Empirical Mode Decomposition: Definition and application to non linear time series

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"Nonlinear dynamical methods and time series analysis workshop"
Udine (Italy), Aug. 29 – Sept. 1, 2006

Outline

- EMD : principle & algorithmic definition
- Equivalent filter bank
- Frequency resolution / components separation
- A simple application to satellite image time series
- Concluding remarks

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Principle

- Objective — Uniquely from the observation $x(t)$, get a AM-FM type decomposition:

$$x(t) = \sum_{k=1}^{K} a_k(t)\Psi_k(t)$$

with $a_k(.)$ amplitude modulating functions and $\Psi_k(.)$ oscillating functions.

- Idea — "signal = fast oscillations superimposed to slow oscillations".
- Operating mode — ("EMD", Huang et al., '98)
  1. identify locally in time, the fastest oscillation
  2. subtract it from the original signal
  3. iterate upon the residual

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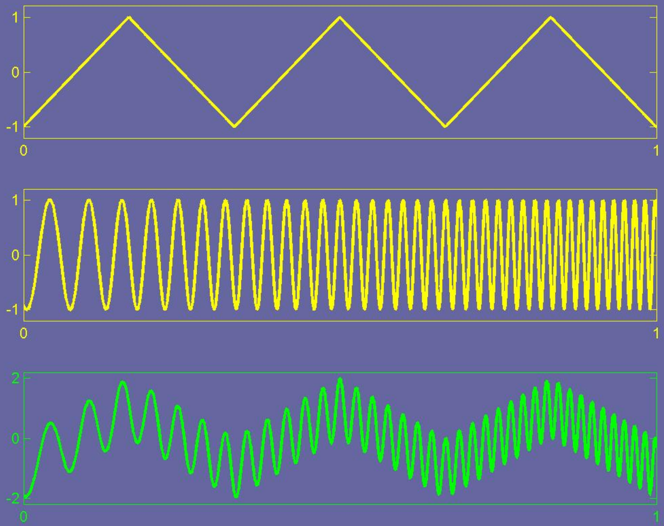
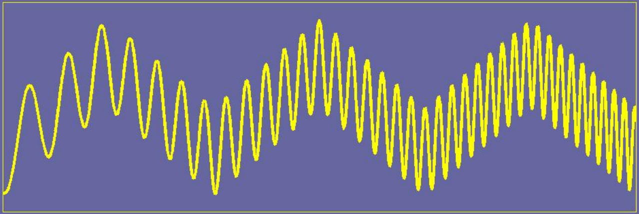


Algorithm - Sifting Process



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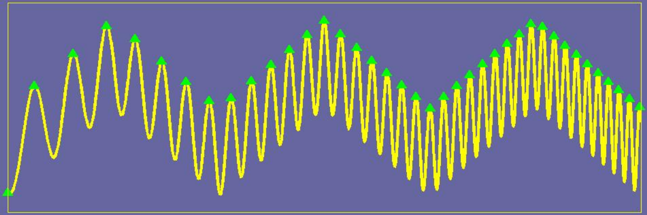

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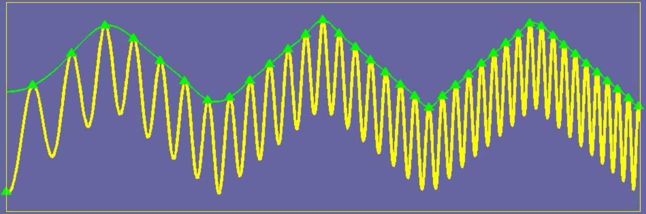

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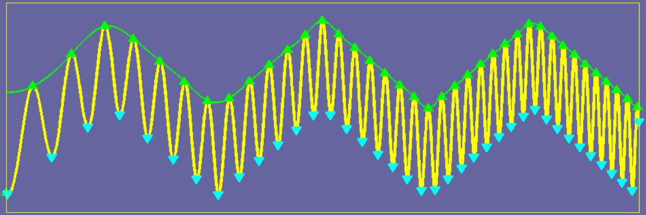

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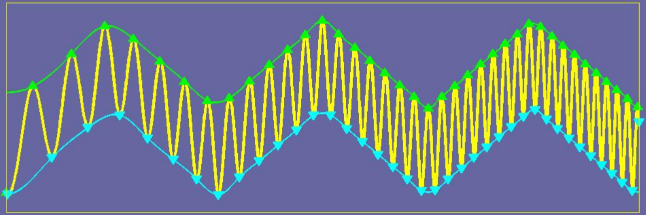

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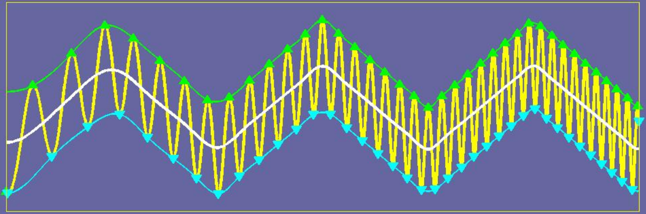

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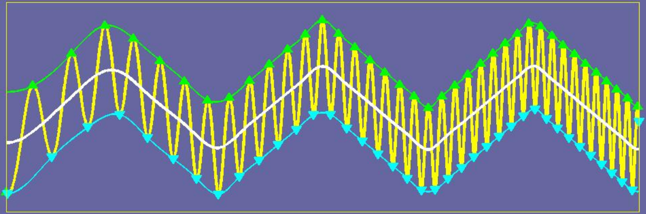

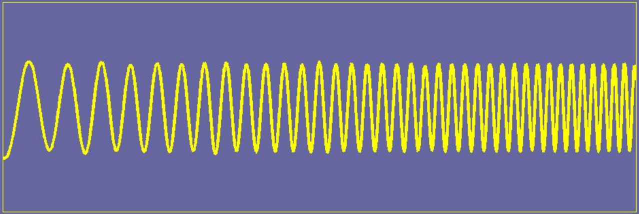

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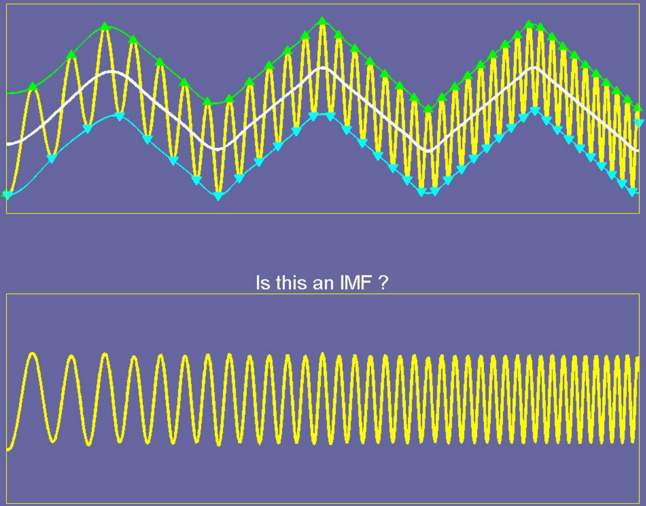

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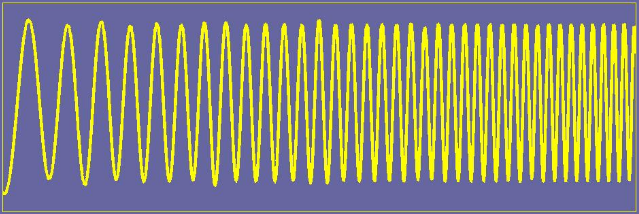

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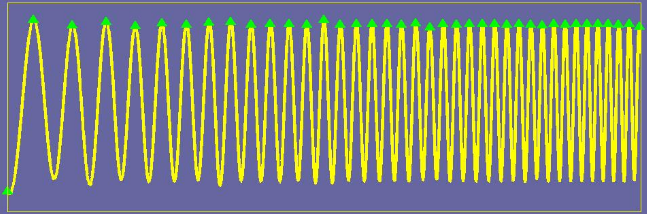

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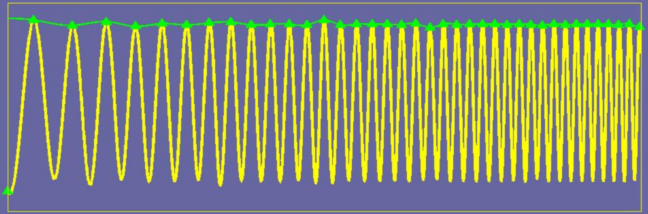

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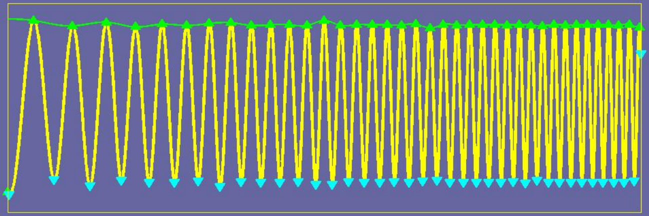

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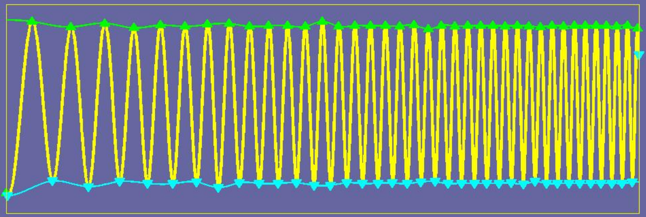

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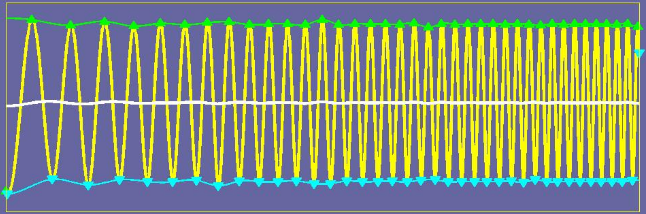

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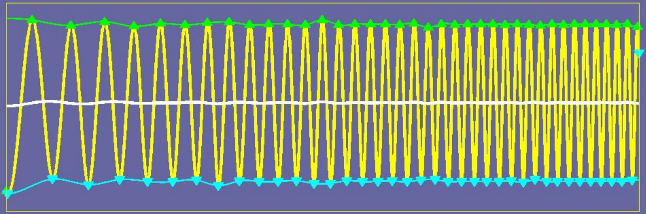

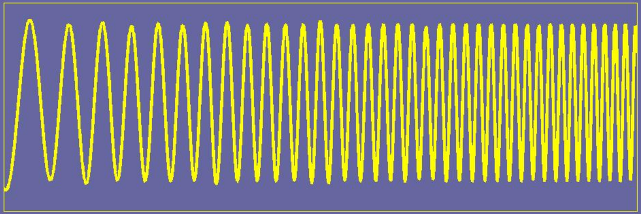

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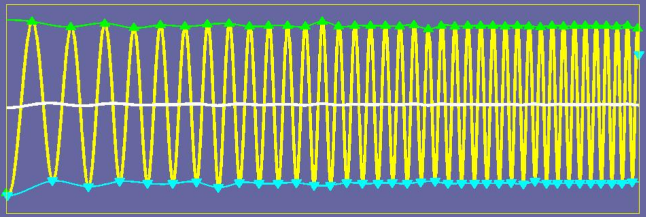

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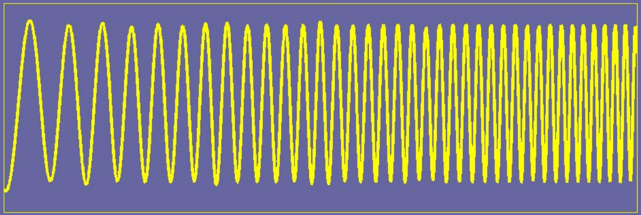

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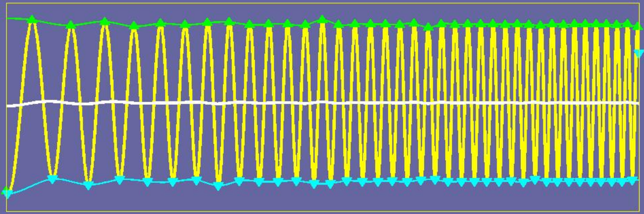

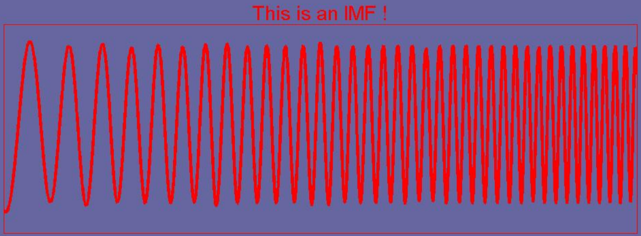

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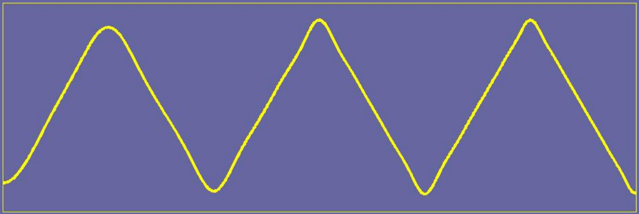

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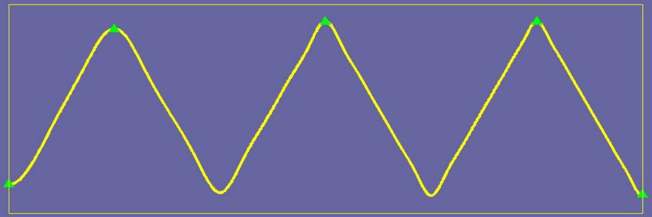

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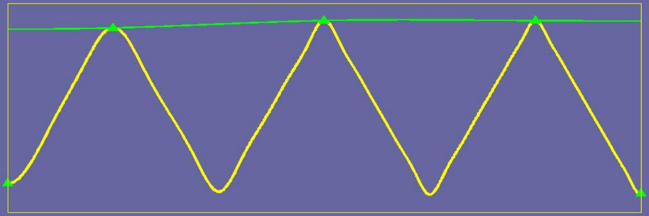

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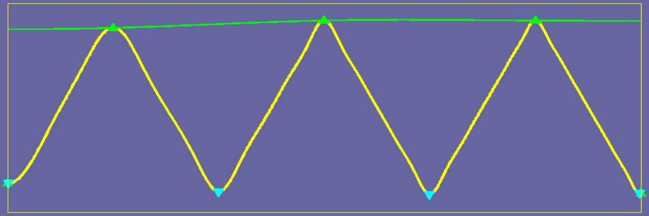

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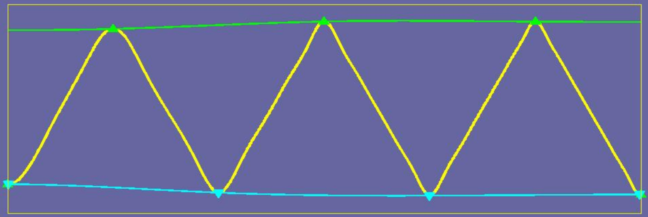

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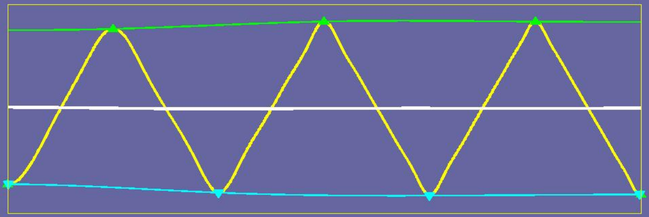

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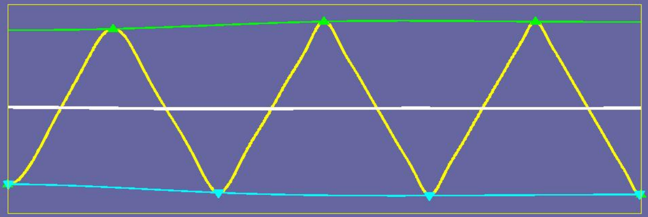

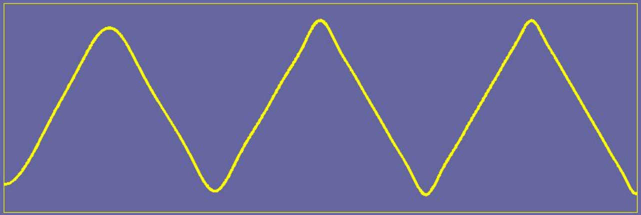

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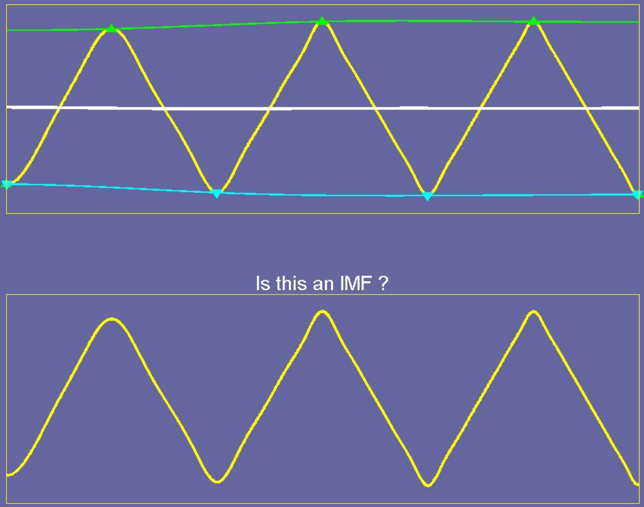

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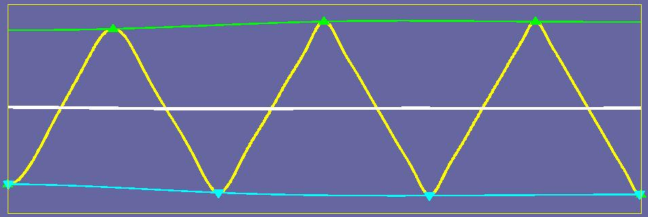

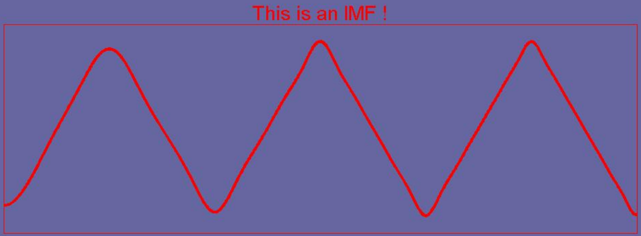

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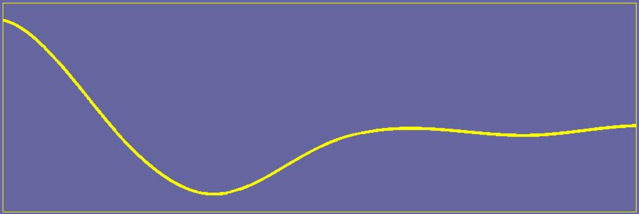

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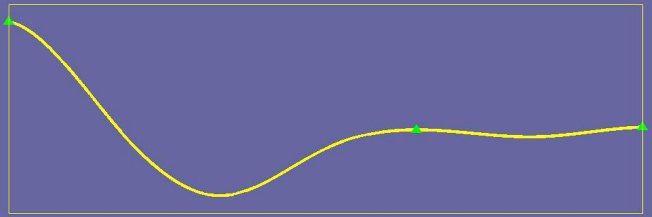

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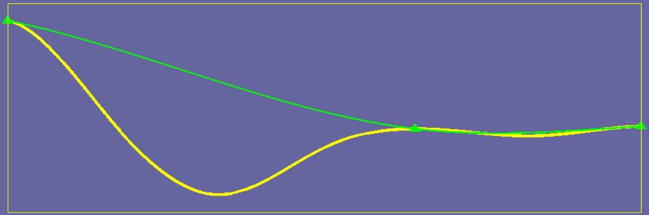

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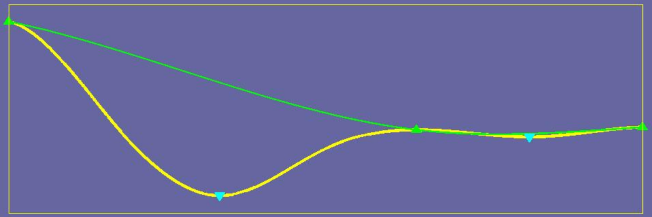

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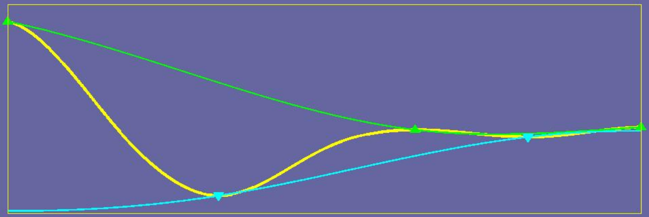

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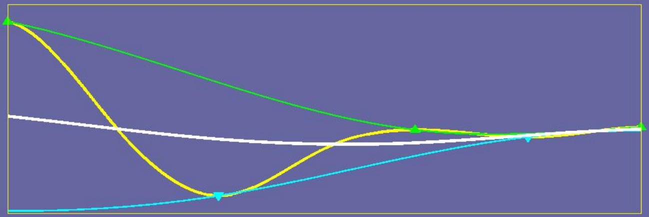

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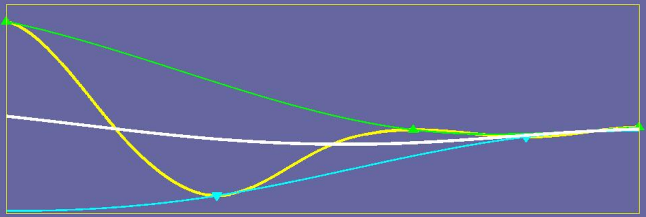

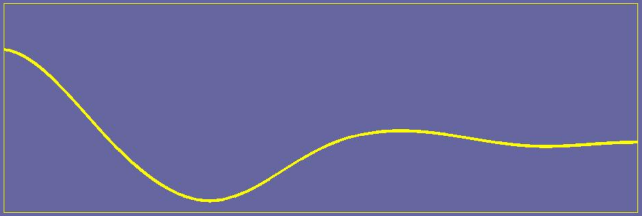

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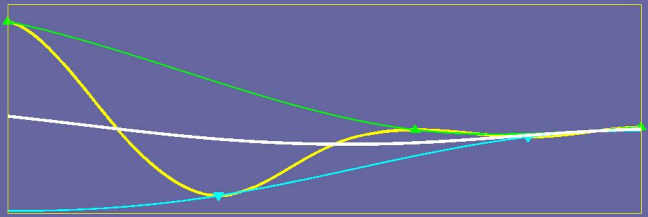

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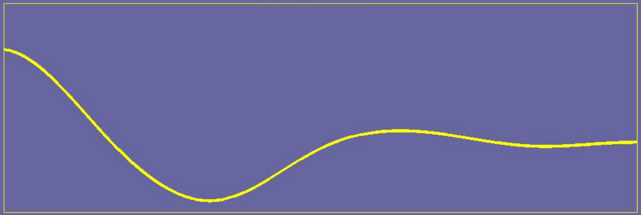

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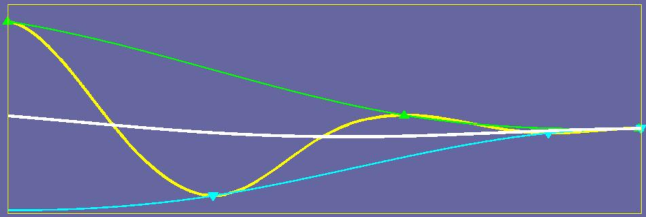

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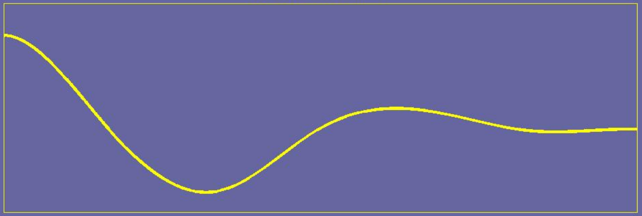

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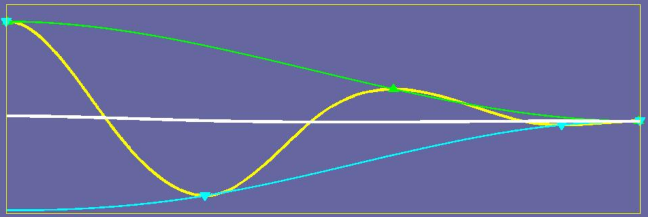

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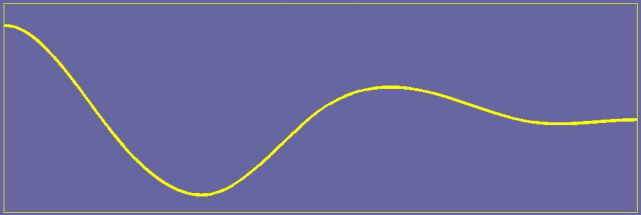

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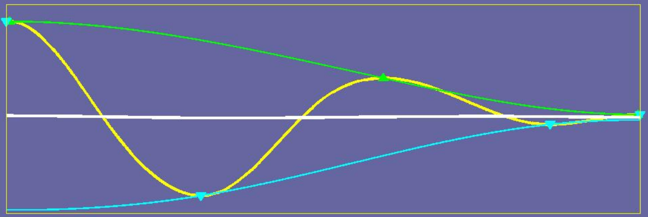

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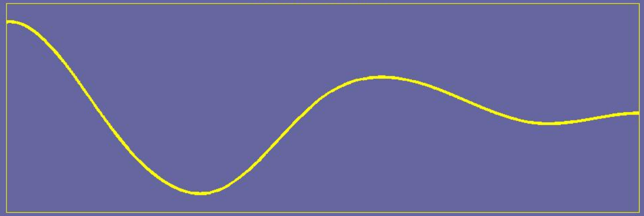

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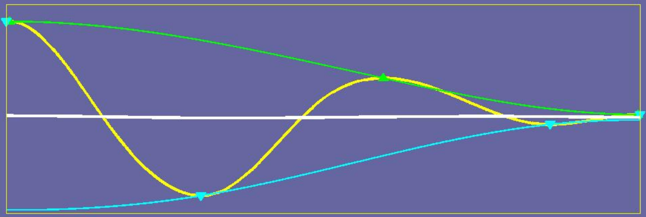

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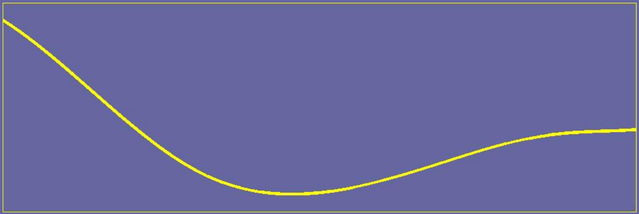

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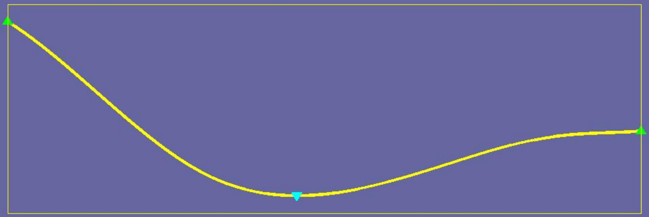

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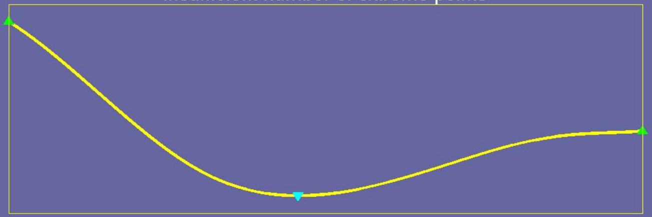
Insufficient number of extreme points



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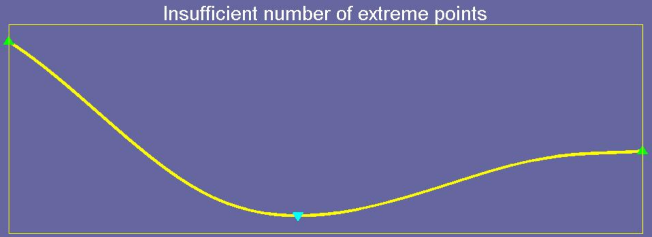

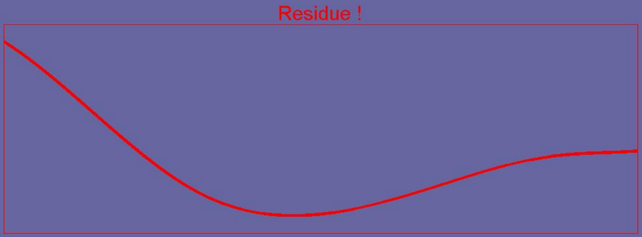

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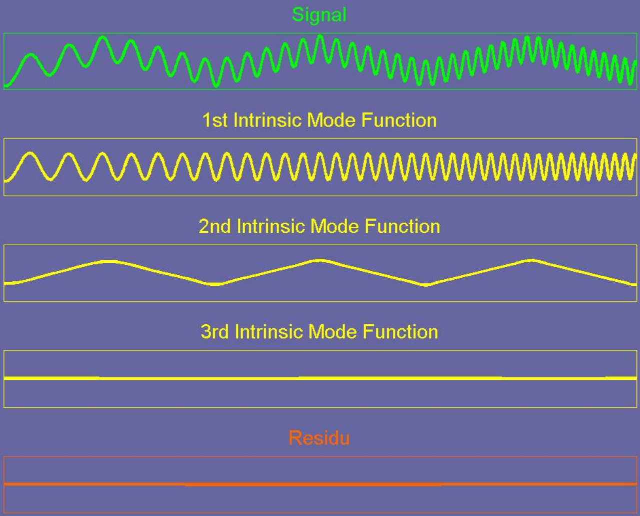

Intrinsic Mode Function

Definition

1. $\#\{\text{zero crossing}\} = \#\{\text{extrema}\} \pm 1$
2. symmetric envelopes around the $y = 0$ axis

- symmetrically oscillating functions
- IMF $\neq$ Fourier mode, and in nonlinear situations IMF $=$ several Fourier modes
- Output of a self-adaptive time-varying filter ($\neq$ standard linear filter)

illustration: 2 sinus FM + gaussian wave packet

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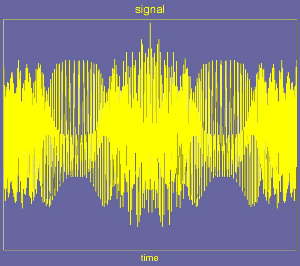
illustration: 2 sinus FM + gaussian wave packet

Self-adaptive filter



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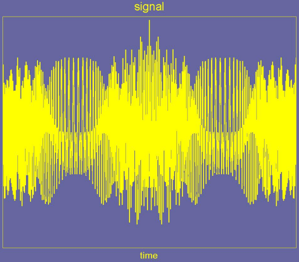

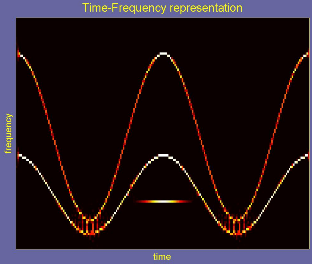

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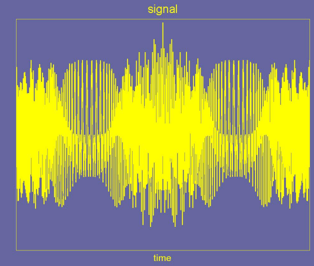
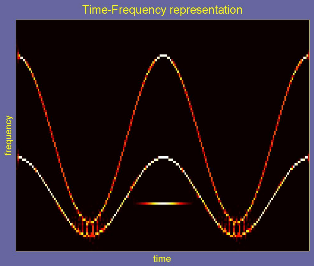
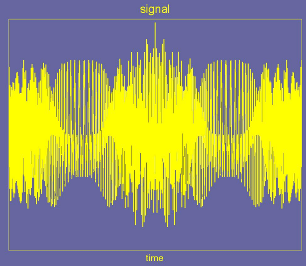
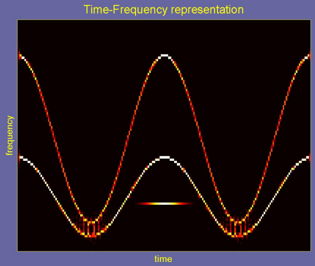


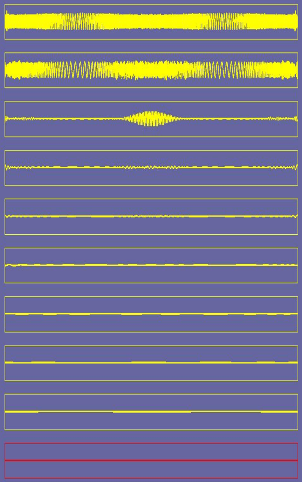


How to separate these time and frequency overlapping components ?

Self-adaptive filter







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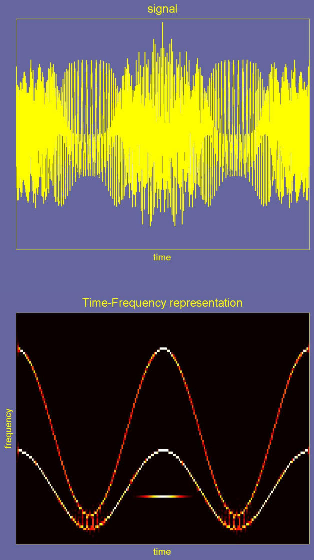

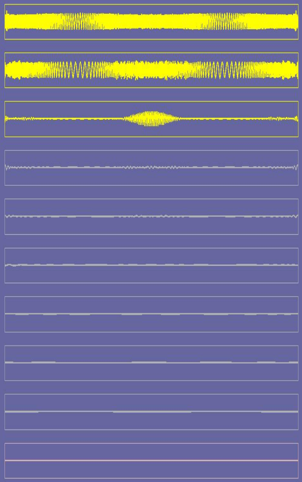

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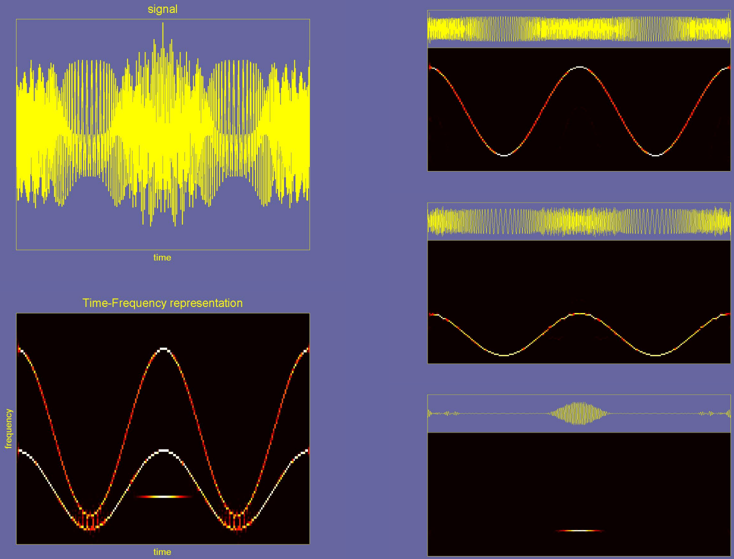

IMF - A mathematical Approach (Vatchev, 2002)

Definition

A twice differentiable function f is an IMF if it is a solution of the self-adjoint ODE:

$$(Pf')' + Qf = 0$$

with $P(t) > 0$ and $Q(t) > 0$ for $t \in [a, b]$

and

$$Q(t) = 1/P(t)$$

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  ensures that $\#\{\text{zero crossings}\} = \#\{\text{extremes}\} \pm 1$

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- $Q(t) = 1/P(t)$

ensures that upper and lower envelopes $U(t)$ and $L(t)$ are symetric

IMF - A mathematical Approach (Vatchev, 2002)

Theorem

If f is solution of a self-adjoint ODE, with

$$Q(t) = 1/P(t) > 0$$

then f is an oscillating function with constant amplitude (harmonic function) !

In practice, to overcome the envelope restriction:

- require that $|U(t) - L(t)| < \varepsilon$, for some prescribed $\varepsilon > 0$
- change the envelope definition (???)

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Equivalent filter bank: a stochastic approach

Goal — Consider EMD as a filter bank and identify for each IMF an "equivalent frequency response"

Model — fractional Gaussian noise (fGn – derivative of fractional Brownian motion fBm) with Hurst exponent $0 < H < 1$:

- $H = 1/2$ white Gaussian noise
- $H < 1/2$ short range correlation
- $H > 1/2$ long range correlation
- power spectral density (PSD) $\sim f^{1-2H}$

Simulations — for $H = 0.1$ to 0.9, PSD of each IMF is estimated over a set of 1000 realizations of $N = 512$ points

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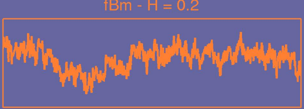

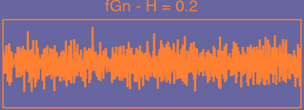

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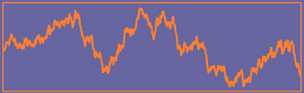

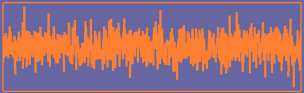

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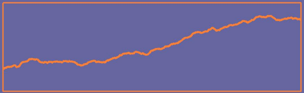

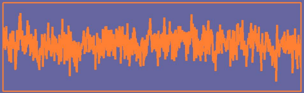

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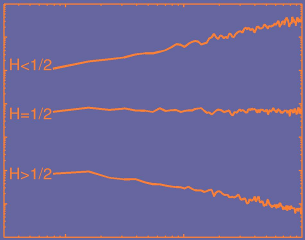

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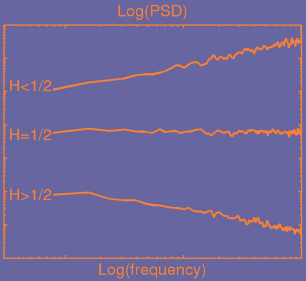

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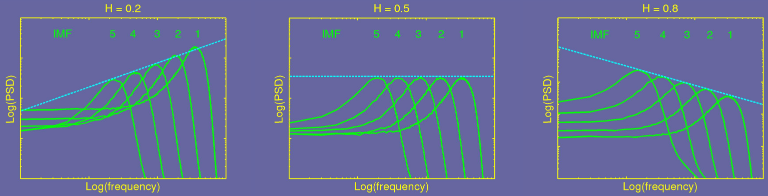

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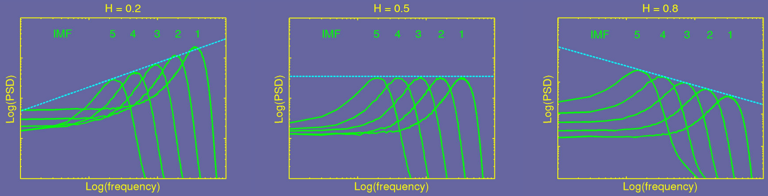

Result

$$S_{k',H}(f) = \rho_H^{(2H-1)(k'-k)} S_{k,H}(f), \textit{ with } \rho_H \approx 2 : \underline{\textit{dyadic filter bank}}$$

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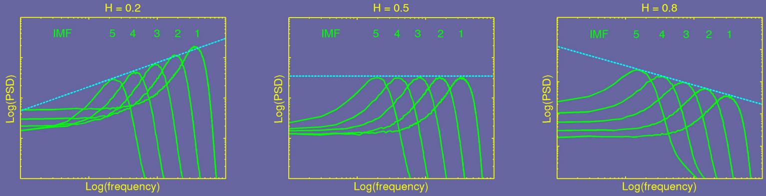

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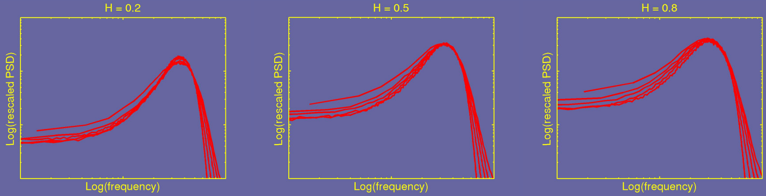

Equivalent filter bank: a deterministic approach

Goal — Consider EMD as a filter bank and identify for each IMF an "equivalent impulse response"

Model — Impulse as the "*limit process*" of the sum

$$s(t) = \delta(t) + \epsilon n(t)$$

with $n(t)$ a white Gaussian noise, and $\epsilon \to 0^+$

Simulations — Compute the average mean for each IMF over a set of 1000 realizations of $s(t)$ with $N = 2048$ points

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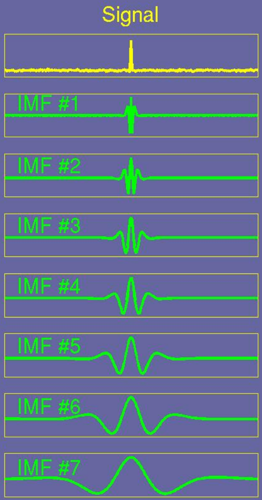

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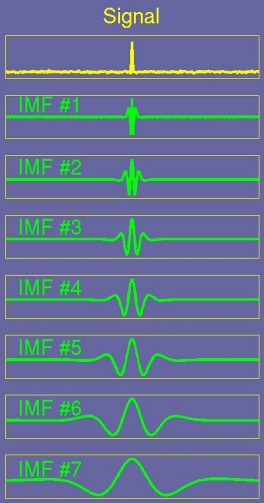

$$\mathrm{IMF}_k = a^k \, \Psi\left(a^k t\right)$$

Wavelet-like multiresolution analysis

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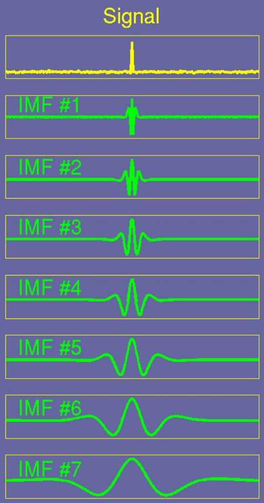

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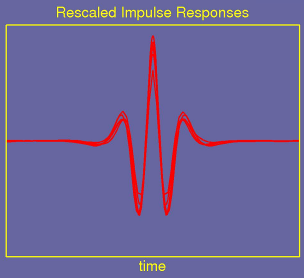

Frequency resolution

Problem — When can EMD separate pure tones in a sum

$$s(t) = \sum_{k=1}^{k=M} a_k \sin(\omega_k t + \phi_k)$$

Obvious exceptions — Fourier series

Model — A sum of two sine waves $s(t) = \underbrace{a_1 \sin(\omega_1 t)}_{s_1(t)} + \underbrace{a_2 \sin(\omega_2 t)}_{s_2(t)}$

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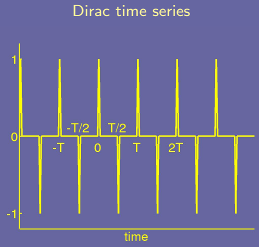

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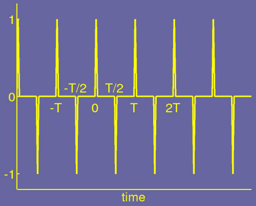
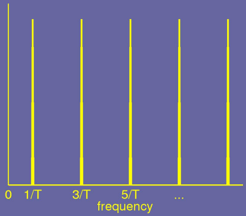


Dirac spectrum



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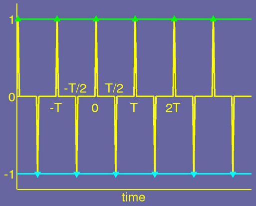

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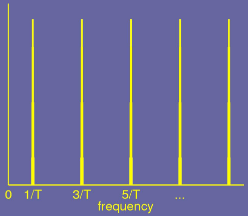

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- $0 < \omega_2 \leq \omega_1$ and $a_1/4 \leq a_2 \leq 4a_1$
- Resolution measure (relative error)

$$e(\omega_1) = \frac{\| s_1(t) - imf_1(t) \|_{l_2}}{\| s_2 \|_{l_2}}$$

- Ignore sampling issues... $(\omega_1 \ll \pi)$

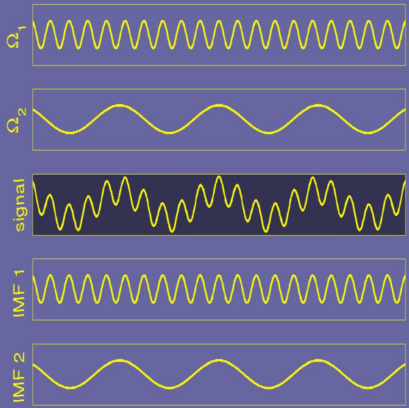
Frequency resolution

varying frequency $\omega_2 \leq \omega_1$

Confusion map

Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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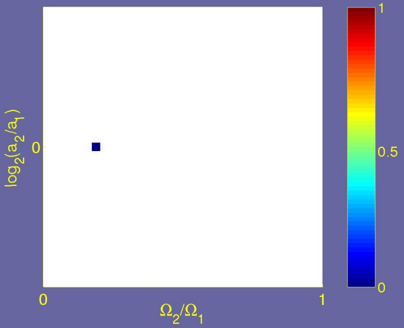

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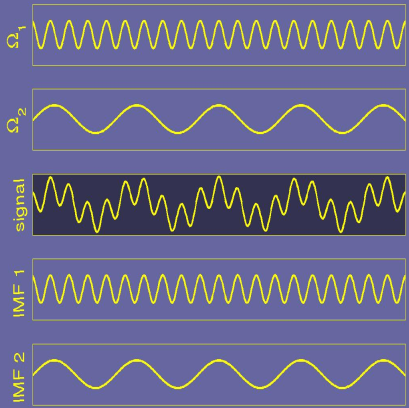

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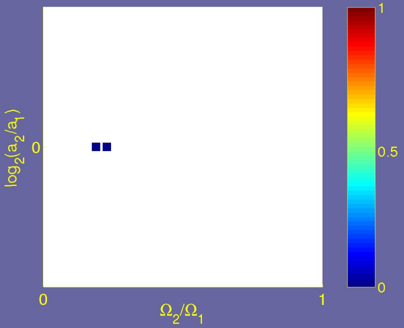

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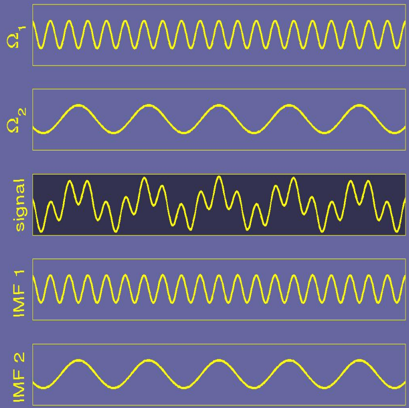

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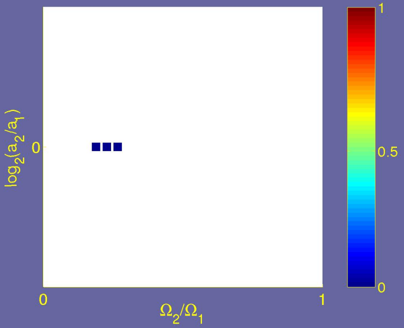
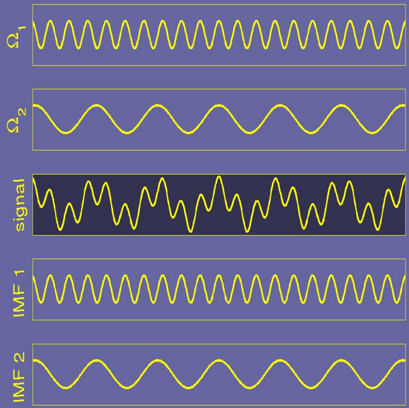
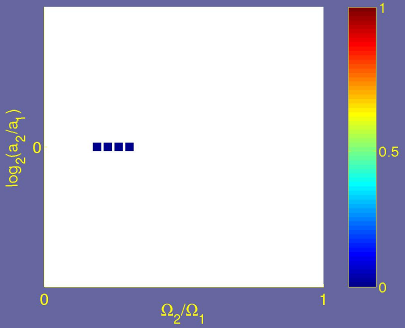


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$

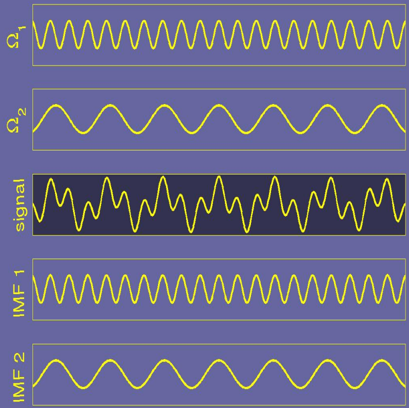


Confusion map

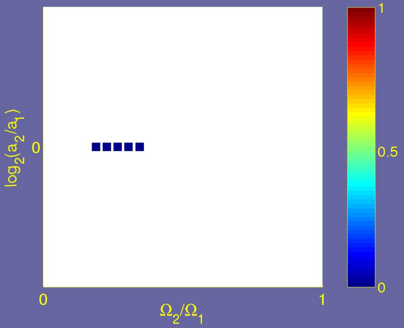


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$

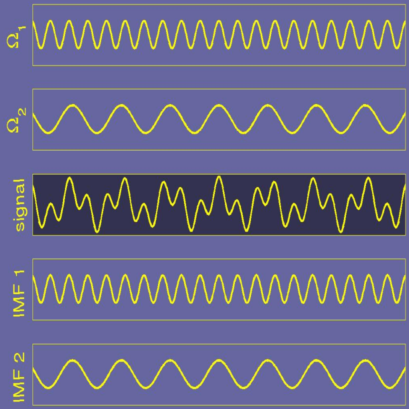


Confusion map

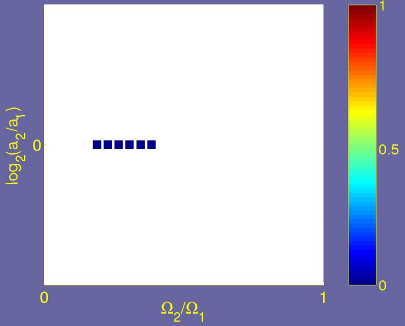


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$

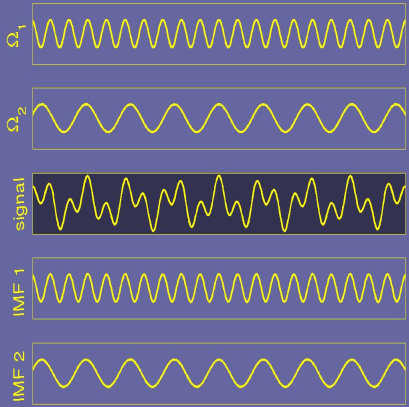


Confusion map



Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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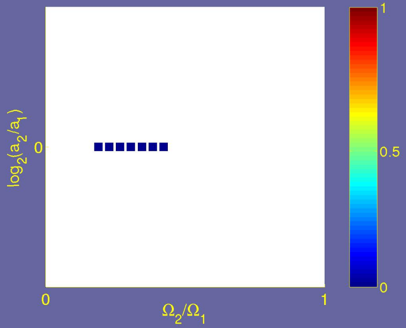
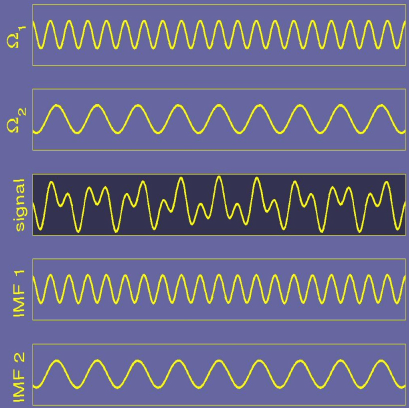


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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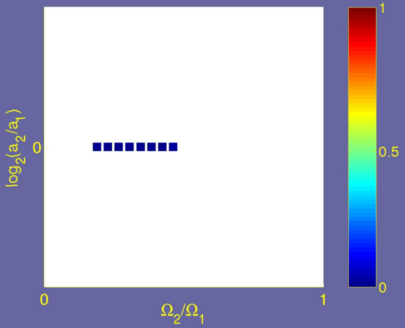
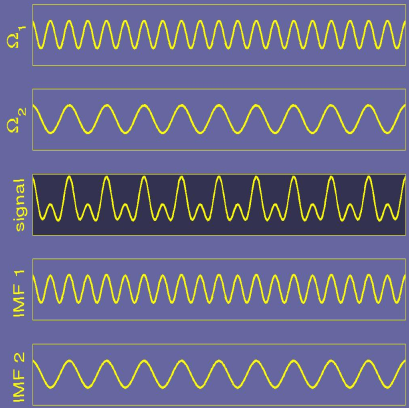


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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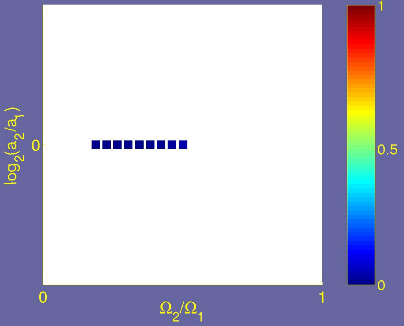
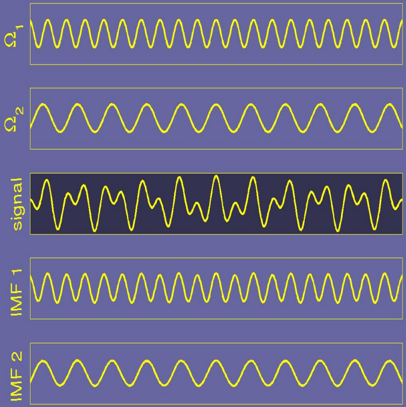


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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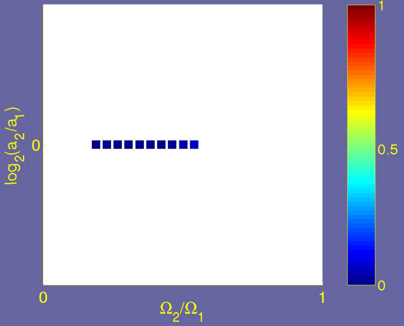
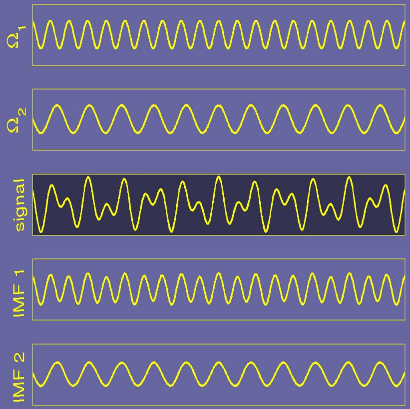


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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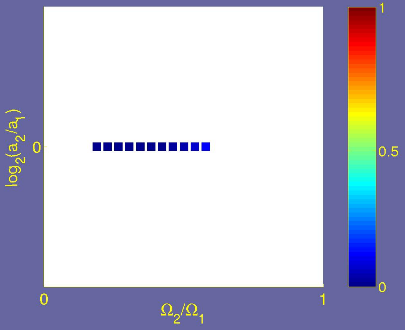
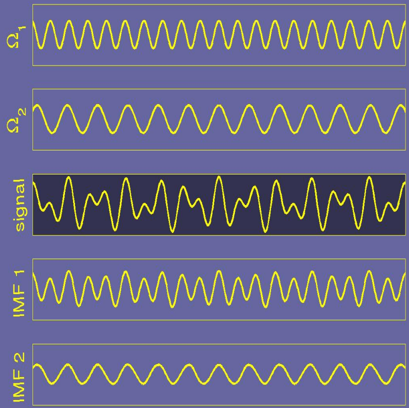


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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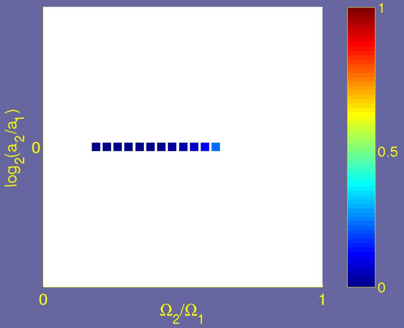
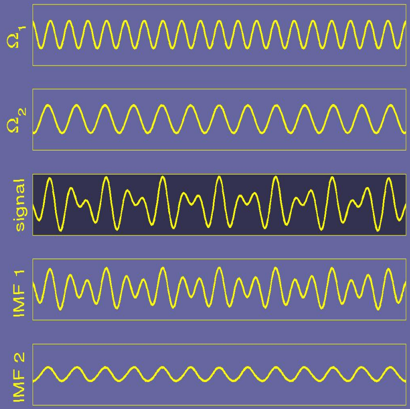


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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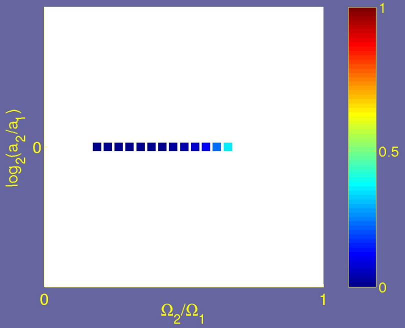
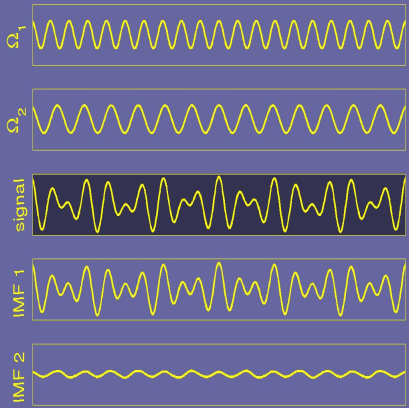
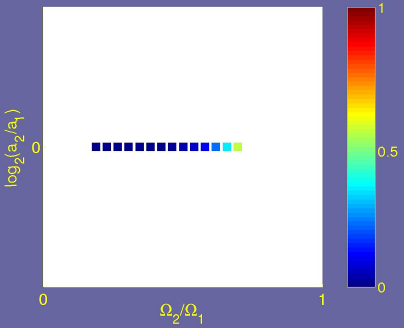


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$

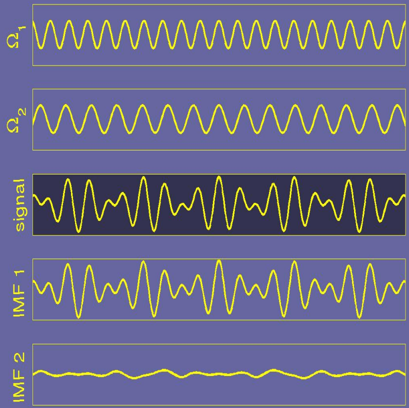


Confusion map



Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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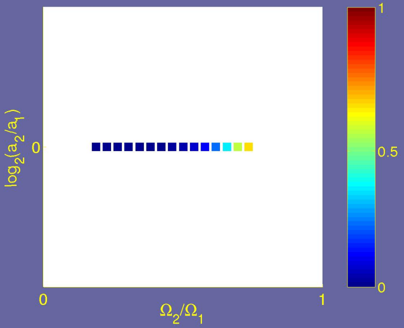
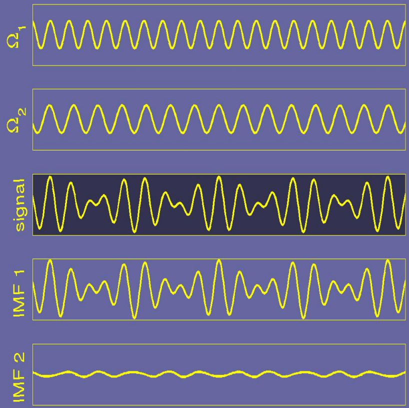


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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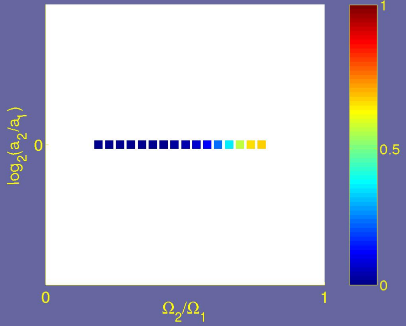
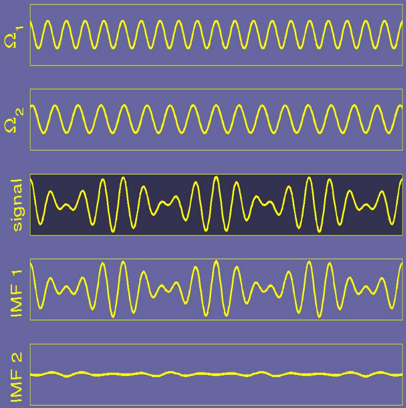


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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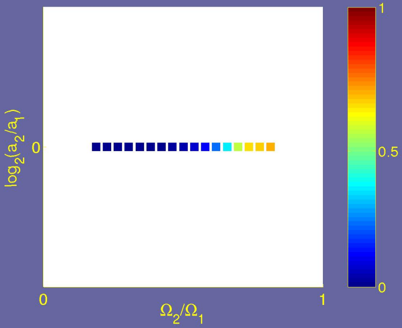
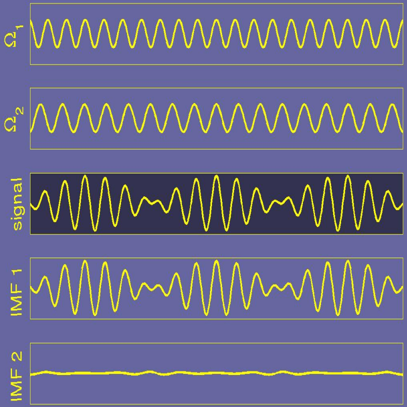


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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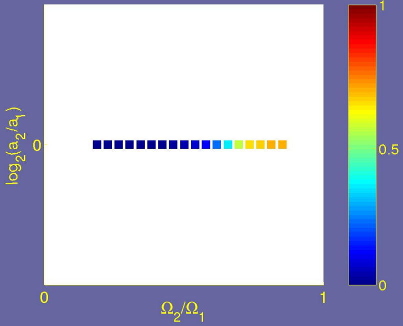
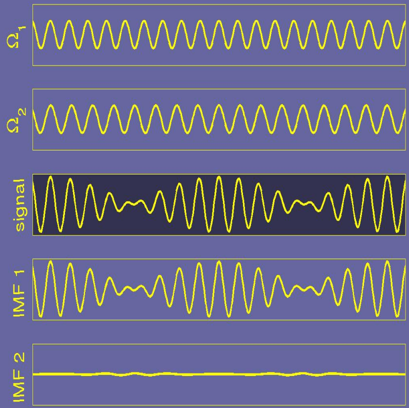


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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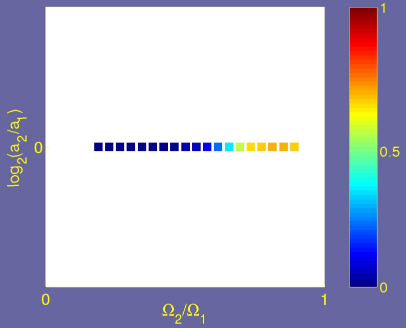
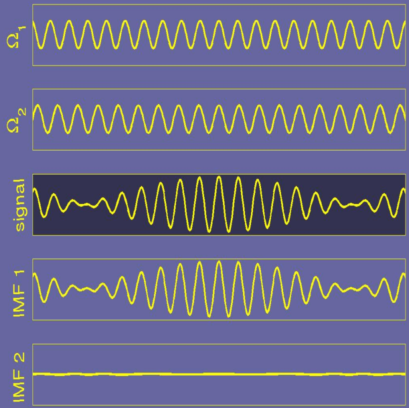


Frequency resolution

varying frequency $\omega_2 \leq \omega_1$



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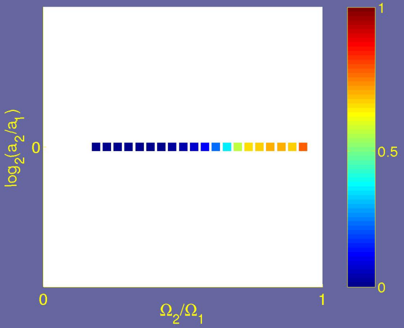
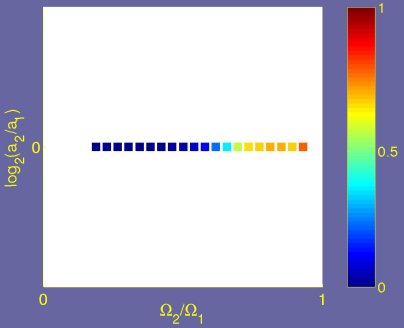


Frequency resolution

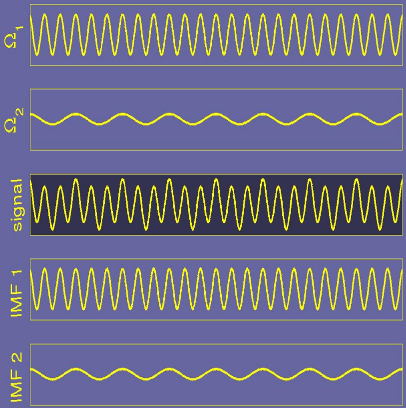
varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

Confusion map

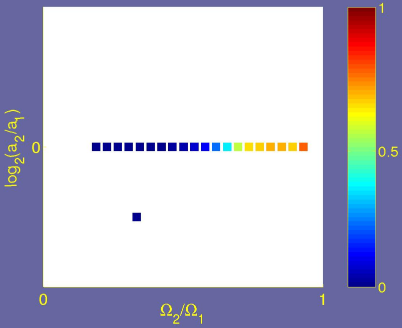


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

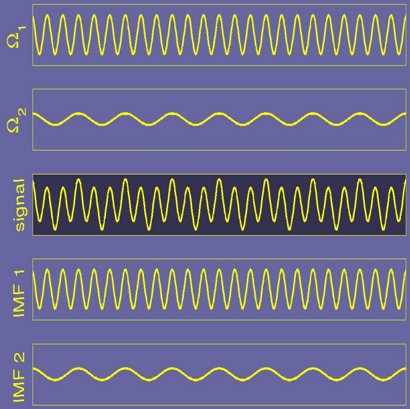


Confusion map



Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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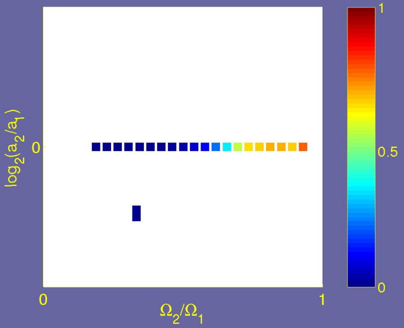
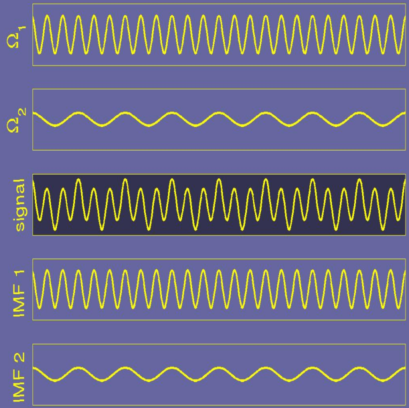


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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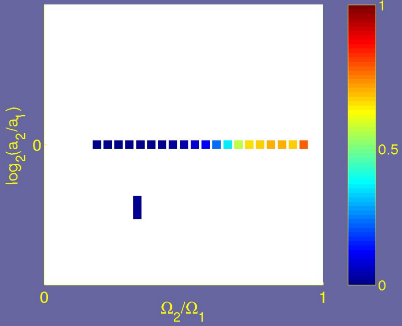
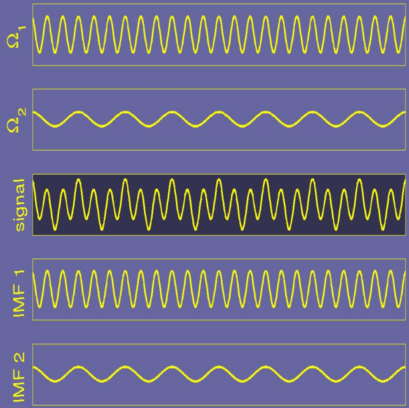


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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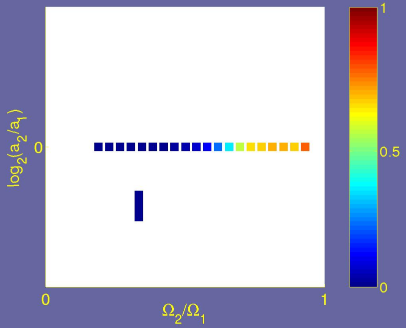
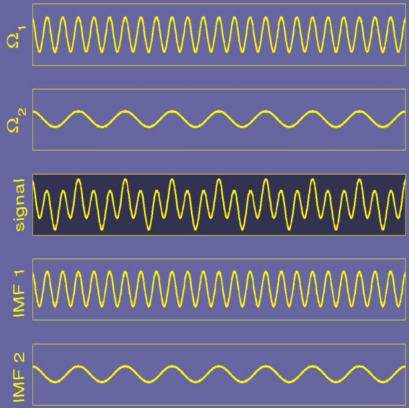


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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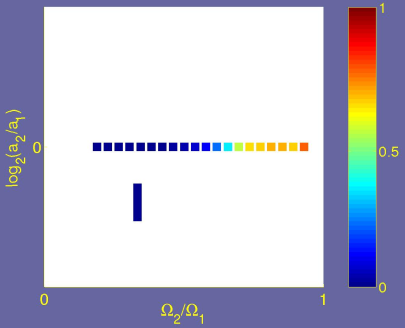
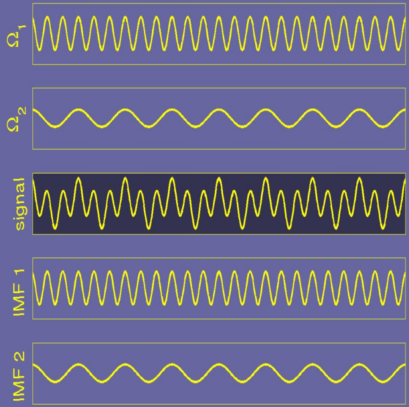


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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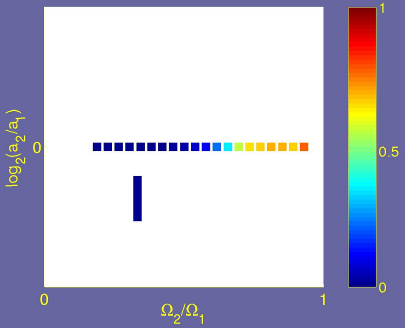
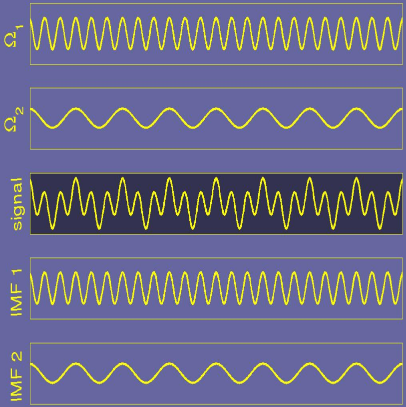


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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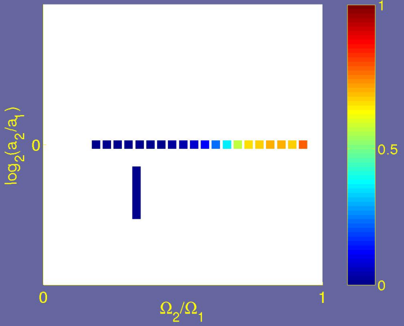
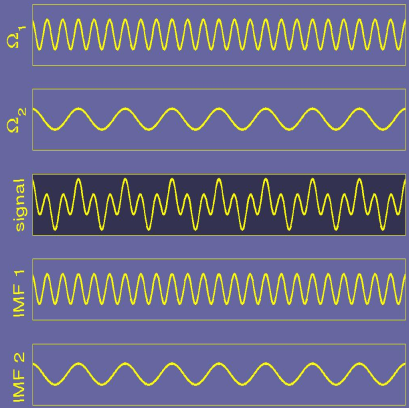


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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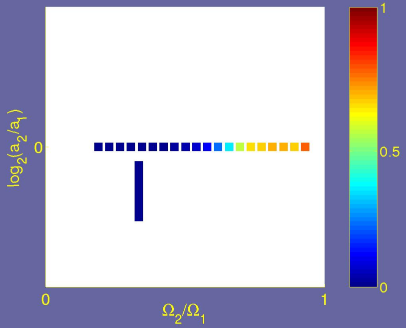
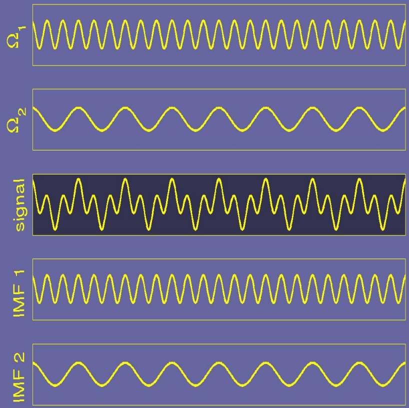
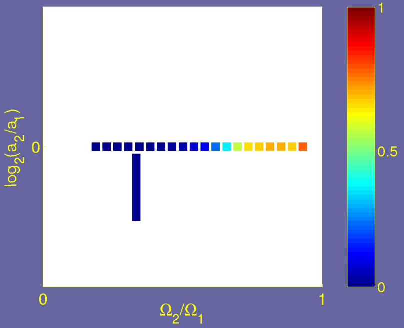


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

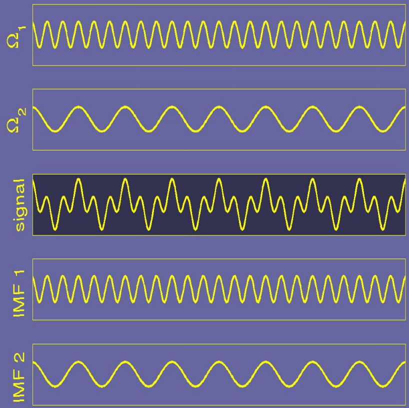


Confusion map

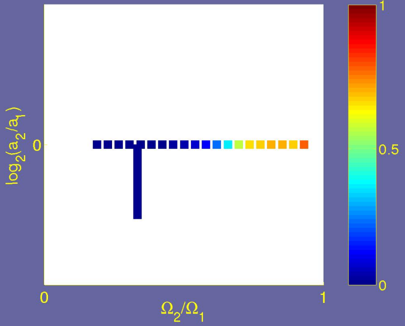


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

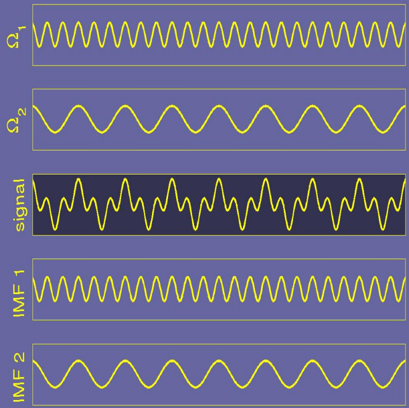


Confusion map

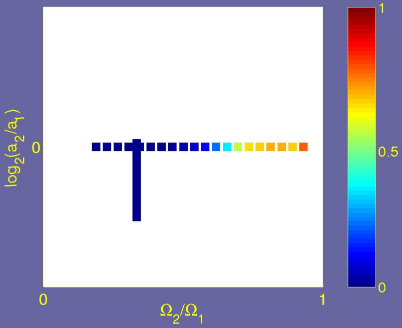


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

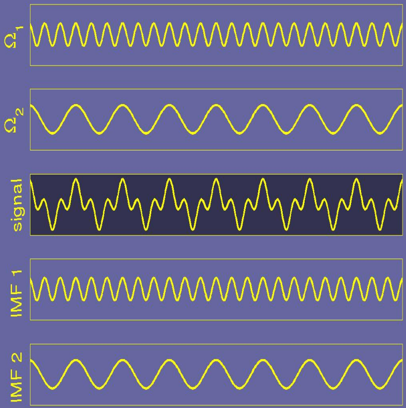


Confusion map



Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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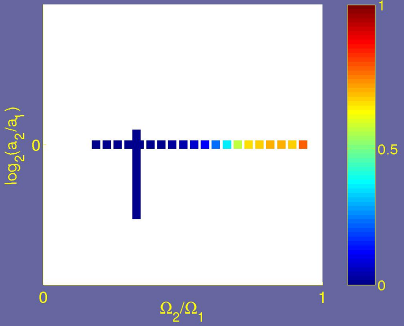
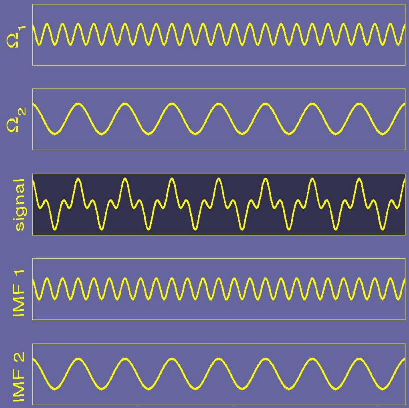


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



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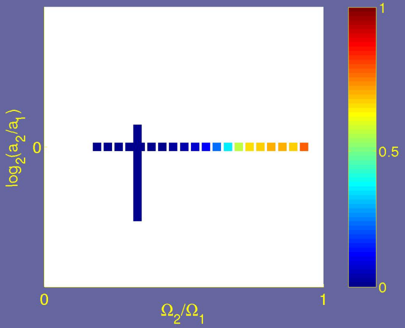
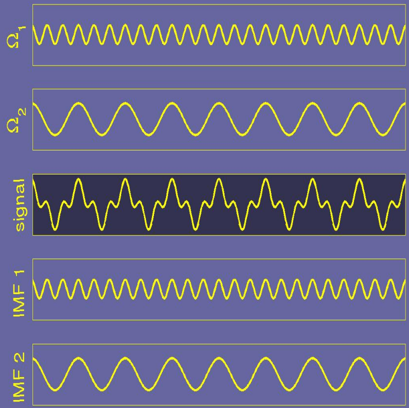
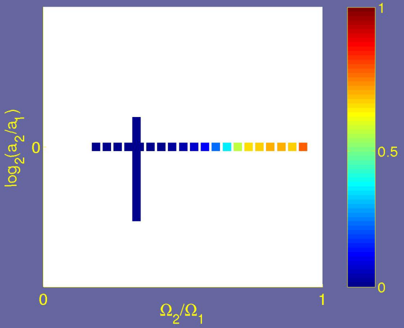


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

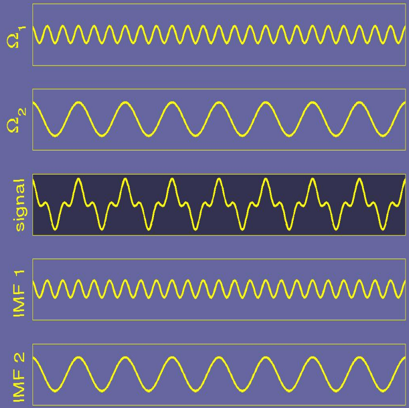


Confusion map

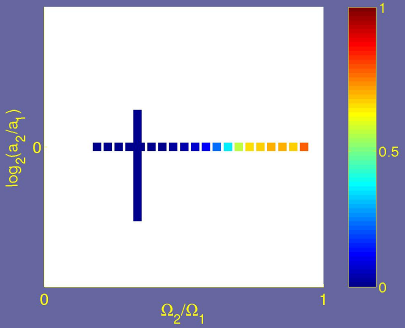


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

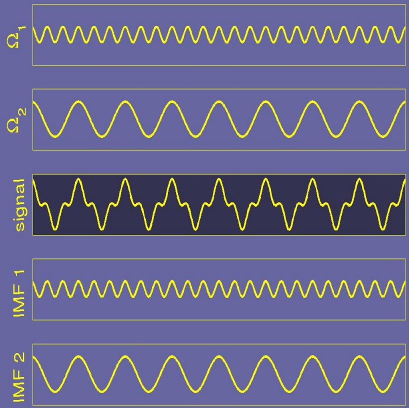


Confusion map

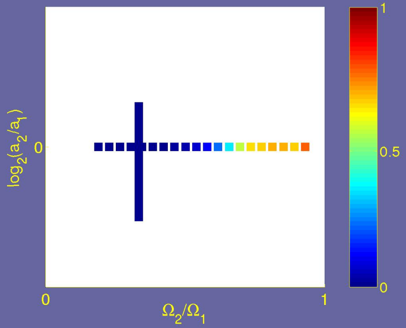


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$

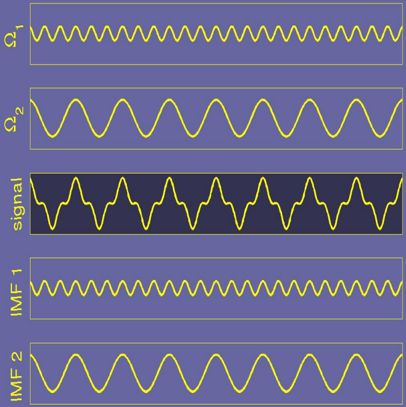


Confusion map

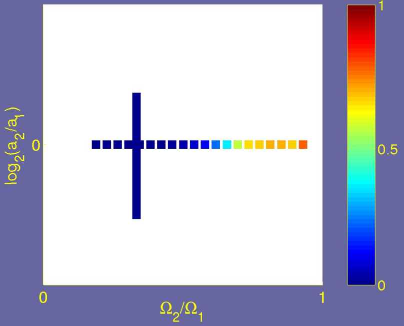


Frequency resolution

varying amplitude $a_1/4 \le a_2 \le 4a_1$

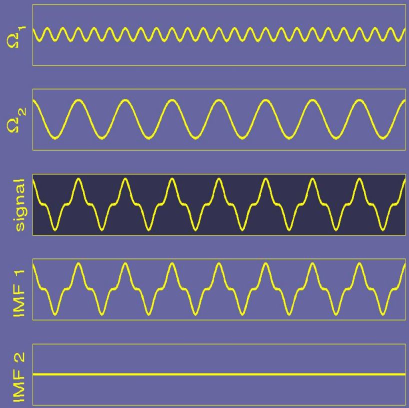


Confusion map

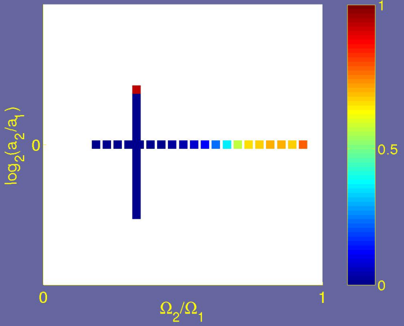


Frequency resolution

varying amplitude $a_1/4 \leq a_2 \leq 4a_1$



Confusion map



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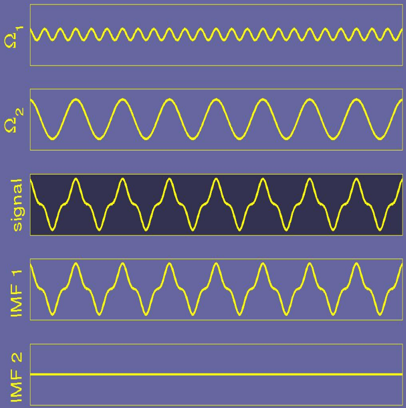

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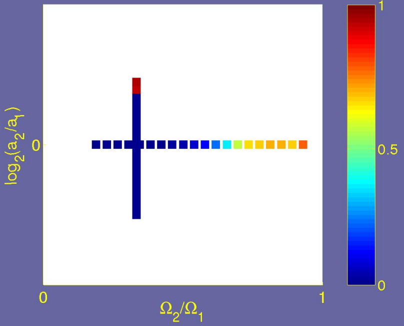

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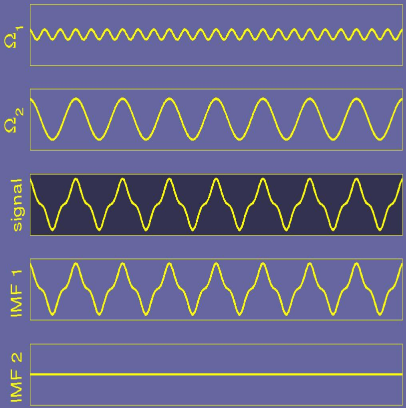

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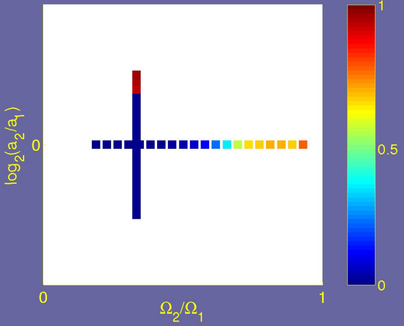

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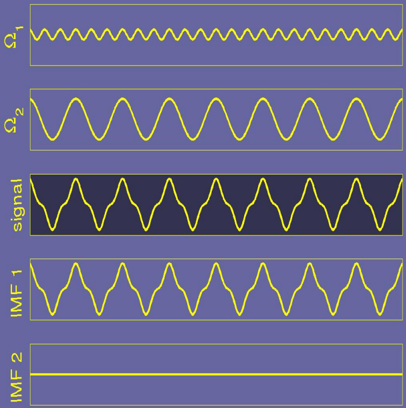

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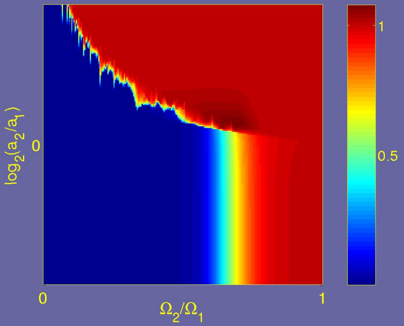

Application to satellite image time series

Sensor — MODIS (NASA). Radiometric sensitivity measured in 7 spectral bands and 2 vegetation indices (NDVI, EVI)

Resolutions — Weekly aquired images at nominal spatial resolution of 500m at nadir. One full year pixels time series: *voxels*

Study area — 9 land cover classes

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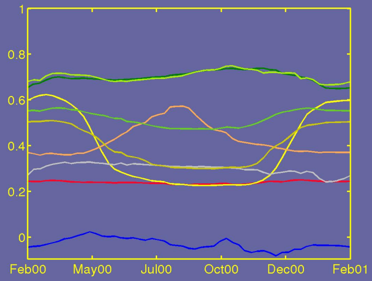

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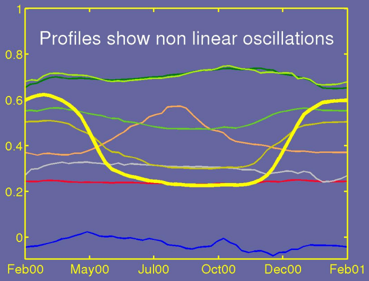

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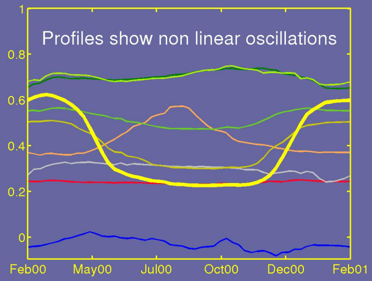
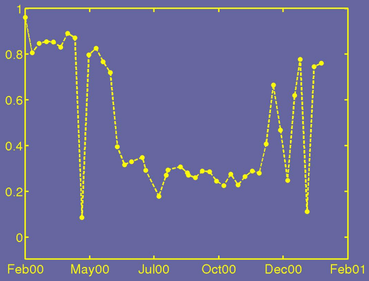


Single voxel



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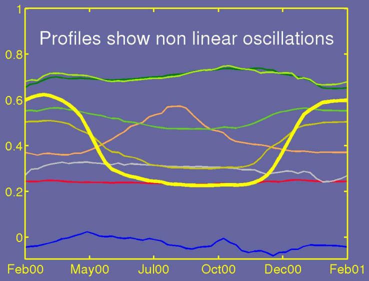

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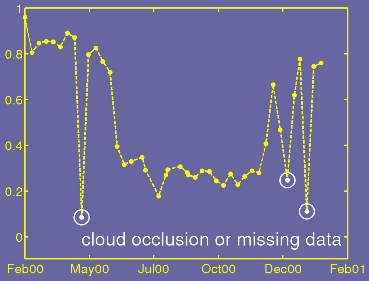
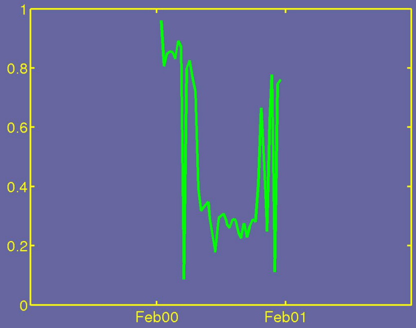


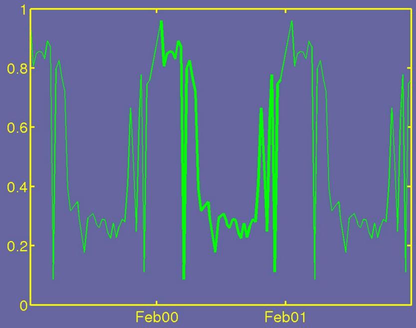
Application to satellite image time series

Annual time series



Application to satellite image time series

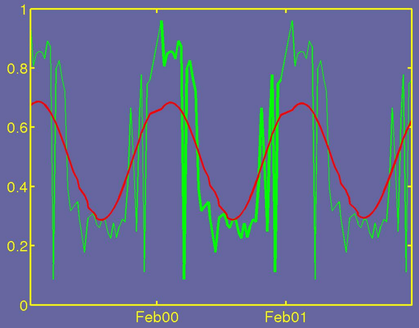
Periodized time series



Assume cyclo-stationarity for time series

Application to satellite image time series

Periodized time series



Linear low-pass filters do not preserve non-linear responses

Application to satellite image time series

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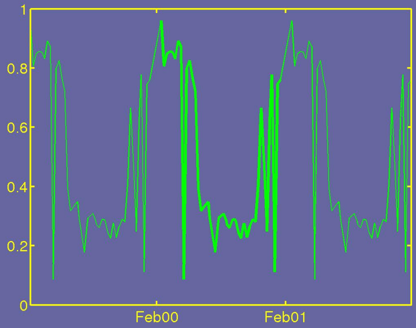
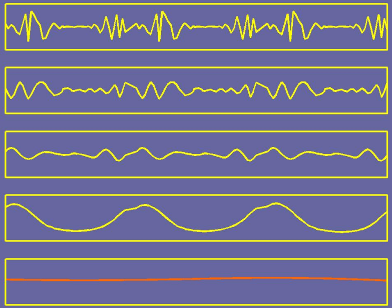


EMD: IMFs and residue



Perform a full EMD of periodized data

Application to satellite image time series

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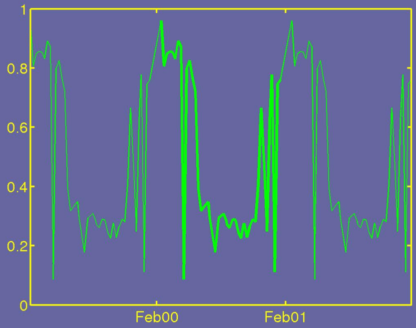

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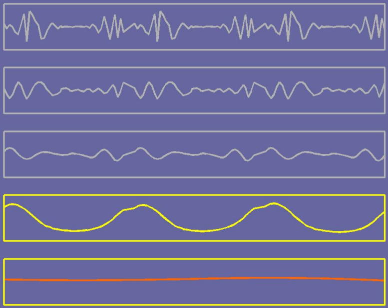

Keep only the one-year period IMF and residue

Application to satellite image time series

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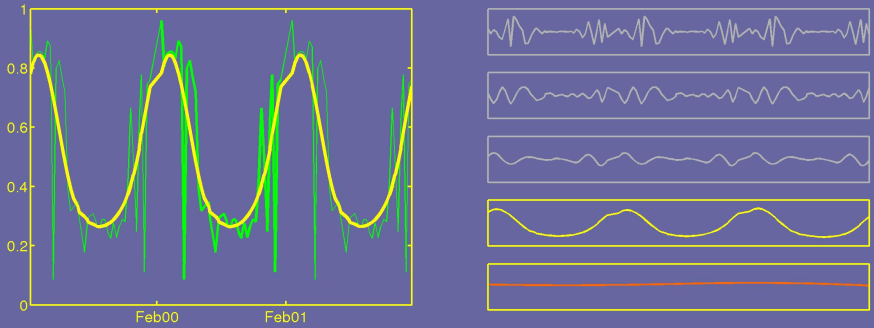

Use partial reconstruction as denoised signal

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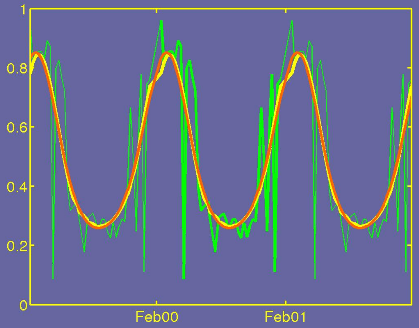

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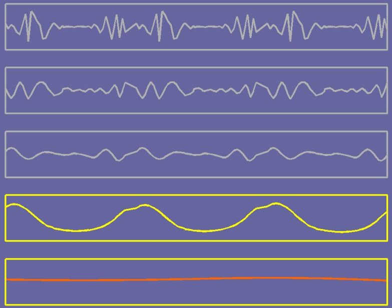

Fit with a non linear model:

$$s(t) = \mu + A\cos(\Omega t + \psi + \alpha\cos(\Omega t + \varphi))$$

Concluding remarks and open issues

Assets

- ✓ *adaptive* — *totally data-driven decomposition*
- ✓ *local in time* — *appealing for non stationary processes*
- ✓ *nonlinear* — *efficient at stressing non harmonic oscillations*
- ✓ *multiscale* — *spontaneous dyadic filterbank structure*

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- ✓ *adaptive* — *few degrees of freedom*
- ✓ *definition* — *consistency of an algorithm-based definition?*
- ✓ *theory* — *mathematical framework beyond numerical simulation: frustration, challenge...*

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