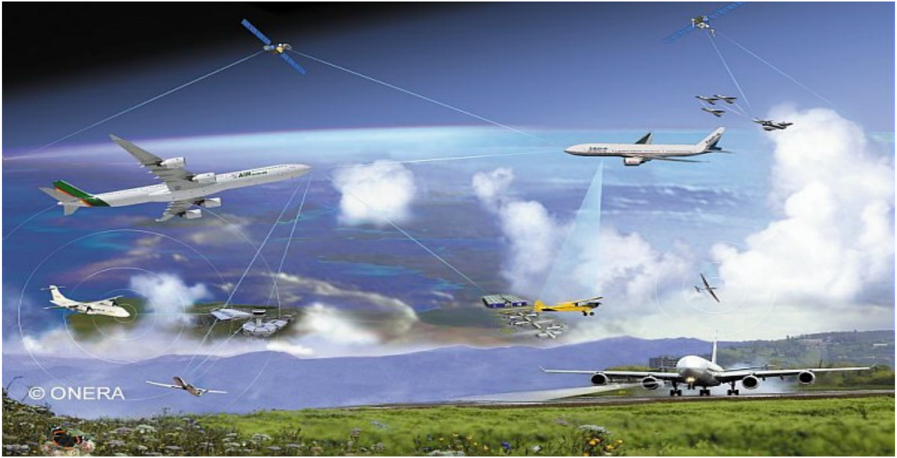

Analysis of critical control systems: combining formal analyses

Michael Dierkes joint work with Rémi Delmas, Pierre Roux, Romain Jobredeaux, Adrien Champion, and Pierre-Loïc Garoche

September 23rd 2013 – FMICS

Typical development cycle of a controller

Differential Equations (plant)

Control theorists

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↪ Continuous controller

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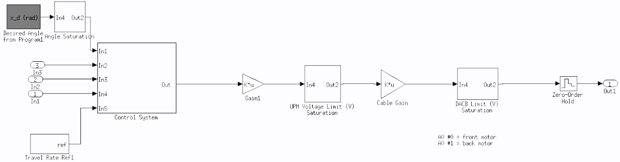

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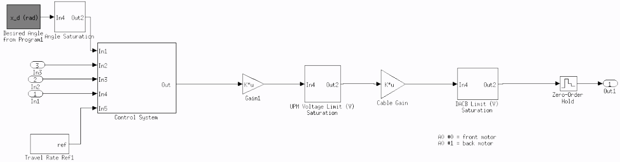

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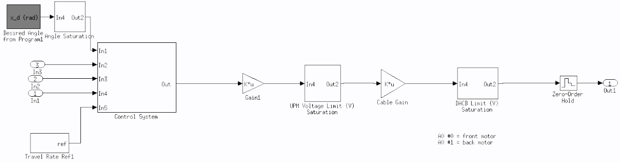

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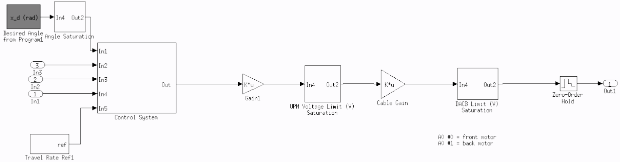

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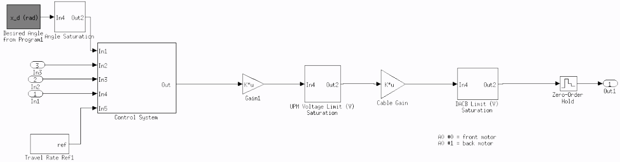

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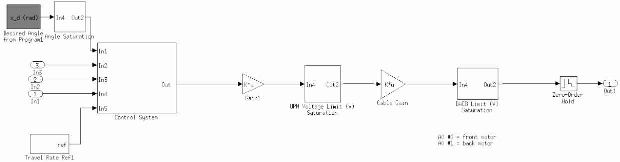

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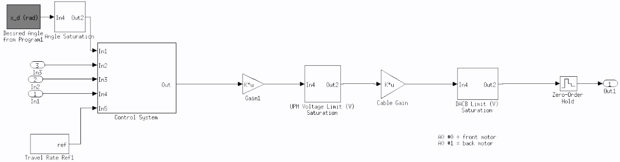

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  - frequency domain proof argument vs state space domain (ie. Lyapunov functions)

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redundancy, validators,
COM/MON...

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Test

Simulation

- Fault tolerance: set of constructs to recover from system/hardware failures
  * is this architecture sound (ie. when there is less than n simultaneaous error, the output is still valid or there will still be a working controler)

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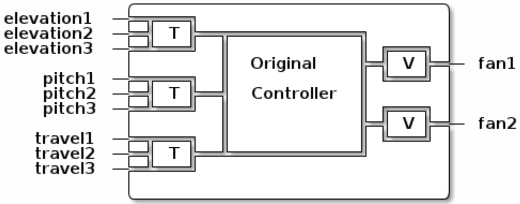

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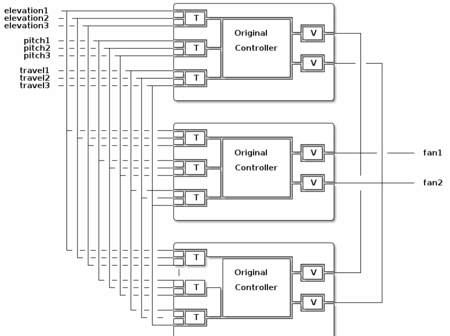

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- Actual implementation:
  - floats not reals
  - pointers, arrays, memory access → potential failure
  - real world: overflows

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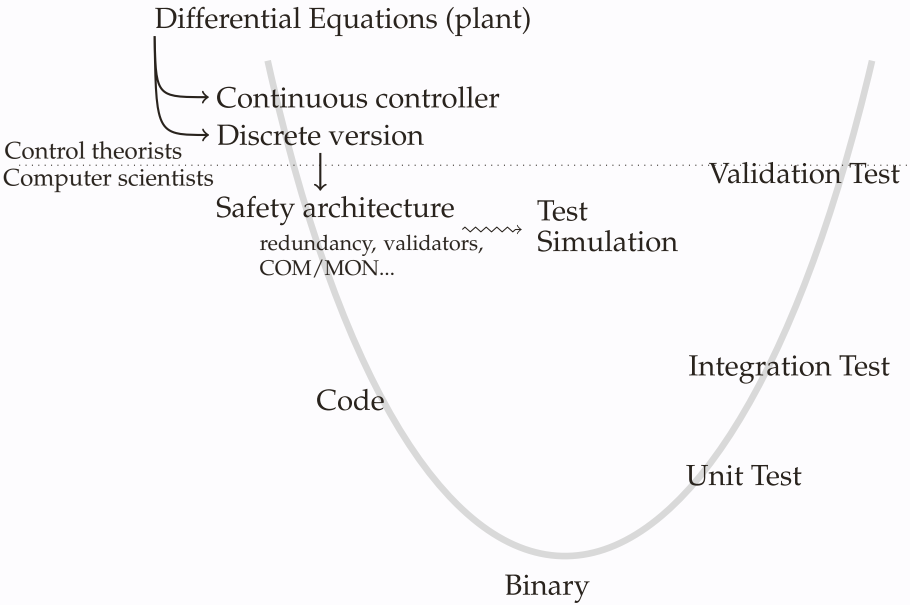

Verification methods used in the industry

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- static analysis of the code/model: compute an abstract representation of reachable state, mainly focuses on numerical accuracy, or data structure topology and manipulation (null pointers access, arrays, …)

Running example

Simple yet hard to analyze controller for a mass-spring damper

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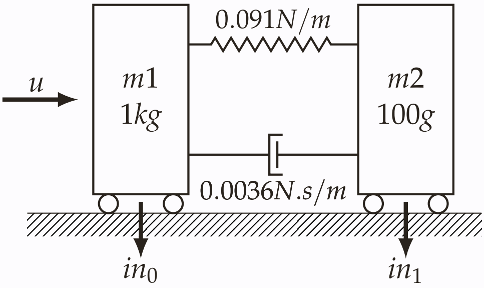
Simple yet hard to analyze controller for a mass-spring damper

System to be controlled:



Running example

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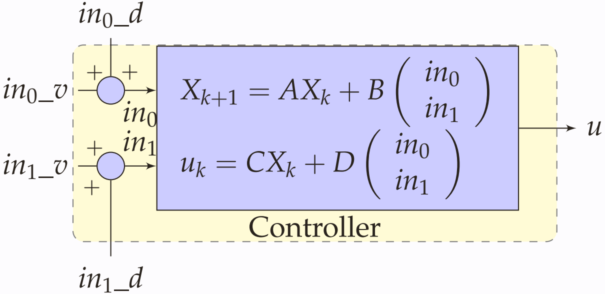
Controller itself:



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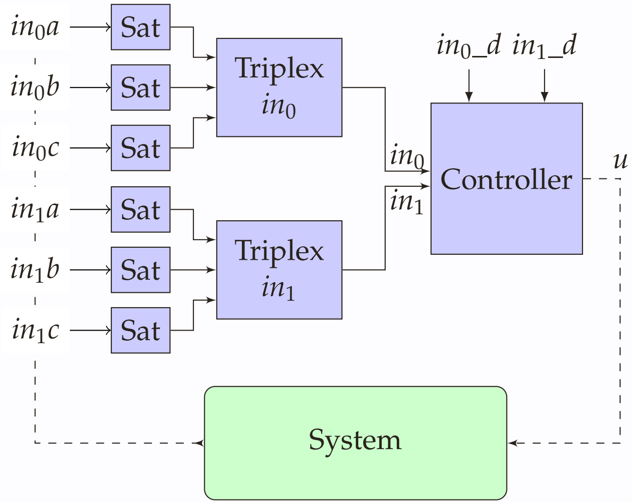
Fault tolerance architecture:



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1. The control flow graph of the controller is identified
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3. The set of reachable states is bounded using the generated ellipsoids.

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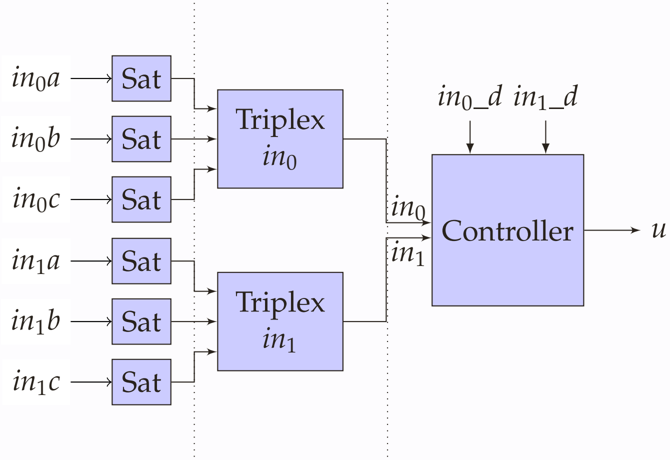
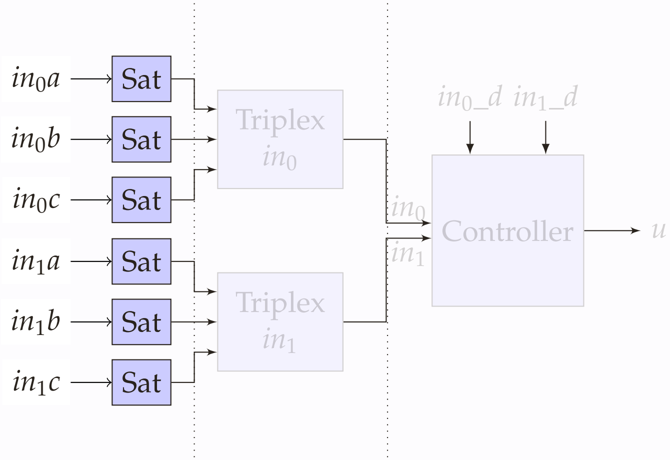
- are restricted to linear inductive or k-inductive properties;
- do not give good results in presence of complex numerical computations

Combining Analyses

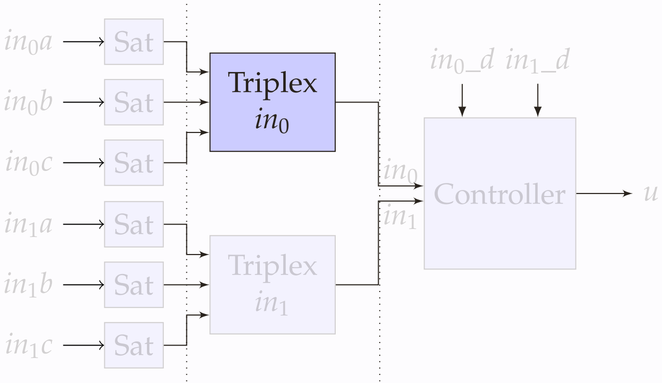


BASIC SATURATIONS



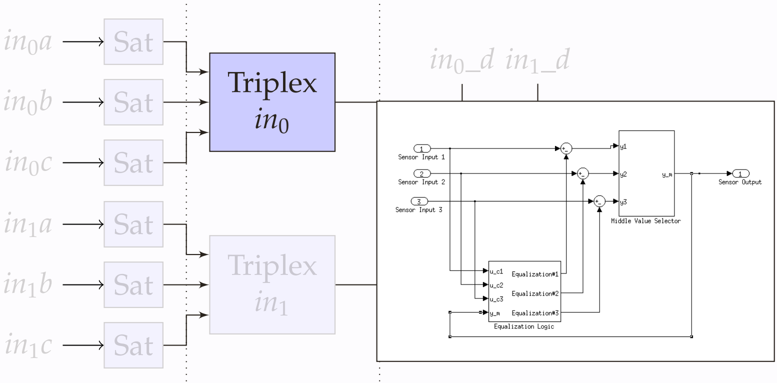
Abstract Interpretation computes a sound bound (1.2) on each ouptut whatever the value of in_xy is.

ANALYSIS OF THE TRIPLEX VOTER



Backward analysis applied on each triplex proves the specification BIBO.

Analysis of the triplex voter



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$$\forall k \in \mathbb{N}, \quad |InA_k| \leq a \wedge |InB_k| \leq a \wedge |InC_k| \leq a \implies |Output_k| \leq 3a \;\wedge$$
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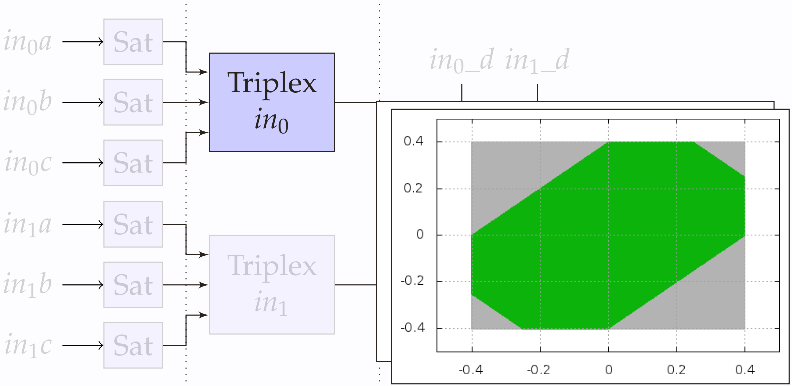

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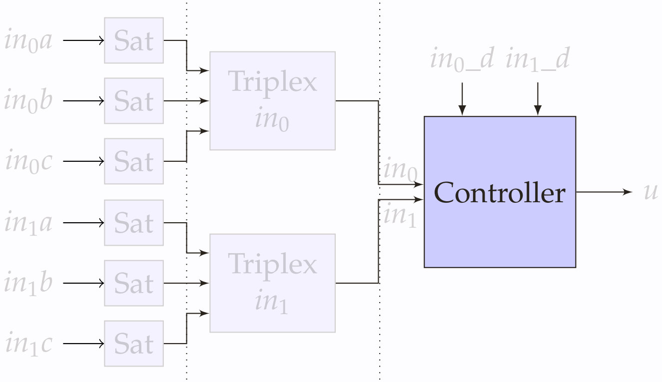
Assuming input is bounded by 1.2, we have output bounded by 3.6.

ANALYSIS OF THE CONTROLLER



Providing a bound on the inputs (3.6) an over-approximation of the output is computed:

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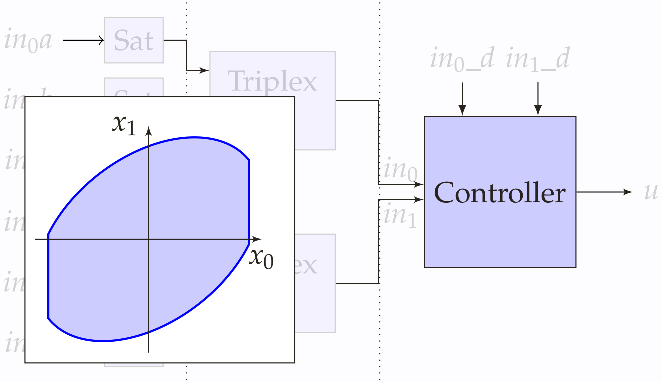
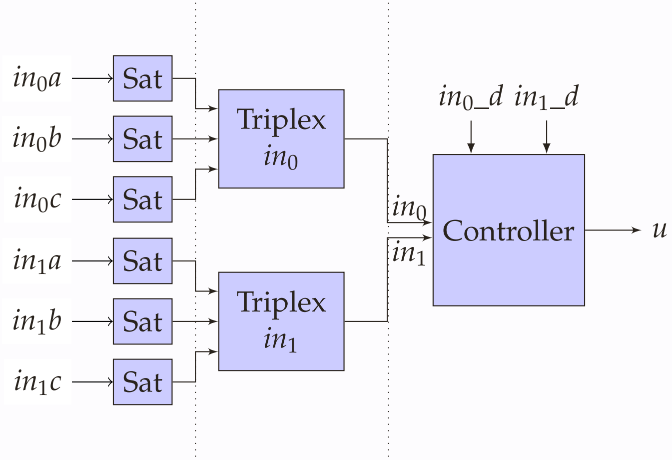


Providing a bound on the inputs (3.6) an over-approximation of the output is computed: $|u| \leq 194.499$.

$$0.098\,x_3^2 - 0.224\,x_3\,x_2 + 0.040\,x_3\,x_1 - 0.026\,x_3\,x_0 + 0.141\,x_2^2 - 0.053\,x_2\,x_1 \\ +0.030\,x_2\,x_0 + 0.024\,x_1^2 - 0.017\,x_1\,x_0 + 0.019\,x_0^2 \leq 14.259$$

REBUILDING THE ANALYSIS



$]-\infty, +\infty[$

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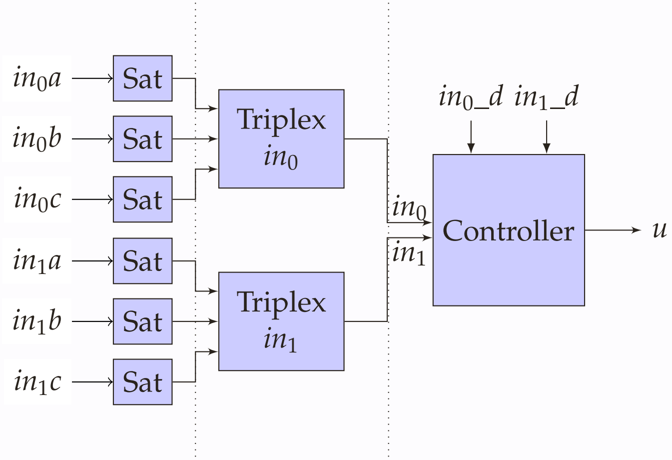

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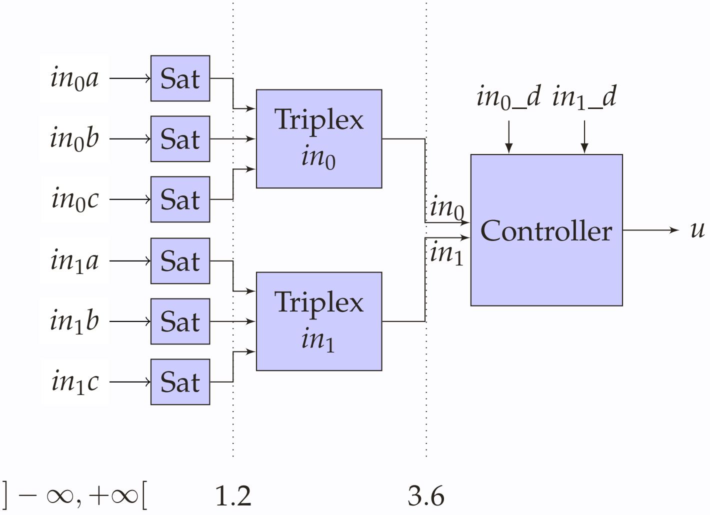
Rebuilding the analysis



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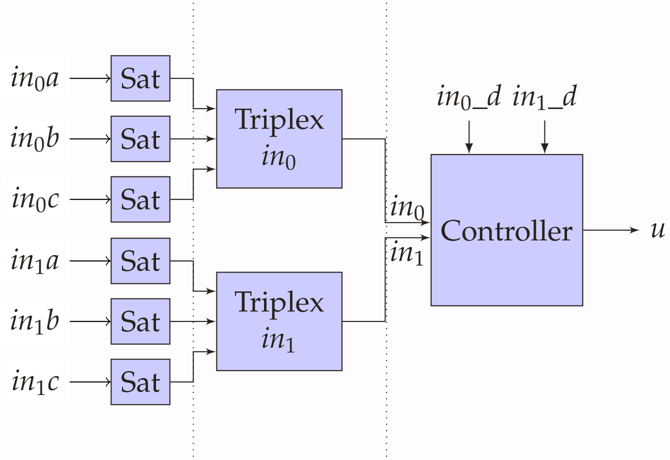

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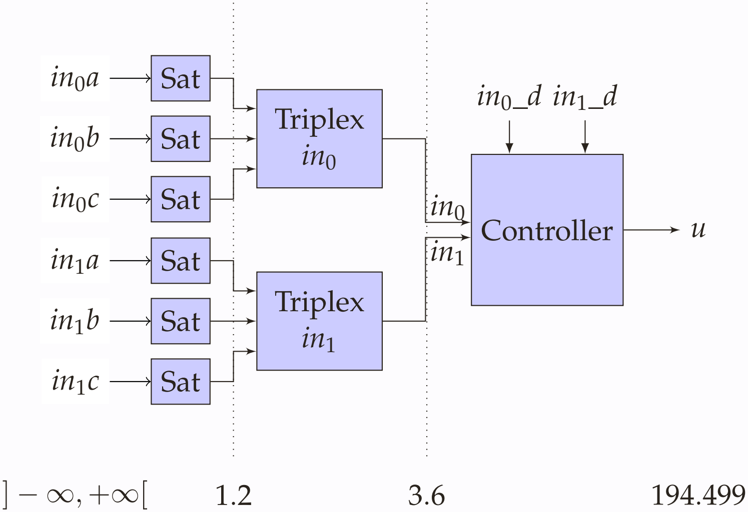

System is bounded!

Conclusion

Successful approach to analyze representative example un-analyzable with a single method.

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Good results on simple usecase. Currently addressing the analysis of industry-level FADEC (collab. with industrial partners) and academic yet representative examples of aircraft controllers (collab. with Polytech Montréal, Georgia Tech and NASA).