

# Master d'Informatique Fondamentale

## Exercices on Games and Types

1<sup>st</sup> semester 2008

### Exercise [GTE 2.4] : Eliminating Dominated Strategies

- a. Eliminate dominated strategies on the following game :

|   | N      | C      | J      |
|---|--------|--------|--------|
| N | 73, 25 | 57, 42 | 66, 32 |
| C | 80, 26 | 35, 12 | 32, 54 |
| J | 28, 27 | 63, 31 | 54, 29 |

is the resulting solution a Nash equilibrium ?

- b. Can a Nash equilibrium to the following game be found by eliminating dominated strategies? If so, describe in what order you delete strategies.

|   | A      | B      | C      | D       | E      |
|---|--------|--------|--------|---------|--------|
| A | 63, -1 | 28, -1 | -2, 0  | -2, 45  | -3, 19 |
| B | 32, 1  | 2, 2   | 2, 5   | 33, 0   | 2, 3   |
| C | 54, 4  | 95, -1 | 0, 2   | 4, -1   | 0, 4   |
| D | 1, -33 | -3, 43 | -1, 39 | 1, -12  | -1, 17 |
| E | -22, 0 | 1, -13 | -1, 88 | -2, -57 | -3, 72 |

### Exercise [GTE 2.7] : An Armaments Game

Two kid gangs fight each other. The general Lebrac of the *Longeverne*<sup>1</sup> has four strategies and his opponent, the general of the *Verlans*, have three counterstrategies. The diagram below show the probabilities that the *Longeverne* beat the *Verlans*. Use the elimination of the dominated strategies to determine a solution of this game.

|            |                 | Verlans |      |            |
|------------|-----------------|---------|------|------------|
|            |                 | run     | hide | hide & run |
| Longeverne | boxing          | 0.30    | 0.25 | 0.15       |
|            | throwing stones | 0.18    | 0.14 | 0.16       |
|            | raining         | 0.35    | 0.22 | 0.17       |
|            | trapping        | 0.21    | 0.16 | 0.10       |

### Exercise (inspired from [GTE 3.3]) : Competition on Main Street

Lorna and Diana decide to set up a fast food on “Grand rue”. They can locate their snack bar on any side of the street at any point between the beginning of the street which is  $-1$  and the end which is  $1$ .

<sup>1</sup>See for instance « La guerre des boutons (roman) » in Wikipédia.

Each fast food will attract the customers between it and the end of the street on its side and the fast food share equally customers who are between the two. Each customer contributes \$1 in profits each day to the superette he visits. By convention, if Lorna and Diana decide to set up their shop on the same point of the street they put them on each side of the street and they share their customers, in other words, their payoff is 1 for each of them.

*To summarize.* If Diana sets her superette at  $x_2$  between the superette of Lorna (at  $x_1$ ) and the end of the street at +1, she gets :

- half of the customers between  $x_1$  and  $x_2$ , i.e.  $\frac{x_2 - x_1}{2}$  plus
- the customers between  $x_2$  and 1, i.e.  $1 - x_2$ .

*Questions.*

- a. Define the actions, strategies and daily payoffs for this game. Is there a Nash equilibrium ? Are there two Nash equilibria ?
- b. Lola starts a third fast food. Show that there is no Nash equilibrium.