Magistère d'Informatique et Modélisation
Réécriture

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Rewriting - TD n°4

Polynomial order and Hilbert's 10^{th} problem

Exercise 1

Let $\Sigma = \{+, s, d, q, 0\}$, where $+$ is a binary predicate, s, d, q are unary and 0 is a constant symbol. Consider the term rewriting system

$$\begin{array}{lcclcl} x+0 & \longrightarrow & x & x+s(y) & \longrightarrow & s(x+y) \\ d(0) & \longrightarrow & 0 & d(s(x)) & \longrightarrow & s(s(d(x))) \\ q(0) & \longrightarrow & 0 & q(s(x)) & \longrightarrow & q(x)+s(d(x)) \end{array}$$

1. Let $\mathcal{A} = \mathbb{N}\backslash\{0,1\}$. Show that the polynomial interpretation $[\![+]\!] = X+2Y$, $[\![s]\!] = X+1$, $[\![d]\!] = 3X$ $[\![q]\!] = X^3$, and $[\![0]\!] = 2$ proves the termination of the given system.
2. Discuss the reduction of the term $q^{p+1}(s^2(0))$.

Exercise 2

Given a term rewriting system

1. $\{f(f(x,y),z) \to f(x, f(y,z)), f(y, f(x,z)) \to f(x,x)\}$,
2. $\{f(f(x,y),z) \to f(x, f(y,z)), f(x,y) \to f(y,x)\}$

find a polynomial interpretation in order to show that the given system terminates.

Exercise 3

1. Discuss the connection between proving termination by polynomial interpretation and Hilbert's 10^{th} problem.
2. Does there exist a polynomial with integer coefficients such that the set of its positive values is exactly the set of prime numbers ?

Exercise 4

Solve "Exercise 2" of last year "examen" (details on the attached sheet).