Rewriting - TD n°1: **Abstract reduction systems**,

18 January 2002

Exercise 1 *Group axioms and rewriting*

Using the axioms (identities) of groups prove:

$$
\begin{aligned}
e * x &= x \\
\overline{x} * x &= e.
\end{aligned}
$$

Represent the corresponding reductions.

Exercise 2 *termination and normalization*

(i) Does termination of a relation imply its normalization?

(ii) Does normalization of a relation imply its termination?

Prove or find counter-examples.

Exercise 3 *confluence*

Prove the following properties:

(i) If a relation is confluent, then every element has at most one normal form.

(ii) If a relation is confluent and normalizing, then every element has a unique normal form.

Exercise 4 *finitely branching and globally finite relations*

(i) Prove the following properties:

(1) A finitely branching terminating relation is globally finite. (Hint: prove this statement using Noetherian induction.)

(2.) A globally finite acyclic relation is terminating.

(ii) Combining the two previous properties state an equivalence (which is referred to as the Konig's Lemma).