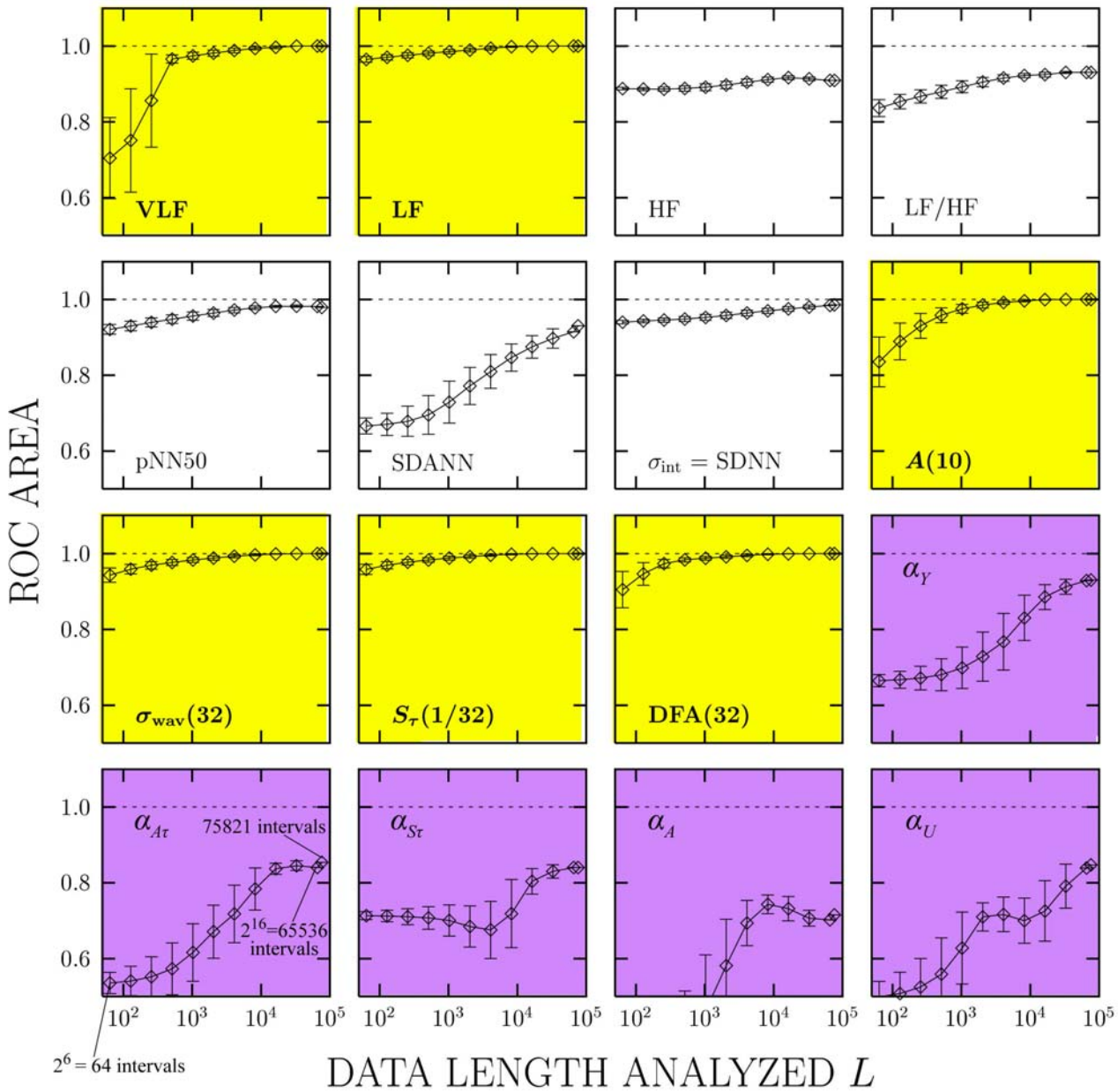

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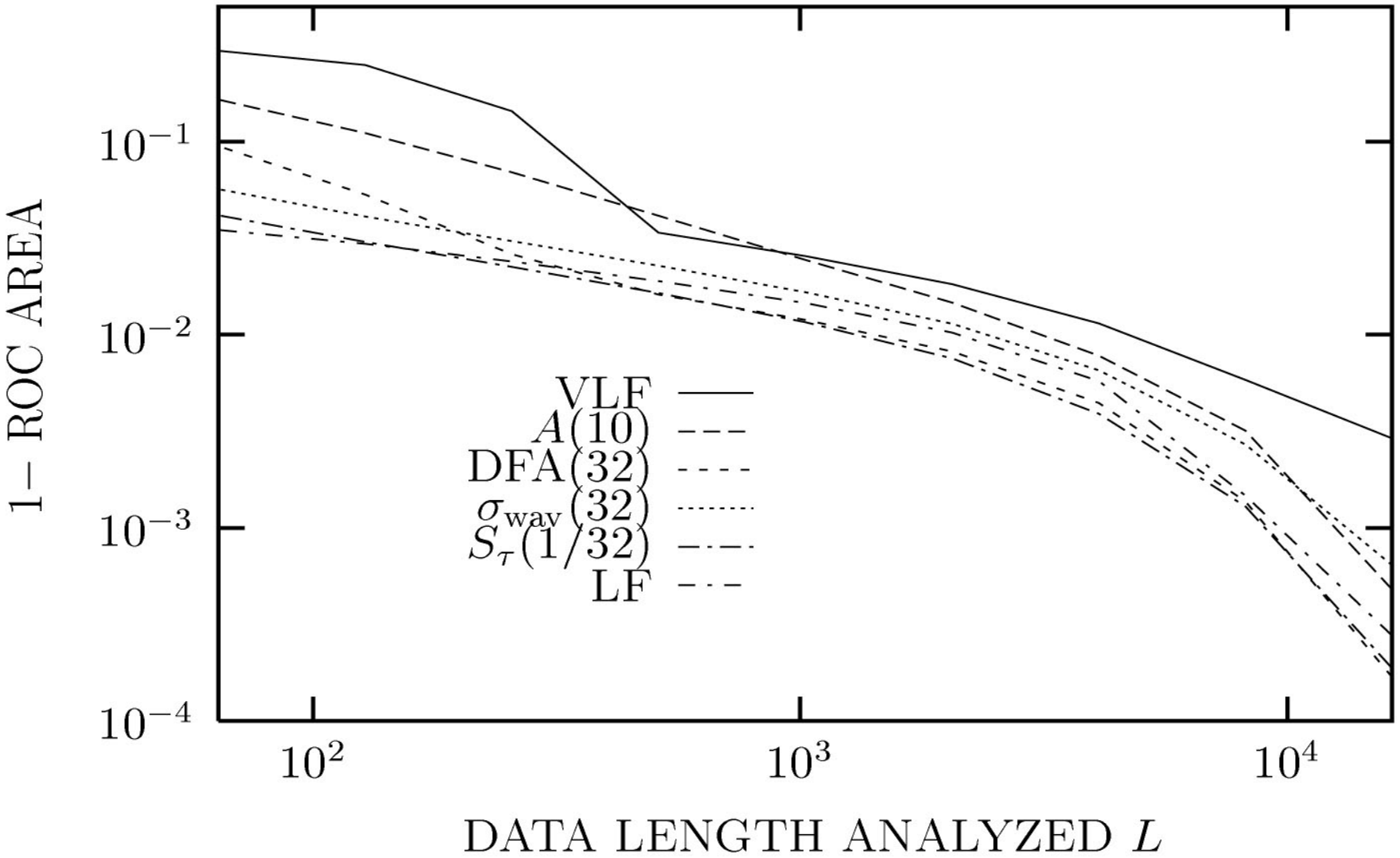

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in *Nonlinear Biomedical Signal Proc*essing, vol **II**, M. Akay, Ed.
(IEEE Press, NY, 2001), pp. 159-213.

Measure	Execution Time (msec)
VLF, LF, HF, and LF/HF	330
pNN50	40
SDANN	160
$\sigma_{\mathbf{int}}$	190
$A(10)$	160
$\sigma_{\mathbf{wav}}(32)$	20
$S_\tau(1/32)$	60
DFA(32)	650,090
α_γ	650,110
$\alpha_{A\tau}$	220
$\alpha_{S\tau}$	920
α_A	610
α_U	570

After Teich, Lowen, Jost, Vibe-Rheymer & Heneghan,
in *Nonlinear Biomedical Signal Proc*essing, vol **II**, M. Akay, Ed.
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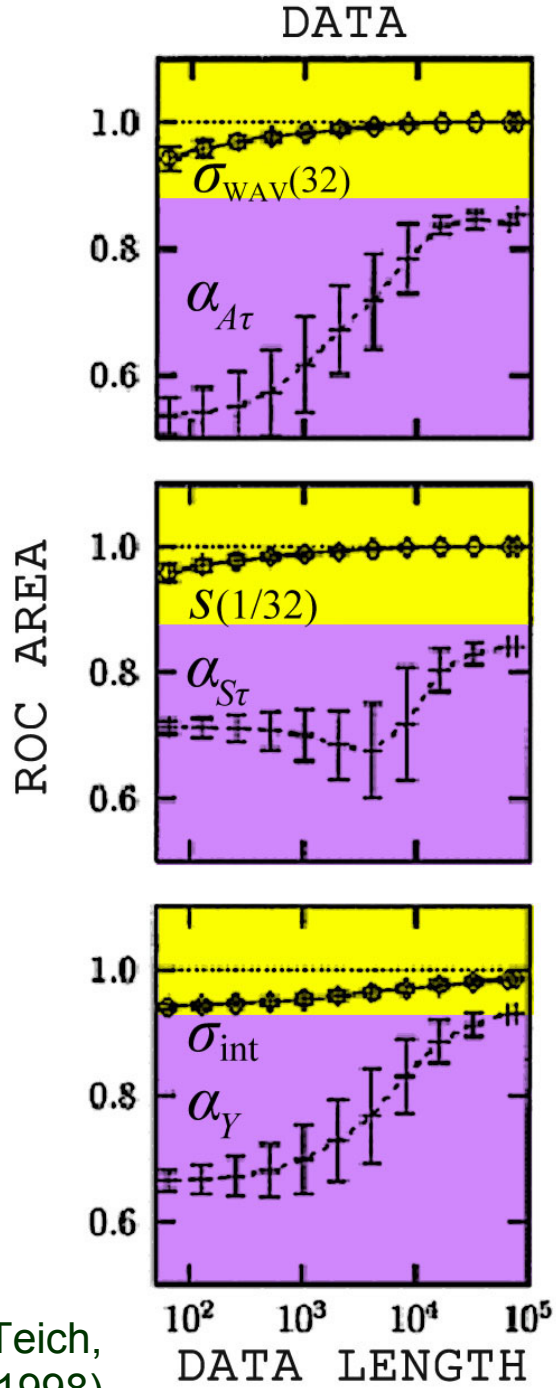
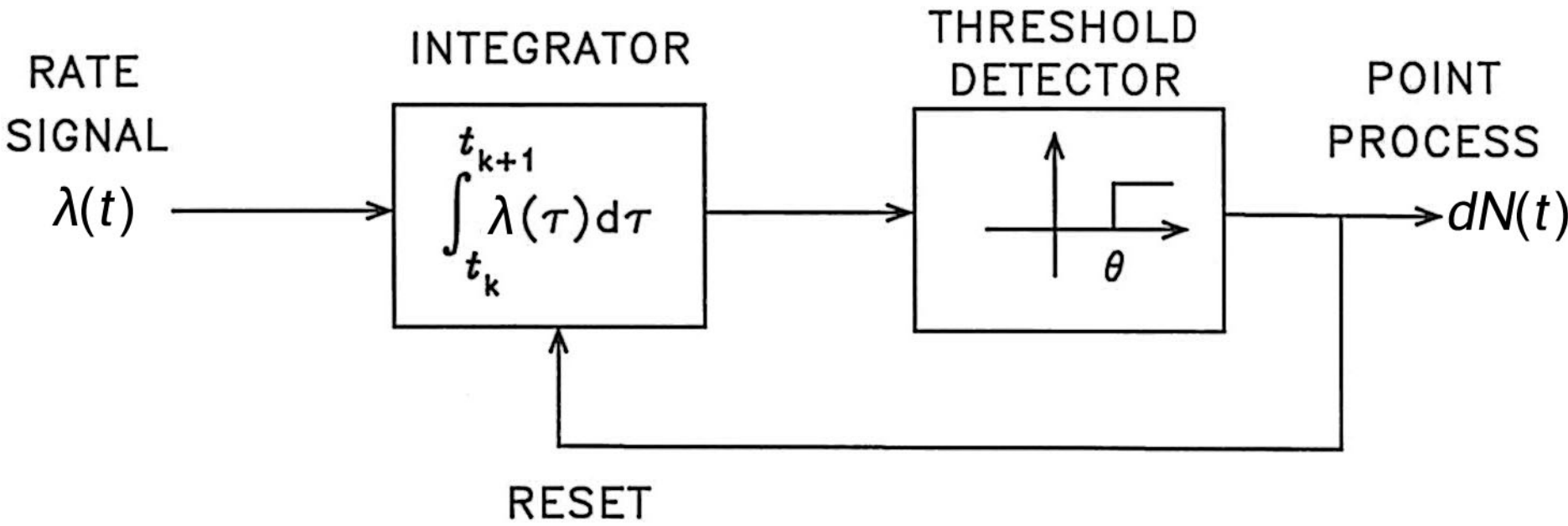


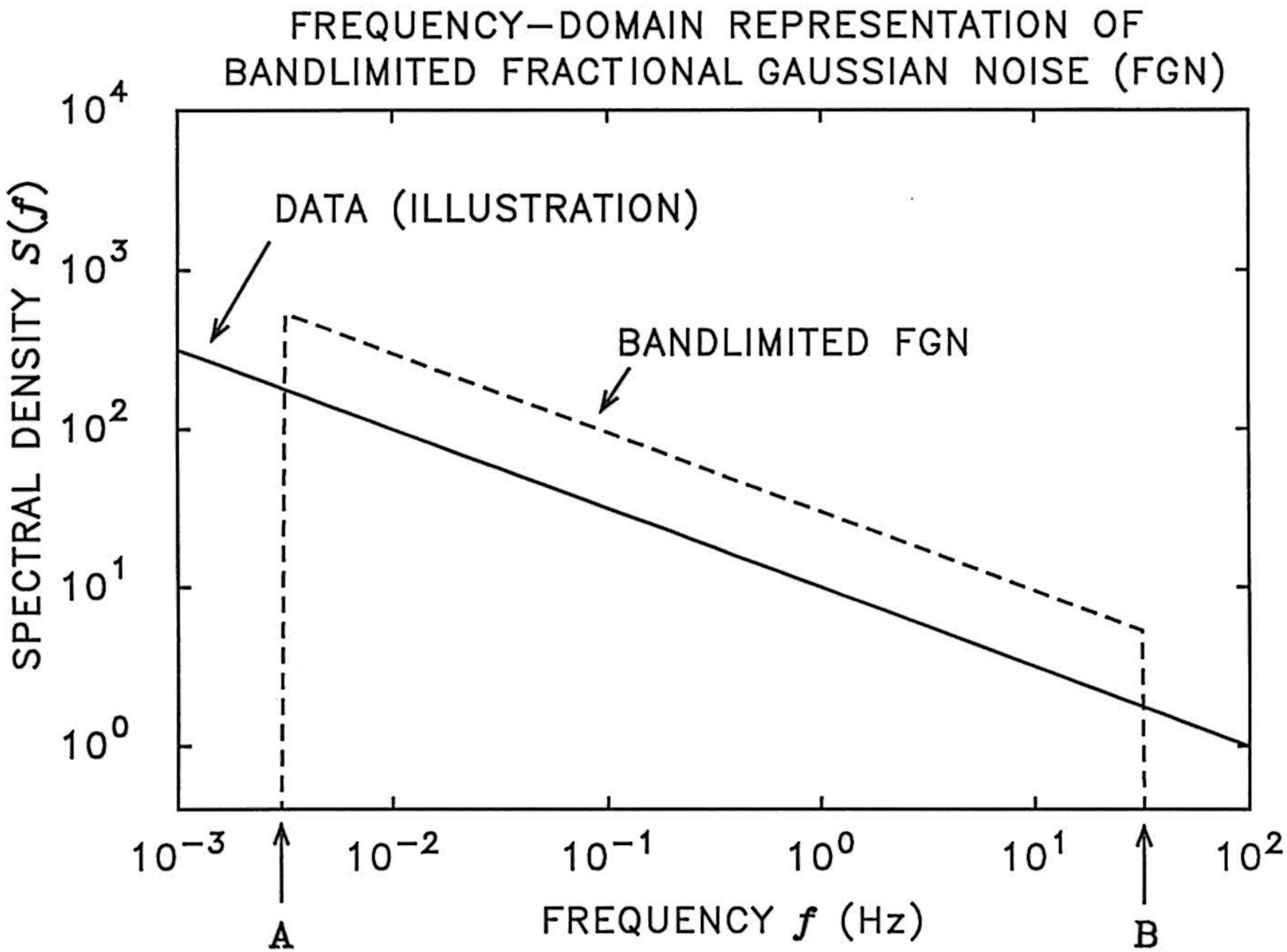
After Thurner, Feurstein, Lowen & Teich,
Phys. Rev. Letters **81**, 5688-5691 (1998).

M. C. Teich 2004

THEORY



After Turcott & Teich, *Ann. Biomed. Eng*. **24**, 269-293 (1996).



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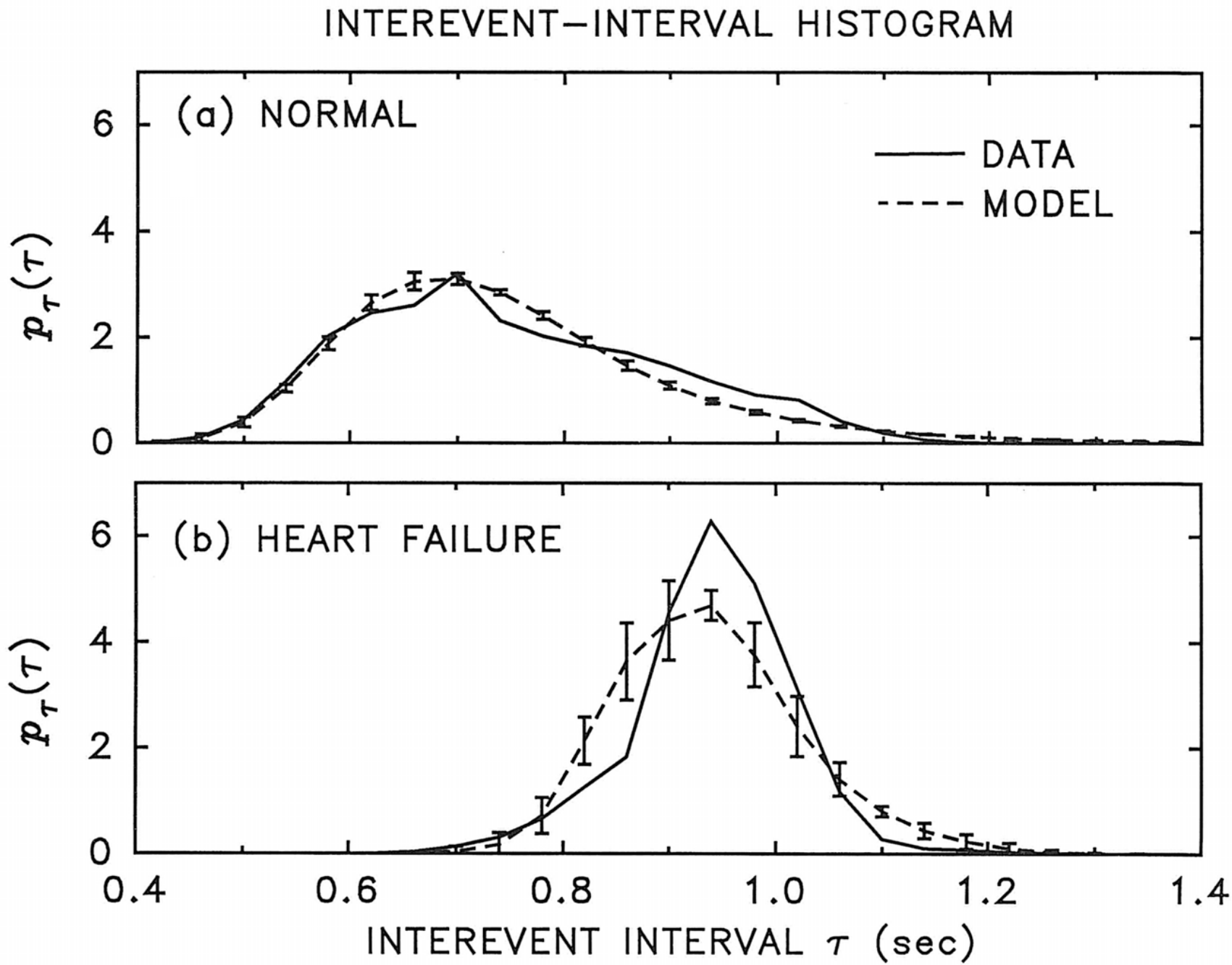

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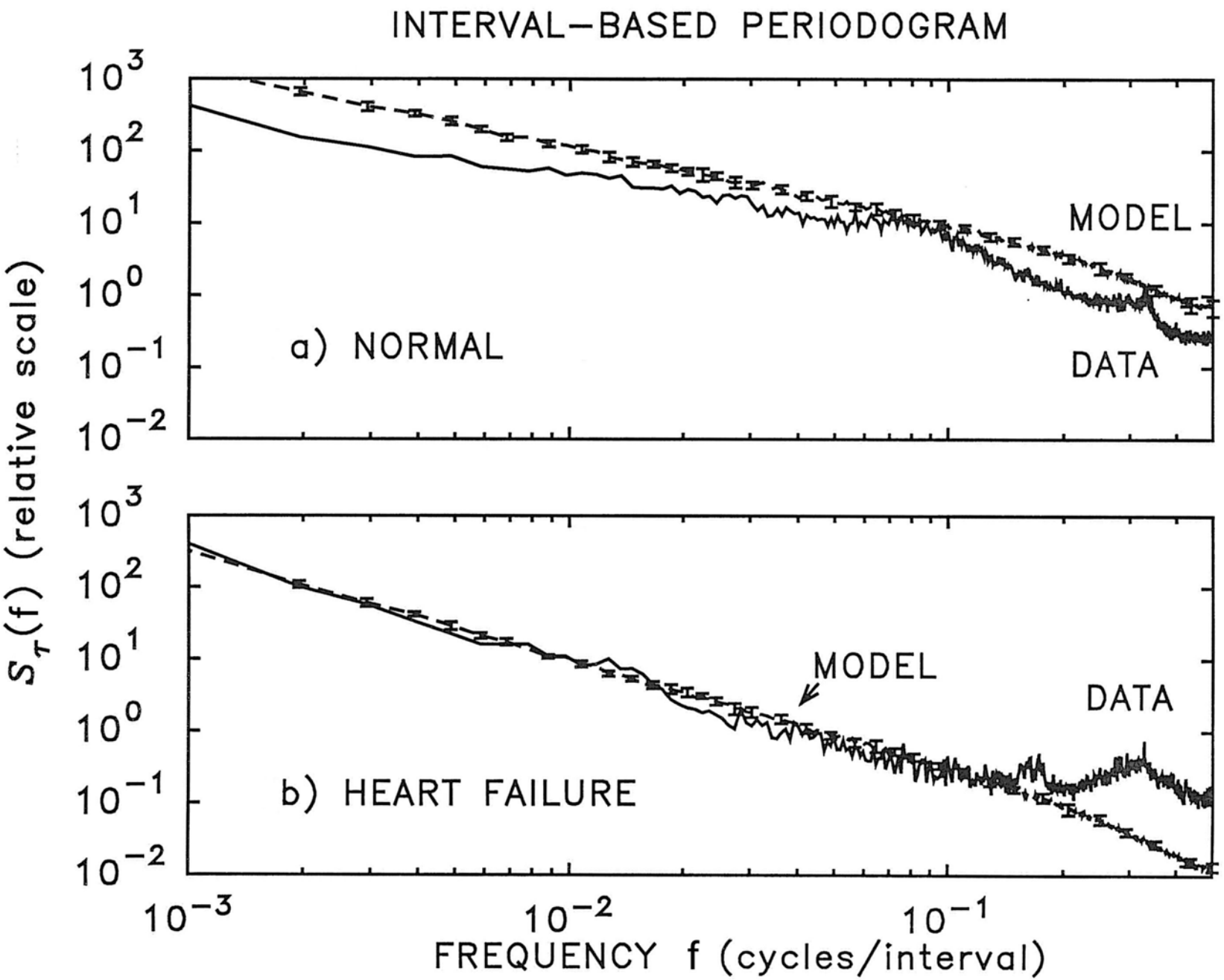

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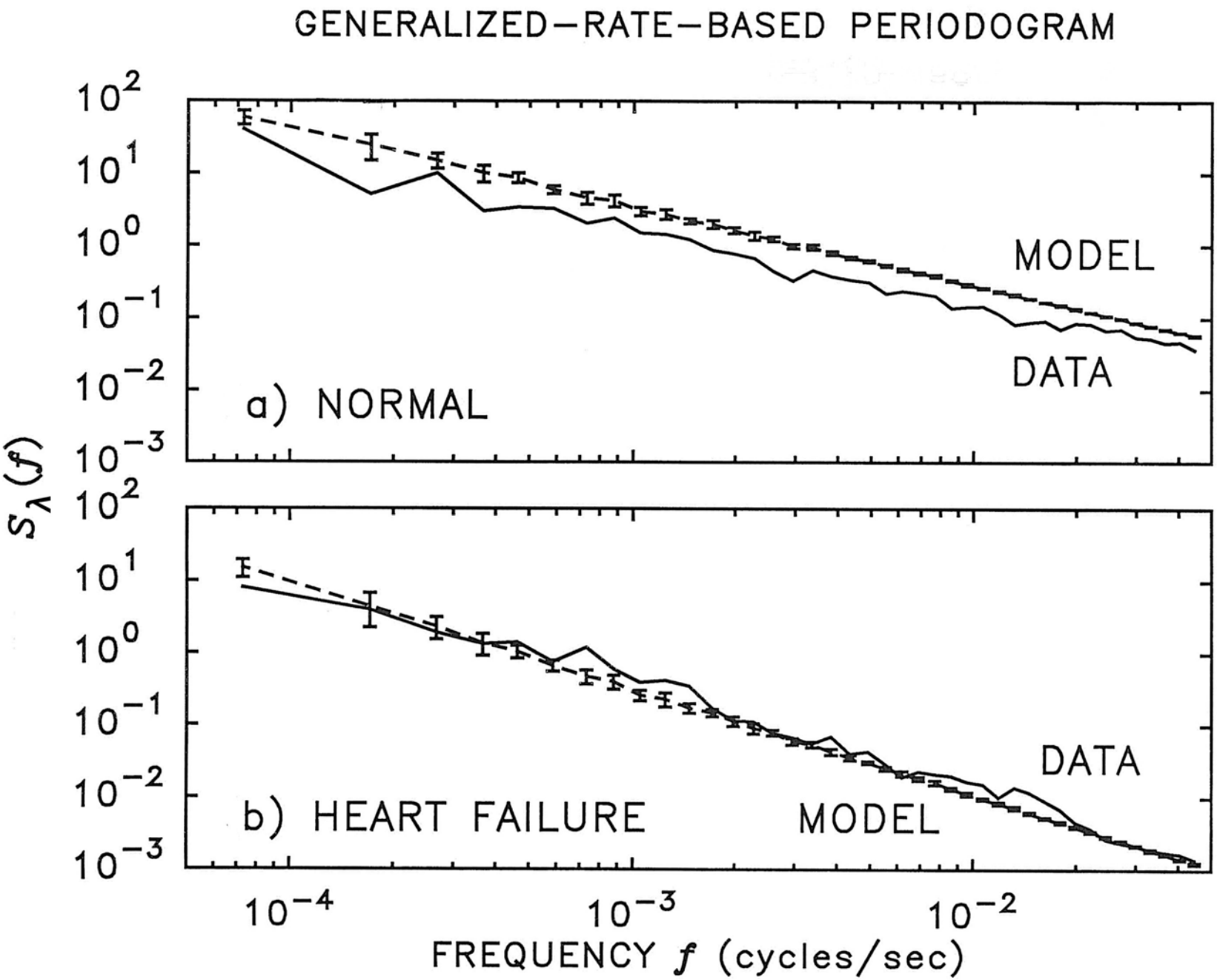

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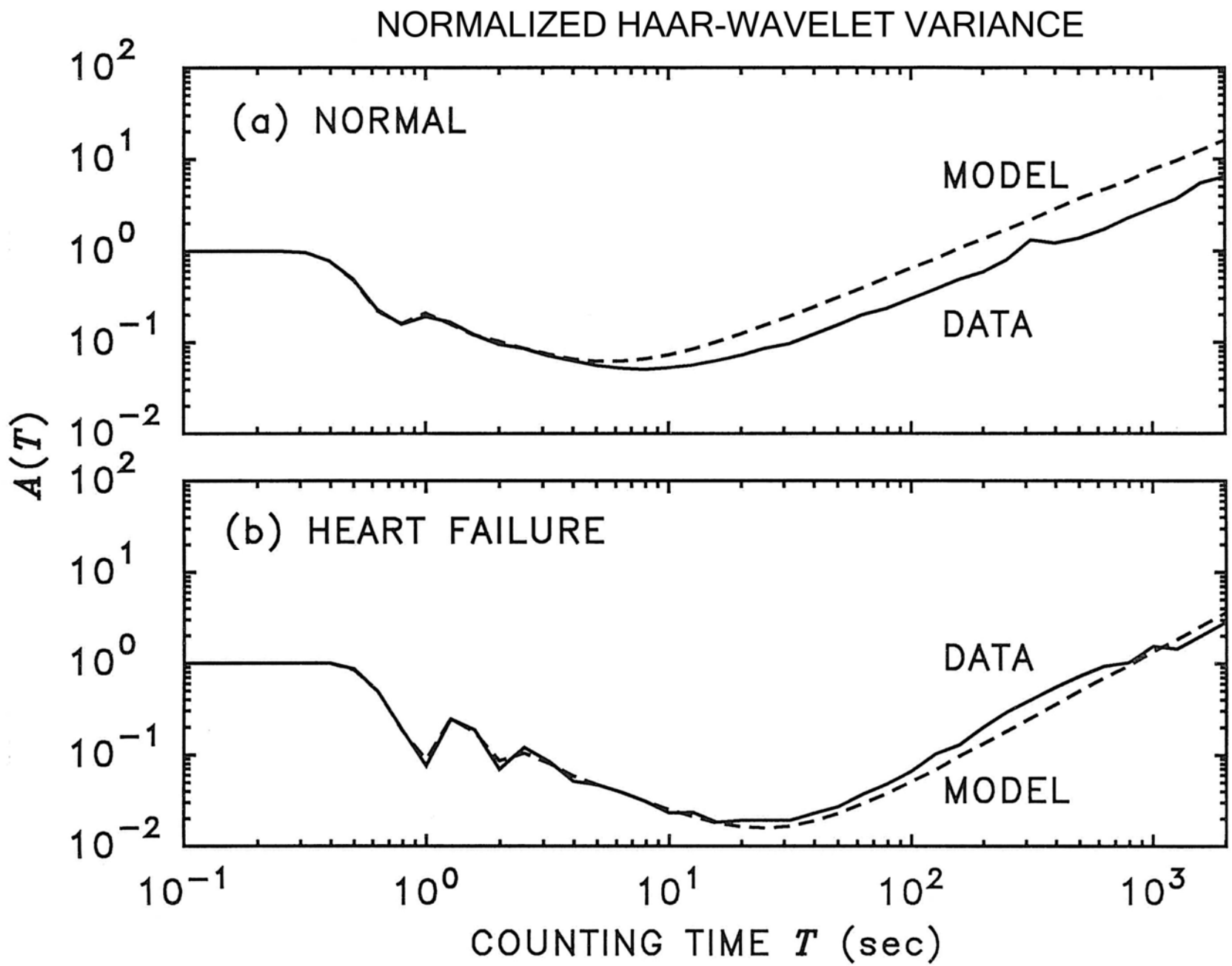
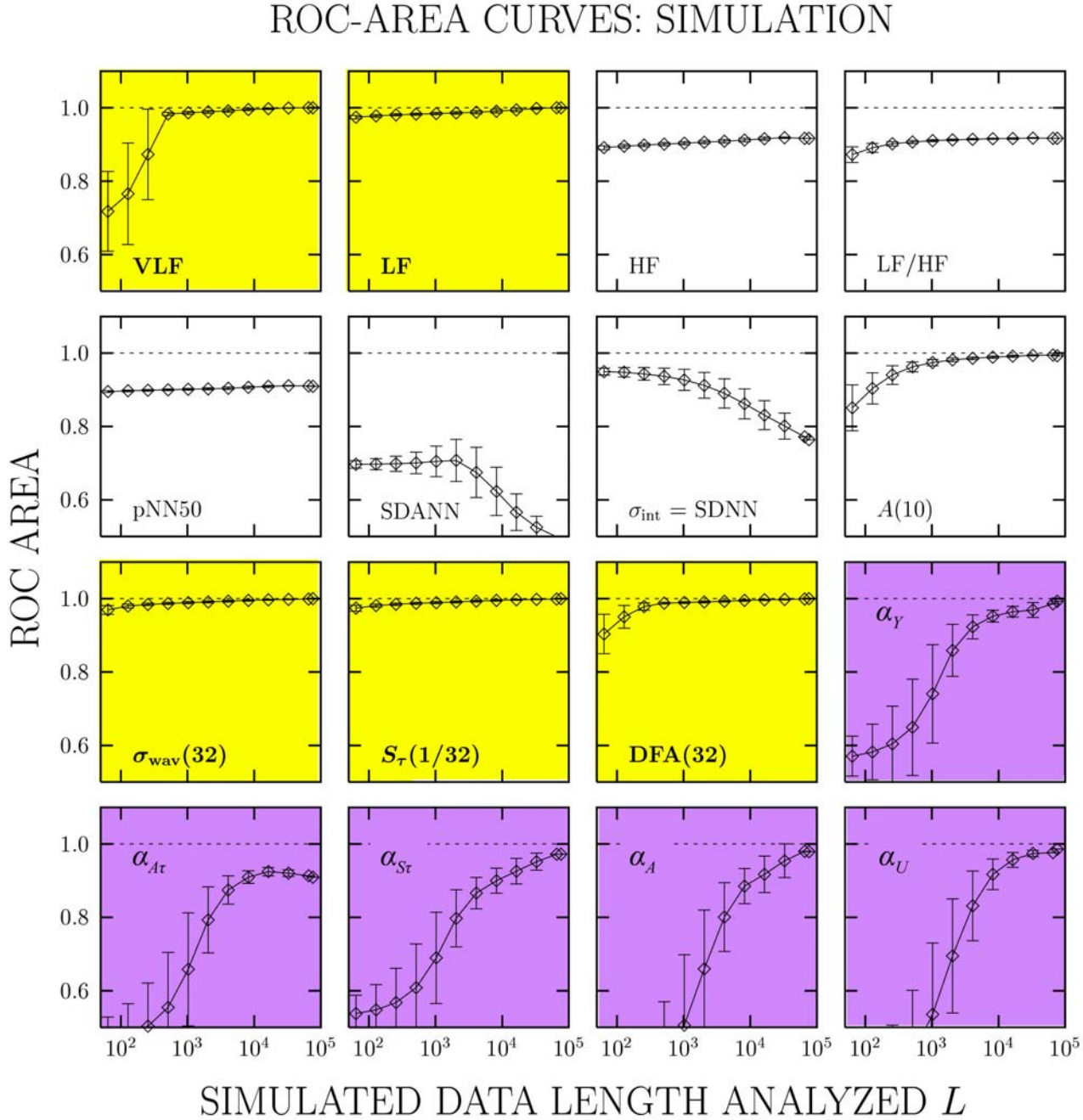


After Turcott & Teich, *Ann. Biomed. Eng.* **24**, 269-293 (1996).



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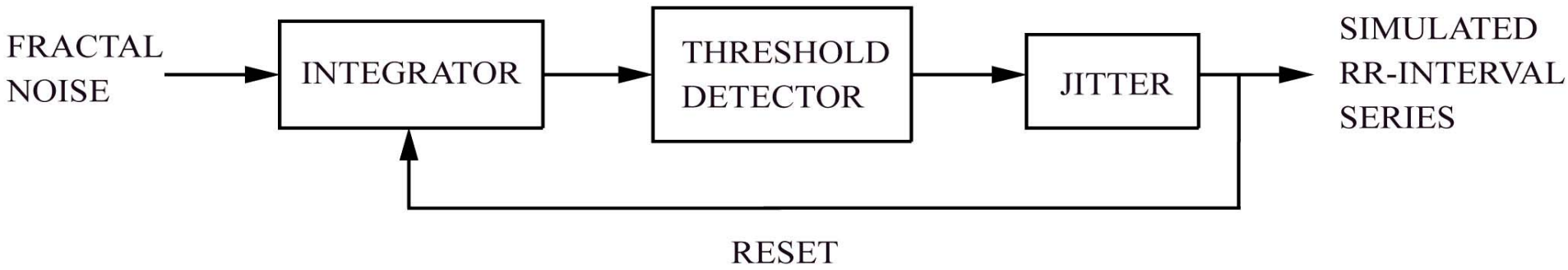

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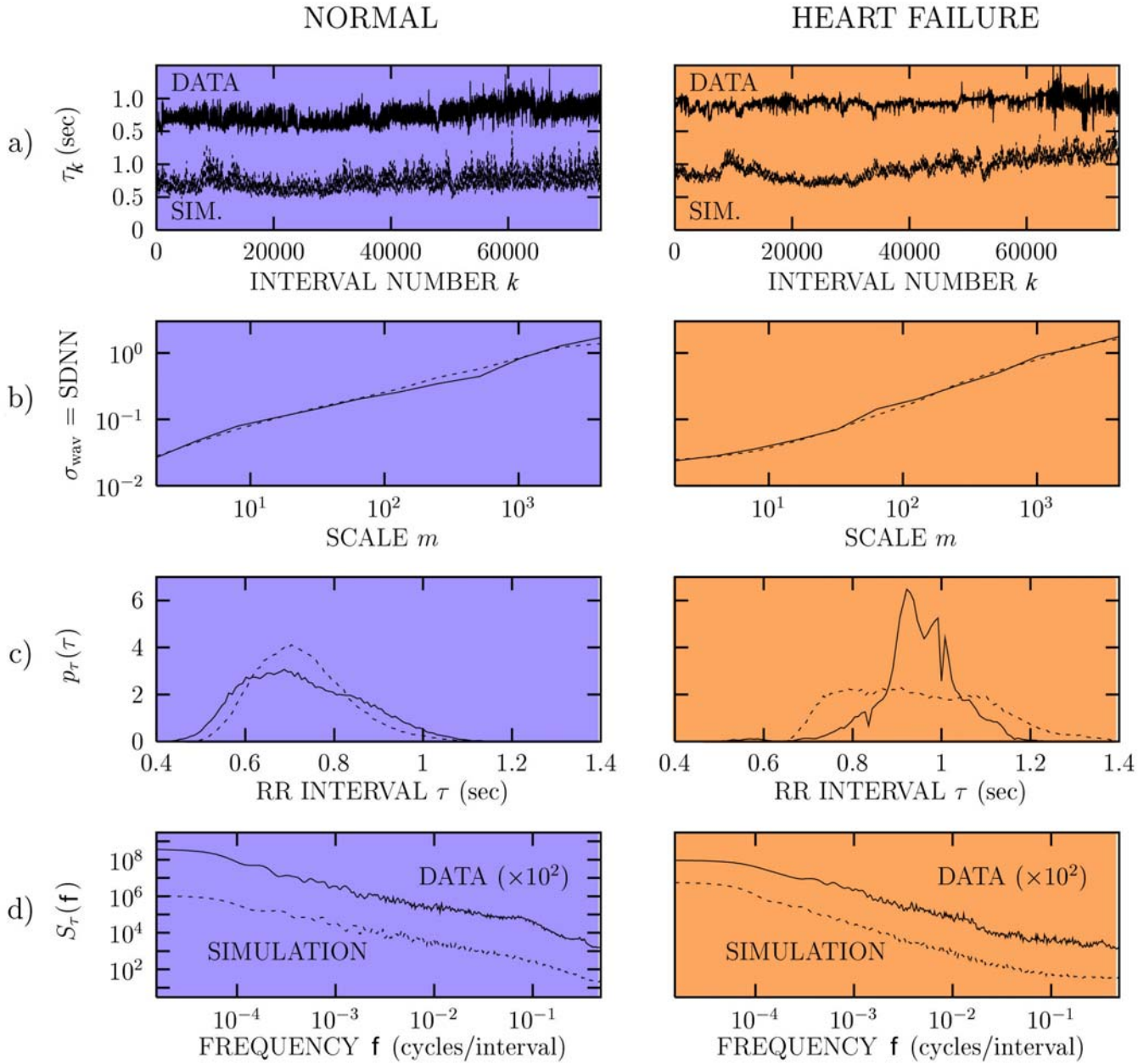

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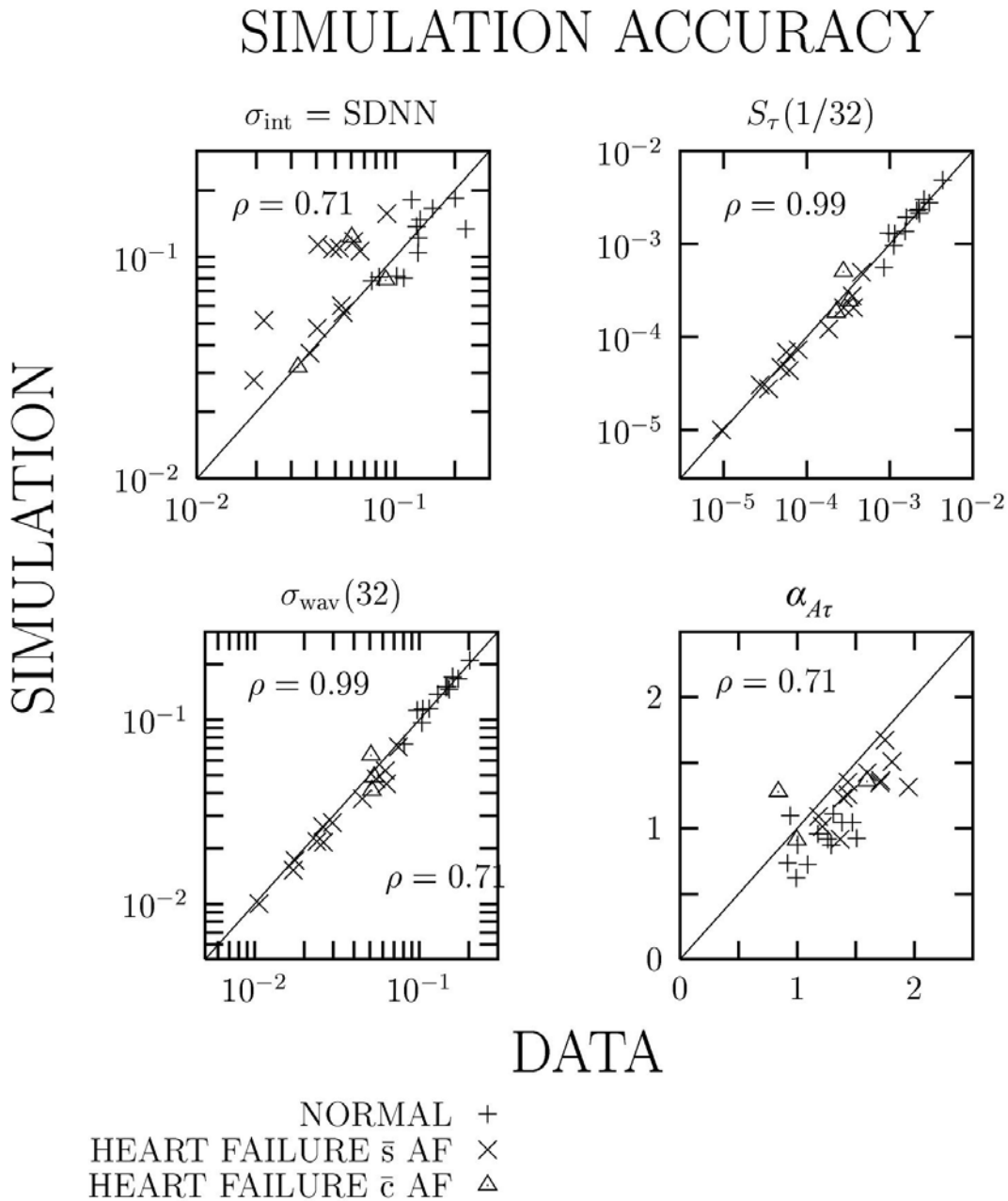

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DOES THE HEARTBEAT REFLECT DETERMINISTIC CHAOS?

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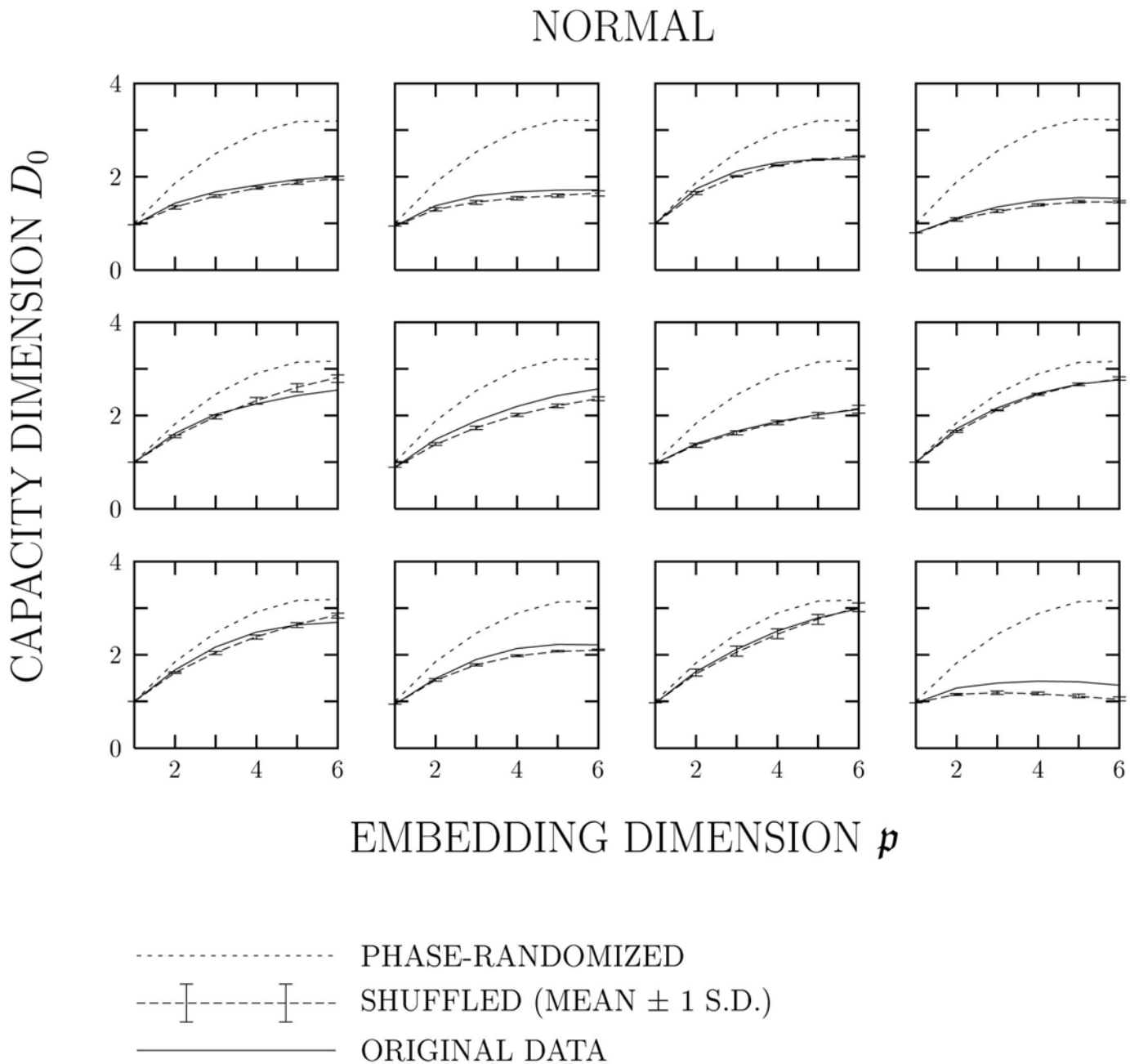

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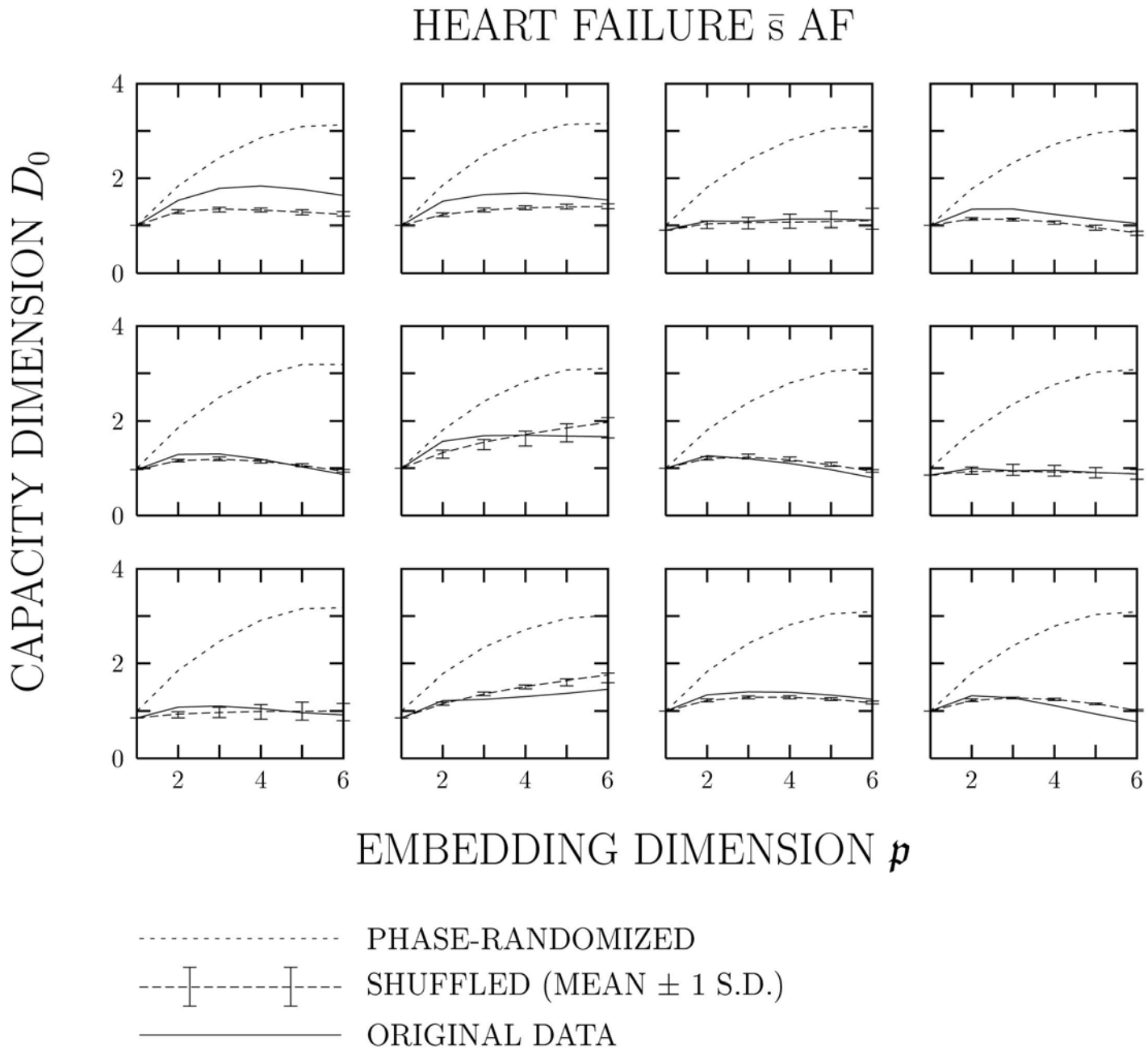

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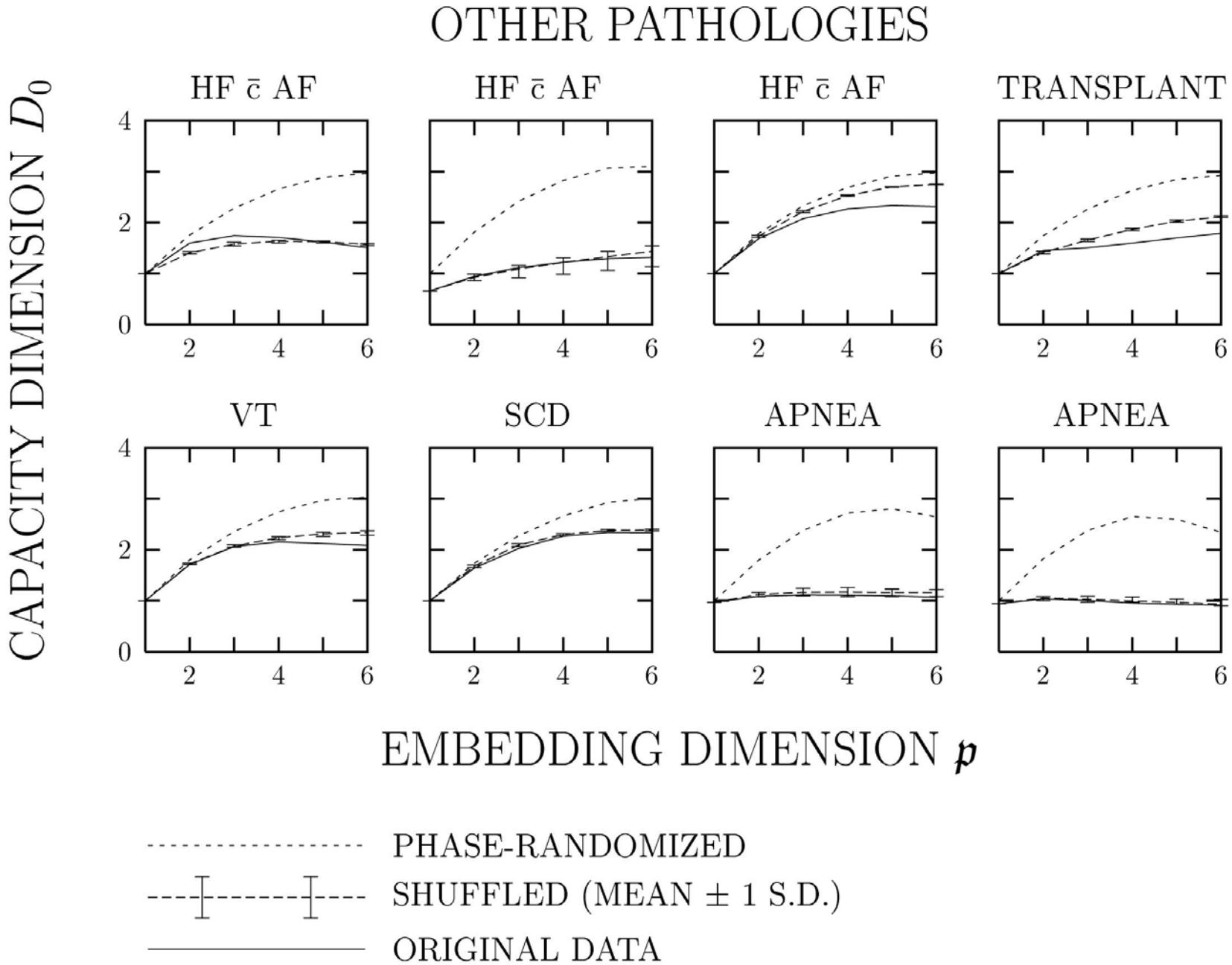

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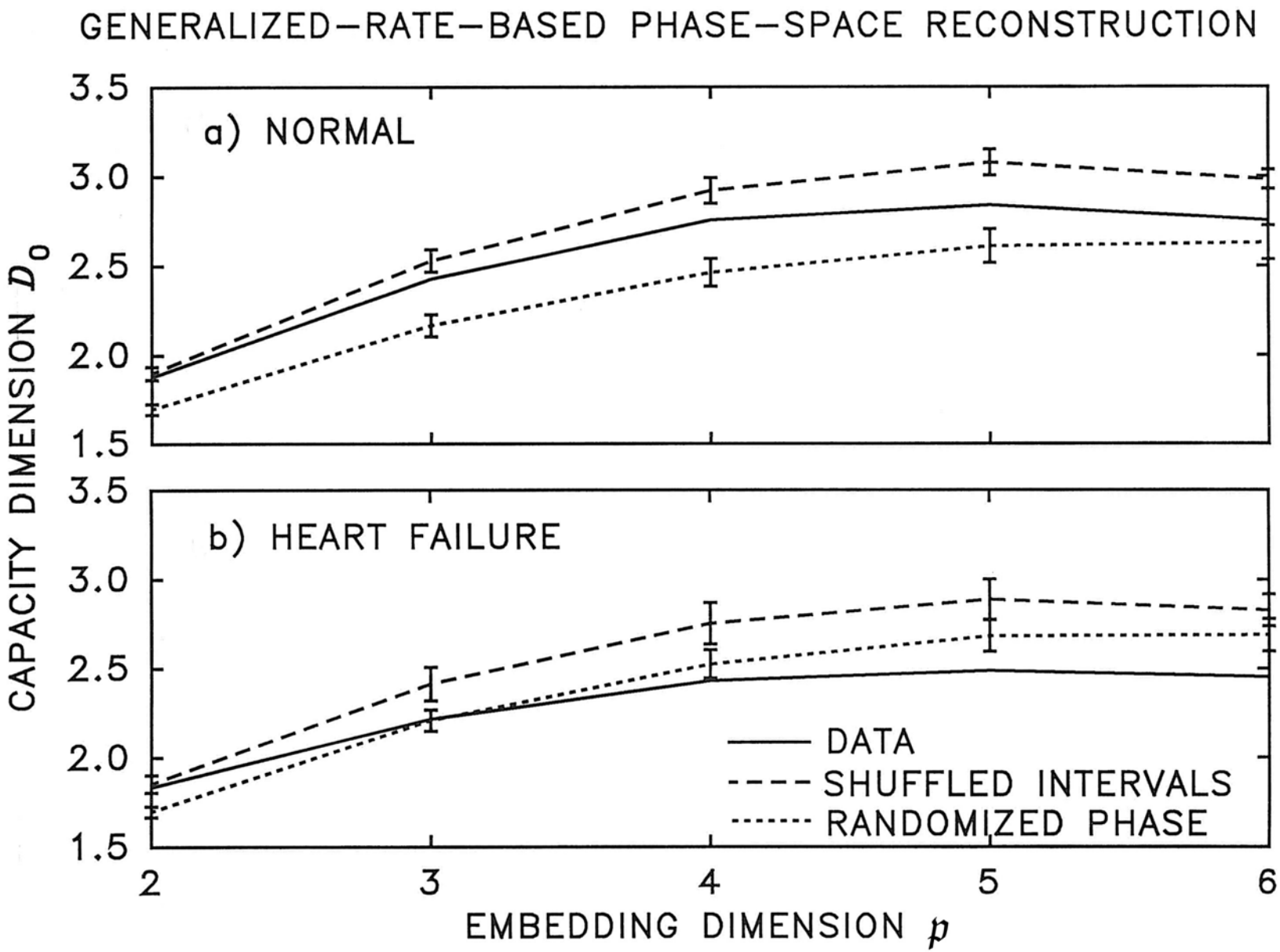

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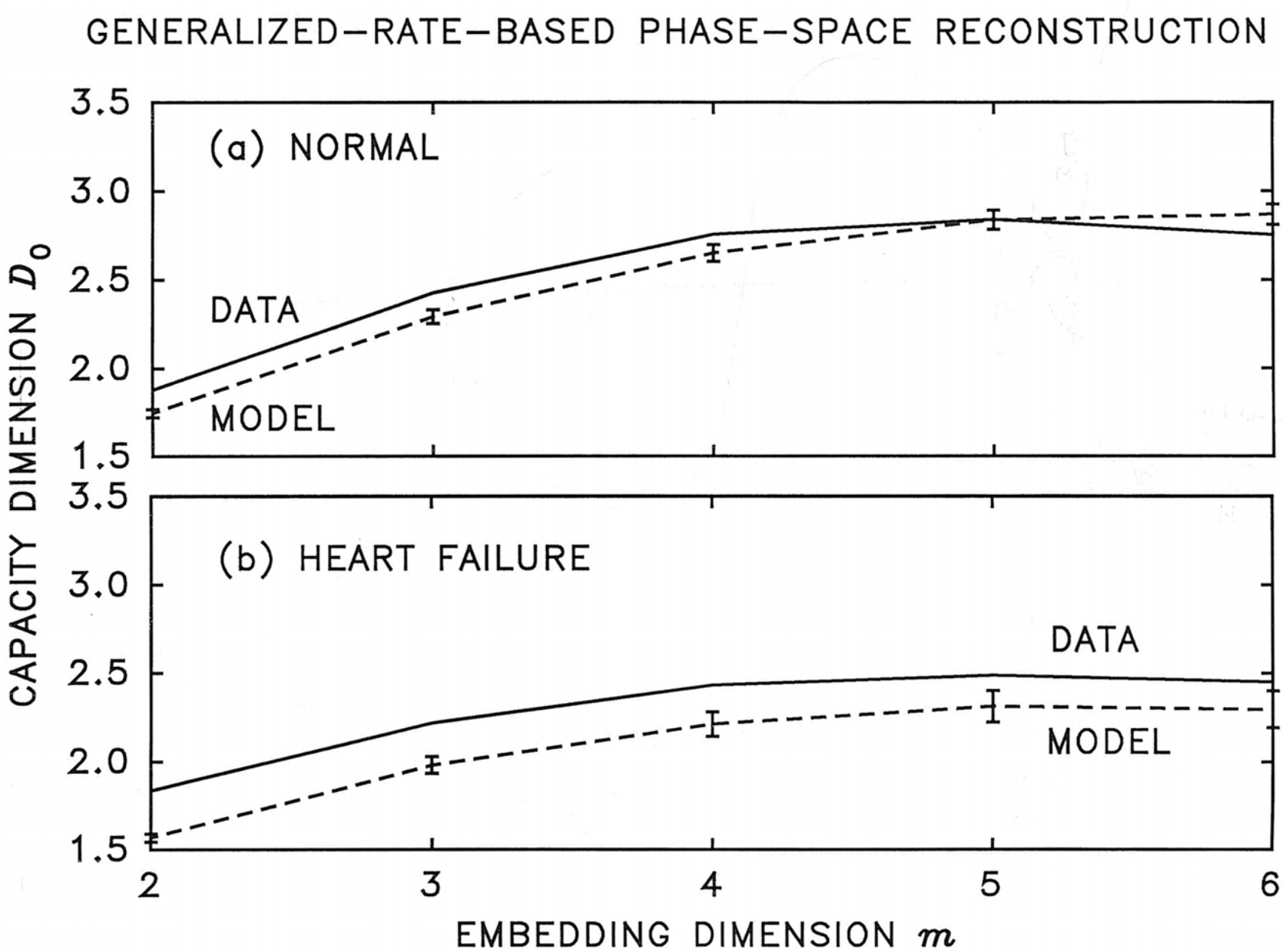

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