We assume a collection $\mathcal{R}$ of rules and a mixture M. The simulation algorithm keeps track of, for each rule r, the set $\Phi_r(M)$ of the *matchings* of its LHS into M and its *apparent* activity $\alpha'_r(M)$. The *total apparent* activity is then just the sum over all rules: $\alpha'(M) := \sum_r \alpha'_r(M)$. The simulation algorithm will select 'rule r to fire next' with probability $\alpha'_r(M)/\alpha'(M)$. It will then select a specific r-*event* $\varphi_r \in \Phi_r(M)$ *uniformly at random*, *i.e.* each r-event gets an equal slice $1/|\Phi_r(M)|$ of the apparent activity of r.

We assume that, given any event $\varphi_r \in \Phi_r(M)$, the simulation algorithm can assign it some $p_{\varphi_r}(M) \in [0,1]$. If $p_{\varphi_r}(M) = 1$, we say that φ_r is an *intrinsically real* event; if $p_{\varphi_r}(M) = 0$, we say that φ_r is an *intrinsically null* event. Otherwise, the event could be real or null; that will be decided, by a '$p_{\varphi_r}(M)$ vs $1 - p_{\varphi_r}(M)$' biased coin toss, during simulation.

Given that rule r has been selected, the probability of 'acceptance of the to-be-chosen r-event' is $p_r(M) := \sum_{\varphi_r} p_{\varphi_r}(M)/|\Phi_r(M)|$. (Note that $p_r(M)$ is not *known* to the simulation algorithm; it is immanent to the engendered stochastic process.) This induces a theoretic, *i.e.* unknown to the simulator, value of $\alpha_r(M) := p_r(M) \cdot \alpha'_r(M)$ for the *actual*, *i.e.* accepted, activity of r in M; note that $\alpha_r(M) \in [0, \alpha'_r(M)]$. The probability that 'the next event chosen is *both* an r-event *and* accepted' is therefore $\alpha_r(M)/\alpha'(M)$.

The *total actual* activity, also unknown to the simulator, is obviously $\alpha(M) := \sum_r \alpha_r(M)$. The probability that 'the next event chosen is accepted' is thus $p(M) := \sum_r p_r(M) \cdot \alpha'_r(M)/\alpha'(M) = \alpha(M)/\alpha'(M)$.

Putting the last two paragraphs together, we find that the probability that 'the next accepted event is an r-event' is $(\alpha_r(M)/\alpha'(M))/(\alpha(M)/\alpha'(M)) = \alpha_r(M)/\alpha(M)$. (An alternative derivation of this comes via summing the geometric series (q^n), with $q := 1 - p(M)$, and multiplying by the probability $\alpha_r(M)/\alpha'(M)$ that 'the next event chosen is both an r-event and accepted'.)

The mixture M and its total apparent activity are unaffected by a *null*, *i.e.* not-accepted, event. We do however advance time by δt by sampling the exponential random variable, $\Pr(\delta t \leqslant t) = F(t) := 1 - e^{-\alpha'(M)t}$, parametrized by the *total apparent* activity $\alpha'(M)$. The total time advance associated with a sequence of n null events followed by a real event is thus the sum of $n+1$ independent exponential random variables, all parametrized by $\alpha'(M)$. The total time advance is therefore distributed as

$$\begin{aligned}
P(t) &= \sum_{n=0}^{\infty} q^n \cdot p(M) \cdot \Gamma(t, n+1, \alpha'(M)) \\
&= \sum_{n=0}^{\infty} q^n \cdot \alpha(M) \cdot \alpha'(M)^n \cdot t^n \cdot e^{-\alpha'(M)t}/n! \\
&= \alpha(M) \cdot e^{-\alpha'(M)t} \cdot \sum_{n=0}^{\infty} ((\alpha'(M) - \alpha(M))^n \cdot t^n/n! \\
&= \alpha(M) \cdot e^{-\alpha(M)t}
\end{aligned}$$