Rewriting - TD n°3

Proving termination

13/15 February 2002

Exercise 1 *basic method*

Let $A = \mathbb{N} \times \mathbb{N}$ be the cartesian product. We define the following two rewrite systems:

$$\begin{array}{rcl} (i+1, j) & \rightarrow & (i, k) \\ (i, j+1) & \rightarrow & (i, j) \end{array}$$

$$\begin{array}{rcl} (i+1, j) & \rightarrow & (i, i) \\ (i, j+1) & \rightarrow & (i, j) \end{array}$$

Is the reduction finitely branching? Does it terminate?

Exercise 2 *lexicographic order*

Show the termination of Ackerman's function.

$$\begin{array}{rcl} a(0, n) & \rightarrow & n+1 \\ a(m+1, 0) & \rightarrow & a(m, 1) \\ a(m+1, n+1) & \rightarrow & a(m, a(m+1, n)) \end{array}$$

Exercise 3 *reduction order*

Let us define the following orders on $T(\Sigma, V)$, where Σ is a signature and V is a countable set of variables.

(i) $s > t$ iff $\mid s \mid > \mid t \mid$, where $\mid s \mid$ is the size of s.

(ii) $s > t$ iff $\mid s \mid > \mid t \mid$ and $\mid s \mid_x \geq \mid t \mid_x$, where $\mid s \mid_x$ is the number of occurrences of x in s.

Are the given orders reduction orders?

Exercise 4 *polynomial order*

Let $\Sigma = \{\oplus, \odot\}$ and let $\mathcal{A} = \mathbb{N} \setminus \{0, 1\}$. Consider the rewrite system

$$\begin{array}{rcl} x \odot (y \oplus z) & \to & (x \odot y) \oplus (x \odot z) \\ (x \oplus y) \oplus z & \to & x \oplus (y \oplus z) \end{array}$$

By taking the polynomial interpretation

$$\begin{array}{rcl} [\![\oplus]\!] & = & 2X + Y + 1 \\ [\![\odot]\!] & = & XY \end{array}$$

show that the given rewrite system terminates.

Exercise 5 *various orders*

Solve the exercise 1 of last year "partiel" (details on the attached sheet).