General Relativity

Einstein suggested several experiments :

• The deflection of the light by the sun

• The perihelion advance of Mercury

The gravitational red shift of spectral lines

 The time delay of an electromagnetic wave travelling in a gravitational field

Time relativity

The deflection of the light by the sun



The deflection of the light by the sun. Theoretical predictions



In 1920 they did not know whether the photons had a mass or not. Thus one can use Newton gravitation to compute the deflections.

Equation of the trajectory
$$\frac{A}{r} = 1 + e \cos(\varphi)$$

e > 1 hyperbole.

with
$$e = \sqrt{1 + \frac{2\Gamma^2 E}{m^3 G^2 M^2}} = \frac{1}{\cos(\alpha)}$$
, $A = \frac{2\Gamma^2}{m^2 G M}$ and $\Gamma = m v d$

Here $E = \frac{1}{2}mv^2$, d = distance of the focus from the asymptote, v = initial velocity.

The deflection angle $\delta \Phi_N = \pi - 2\alpha$ so if v = c and d = distance of the light rays from the sun center:

$$\delta \Phi_N = 2 \frac{GM}{c^2 d}$$

The deflection of the light by the sun. Theoretical predictions

From Newton gravitation

$$\delta \Phi_N = 2 \frac{GM}{c^2 d}$$

From General Relativity

$$\delta \Phi_R = (1+\gamma) \ 2 \ \frac{GM}{c^2 d}$$

where $\gamma = 1$ for General Relativity

 $\begin{array}{l} M=solar\ mass=2\ 10^{30}\ kg\ ,\ G=6.7\ 10^{-11}\ m^3\ Kg^{-1}\ s^{-2}\ ,\\ d=ds=\ radius\ of\ the\ sun=\ 7\ \ 10^8\ m,\ dTS=\ Earth\ sun\ distance\ =1.5\ 10^{11}\ m \end{array}$

 $\delta \phi_{\rm R} = 2.13 \ 10^{-6} = 1.75$ "

Expected deflection

$$\frac{\mathrm{ds}}{\mathrm{dTS}} = 4.7 \ 10^{-3} = 16' = 962 \ ''$$

Sun radius seen by the earth



Robertson (1991) Shapiro (1995)

γ=1 +**-** 0.001

Eclipse method

This method consits in comparing by accurate measures a photograph of the stars surrounding the eclipsed sun with a photograph of the same stars taken either at night or in another time of the year when the sun is in another part of the sky.

Problems:

- Distorsion of the reprodution of the star field by the optical instruments. (temperature change during the eclipse)
- Disturbing effect of the uneven illumination of the background
- Abnormal refraction of the earth and solar atmosphere
- Correction of stellar aberrations (20")

Campbell and Trumpler (1922)



Eddington 1919

				Co-ord	inates.	Gravitational displacement.					
No).	Names.	Photog. Mag	Unit	= 50'.	Sok	oral.	Principe.			
			o.	x.	y.	x.	у.	<i>x</i> .	<i>y</i> .		
			m.			"	"	"	"		
		B.D., 21°, 641	$\begin{array}{c c} 7 \cdot 0 \\ 5 \cdot 8 \end{array}$	+0.026 +1.079	$-0.200 \\ -0.328$	$ -1 \cdot 31 $ +0.85	+0.20 -0.09	-1.04 + 1.02	-0.16		
3		κ^2 Tauri	$5 \cdot 5$ $4 \cdot 5$	+0.348 +0.334	$+0.360 \\ +0.472$	$-0.12 \\ -0.10$	+0.87 +0.73	$-0.28 \\ -0.21$	$+0.81 \\ +0.70$		
5		Piazzi, IV, 61	6·0	-0.160	-1.107 +1.099	-0.31 +0.04	-0.43 +0.40	-0.31 +0.01	-0.38 + 0.41		
7	1971 1971 1971 1971	B.D., 20°, 741	$\begin{bmatrix} \pm 0\\ 7 \cdot 0\\ 7 \cdot 0 \end{bmatrix}$	-0.707	-0.864	-0.38	-0.20	-0.35	-0.17		
8		B.D., 20°, 740 Piazzi, IV, 53	7.0	-0.727 -0.483	-1.040 -1.303	-0.33 -0.26	-0.30	-0.20 0.26	-0.20 -0.27		
		72 Tauri	5 · 5 5 · 5	+0.860 -1.261	$+1 \cdot 321 \\ -0 \cdot 160$	$ +0.09 \\ -0.32$	+0.32 +0.02	-0.30	+0.34 +0.01		
		53 Tauri ,	5·5 8·0	-1.311 + 0.089	-0.918 +1.007	$ -0.28 \\ -0.17$	-0.10 + 0.40	$-0.26 \\ -0.14$	$\begin{vmatrix} -0.09 \\ +0.39 \end{vmatrix}$		
	<u> </u>								<u> </u>		
* 'Monthly Notices, R.A.S.,' LXXVII, p. 445.											













Tables of deflections

TABLE II.-Eclipse Plates-Scale.

No. of	I		11.		ш.		г	7.	v.		VII.		VIII.	
Star.	Dæ.	Dy.	Dz.	Dy.	Dz,	Dy.	Dr.	Dy.	Dr.	Dy.	Dr.	Dy.	Dx.	Dy.
11 5 4 3 6 10 2	$\begin{array}{c} r \\ -1 \cdot 411 \\ -1 \cdot 048 \\ -1 \cdot 216 \\ -1 \cdot 237 \\ -1 \cdot 342 \\ -1 \cdot 289 \\ -0 \cdot 789 \end{array}$	-0.554 -0.338 +0.114 +0.150 +0.124 +0.205 +0.109	r -1.416 -1.221 -1.054 -1.079 -1.012 -0.999 -0.733	τ 1.324 0.944 0.862 0.932 0.932 0.948 1.019	7 +0-592 +0-756 +0-979 +0-958 +1-052 +1-157 +1-256	7 +0.956 +0.843 +1.172 +1.244 +1.197 +1.211 +0.924	r +0-563 +0-683 +0-849 +0-861 +0-894 +0-934 +1-177	r +1.238 +1.226 +1.524 +1.587 +1.587 +1.564 +1.522 +1.373	7 +0.406 +0.468 +0.721 +0.733 +0.798 +0.864 +0.995	7 +0.970 +0.861 +1.167 +1.334 +1.30 +1.19 +0.935	$-1 \cdot 456$ $-1 \cdot 267$ $-1 \cdot 028$ $-1 \cdot 010$ $-0 \cdot 888$ $-0 \cdot 820$ $-0 \cdot 768$	7 +0.964 +0.777 +1.142 +1.185 +1.125 +1.072 +0.892	r 1.285 1.152 0.927 0.838 0.768 0.585	r 1.195 1.332 0.930 0.894 0.937 0.964 1.166
	-1.500*	0·55£	-1.500	-1.324	+0.200	+0-843	+0.200	+1.226	+0.400	+0.921	-1.500	+0.777	-1.300	-1.322

COMPARISON Plates-Scale.

No. of	14 ₅₅ .		14	1423. 15		i ₁ . 15 ₂ .		171.		172.		182.		
Star.	Dz.	Dy	Dz.	Dy.	Dg.	Dy.	Dz.	Dy.	Dz.	Dj.	Dr.	Dy.	Dz.	Dy.
11 5 4 3 6 10 2	-0.478 -0.544 -0.368 -0.350 -0.317 -0.272 -0.396 -0.396	r -0-109 -0-204 -0-136 -0-073 -0-144 -0-146 -0-182 -0-206	r +0.967 +1.013 +1.030 +1.044 +0.980 +0.997 +1.102 +0.967	7 +1.170 +1.192 +1.249 +1.305 +1.319 +1.327 +1.289 +1.170	r +1.098 +0.899 +1.133 +1.164 +1.244 +1.249 +0.969 +0.899	$+1 \cdot 228$ +1 \ 232 +1 \ 036 +1 \ 114 +1 \ 012 +0 \ 960 +1 \ 052 +0 \ 960	r +0.725 +0.692 +0.725 +0.732 +0.732 +0.714 +0.722 +0.734 +0.690	+0.830 +0.938 +0.854 +0.893 +0.824 +0.831 +0.831 +0.941 +0.941	7 -1.073 -1.296 -1.296 -1.278 -1.375 -1.424 -1.236 -1.424	r 1 · 530 1 · 675 1 · 631 1 · 614 1 · 652 1 · 636 0 · 909 1 · 530	7 +1.242 +1.161 +1.354 +1.342 +1.363 +1.370 +1.278 +1.161	7 0.302 0.224 0.281 0.261 0.390 0.423 0.328 0.423	7 -1.188 -1.195 -1.165 -1.178 -1.165 -1.164 -1.164 -1.195	7 -1.572 -1.432 -1.454 -1.394 -1.473 -1.476 -1.335 -1.572

* The numbers -1.500, -0.554, &c., given below the line, were taken out to make the values of 12, Dy small and positive for arithmetical convenience.

The units here are in the division of the micrometric screw: 1 div = 6.25"

Star shift Campbell and Trumpler (1922)

They consider 99 stars

This allows them to check the 1/r law

RESULTS

The residuals, after application of these plate corrections, represent the observed star displacements due to light deflection. We shall call them the relative displacements, because they are derived by comparing the stars near the Sun with the reference stars of the outer portions of the plate, and the plate constants are so determined as to make the average displacements of the reference stars zero. To convert the relative values into absolute ones an additional scale correction is needed.

In forming the means of the results of both observers for each plate the weight of any measure made only by one observer (plate AB17) was reduced



-48-

Data analysis

Only 7 of the 13 stars are useful Least square method $\Delta x_{i,j} = a_j \ x_i + b_j y_i + c_j + \alpha_j E_{x_i}$ $\Delta y_{i,j} = d_j \ x_i + e_j y_i + f_j + \alpha_j E_{y_i}$ Eddington (1919) *j* is the plate index and *i* the star index. α is the deflection at 50'

 E_{y_i} and E_{x_i} are the computed deflections for the considered stars normalized to the deflections at 50' from the sun.

You may further assume that when there is no deflection the Δ are pure rotations and shift thus: a = e and d = -b.

Using the data of the 8 plates during the eclipse and the 8 plates without sun he may compute α

Computed values of $\boldsymbol{\alpha}$

Right Asc	cension.	Declination.					
Eclipse – Scale.	Comparison — Scale.	${f Eclipse}-{f Scale}.$	Comparison — Scale.				
r +0.098 +0.126 +0.107 +0.148 +0.140 +0.073 +0.145		$ \begin{array}{r} r \\ +0.126 \\ +0.139 \\ +0.114 \\ +0.111 \\ +0.137 \\ +0.139 \\ +0.136 \\ \end{array} $	$ \begin{array}{r} r \\ +0.044 \\ +0.007 \\ +0.021 \\ +0.010 \\ +0.040 \\ +0.060 \\ +0.036 \end{array} $				
Mean +0.120		+0.129	+0.031				

 $\alpha_{AR} = 0.105 div$ and $\alpha_{decl} = 0.098 div$

mean value $\alpha = 0.1 div = 0.625''$.

Computed values of $\boldsymbol{\alpha}$

 $\alpha = 0.625''$

This corresponds to a distance of the light rays from the sun of d = 50'

The sun radius was during the eclipse Rs= 15.8'. Thus the defelction for $d\simeq Rs$ is : 1.98''

which gives $\gamma \simeq 1.26$

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

LICK OBSERVATORY BULLETIN

NUMBER 346

OBSERVATIONS ON THE DEFLECTION OF LIGHT IN PASSING THROUGH THE SUN'S GRAVITATIONAL FIELD

MADE DURING THE TOTAL SOLAR ECLIPSE OF SEPT. 21, 1922

The observations forming the subject of this paper represent a part of the program of the Wm. H. Crocker Eclipse Expedition to Wallal, Western Australia, of which Mr. Campbell has given a detailed report in another publication.³ That report also contains a full account of the previous efforts made to determine the light deflection by the Sun's gravitational field during total solar eclipses, and of the results obtained.

Since the British expeditions of 1919 published their measures⁹ on this subject, several physicists and astronomers have pointed out various sources of error that might enter into this problem, which consists in comparing by accurate measures a photograph of the stars surrounding the eclipsed Sun with a photograph of the same stars taken at night several months before or after the eclipse, when the Sun is in another part of the sky and has no disturbing influence on the apparent positions of these stars. The most important of the sources of error are:

 Distortion in the reproduction of the star field by the optical parts of the instruments. In view of the differential character of the measures such distortions are only to be feared in case they are different for the cellpse photographs and the night comparison plates. This is most likely to occur if the optical parts of the instrument contain mirrors. The experience of the British expedition to Sobral in 1019 has shown such mirrors to be easily deformed in consequence of the inevitable temperature changes during a total celipse. 2. Disturbing effect of the uneven illumination of the background,⁵ the intensity of which is gradually diminishing with increasing distances from the Sun. There may be a physiological effect in bisecting a star image seen on such a background, but there may also be a real shifting of the center of a star's image if this is photographed on a background of uneven intensity. In view of the gentle gradation, hardly noticeable in the field of the measuring microscope, this effect can be but small and may be negligible.

3. Systematic distortion of the photographic film during the process of drying.⁴ According to Ross the blackened part of the plate corresponding to the corona should dry the more rapidly, and we should expect contraction of the film toward the inner corona. Such distortion is favored by the high temperatures encountered by expeditions to tropical countries; it should have the tendency to make the measured light deflections too small.

 Abnormal refraction in the Earth's atmosphere accompanying the changes of temperature produced by the passing of the Moon's shadow.⁵

It has been stated, also, that other causes than the Sun's gravitational field may be responsible for all or a part of the observed light deflections. As such should be mentioned:

5. Refraction in an extended solar atmosphere."

² K. F. Bettlinger: Die astronomischen Pr
üfungsmöglichkeiten der Estativitätstheorie, Jahrbuch für Esdivaktivität und Elektronik. 17, 149 and 152, 1920. M. Welf: A. N., 212, 181, 1920.

4 P. Boss: Image Contraction and Distortion on Photographic Plates. Ap. J., 52, 98; Silberstein: M. N., 80, 600, 1020.

* A. Anderson: Nature, 104, 354, 372, 333, 436, 468, 503, 1919-20.

⁴ H. F. Newall: M. N., 80, 22, 1919; The Observatory, 42, 428, 1919; 43, 145, 1920. W. Anderson: A. N., 213, 251, 1923.

¹ Publ. A. S. P., 25, 11, 1923.

² F. W. Dyson, A. S. Eddington, C. Davidson: A Determination of the Deflection of Light by the Sus's Gravitational Field, from Observations made at the Total Solar Eclipse of May 29, 1919. Men. R. A. S., 62, Appendix, 1930.

Star deflection Campbell and Trumpler (1922)

They use a different strategy

They compute the residuals once the rotation and shifts have been subtracted from the values of Δ

The agreement with the prediction is again very satisfactory, only one of the seven results differing from it by more than its probable error. The plate means were formed by giving equal weight to both observers. The means for each observer are based on the weights of the separate least squares solutions. When taking the mean of the four plates, the first three plates received weight 1, the last plate, measured only once, weight 0.9 (see page 48).

For every plate the measures of the two observers were reduced independently, even as to the weighting of the stars according to the estimates of each observer, and as to all the plate constants determined from the measures. No measures once entered in the observing book were rejected during the reductions, except when a remark suggesting that this be done had been made during the measures (two cases), or if the star image on one of the plates was within 10 mm. from its edge, and further in two cases of measures of very faint and uncertain images, giving residuals exceeding 175.

The star field was further divided into four quadrants so situated that the Sun's equator passed through the middle of two of the quadrants and the Sun's axis of rotation through the middle of the other two. For the stars of each quadrant a separate least squares solution with the formula of page 51 was made, using the mean radial displacements D₁ of TABLE II.

Quadrant	No. of stars	Light deflection at Sun's limb
recoding Sun's equator	24	$1'.61 \pm '.28$
ollowing Sun's equator	29	$1.68 \pm .25$
t Sun's North pole	18	$1.76 \pm .26$
t Sun's South pole	21	$1.73 \pm .24$
fean of equatorial quadrants	53	$1.63 \pm .15$
fean of polar quadrants	39	1.76 _{ab} .18







Conclusions on eclipse experiments

The results of these experiments in 1920-1930 can be summarized:

• The Newton gravitation is excluded. It is clearly outside the error bars.

• It is difficult to say that General Relativity is the only possible theory with an error of 30%

The results of the measurements in 1971 were better than these of about 10% of error bars. However another technique appeared

Radio waves technique and the Very Large Base Interferometer (VLBI)

- Principle of the measurement
- Corrections for the sun and earth ionospheres
- Results



Principle of VLBI

 $D = B\sin(\theta)$

dephasing
$$\delta \phi = \frac{2\pi D}{\lambda} = \frac{2\pi B \sin(\theta)}{\lambda}$$

$$S_o = s_1^2 + s_2^2 + 2 s_1 s_2 \cos(\frac{2\pi B \sin(\theta)}{\lambda})$$



Principle of VLBI

 $D = B\sin(\theta)$

dephasing
$$\delta \phi = \frac{2\pi D}{\lambda} = \frac{2\pi B \sin(\theta)}{\lambda}$$

$$S_o = s_1^2 + s_2^2 + 2 s_1 s_2 \cos(\frac{2\pi B \sin(\theta)}{\lambda})$$

VLBI data

$$B=20km$$
 $f=10^{10}Hz$, $\lambda=2~10^{-2}m$

declared sensitivity 0.01 fringes which means for $\theta\simeq45$

$$\delta \theta_{min} = 10^{-8} \text{rad} = 2.2 \ 10^{-3}$$
" arc



Three stars have been chosen. They are almost collinear and separated by almost 10°

The A source (0116+08) is covered by the sun the 11 april 1976

In contrast to eclipse method this experiment can be repeated

The measuring sequence is : CABAC

Two freqencies have been chosen 2.7GHz and 8.1GHz.

The energy flux $10^{-25} \frac{W}{m^2 Hz}$ which with an antenna of $100m^2$ and an output impedence of 50 Ω gives about 2 $10^{-11} V Hz^{-1/2}$

Problems

The radio waves crosses the solar corona which has an effectif index of refraction:

$$n(r)^2 = 1 - \frac{e^2 N(r)}{\epsilon_0 m \omega^2}$$
 with $N(R)$ =charge density

The two frequencies measure solves the problem:

The dephasing
$$\delta \phi_i = \frac{2\pi D \sin(\theta) f_i}{c_i} = \phi_i f_i$$

the measured dephasing is $\Psi_i = \phi_i f_i (1 + \alpha_c f_i^{-2})$ with i = 1, 2.

As a consequence α_c can be measured :

$$\alpha_c = \left(\frac{\Psi_1}{f_1} - \frac{\Psi_2}{f_2}\right) \left(f_1^{-2} - f_2^{-2}\right)^{-1}$$





Relativity of time

- Four atomic clocks
- Travel around the world
- Measure of the time difference

Test of the clocks

Fig. 1. Time differences between each clock in the flying ensemble and the Naval Observatory clock MEAN(USNO) at hourly intervals throughout the data period. The hour count along the abscissa begins at 0^h U.T., 25 September 1971. The trace of each clock is labeled with the clock's serial number; and the trace labeled Average is the average of the four time differences.





Fig. 2. Magnified views of the average time difference data of Fig. 1 in the vicinity of (a) the eastward trip and (b) the westward trip. The small step down for the eastward trip indicates an average time loss, and the somewhat larger step up for the westward trip indicates an average time gain. The indicated time differences follow from application of the average rate method with a fitting interval of 25 hours.



Direct measurement

Measure of the frequency shift of a maser source transported by a spacecraft launched at 10⁷ m by a rocket.

$$V_{in} = 6000 m/s$$
 so $V/C = 2 \ 10^{-5}$

$$\frac{\Phi_A - \Phi_B}{C^2} \simeq 4.5 \ 10^{-10}$$

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Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser

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and

R. Decher, P. B. Eby, C. R. Baugher, J. W. Watts, D. L. Teuber, and F. D. Wills George C. Marshall Space Flight Center, Huntsville, Alabama 35812 (Received 19 August 1980)

Predictions for second order doppler during the flight





Simplified diagram



 f_1 =frequency emitted by the satellite and received by the earth station

 f_2 =frequency emitted by the earth station and received by the satellite

 f_3 =frequency retransmitted by the satellite and received by the earth station

$$\frac{M}{N} = \frac{240}{221} \qquad \frac{P}{Q} = \frac{76}{49} \qquad \frac{R}{S} = \frac{82}{55}$$

These ratios satisfy the following equation with an accuracy of 2 10^{-5}

$$\frac{P}{Q} - \sqrt{2} \frac{R}{S} (1 + (\frac{N}{M})^2)^{-1/2}$$





FIG. 3. Frequency residuals and predicted effect during mission.

