Erratum on "The Biequivalence of Locally Cartesian Closed Categories and Martin-Löf Type Theories"

In [1], pseudo cwf-morphisms (2-cells in the 2-category of cwfs) are defined as follows

Definition 1 (Pseudo cwf-transformation) *Let (F,σ) and (G,τ) be two cwf-morphisms from $(\mathbb{C},T)$ to $(\mathbb{C}',T')$. A* pseudo cwf-transformation *from (F,σ) to (G,τ) is a pair (ϕ,ψ) where $\phi : (F,\sigma) \Rightarrow (G,\tau)$ is a natural transformation, and for each Γ in $\mathbb{C}$ and $A \in \mathrm{Type}(\Gamma)$, a morphism $\psi_{\Gamma,A} : FA \to GA[\phi_\Gamma]$ in $\mathbf{T}'(F\Gamma)$, natural in A and such that the following diagram commutes:*

$$\begin{tikzcd}
FA[F\delta] \arrow[r, "\mathbf{T}'(F\delta)(\psi_{\Gamma,A})"] \arrow[d, "\theta_{A,\delta}"] & GA[\phi_\Gamma F(\delta)] \arrow[d, "\mathbf{T}'(\phi_\Delta)(\theta'_{A,\delta})"] \\
F(A[\delta]) \arrow[r, "\psi_{\Delta,A[\delta]}"'] & G(A[\delta])[\phi_\Delta]
\end{tikzcd}$$

where θ and θ' are the isomorphisms witnessing preservation of substitution in types in the definition of pseudo cwf-morphism.

There is a glitch with this definition: the component ψ is not constrained enough by ϕ. This causes a mismatch with the 2-cells in **LCC** (where only the ϕ remains), and in consequence the family of cwf-transformations ϵ used in the biequivalence (see [1]) fails a condition of pseudonatural transformations.

Missing from this definition is the following coherence diagram:

$$\begin{tikzcd}
F(\Gamma.A) \arrow[rr, "\phi_{\Gamma.A}"] \arrow[d, "\rho^F_{\Gamma,A}"'] & & G(\Gamma.A) \arrow[d, "\rho^G_{\Gamma,A}"] \\
F\Gamma.\sigma_\Gamma A \arrow[r, "\psi_{\Gamma,A}"] & F\Gamma.\sigma_\Gamma A[\phi_\Gamma] \arrow[r, "\langle \phi_\Gamma \mathrm{p}, \mathrm{q}\rangle"] & G\Gamma.\tau_\Gamma A
\end{tikzcd}$$

This means that ψ becomes redundant, and can be defined from ϕ – one could get rid of ψ and adopt natural transformations $\phi : F \Rightarrow G$ as 2-cells from (F,σ) to (G,τ). We refrain from doing that because pseudo cwf-morphisms is most naturally presented with the ψ, reflecting the second components of cwfs and cwf-morphisms.

Finally, we finish this erratum with two remarks:

(1) The pseudofunctor H of [1] yields cwf-transformations satisfying this diagram: in fact they are defined in this way.

(2) The coherence diagram in the original definition above follows from this, as is established in a straightforward adaptation of Lemma 5 in [1] (the proof uses the fact that G preserves finite limits which might not be the case in general, but as a pseudo cwf-morphism it always preserves the substitution pullback used in the proof).

References

[1] Pierre Clairambault and Peter Dybjer. The biequivalence of locally cartesian closed categories and martin-löf type theories. *Mathematical Structures in Computer Science*, 24(6), 2014.