

# Lambda definability is decidable for regular forth order types

Marek Zaionc

Computer Science Department, Jagiellonian University,  
Nawojki 11, 30-072 Krakow, Poland  
E-mail [zaionc@ii.uj.edu.pl](mailto:zaionc@ii.uj.edu.pl)

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**Abstract:** It has been proved by Loader [2] that Statman-Plotkin conjecture (see [5] and [4]) fails. The Loader proof was done by encoding the word problem in the full type hierarchy based on the domain with 7 elements. Later Loader refined the result by establishing that the problem is undecidable for every finite domain with 3 ground elements [3]. The remaining non trivial case with 2 ground elements has been shown to be undecidable by Thierry Joly (2003) in [1]. The aim of this note is to show that the lambda definability problem limited for regular forth order types is decidable in any finite domain. Obviously  $\lambda$  definability is decidable for 1,2 and 3 order types in any finite domain. As an additional effect of the result described we may observe that for certain types there is no finite context free grammar generating all closed terms.

## References

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