Multicolouring of posets online

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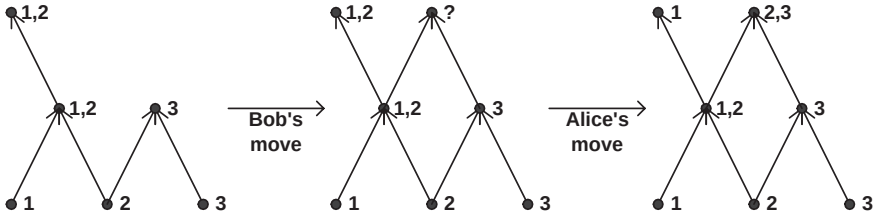
Multicolouring of upgrowing posets on-line can be viewed as a two-person game. We call the players Alice and Bob. Alice represents algorithm and Bob represents an adaptive adversary. In game for width k orders, Bob builds an on-line order of width at most k and Alice maintains a multicolouring of the order. The game is played in rounds. During round i Bob itroduces a new point x to the order and describes the comparabilities between x and the points from previous rounds. Alice responds by "colouring" x by some, not empty set of colours. Alice can also substract (but never add) some colours from other points. As an invariant of multicolouring we formulate two properties:

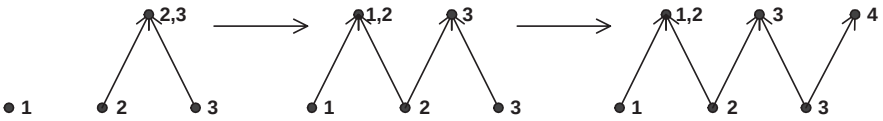
- for each colour: points which has this colour create a chain
- every point must have at least one colour

Picture 1.

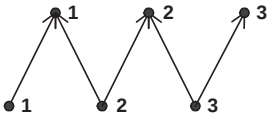


Bob using the lack of knowledge of Alice about the further extension of the poset at the game can force her to use greater number of colours than she (Alice) could use if she would know what would be next. For example:

Picture 2a.



Picture 2b.



Bob will be forced to use colour 4 (pic.2a). However, poset created during the game can be coloured by 3 colours (pic.2b).

Theorem

There is no online algorithm for adaptive coloring of upgrowing posets with competitive ratio strictly smaller than 2, *i.e. for* $c < 2$ *there is no algorithm that uses only* $c \cdot d$ *colors, where* d *is the width of the poset.*