Flows, Subset Sums, Permanent and Graph Decompositions

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LIP - ENS LYON

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Balanced Directed Graph

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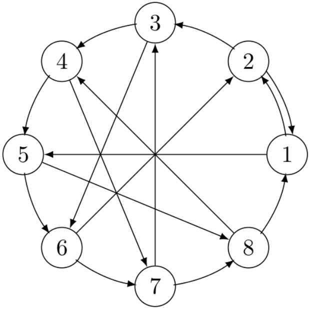

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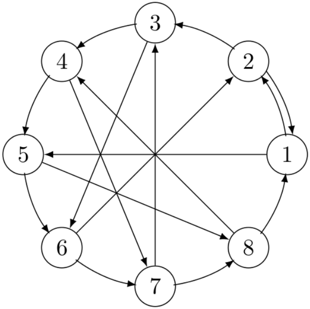

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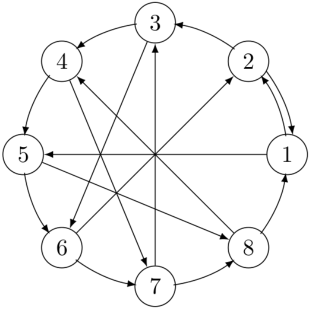

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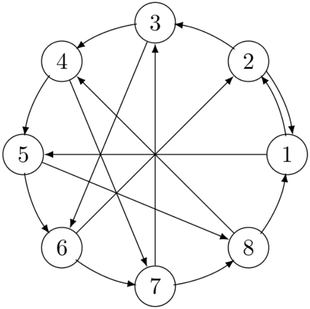

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This is a characterization of balanced directed graphs.

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We assume our graphs connected from this point.

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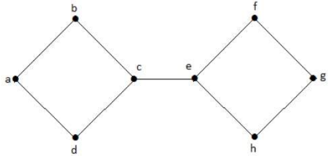

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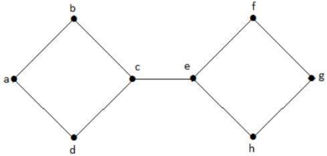

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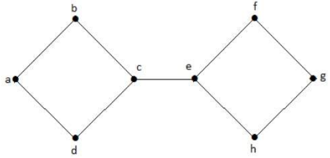

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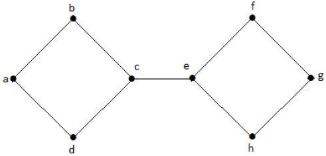

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Conjecture (Tutte 1954)

Every bridgeless graph has a 5-flow.

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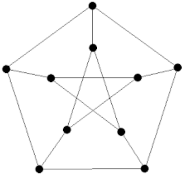

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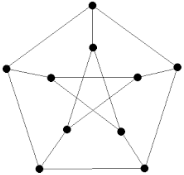

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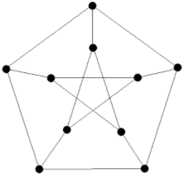

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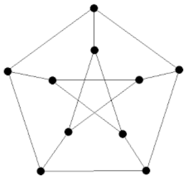

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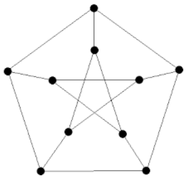

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Can we make a weaker version of Tutte's conjectures?

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Every $2k$-edge connected graph has k edge-disjoint spanning trees.

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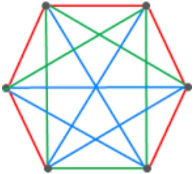

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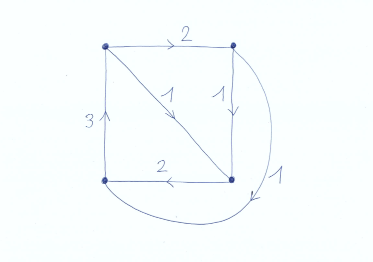

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Easier to find a $(\mathbb{Z}/2\mathbb{Z})^2$-flow than a 4-flow.

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Seymour 1981: Every bridgeless graph has a 6-flow.

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Modular 3-flows are special, since reversing an arc with flow 2 turns it into 1, we can assume that all arcs have flow 1.
Hence G has a 3-flow if and only if it has an orientation such that $d^+(v) = d^-(v)$ mod 3.

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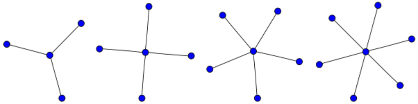

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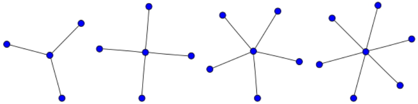

Examples of k-stars, when $k = 3, 4, 5, 6$.

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Our proof implicitely uses the case of k-stars

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Same question with coefficients in $-1, 1$.

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If subset sum conjecture holds, the vector 0 is achievable via a linear combination with coefficients -1 or 1.
This is exactly a 3-flow.

And at last the permanent

Permanent Lemma (Alon)

Let M be a $n \times n$ matrix with entries in $\mathbb{F}_3$ with non zero permanent. Let x be any vector of $(\mathbb{F}_3)^n$. Then there is a linear combination v of M in -1,1 such that x and v differs on all coordinates.

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Would imply the subset sum problem.