Failure time and microcrack nucleation

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Abstract

The failure time of samples of heterogeneous materials (wood, fiberglass) is studied as a function of the applied stress. It is shown that in these materials the failure time is predicted with a good accuracy by a model of microcrack nucleation proposed by Pomeau. It is also shown that the crack growth process presents critical features when the failure time is approached.

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It is very well known that different materials subjected to a constant load may break after a certain time, which is a function of the applied load [1-5]. Many models have been proposed to predict this failure time, but the physical mechanisms remain often unclear [4, 5]. Very recently Pomeau proposed a model [6], which explains quite well the failure time of microcrystals [7] and gels [8] submitted to a constant stress. This model is based on the interesting idea that a nucleation process of microcracks has to take place inside the materials, in order to form the macroscopic crack. This nucleation process is controlled by an activation law, as the coalescence of a phase into another in a liquid-solid transition. Based on this prediction [6], L. Pauchard et al.

[7] found that if a constant load is applied to a bidimensional microcrystal, it breaks after a time τ given by the equation $\tau = \tau_o e^{P_o^2/P^2}$, where P is the applied pressure, and τ_o and P_o are constants. Bonn et al. [8] found a similar law for gels. Pomeau predicted that for three-dimensional microscopic systems the life-time should be

$$\tau = \tau_o e^{(P_o/P)^4} \tag{1}$$

where τ_o is a characteristic time and P_o a characteristic pressure, which mainly depend on the material characteristics, the experimental geometry and temperature. This idea is quite interesting and it merits to be checked experimentally in hetrogeneous materials, such as fiber glass and wood pannels. Indeed, in two recent papers [9, 10], we have shown that in these materials the microcracks, preceding the main crack form something like a coalescence around the final path of the main crack.

Driven by these observation we decided to study the behaviour of these materials as a function of time and to check whether eq.1 could be useful in order to predict the sample life time. To do this we monitor the acoustic emission (AE) released before the final break-up by a sample placed between two chambers between which a pressure difference P is imposed. In Fig. 1 a sketch of the apparatus is shown. We have prepared circular wood and fiberglass samples with a diameter of 22 cm. and a thickness of 4 mm (wood) and 2 mm (fiberglass). In our samples the AE consists of ultrasound bursts (events) produced by the formation of microcracks inside the sample. For each AE event, we record the energy ε detected by the four microphones, the place where it was originated, the time at which the event was detected and the instantaneous pressure and deformation of the sample. The energy is defined as the integral of the sum of the squared signals. A more detailed description of the experimental methods can be found in [9, 10].

We first investigate the behaviour of the samples as a function of time when they are submitted to a constant load. Our interest is focused on the life-time of the sample, on the behaviour of the released acoustic energy near the fracture and on the distributions of energy and time elapsed between two consecutive events. The behaviour of the energy as a function of time for a system submitted to a constant load has been studied by geologists, but they were not specially interested in what happened near the fracture [11, 12, 13].

We first imposed a constant strain to our samples, as it has been made for crystals. As strain is fixed, every microcrack leads to a pressure decrease, so the system reaches a stationary state. This is because a microfracture weakens the material. In the absence of microcraks the pressure remains constant. One sample was submitted to a large deformation (close to the fracture) and it did not break after three days. Therefore, at imposed strain, the effect observed in microcrystals is not valid for heterogeneous materials. On the other hand, if a constant stress is applied to the system, it will break after a certain time which depends on the value of the applied pressure. The reason for this is that after every single microcrack the same load must be endured by the weakened sample, so it becomes more and more unstable. We have submitted several samples to different constant pressures and we have measured the time until the break-up (the life-time τ). The values obtained are well fitted by eq.(1), that is the exponential function predicted by Pomeau; the life-time expression $\tau = ae^{-bP}$ proposed by Mogi [11], on the other hand, does not conform to our data[14]. In Fig. 2a τ is plotted versus $\frac{1}{P^4}$ in a semilog scale, and a straight line is obtained. Even if the pressure difference is very small the sample will eventually break, although the life-time can be extremely long. For example, using eq. (1) and the best fit parameters of fig. 2a, one estimate $\tau \simeq 5000$ s at $P = 0.43$ atm. Halving the imposed pressure causes τ to become extremely large : $\tau = 4.4 \cdot 10^{37}$ years at $P = 0.21atm$).

When a constant pressure is applied to the sample, the acoustic emission of the material is measured as a function of time. We find that the cumulated acoustic energy E diverges as a function of the reduced time $\frac{\tau-t}{\tau}$, specifically $E \propto (\frac{\tau-t}{\tau})^\gamma$ with $\gamma = 0.27$ (see Fig. 2b). Notably, the exponent γ, found in this experiment with a constant applied pressure, is the same than the one corresponding to the case of constant stress rate [9]. Indeed it has been shown [9, 10] that if a quasi-static constant pressure rate is imposed, that is $P = A_p t$, the sample breaks at a critical pressure P_c and E realesed by the final crack precursors (microcrcaks) scales with the reduced pressure or time (time and pressure are proportional) in the following way:

$$E \propto (\frac{P_c - P}{P_c})^\gamma = (\frac{\tau - t}{\tau})^\gamma \tag{2}$$

where $\tau = P_c/A_p$ in this case. Thus it seems that the real control parameter of the failure process is time, regardless of the fact that either a constant pressure rate or a constant pressure is applied.

To find a general law, which is valid for a time dependent imposed stress, we intend to generalize the eq.1 which is valid only for a constant imposed pressure. In the case where the pressure changes with time, it is reasonable to consider the entire history of the load. Therefore we consider that

$$\frac{1}{\tau_o}\exp(-(\frac{P_o}{P})^4)$$

is the density of demage per unit time. The certitude of breaking is obtained after a time τ such that:

$$\int_0^{\tau} \frac{1}{\tau_o} e^{-(\frac{P_o}{P})^4} dt = 1 \qquad (3)$$

where τ_o and P_o have the previously determined value. Notice that this equation is equivalent to eq. 1 when a constant pressure is applied.

To test this, we have applied the load to the sample following different schemes. We have first applied successive pressure plateaux in order to check whether memory effects exist. In fig.3a the pressure applied to the sample is shown as a function of time. A constant load has been applied during a certain time τ_1, then the load is suppressed and then the same constant load for a time interval τ_2 is applied again.The sample breaks after a loading a time $\tau_1 + \tau_2$ which is equal to the time needed if the same load had been applied continuosly without the absence of load during a certain interval. Therefore there is a memory of the load history. The life-time formula (eq. 3) is also valid if different constant loads are applied successively (fig. 3b). This concept can explain the violation of the Kaiser effect in these materials [10].

If the load is not constant, the life-times resulting from the proposed integral equation are still in good agreement with experimental data. A load linearly increasing at different rates A_p has been applied to different samples. The measured breaking times are plotted in fig.4 along with a curve showing the values computed from eq.3. Even if a quasi-static load is applied erratically (fig. 3d), the calculated life-time agrees with the measured one. These experiments show that eq. 3 describes well the life-time of the samples submitted to a time dependent pressure.

The question is to understand why eq.(1) and (3) works so well for a three dimensional heterogeneous material. Indeed, in the Pomeau formulation

$$P_o = G \left(\frac{\eta^3 Y^2}{kT} \right)^{1/4} \tag{4}$$

where Y is the Young modulus,T the temperature, K the Boltzmann constant and η the surface energy of the material under study. G is a geometrical factor which may depend on the experimental geometry, on defect shape and density.

In our experiment, we found $P_o = 0.62$ atm for wood, which has Y=$1.8 \cdot 10^8$ N/m^2, and $P_o = 2.91$ atm for fiberglass, which has Y=10^{10} N/m^2. Thus the ratio between the values of P_o found for the two materials is closed to the ratio of the square root of their Young modula.

In contrast temperature does not seem to have a strong influence on τ. In fact we changed temperature, from $300K$ to $380K$ which is a temperature range where the other parameters, Y and η, do not change too much. For this temperature jump one would expect a change in τ of of about 50% for the smallest pressure and of about 100% for the largest pressure. Looking at fig.4 we do not notice any change of τ within experimental errors which are about 10%. In order to maintain the change of τ within 10% for a temperature jump of $80K$ one has to assume that the effective temperature of the system is about $3000K$. Notice that this claim is independent on the exact value of the other parameters and G.

These observations seem to indicate that the nucleation process of microcraks is activated by a noise much larger than the thermal one. Such a large noise can be probably produced by the internal random distribution of the defects in the heterogeneous materials that we used in our experiments. This internal random distribution of material defects evolves in time because of the appearance of new microcracks and the deformation of the sample. Therefore this internal and time dependent disorder of the material could actually be the mechanism that activates the microcrak coalescence and play the role of a very high temperature. Similar conclusions about a disordered induced high temperature have been reached in other disordered systems [15]. This is an important point that merits to be deeply explored. Simple numerical simulations which we performed in fuse networks seem to confirm this hypothesis.

As a conclusion we have shown that a model based on the nucleation of microcracks is in agreement with experimental data of failure of two heterogeneous materials. This model seems to be quite general because it also explains the failure of gels [8] and microcrystals [7]. It will be certainly interesting to test it in other materials. However many questions remain open. The first one concerns the high temperature of the system. The second is related to the behaviour of acoustic energy close to the failure time. Indeed if AE is considered as a susceptibility [16] it is not easy to put together the observed critical divergency with a nucleation process. Probably the standard phase transition description can be only partially applied to failure because of the intrinsic irreversibility of the crack formation. All of these are very interesting aspects of this problem which certainly merit to be clarified in the future.

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Figure Captions

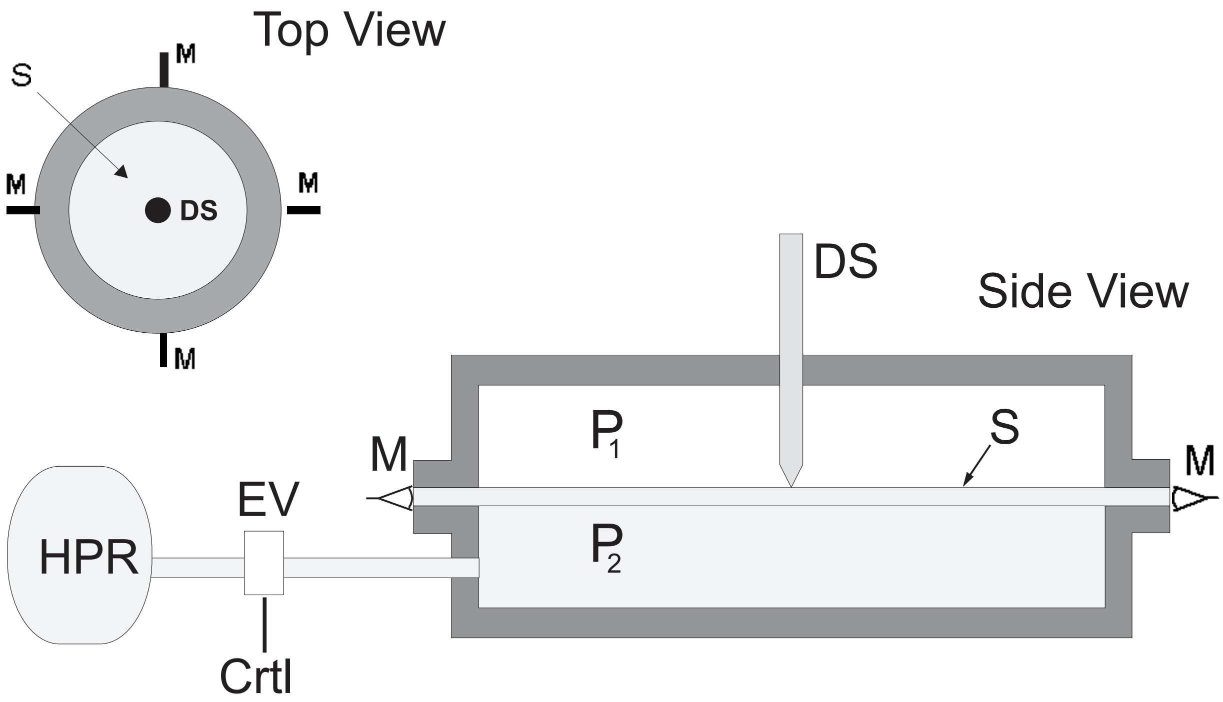
1. Sketch of the apparatus. S is the sample, DS is the inductive displacement sensor (which has a sensitivity of the order of 1 μm). M are the four wide-band piezoelectric microphones. $P=P_1-P_2$ is the pressure supported by the sample. P is measured by a differential pressure sensor (sensitivity = 0.002 atm) that is not represented here. EV is the electronic valve which controls P via the feedback control system Ctrl. HPR is the high-pressure air reservoir.

2. Measures on wood samples: **a)** The time τ needed to break the wood samples under an imposed constant pressure P is here plotted as a function of $1/P^4$ in a semilog scale. The dashed line represents the solution proposed by Mogi [11]($\tau = ae^{-bP}$). The continuous line is the solution proposed by Pomeau for microcrystals ($\tau = \tau_o e^{(P_o/P)^4}$). In the plot $\tau_o = 50.5$ s and $P_o = 0.63$ atm. Every point is the average of 10 samples. The error bar is the statistical uncertainty. For the fiberglass samples, we find $\tau_o = 44.6$ s and $P_o = 2.91$ atm. **b)** The cumulated energy E, normalized to E_{max}, as a function of the reduced control parameter $\frac{\tau-t}{\tau}$ at the neighborhood of the fracture point (Case of imposed constant pressure). The circles are the average for 9 wood samples. The solid line is the fit $E = E_0\left(\frac{\tau-t}{\tau}\right)^{-\gamma}$. The exponent found, $\gamma = 0.26$, does not depend on the value of the imposed pressure. In the case of a constant pressure rate the same law has been found [10, 9].

3. The imposed time dependent pressure (bold dotted line) is plotted as a function of time in the case of wood samples. The continuous line is the integral in time of the function $f(P) = \frac{1}{\tau_o}e^{-P_o^4/P^4}$. On the basis of eq. 3 the predicted breaking time τ is obtained when the integral of $f(P)$ is equal to 1. The horizontal distance between the two vertical dashed lines in each plot represent the difference between the predicted and the measured breaking time. In **a)** a constant pressure has been applied during about 700 s, then the load is suppressed and then the same constant load is applied again. The difference between the lifetime predicted by (eq. 3) and the experimental result is of 3%. **b)** Here two pressure plateaux of different value are successively applied to the sample. The difference between the measured and the predicted

life-time is of 5%. In **c)** an erratic pressure is applied to the sample. Here the error is of 10%.

4. A load linearly increasing at different rates A_p has been applied to different samples. The measured breaking times are plotted as a function of A_p in a loglog scale; circles and squares represent the measures on wood and fiberglass samples respectively at T=300 K. Bold triangles represent measures on wood samples at T=380 K. The lines are the life time calculated from eq.3 using the best fit values for P_o and τ_o. These experiments show that eq. 3 describes well the life-time of the samples submitted to a time dependent pressure.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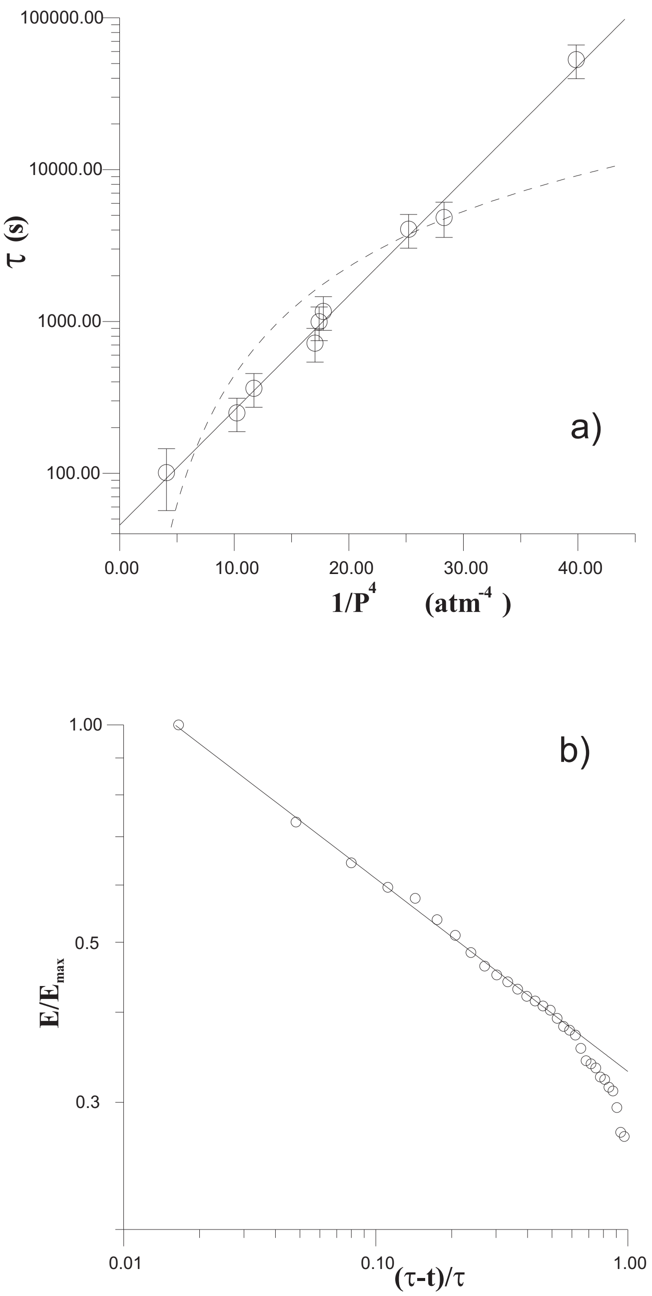
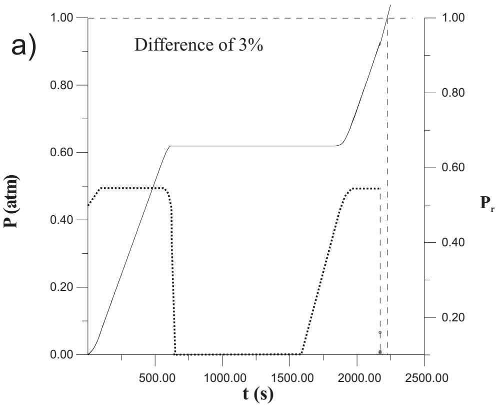
M
M
M
DS
M
DS
Side View
P1
S
M
M
EV
HPR
P2
Crtl


a)
τ (s)
1/P^4 (atm^-4)
100000.00
10000.00
1000.00
100.00
0.00
10.00
20.00
30.00
40.00
b)
E/E_max
(τ-t)/τ
1.00
0.5
0.3
0.01
0.10
1.00


a)
Difference of 3%
P (atm)
P_r
t (s)
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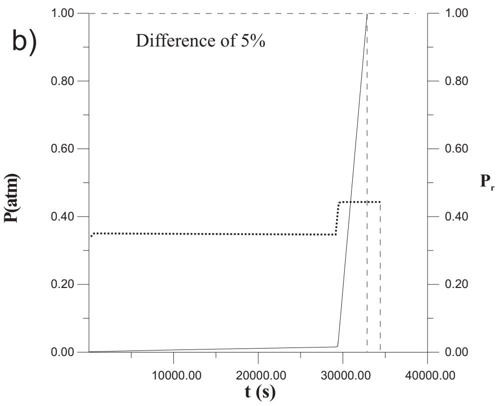
0.80
0.60
0.40
0.20
0.00
500.00
1000.00
1500.00
2000.00
2500.00


b)
Difference of 5%
P(atm)
P_r
t (s)
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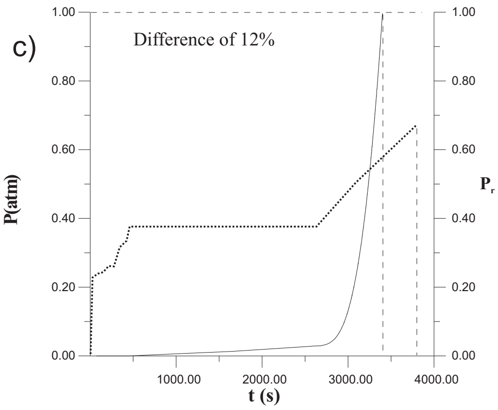
0.80
0.60
0.40
0.20
0.00
10000.00
20000.00
30000.00
40000.00


c)
Difference of 12%
P(atm)
P_r
t (s)
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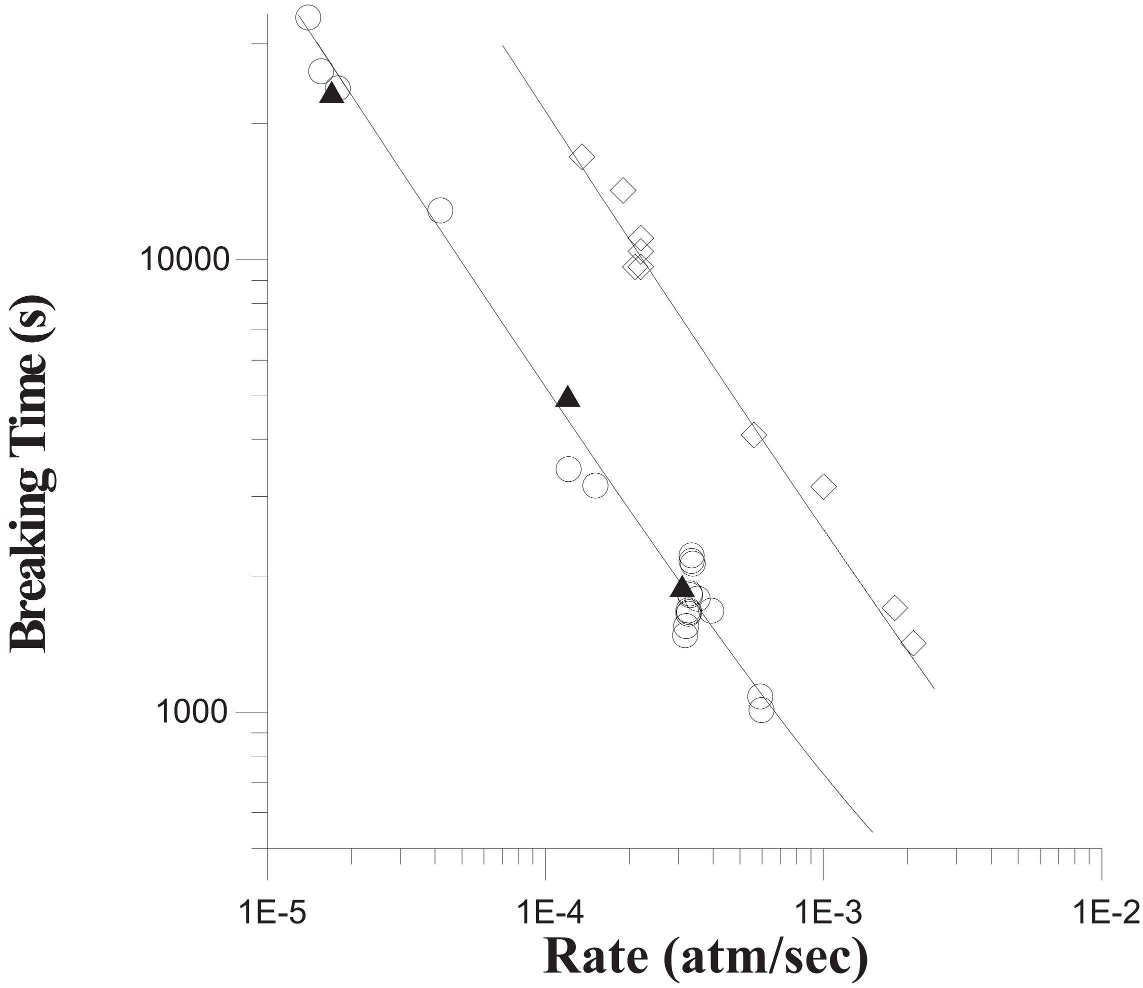
0.60
0.40
0.20
0.00
1000.00
2000.00
3000.00
4000.00



Breaking Time (s)
10000
1000
1E-5
1E-4
1E-3
1E-2
Rate (atm/sec)