Introduction to rheology

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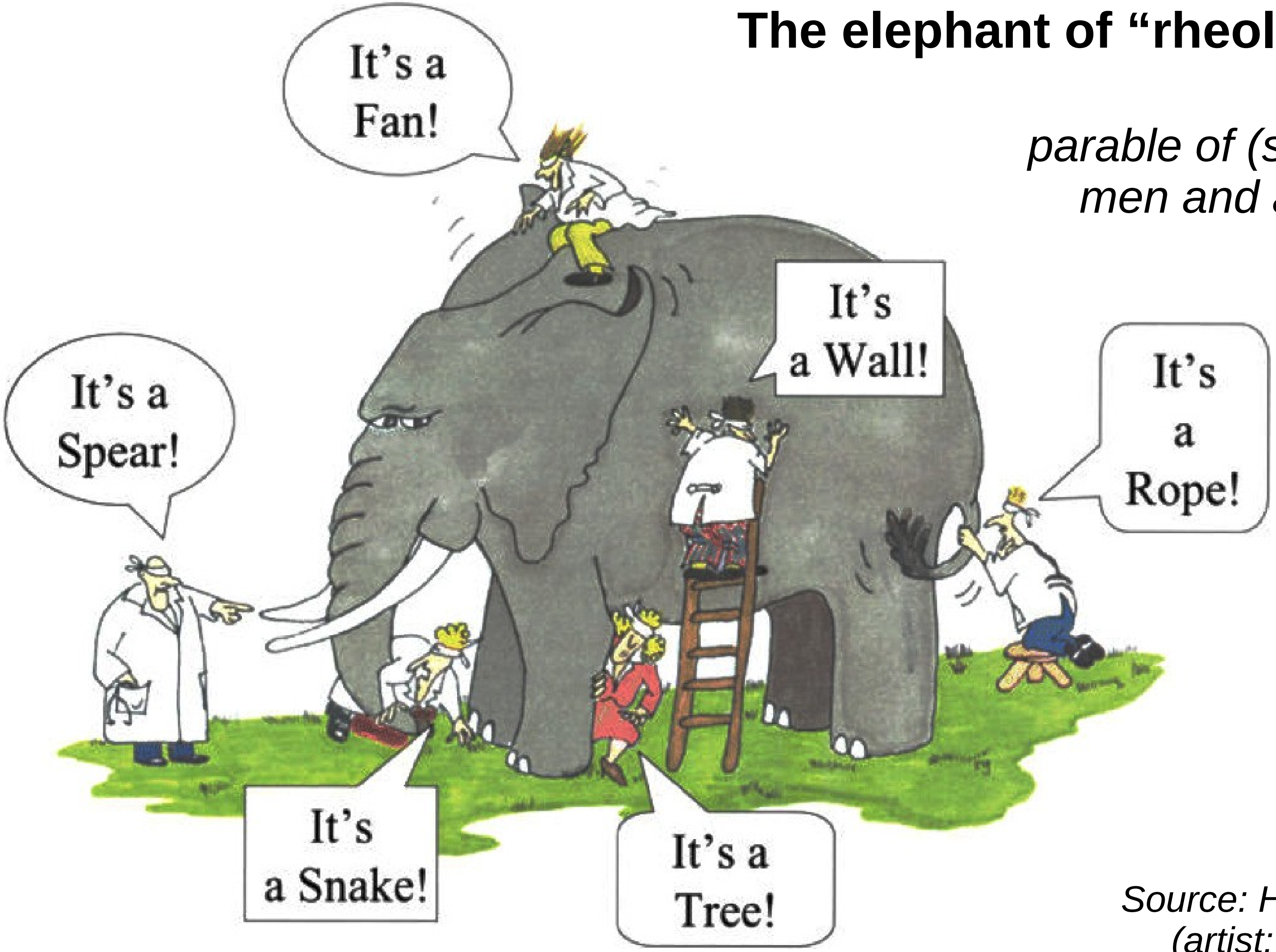
The elephant of “rheology”

parable of (scientific) blind men and an elephant



Source: Himmelfarb et al 2002 (artist: G. Renee Guzlas)

Introduction to rheology

1. Introduction → definition → why does rheology matter for the Earth ?
→ why is difficult to characterize?

2. Investigating and representing rheology for Earth materials

- stress, strain, strain rate
- lab and numerical experiments
- (micro-)mechanical processes
- flow “laws” and dependencies

3. Contemplating mantle rheology at long geological time-scales

- from flow laws to “effective” viscosity
- feedbacks between deformation and dynamics
- independent constraints on Earth’s upper mantle viscosity

4. (open) Conclusion : complexify or simplify ?

+ round-table on upper mantle rheology
Tuesday 13^{th} July

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What do we call rheology?

A- LOGOS (study) of RHEO (flow)

= *the science that deals with the* ***deformation and flow of materials,*** *both solids and liquids (Wikipedia)*

B- Now rheology is also used to name "the response of material to an applied force"

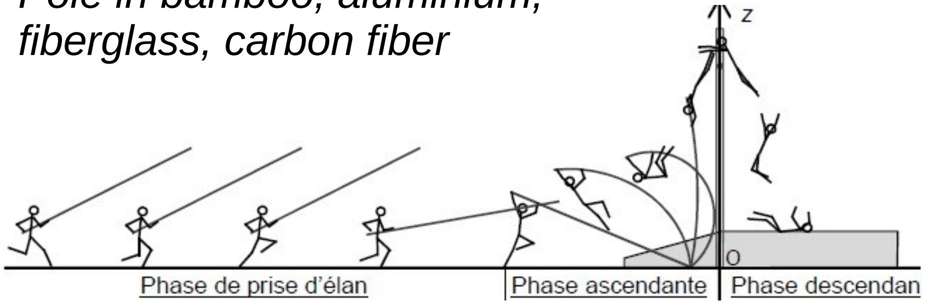
- *Experimentalists produce rheological parameterisations*
- *Thermo-mechanical modelers use rheology as an input to predict plate and mantle dynamics*

Various responses of solids to an applied force

Reversible deformation

> **elastic**

Pole in bamboo, aluminium, fiberglass, carbon fiber



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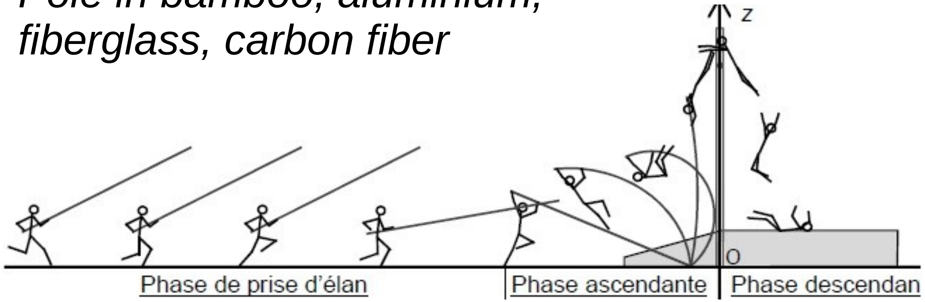

Irreversible deformation
> **plastic**

mechanical rupture
> **brittle**

continuous deformation
> **ductile**

For fluid and solid: viscous flow

Pitch drop exp. : *9 drops in 84 years !*

Credit : John Mainstone

→ if you wait long enough, everything flows !

dimensionless Debora number
"the mountains flowed before the Lord"

Viscosity = **measure** of material resistance to flow/deformation over time (> strain rate)

= resulting from "internal" friction

- *dynamic viscosity η or μ in Pa.s*

- *kinematic viscosity ν in m^2/s*
  "momentum diffusivity"

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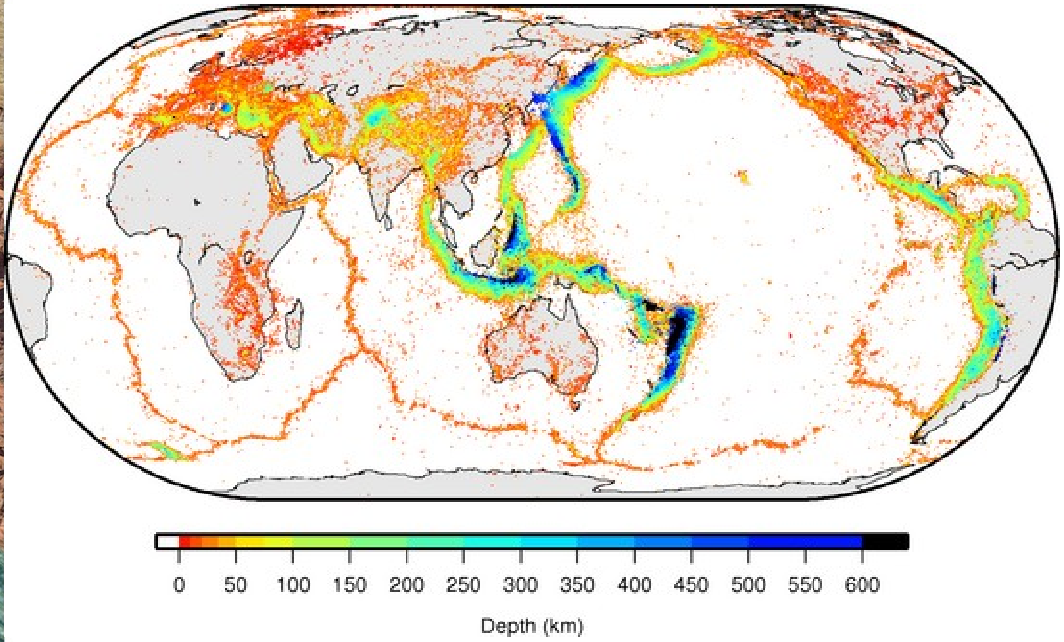
Viscosity – orders of magnitude

Water ~ 10^{-3} Pa.s,
Oil ~ 10^{-2} Pa.s

Maple syrup ~ 1 Pa.s
Peanut butter ~10^{2} Pa.s

Pea ~ 10^{11} Pa.s
Salt diapir ~ 10^{18} Pa.s
Rock ~ 10^{19-28} Pa.s

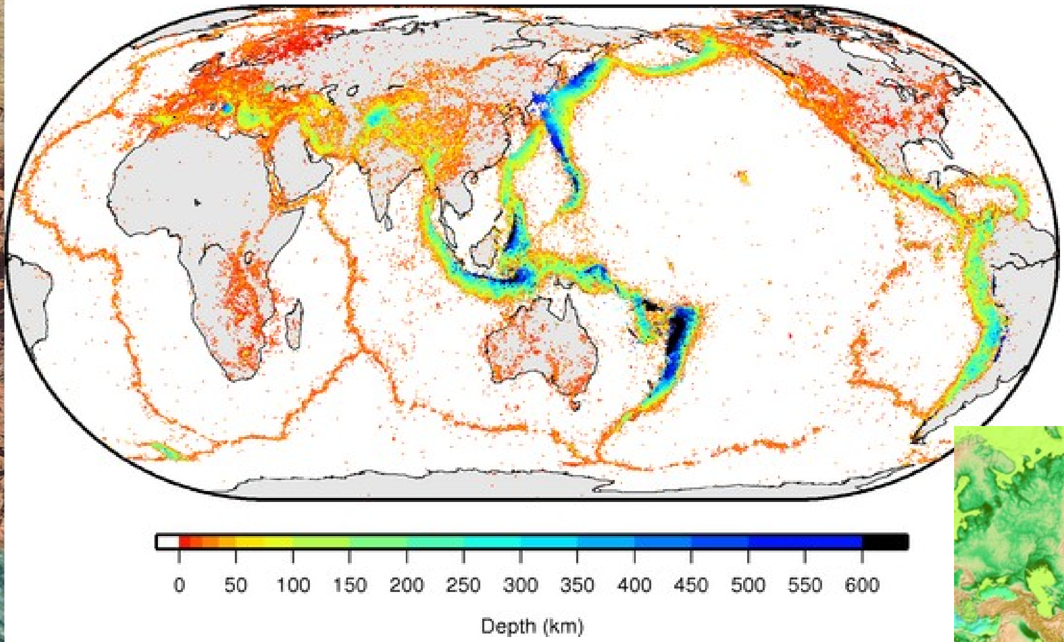
Why does rheology matter?



Earthquake hazard

EQ = rock brittle deformation in response to an applied stress

Why does rheology matter?



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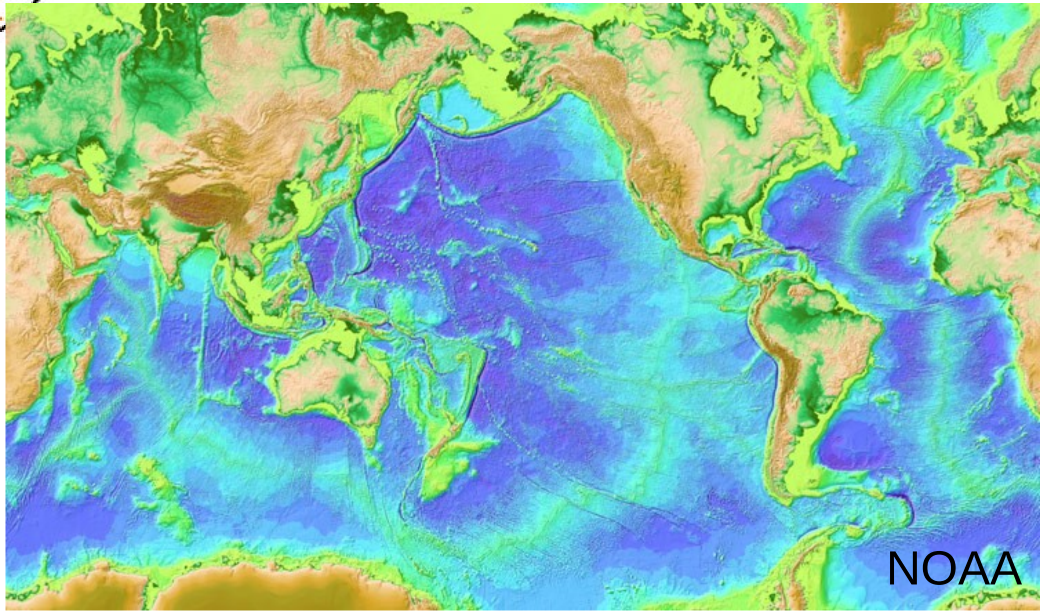
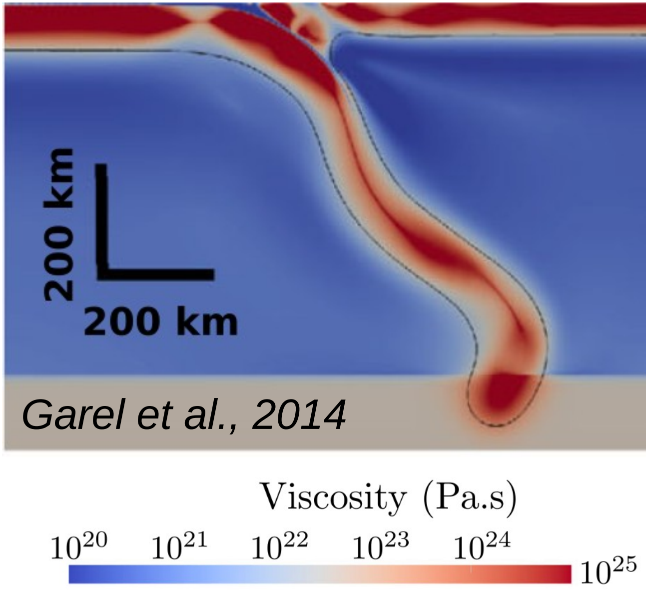
Explaining Earth topography and surface motion

→ lithospheric flexure under loading (ice sheet, volcano, collisional thickened crust, sediments...)



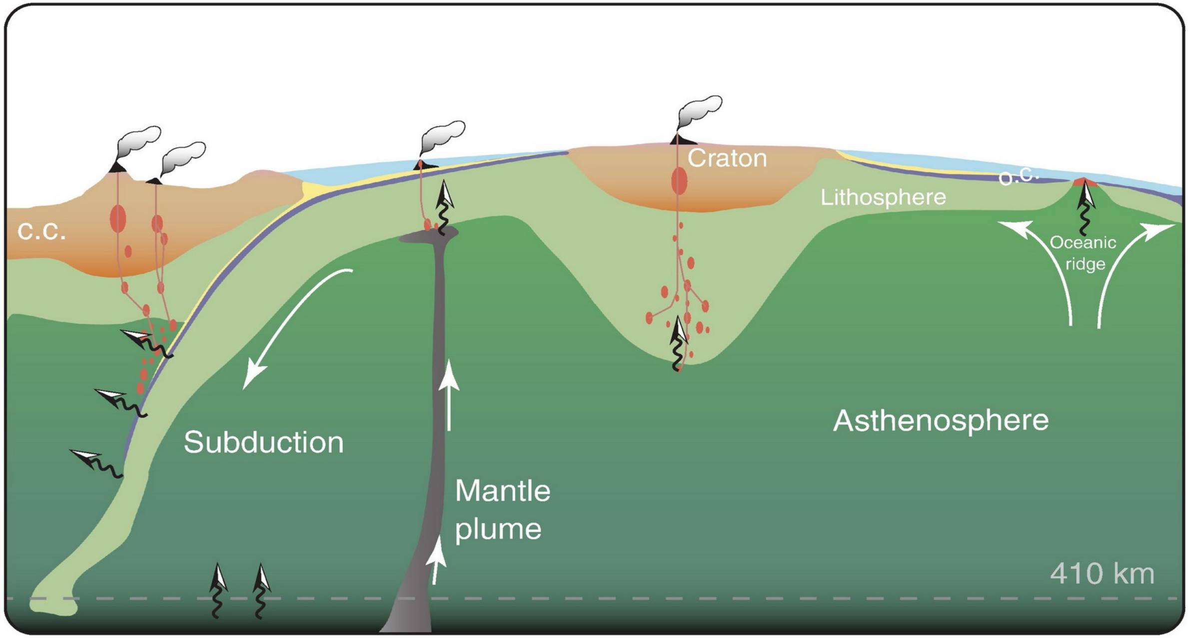
Why does rheology matter?



- Influence of mantle flow and plate deformation on **surface topography**

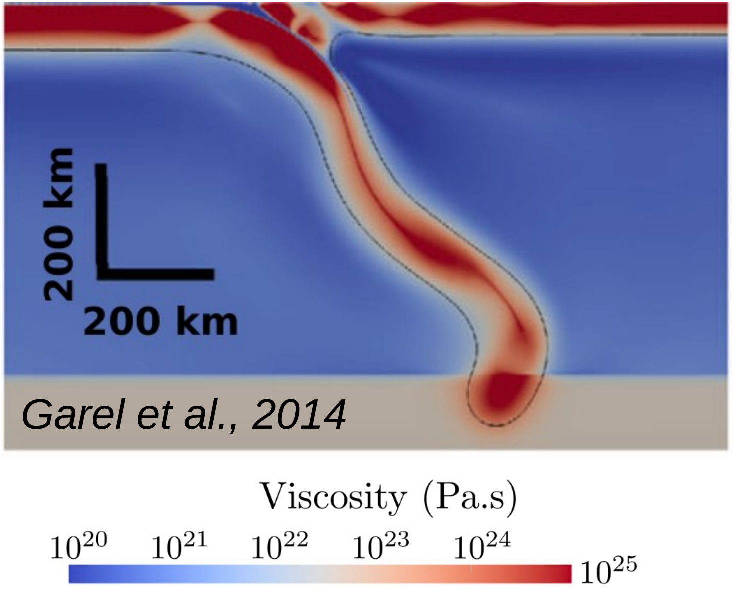
- **Plate tectonics**: force budget, formation of new boundaries, orogenic cycle

- Earth's secular **cooling** and differenciation



Demouchy & Bolfan-Casanova, 2016

Why does rheology matter?



Conservation equations for fluid mechanics
> viscosity !

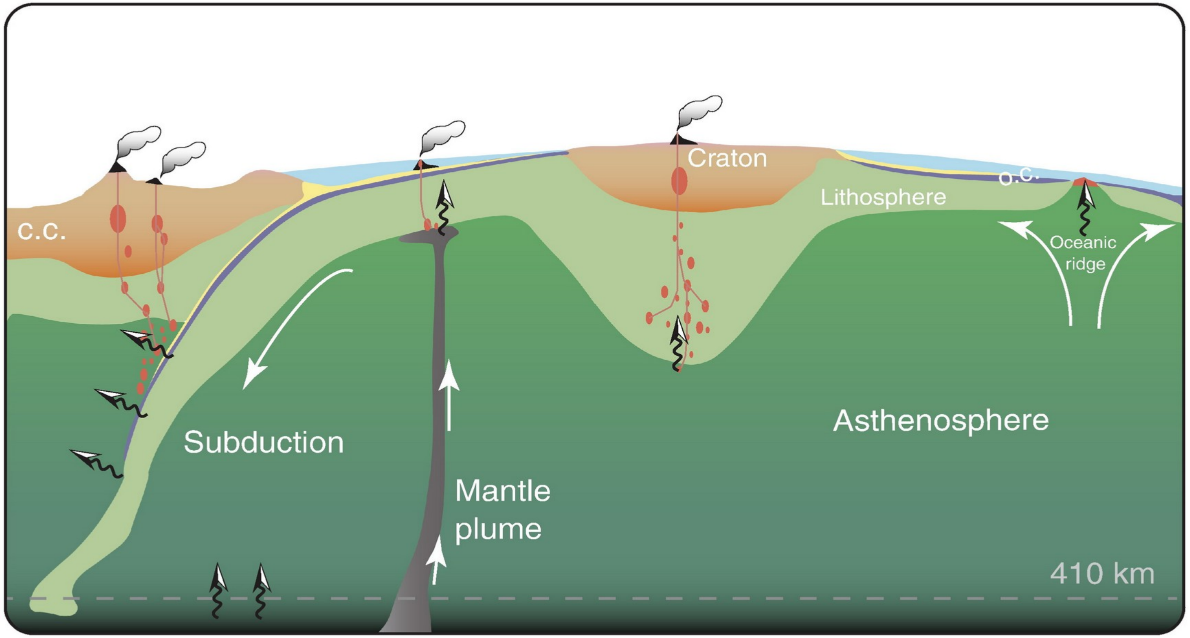
$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\rho_0 \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\nabla P + \rho \boldsymbol{g} + \eta \nabla^2 \boldsymbol{u},$$

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \kappa \nabla^2 T,$$

Rayleigh number

$$\mathrm{Ra} = \frac{\rho g \alpha \Delta T h^3}{\eta \kappa}$$



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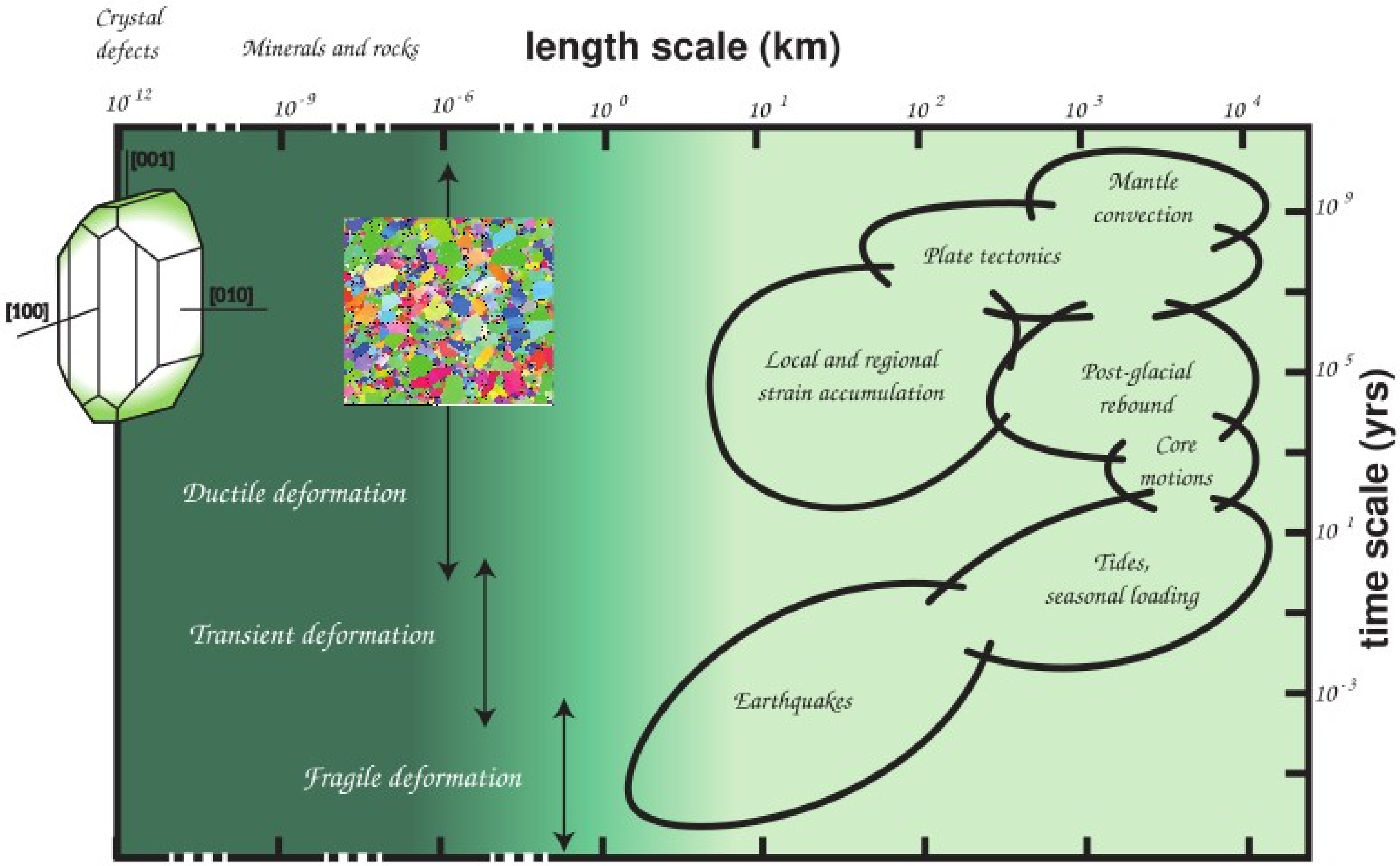
So let's get going…?



Modified from K. Lambeck (1988)

So let's get going…?

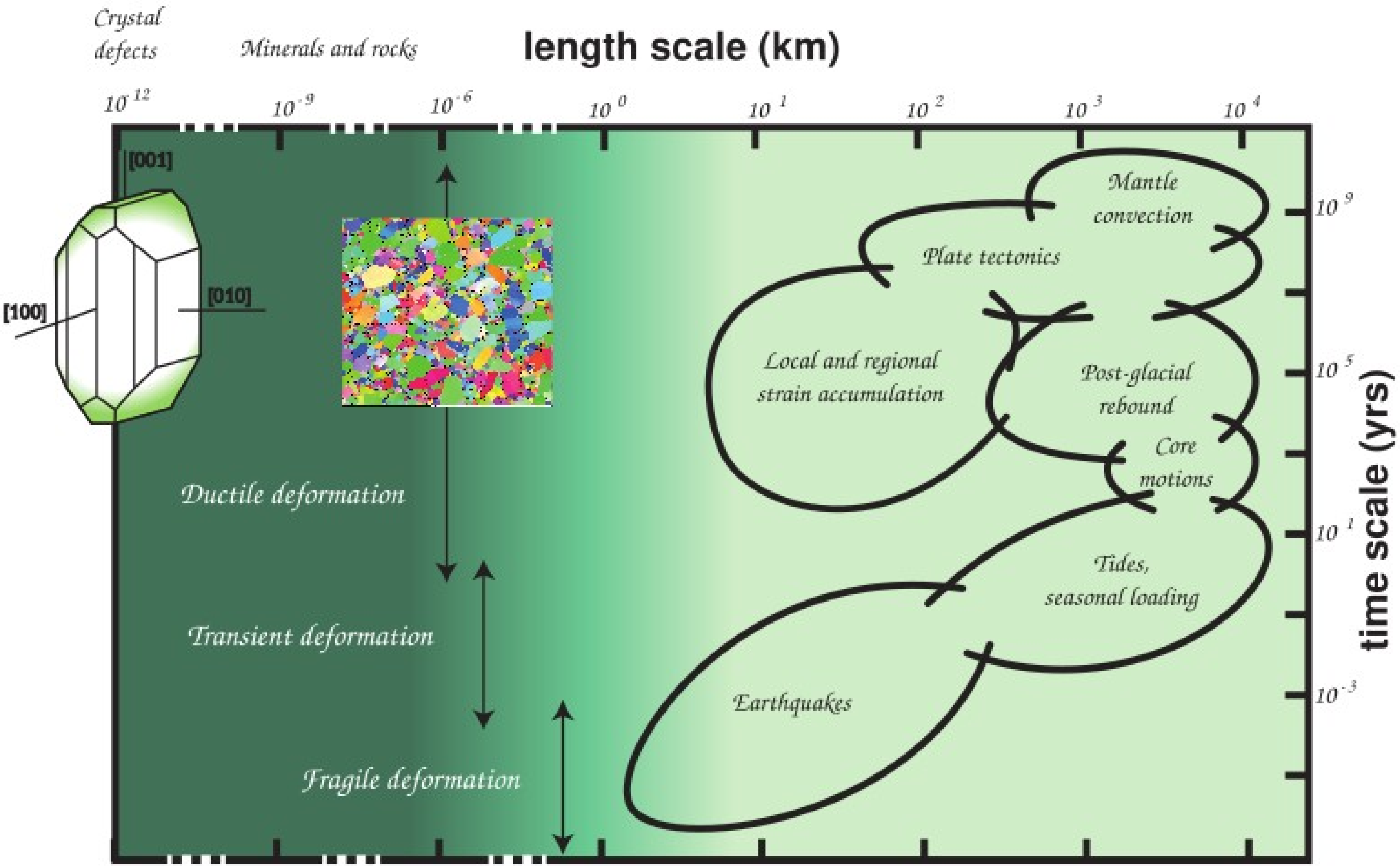
- **identify & parameterize** deformation processes

+ how do they coexist/interact?

- **upscaling to nature**

> from crystal to plate

> from lab rate to convection rate



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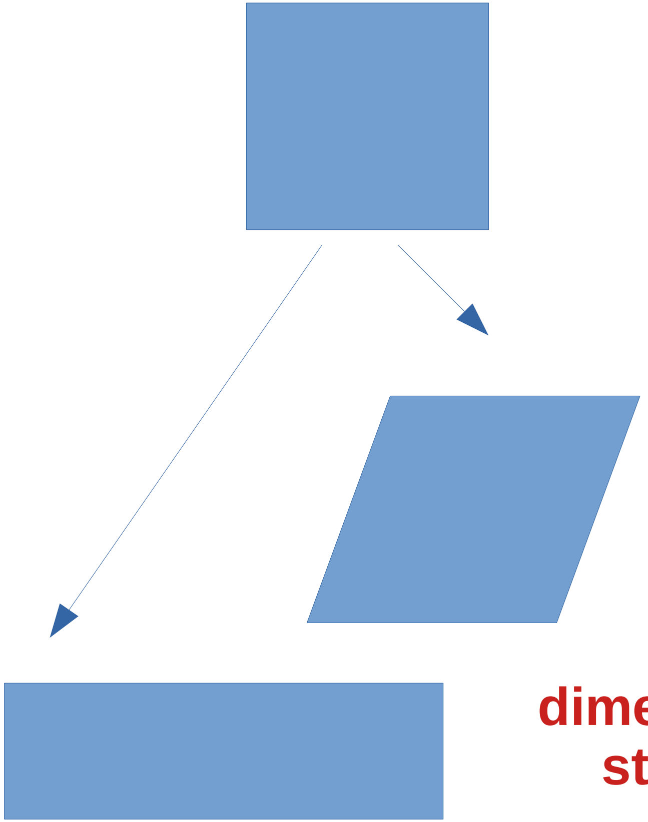
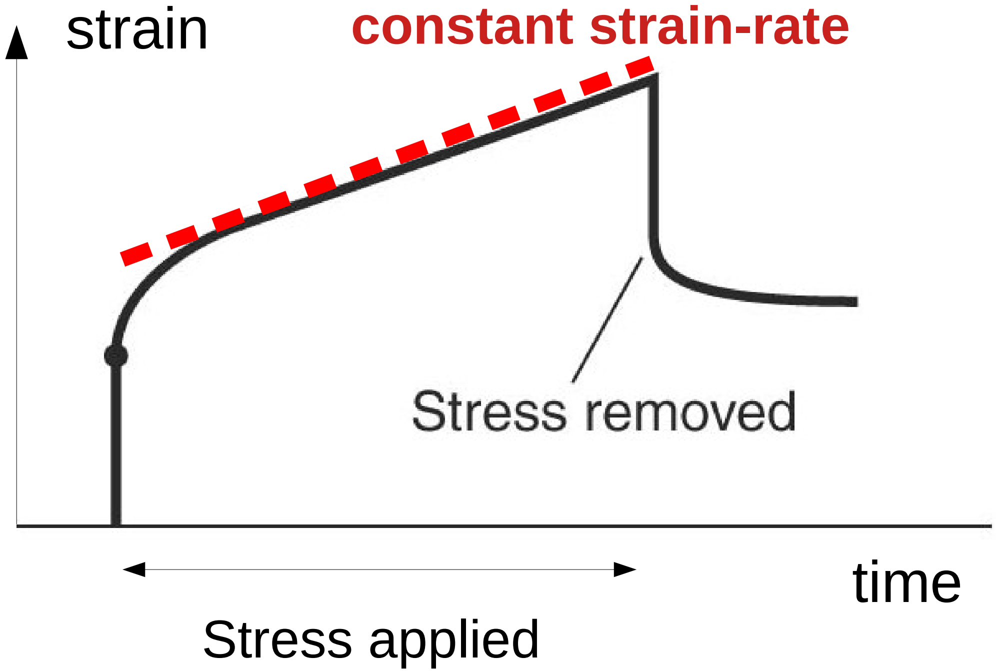
Rheology = deformation (strain, strain rate) of material under a mechanical stress

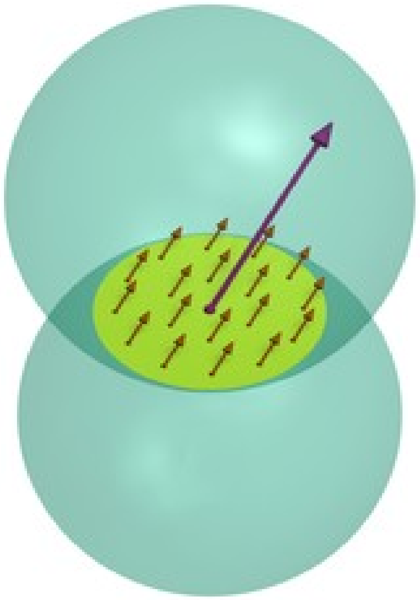


dimensionless strain (%)



Rheology = deformation (strain, strain rate) of material under a mechanical stress

Stress = unit force exerted on a unit surface
(in $N.m^{-2}$ =bar or Pa)

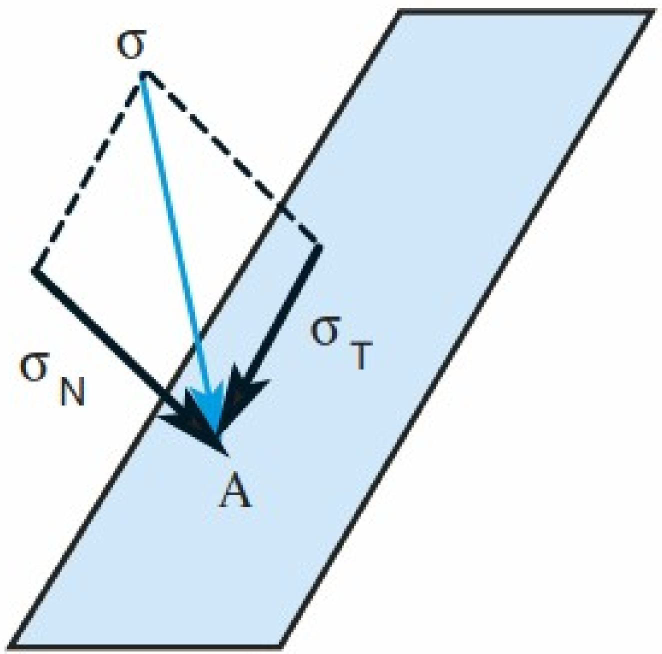


Credit : Jorge Stolfi

Snow racket vs. high-heel shoe :
same force (weight)
but different surfaces

Rheology = deformation (strain, strain rate) of material under a mechanical stress

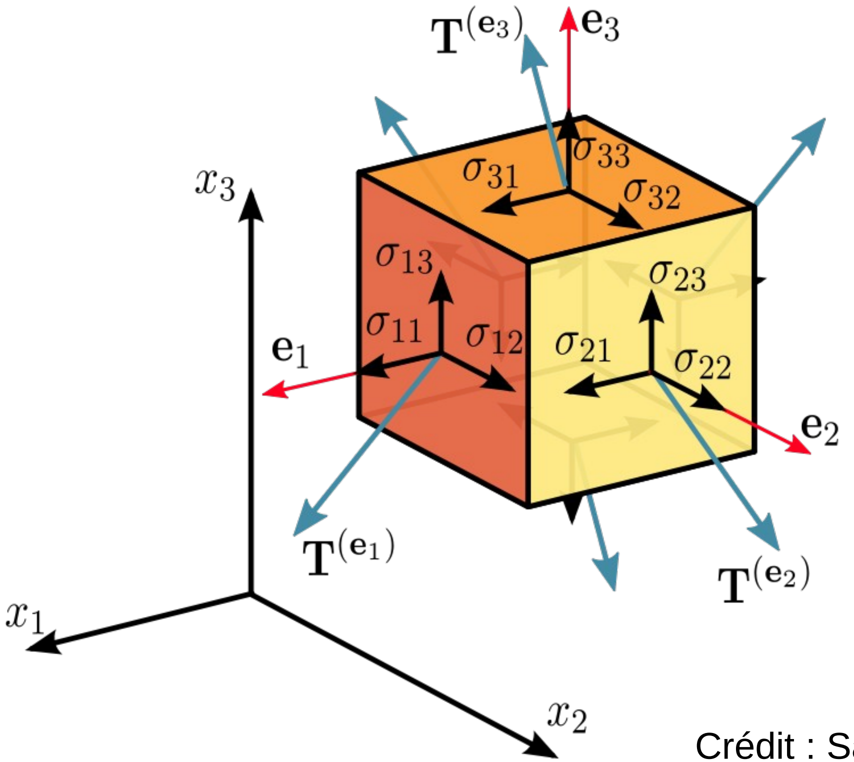
Normal vs. shear (tangential) stress



Géologie BCPST tout-en-un

Stress tensor σ

(6 ind. Components)



Crédit : Sanpaz

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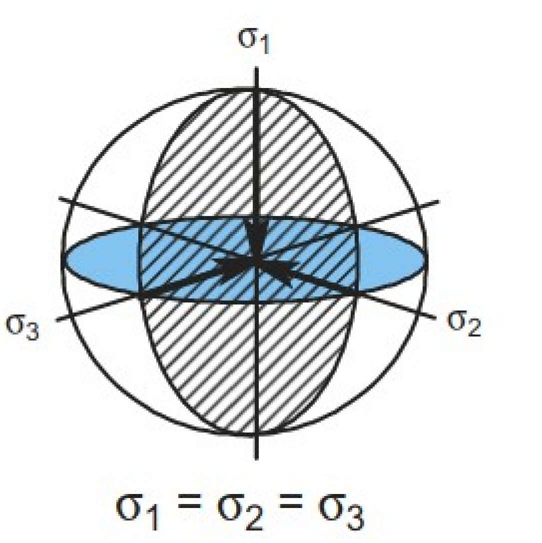
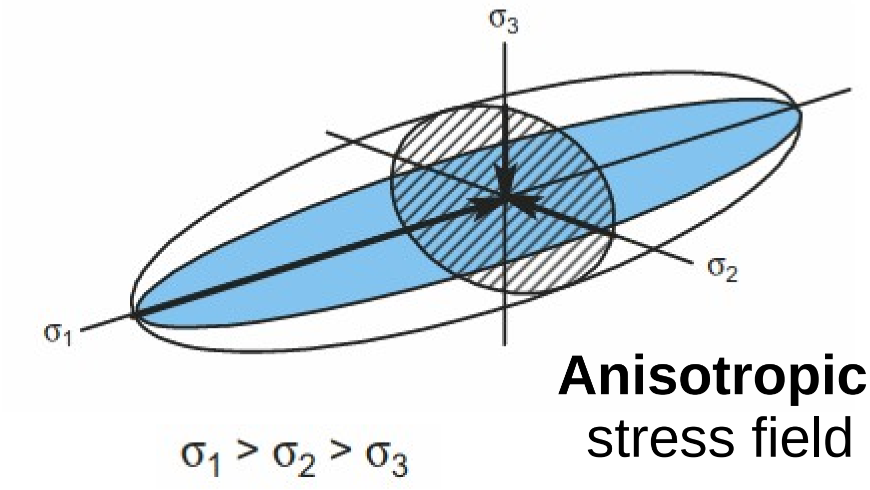


Isotropic stress field
→ no deformation

Lithostatic stress
$\sigma_1 = \sigma_2 = \sigma_3$



Anisotropic stress field

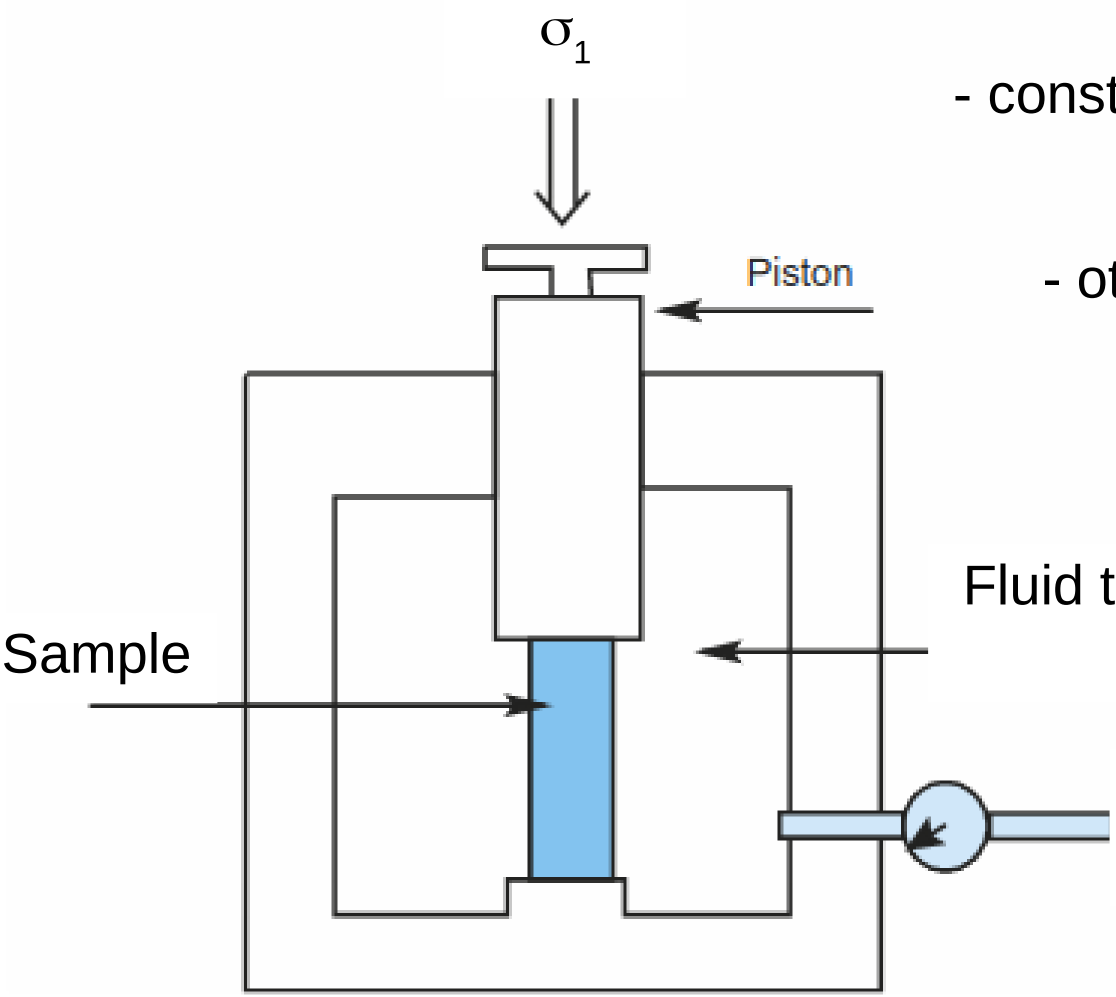
Pressure = mean stress
$P = 1/3\ (\sigma_1+\sigma_2+\sigma_3)$

Deviatoric stress tensor = σ - P

Differential stress : $\sigma_d = \sigma_1 - \sigma_3$

Images : Géologie BCPST tout-en-un

Laboratory experiments of crystal, aggregate or rock deformation



- constant-stress or constant-strain-rate
- compression or tension
- other apparatus for torsion and shearing experiments

Fluid transmitting confinement pressure : $\sigma_2 = \sigma_3$

Compressor tuning the confinment pressure σ_1

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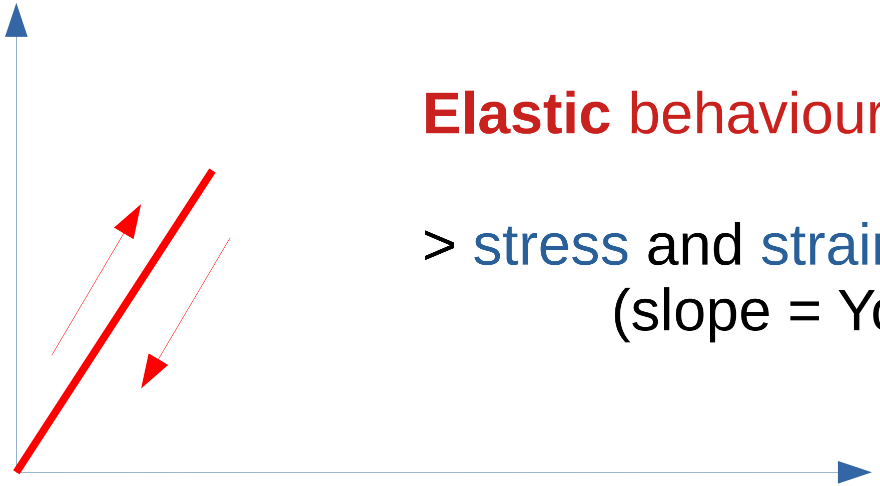
Laboratory experiments of crystal, aggregate or rock deformation

Differential stress $\sigma_d = \sigma_1 - \sigma_3$



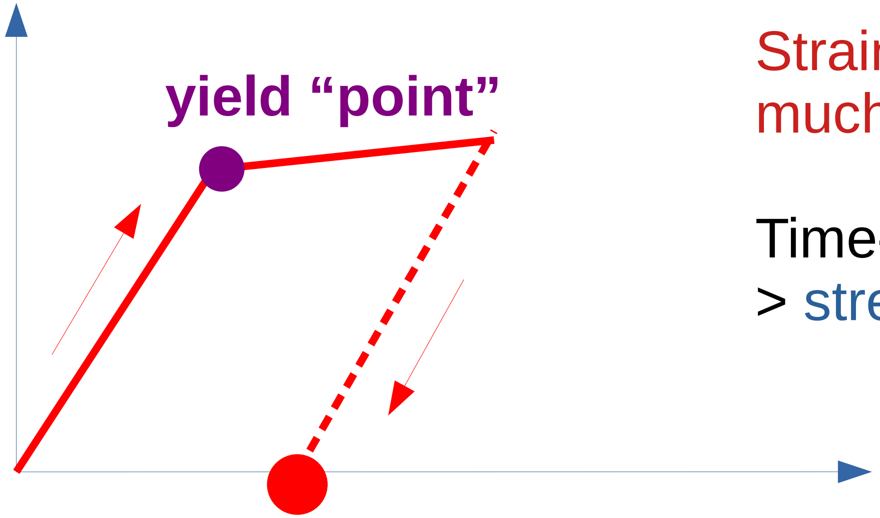
Elastic behaviour

> stress and strain are proportional (slope = Young modulus *E*)

Uniaxial strain γ (shortening in %)

Laboratory experiments
of crystal, aggregate or rock deformation

Differential stress $\sigma_d = \sigma_1 - \sigma_3$



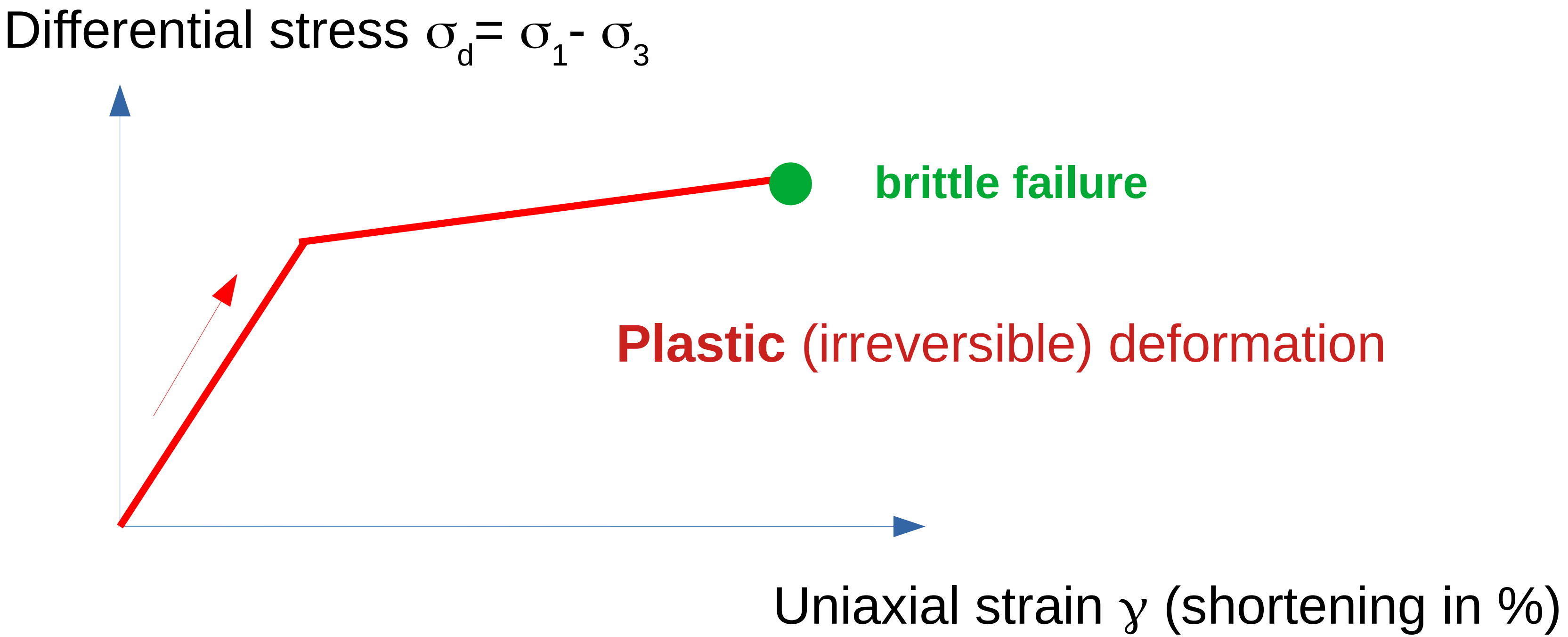
Plastic (irreversible) deformation

Uniaxial strain γ (shortening in %)

Strain increases rapidly without much stress increase (= **creep**)

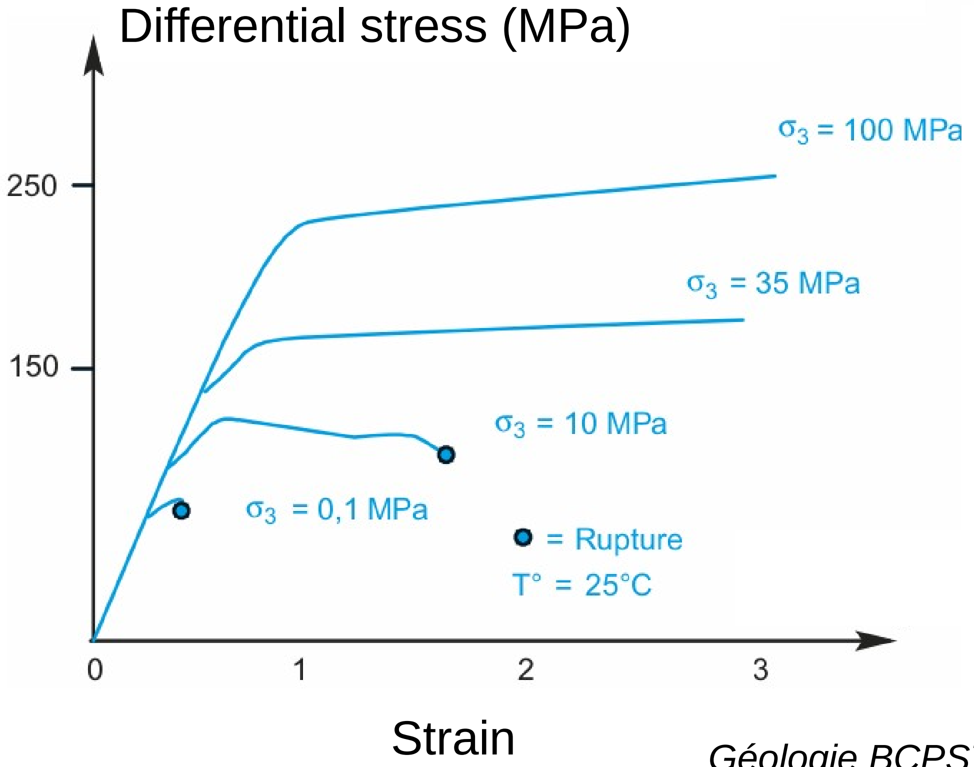
Time-dependent deformation
> stress related to strain rate

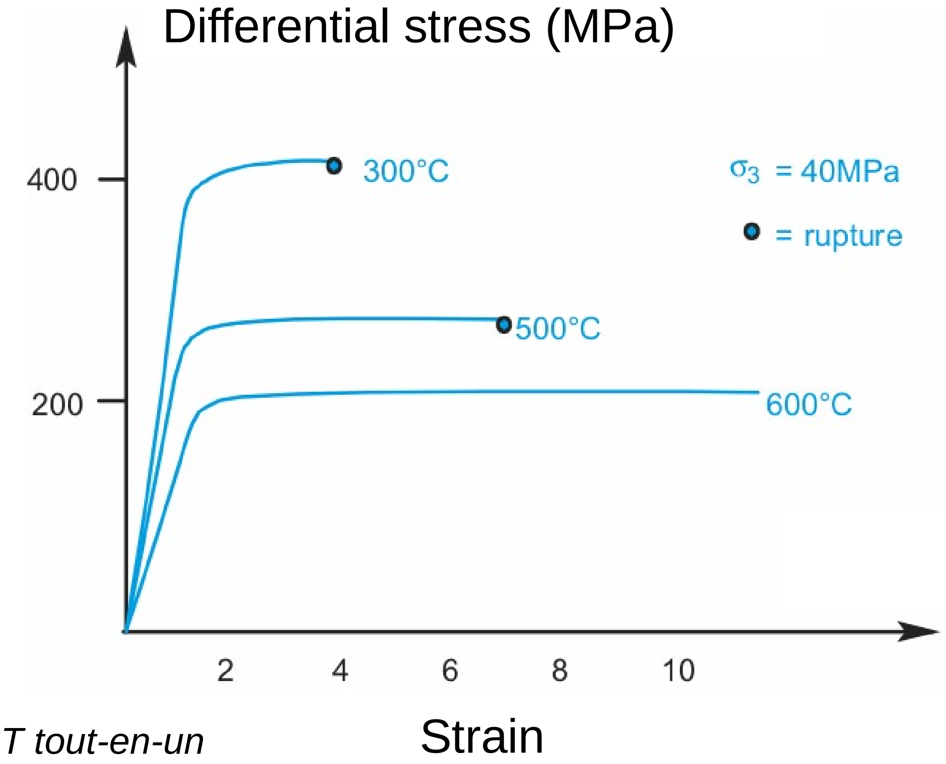
Laboratory experiments of crystal, aggregate or rock deformation



Laboratory experiments
of crystal, aggregate or rock deformation

Influence of confinment pressure + temperature





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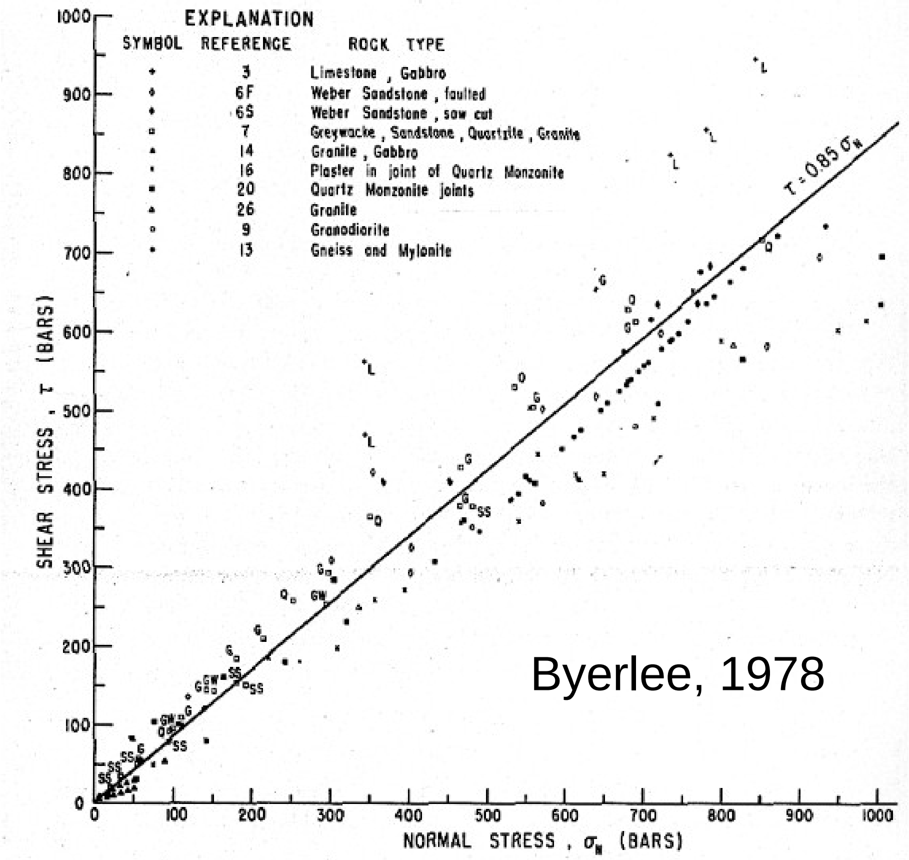
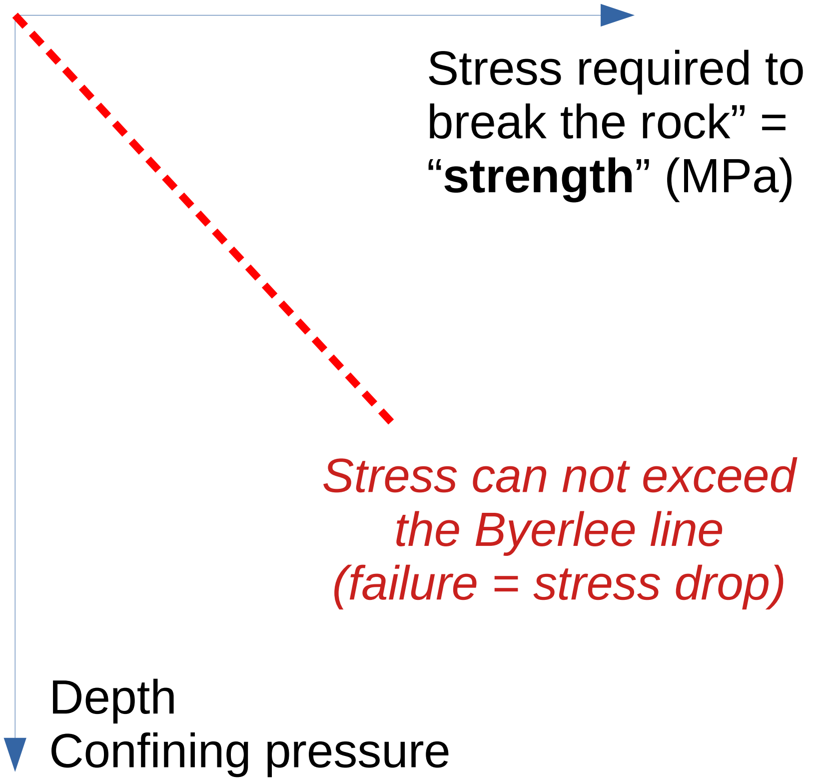
Laboratory experiments of crystal, aggregate or rock deformation

Byerlee’s “law” of friction and rheology profile

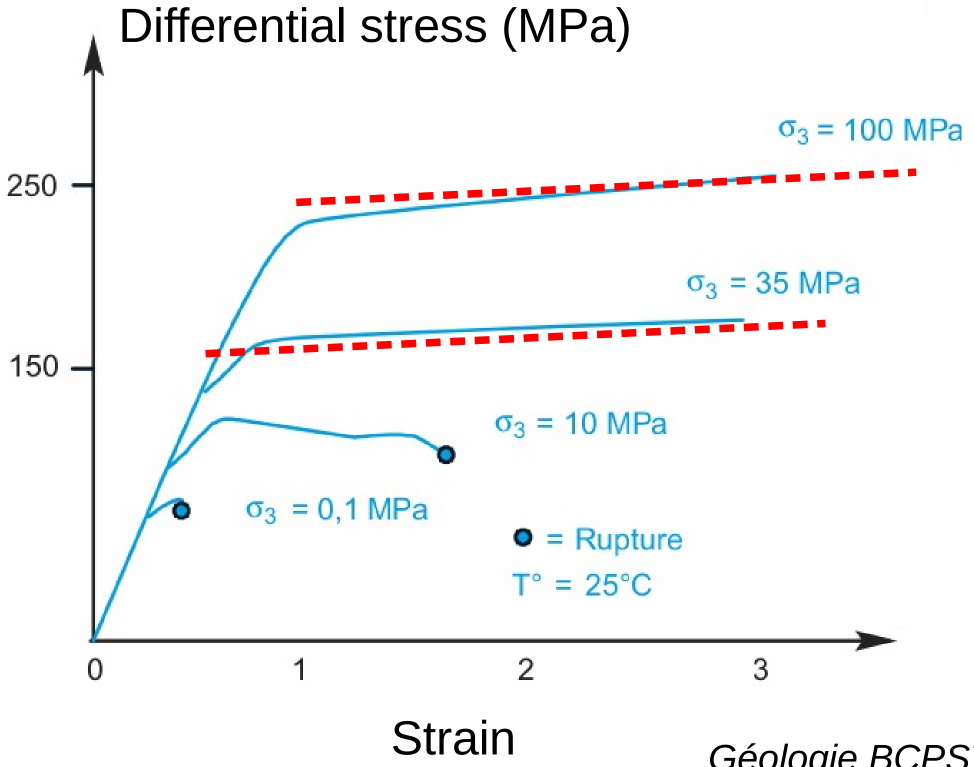


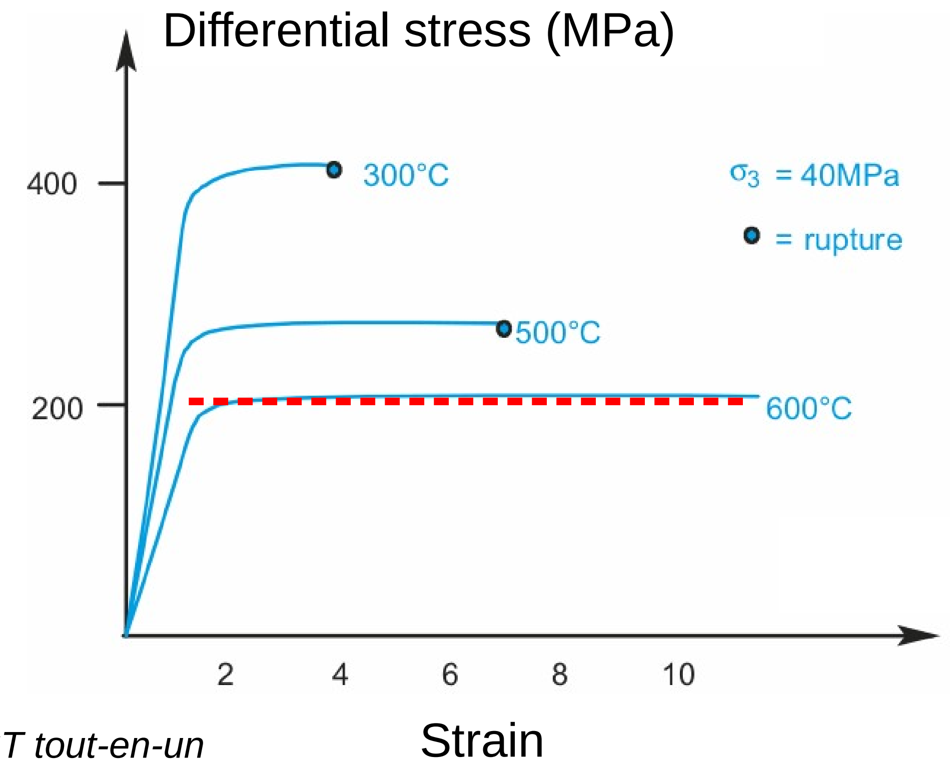
Byerlee, 1978



Laboratory experiments
of crystal, aggregate or rock deformation

- Creep without failure at high P and high T (> mantle)
- infinite strain without additionnal stress > **“yield stress”**





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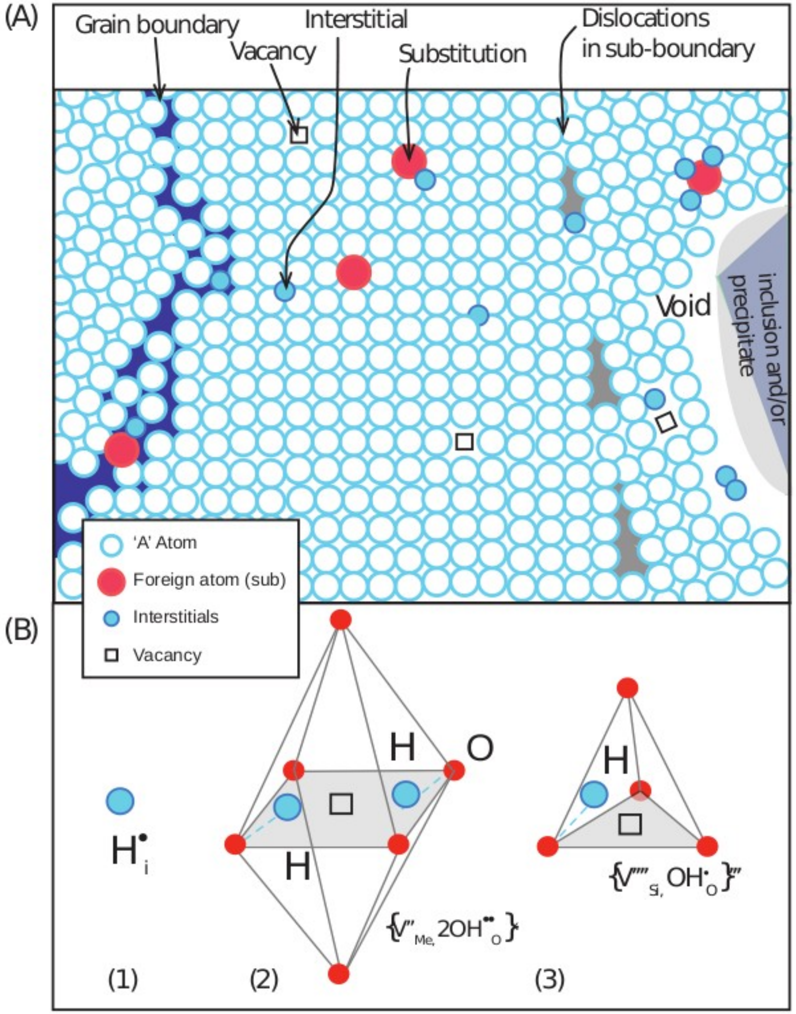
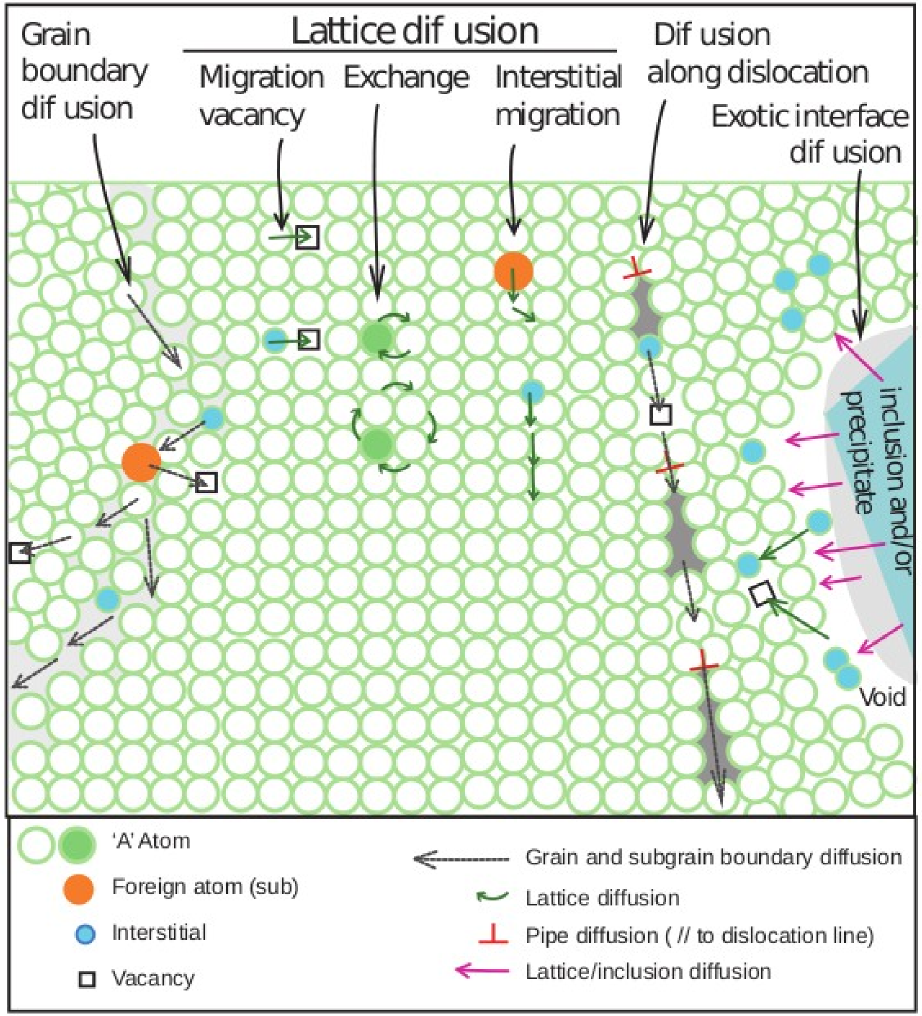
Deformation by defects motion at micro-scale





Courtesy of S. Demouchy

Diffusion creep flow' law

Arrhenius-type law
(thermally-activated)

$$\dot{\varepsilon} = A_{\mathrm{diff}}\,\sigma_d \exp\left(-\frac{E_a + PV_a}{RT}\right)$$

activation energy

lithostatic pressure

activation volume

strain rate

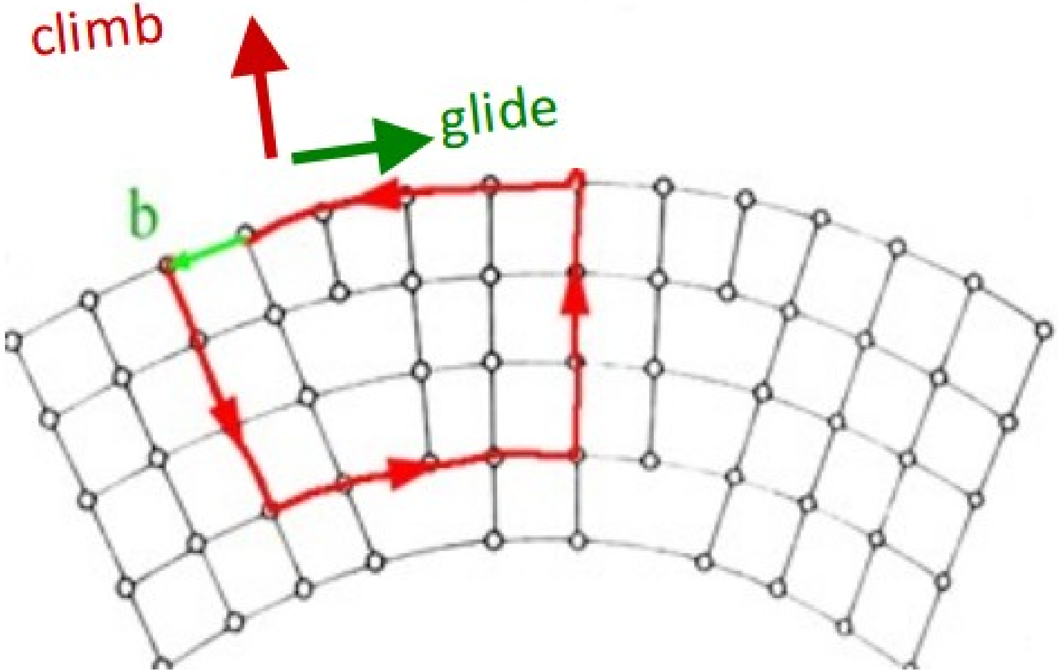
pre-factor

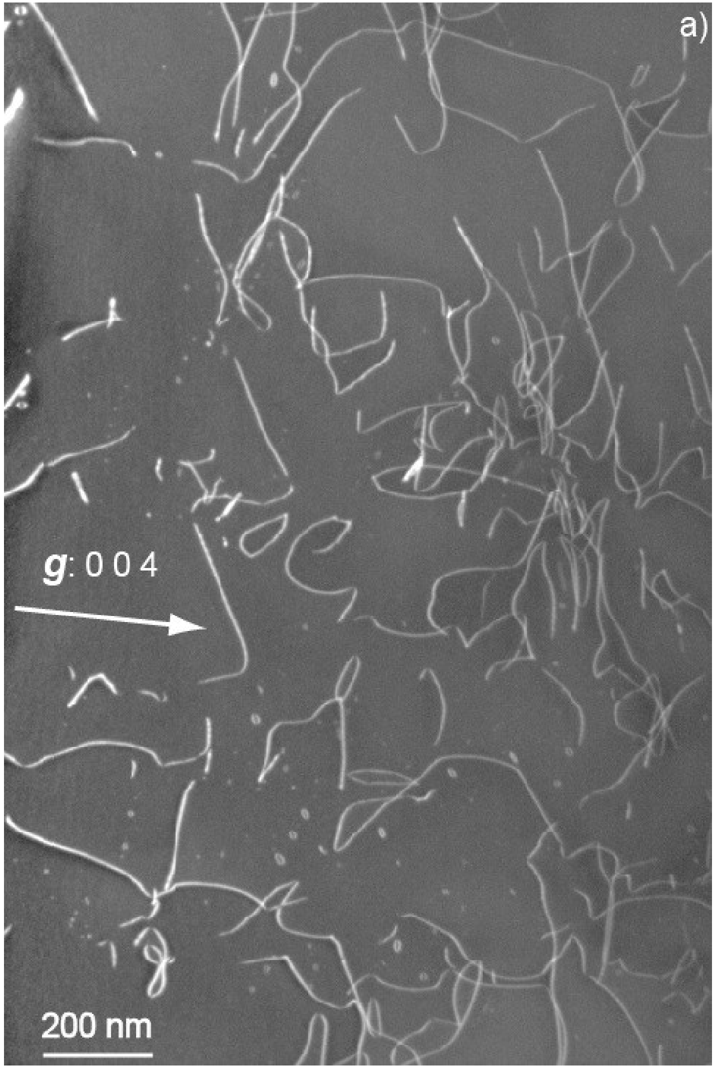
stress

temperature

Dependencies other than P and T : grain size, mineralogy, chemistry/water
(cf Y. Ricard & S. Demouchy's lectures)

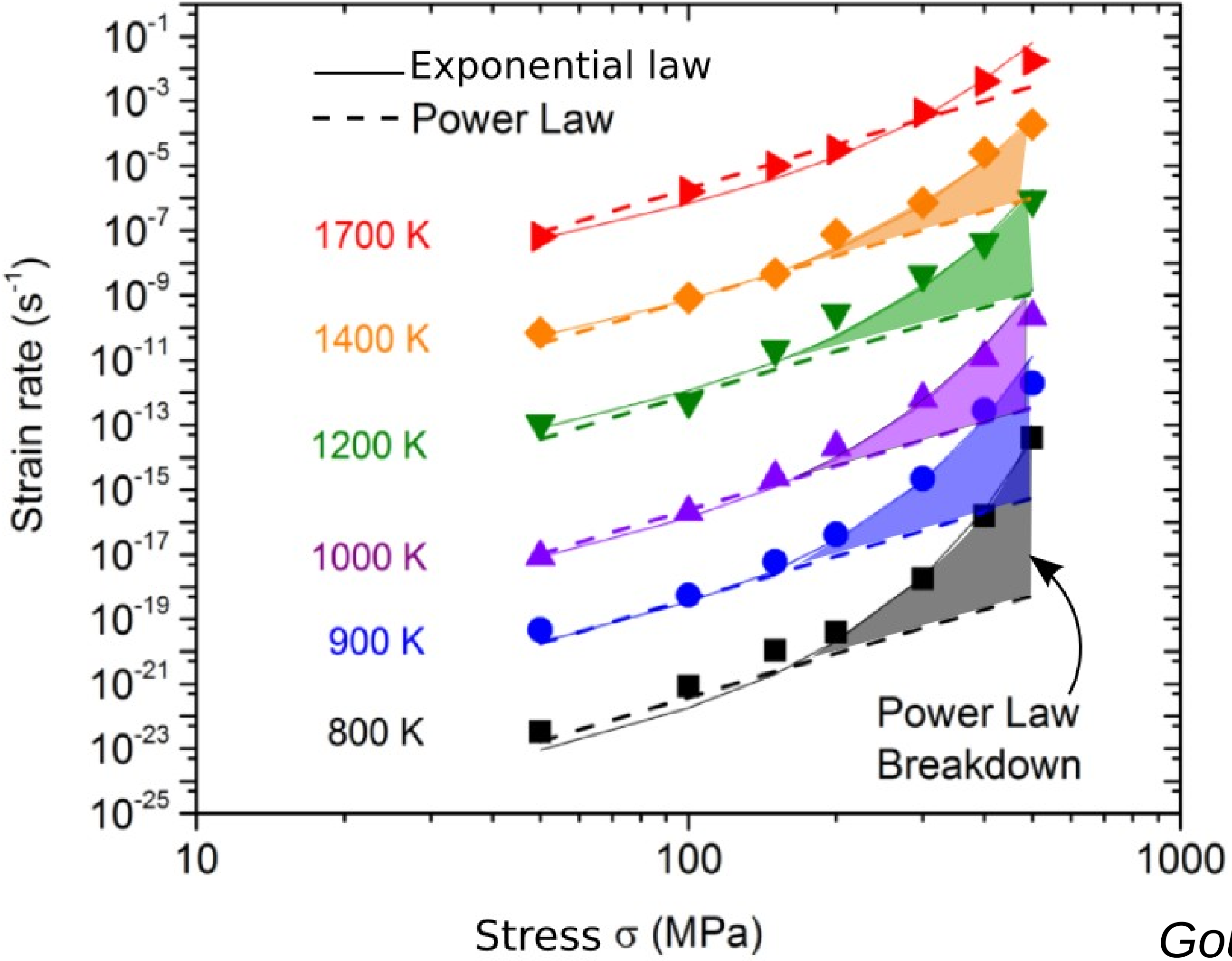
Deformation by defects motion at micro-scale





Dislocations
in olivine from TEM
(*Mussi et al., 2015*)

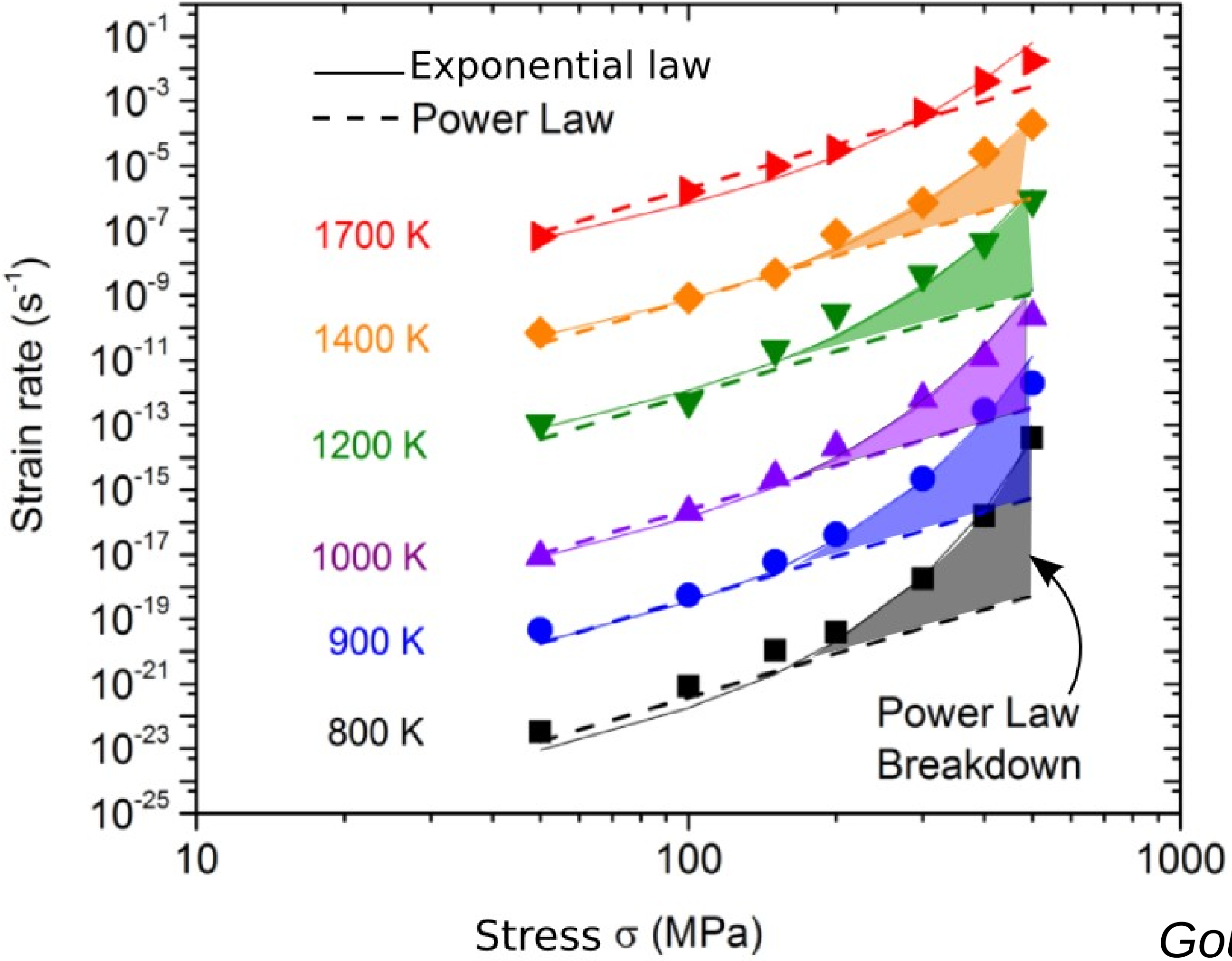
2.5-D dislocation dynamics (DD) *numerical experiments on olivine*



→ continuity of dislocation dynamics (glide + climb) from 800 to 1700 K and from low to high stresses

Gouriet et al., EPSL, 2019

2.5-D dislocation dynamics (DD) *numerical experiments on olivine*



- **Power-law** between strain rate and stress valid at low stress and high T

- **power-law breakdown** at low T, high stress

Gouriet et al., EPSL, 2019

2.5-D dislocation dynamics (DD) *numerical experiments on olivine*

$$\dot{\varepsilon} = A_{\mathrm{disl}}\sigma_d^{\mathbf{n}} \exp\left(-\frac{E_a + PV_a}{RT}\right)$$

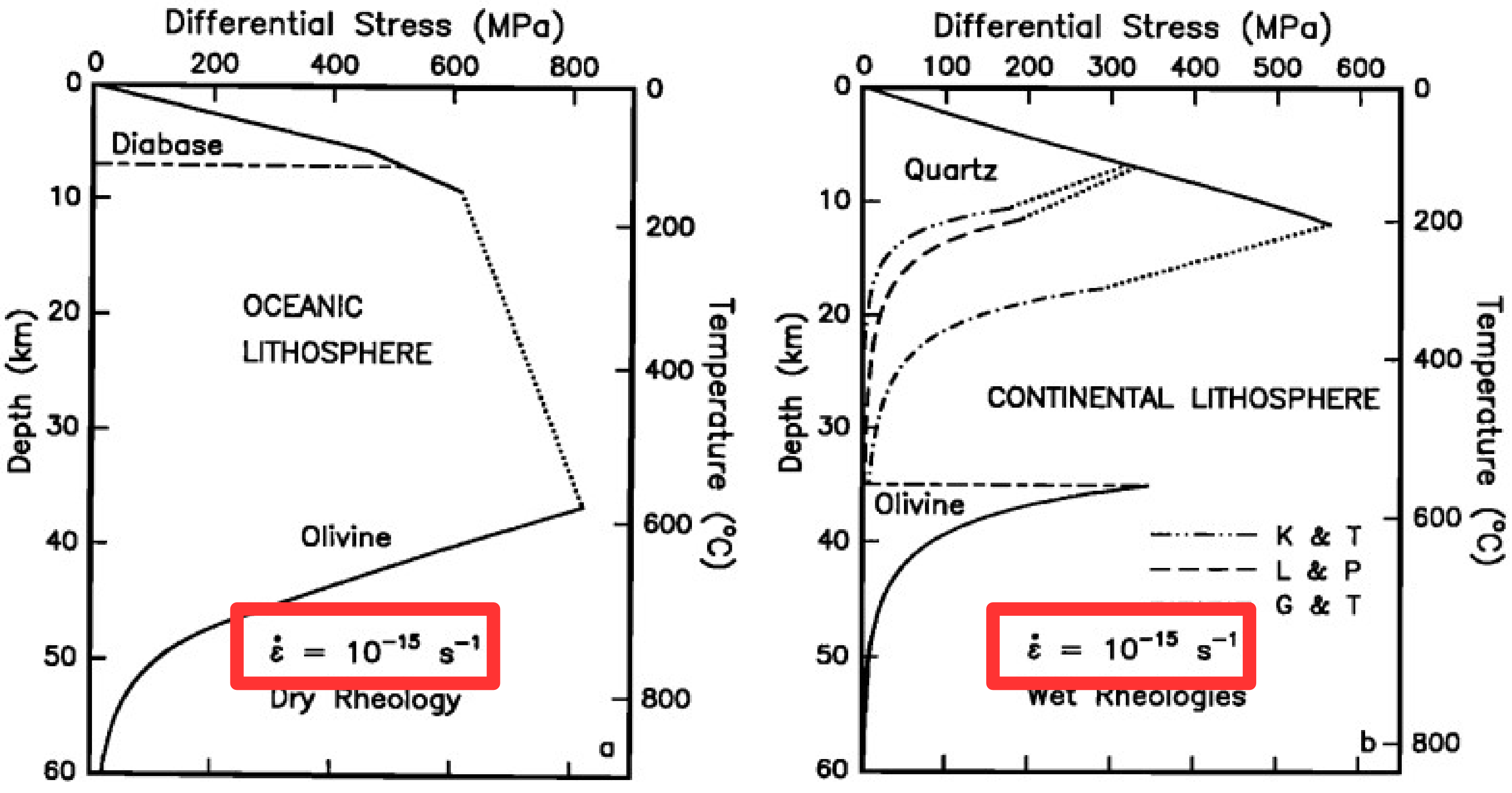
- **Power-law** between strain rate and stress valid at low stress and high T

$$\dot{\varepsilon} = A\sigma^{\mathbf{n}} \exp\left[-\frac{E_a}{RT}\left(1-\left(\frac{\sigma}{\sigma_P}\right)^p\right)^q\right]$$

- **power-law breakdown** at low T, high stress

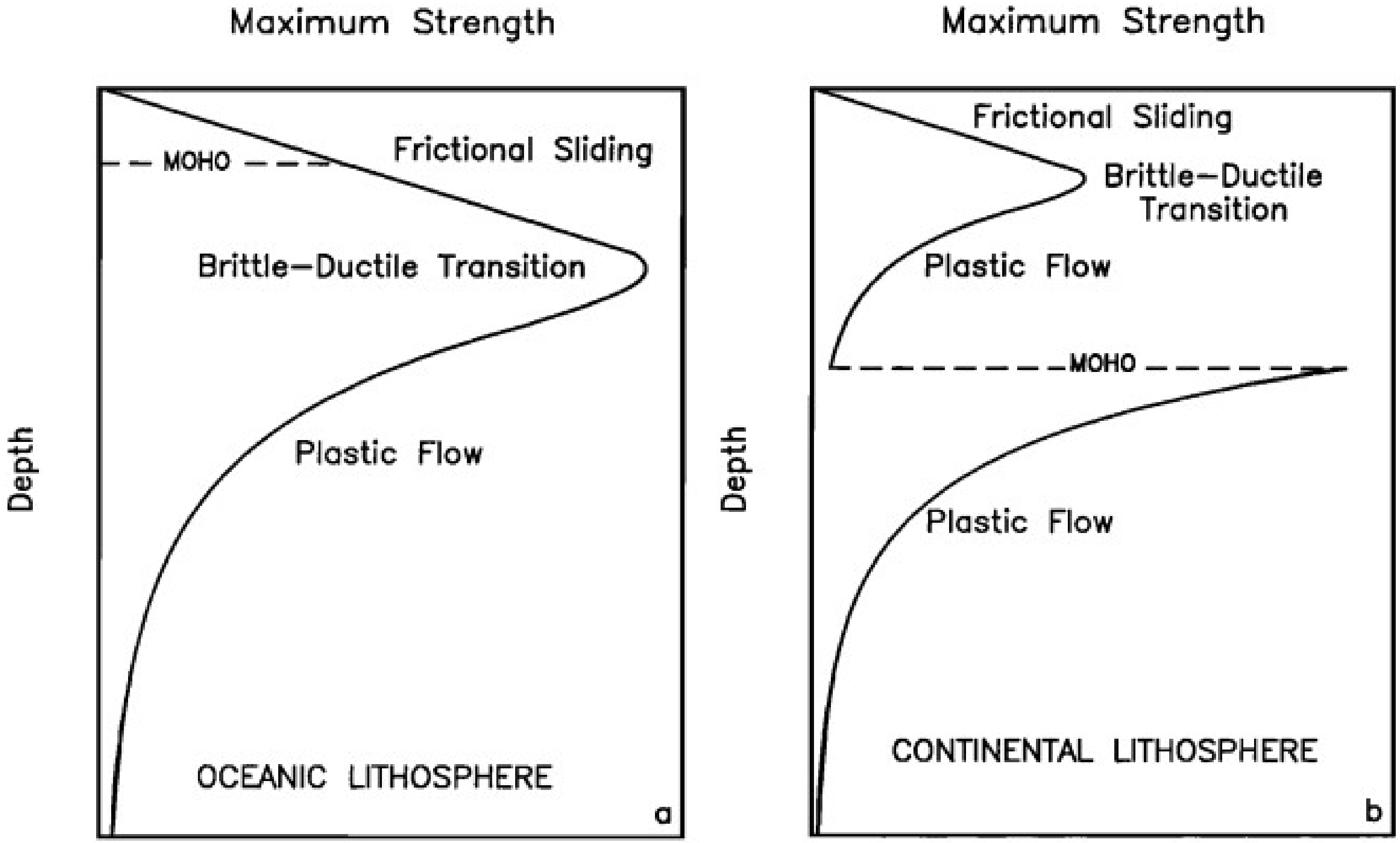
Frost and Ashby, 1982

Rheology “Christmas tree” profiles



Kohlstedt et al., 1995

Rheology “Christmas tree” profiles



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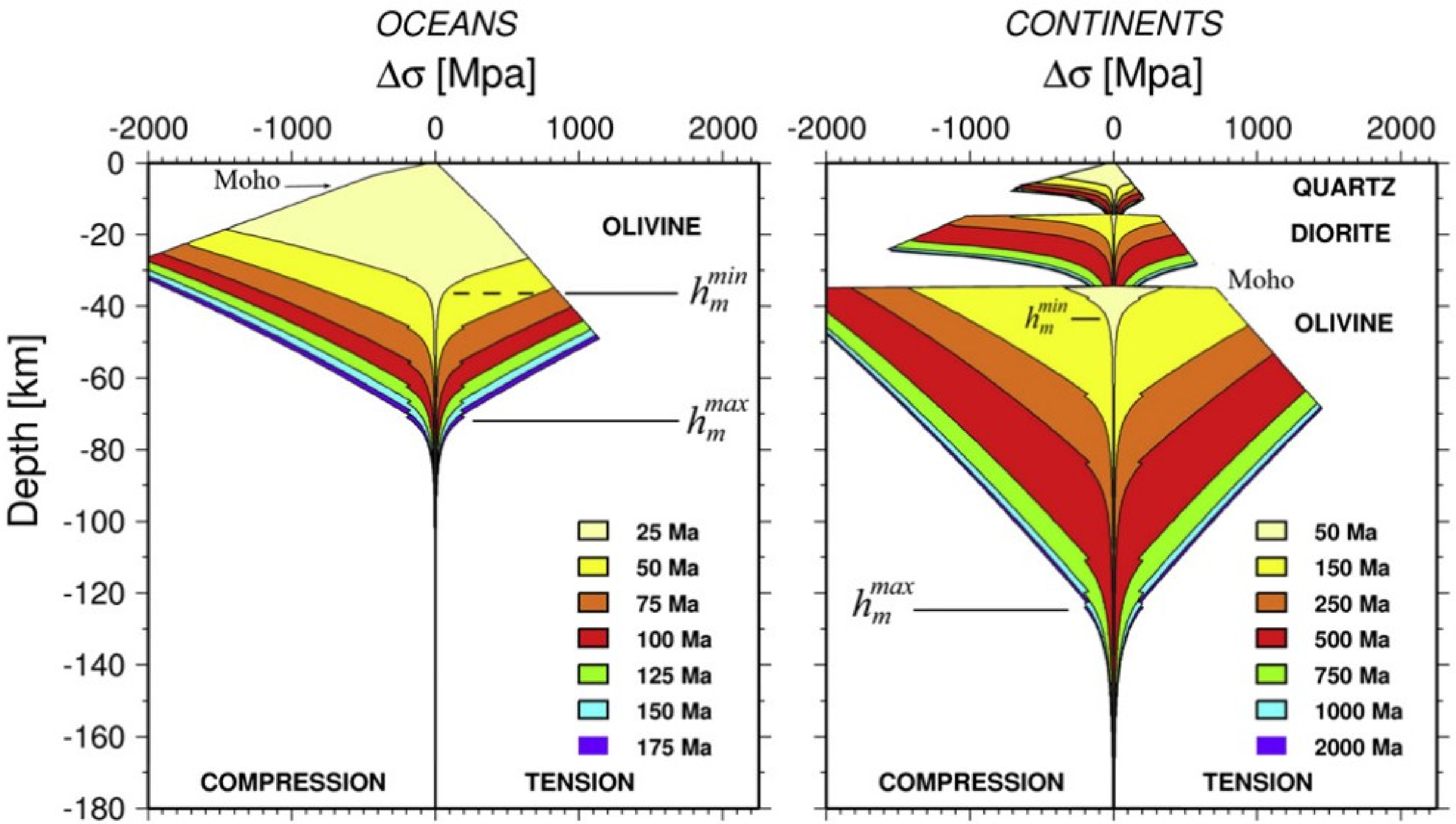
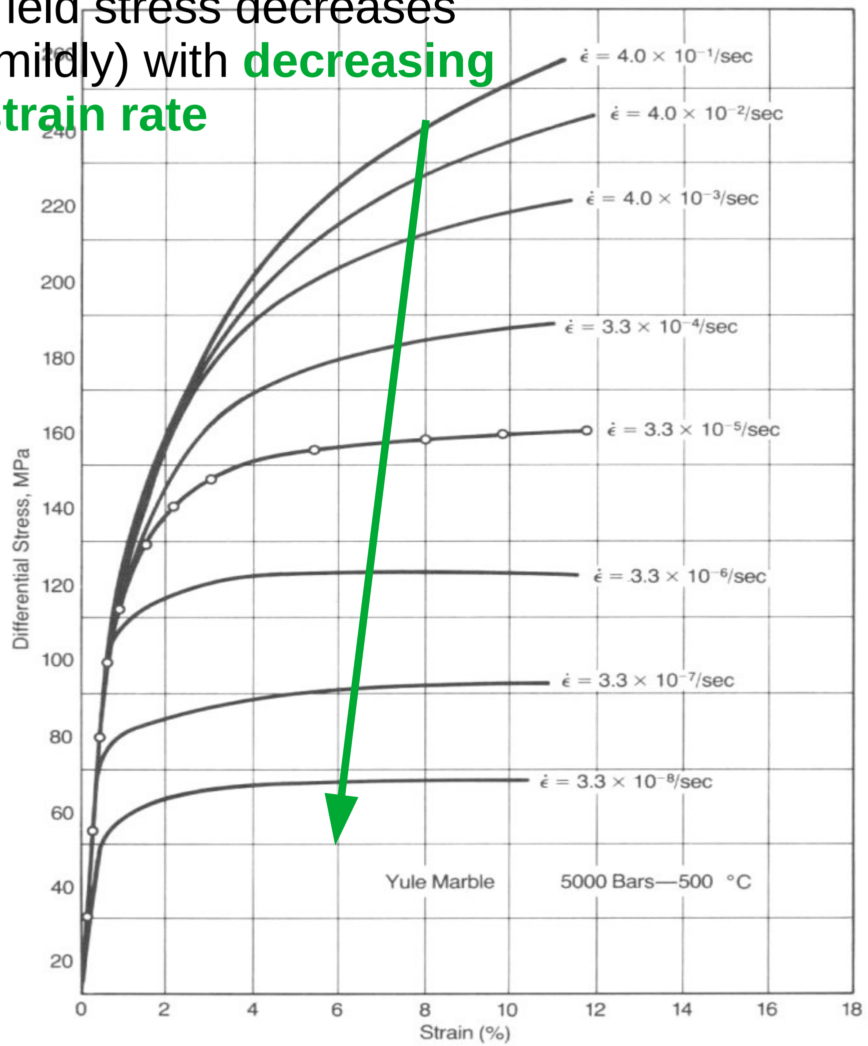
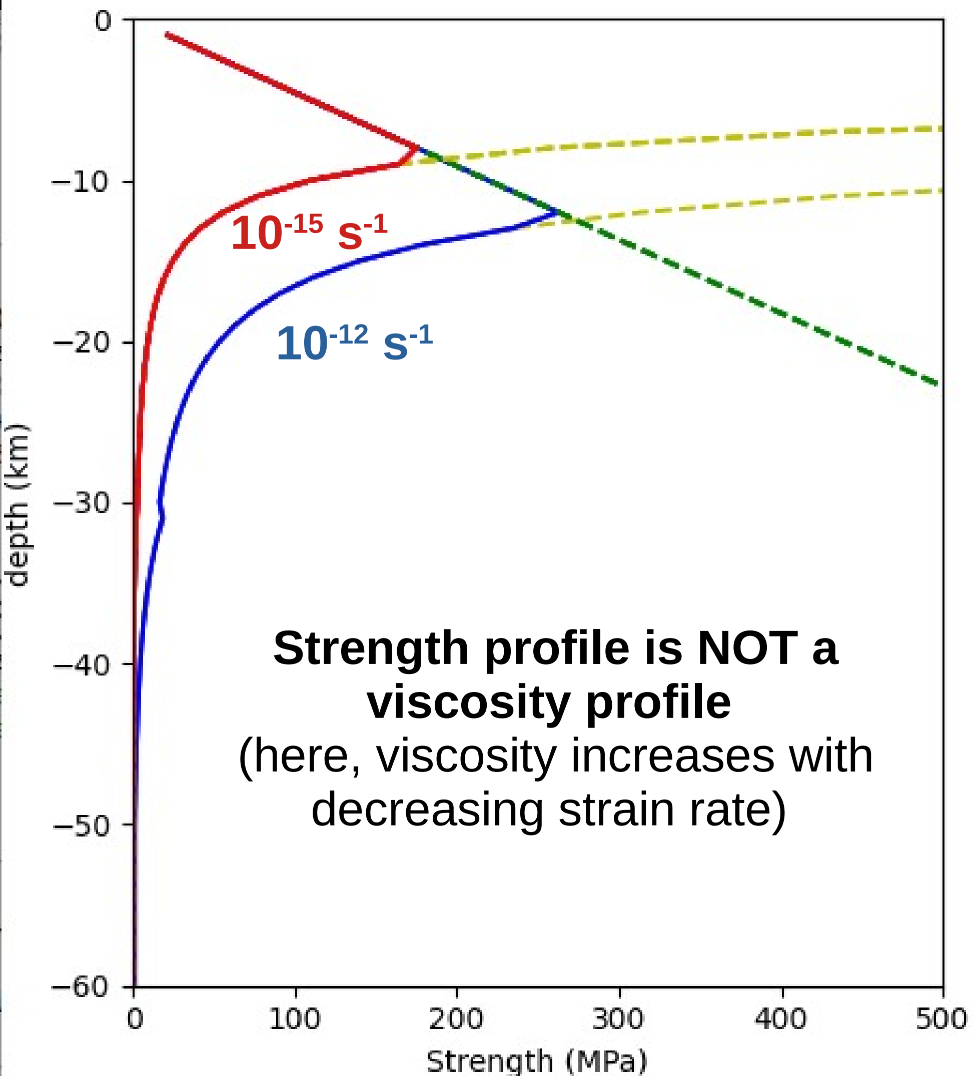


Burov, 2011

Rheology “Christmas tree” profiles

Yield stress decreases (mildly) with **decreasing strain rate**





Strength profile is NOT a viscosity profile
(here, viscosity increases with decreasing strain rate)

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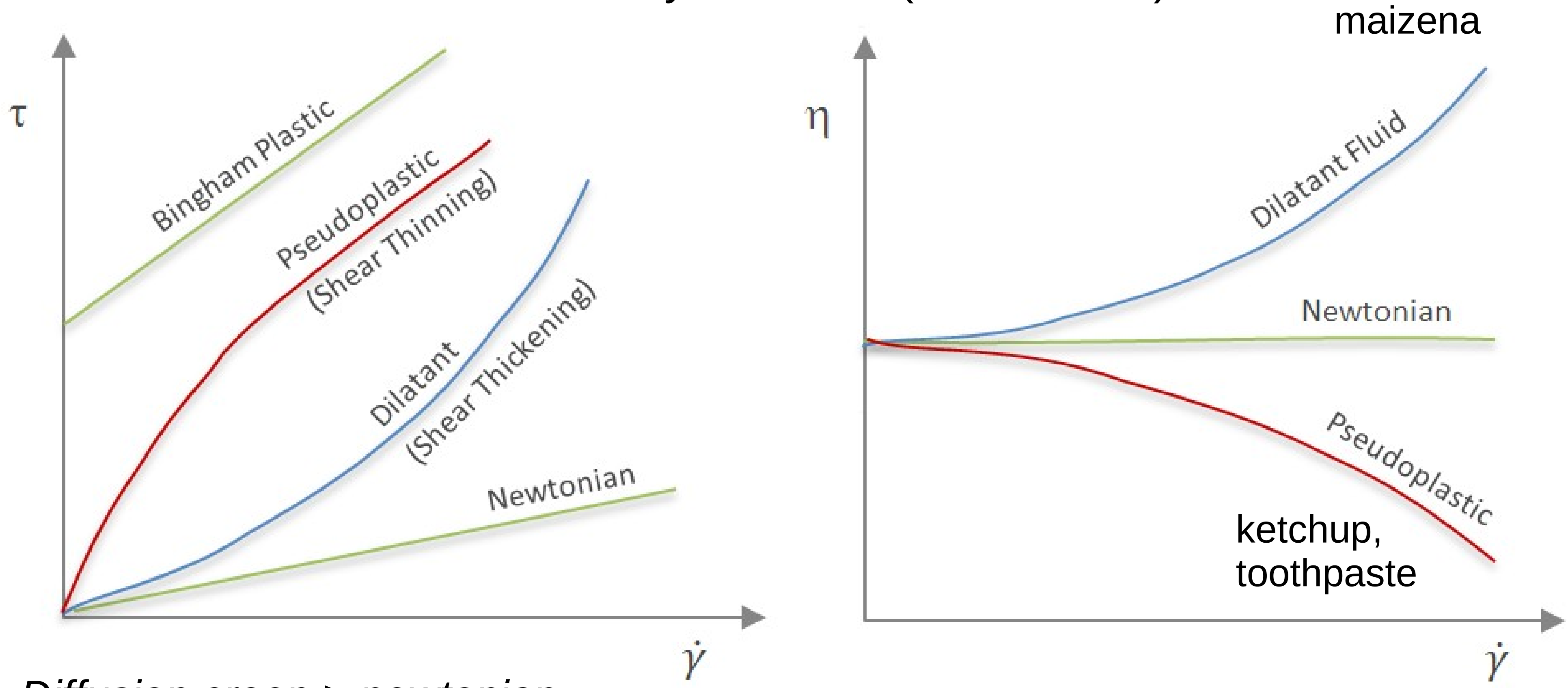
From flow “laws” to viscosity

At first order : viscosity = stress / (2 strain rate)



Diffusion creep > newtonian
Dislocation creep, yield-stress rheology > pseudoplastic

polymerdatabase.com

From flow "laws" to viscosity

At second order… uni-axial experiments are not nature !

In experiments :

- Uniaxial differential stress > $\sigma_d = \sigma_1 - \sigma_3$
- Measured deformation = uniaxial strain in the direction of compression/tension
- Very high strain rates ~ 10^{-6} s^{-1} compared to expected mantle ones ~ 10^{-15} s^{-1}

In the Earth

- 3-D → deviatoric stress = stress – pressure

Pressure = (negative) mean stress ≠ minimal stress σ_3

- 3-D strain rate tensor

→ corrections to incorporate experimental flow laws into dynamical models

cf Schmalholz et al., 2019 ; Gerya, 2010

From flow “laws” to viscosity

Several coexisting deformation mechanisms → <u>minimal viscosity</u> ?

From flow “laws” to viscosity

Several coexisting deformation mechanisms → <u>addition of strain rates</u> ?

$$
\begin{aligned}
\dot{\varepsilon}_{\mathrm{tot}} &= \dot{\varepsilon}_{\mathrm{el}} + \dot{\varepsilon}_{\mathrm{plastic}} \\
\dot{\varepsilon}_{\mathrm{plastic}} &= \dot{\varepsilon}_{\mathrm{brittle}} + \dot{\varepsilon}_{\mathrm{ductile}} \\
\dot{\varepsilon}_{\mathrm{ductile}} &= \dot{\varepsilon}_{\mathrm{disl}} + \dot{\varepsilon}_{\mathrm{diff}}
\end{aligned}
$$

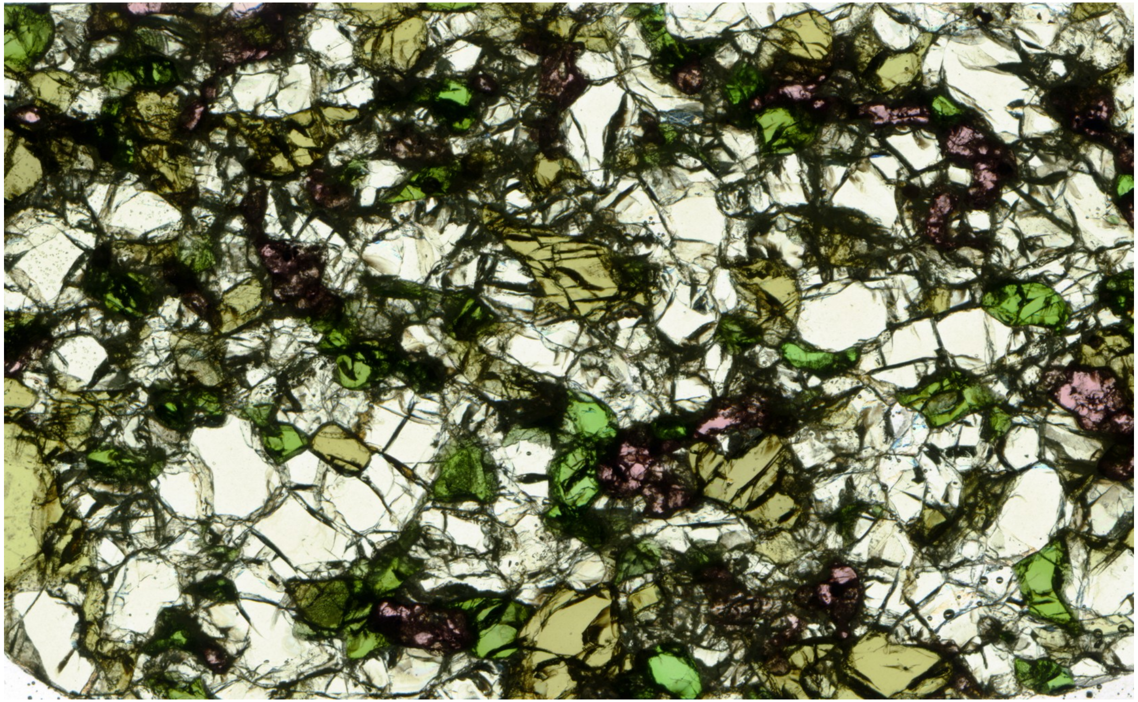
From flow “laws” to viscosity

Several coexisting deformation mechanisms → <u>addition of strain rates</u> ?

$$
\begin{aligned}
\mu_{\mathrm{eff}} &= \frac{\tau_{II}}{2\dot{\varepsilon}_{II}} \\
\frac{1}{\mu_{\mathrm{eff}}} &= \frac{2\dot{\varepsilon}_{II}}{\tau_{II}} \\
&= \frac{2\dot{\varepsilon}_{\mathrm{disl}}}{\tau_{II}} + \frac{2\dot{\varepsilon}_{\mathrm{diff}}}{\tau_{II}} \\
&= \frac{1}{\mu_{\mathrm{disl}}} + \frac{1}{\mu_{\mathrm{diff}}} \\
\mu_{\mathrm{eff}} &= \left(\frac{1}{\mu_{\mathrm{disl}}} + \frac{1}{\mu_{\mathrm{diff}}}\right)^{-1}
\end{aligned}
$$

pseudo-harmonic mean for viscosity

From flow "laws" to viscosity



Peridotite thin section - courtesy of S. Demouchy

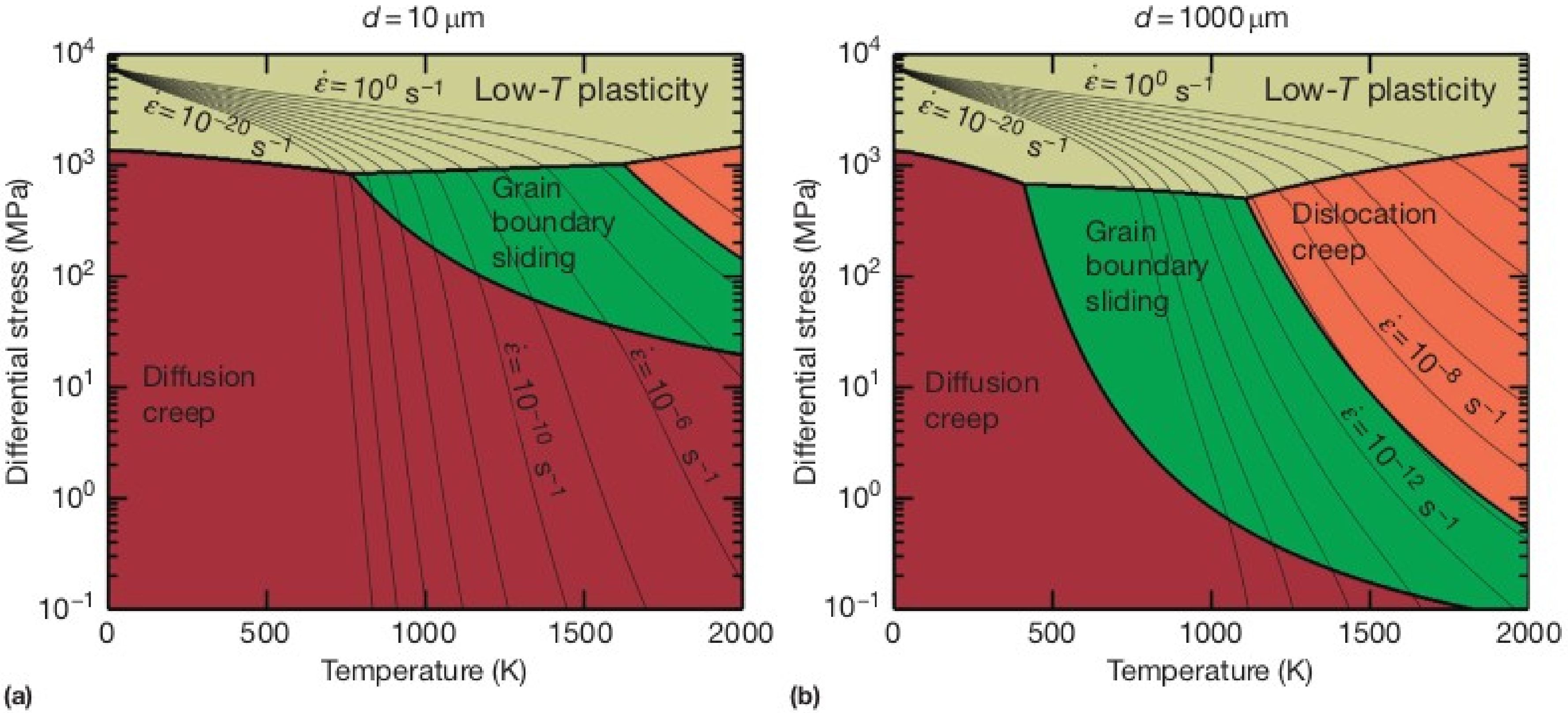
+ rock-scale viscosity of a mineralogic assemblage ? (weak vs. strong phase, grain-boundary sliding, etc.)

+ influence of shape of crystals ?

3-D viscosity tensor
cf A. Tommasi's lecture on anisotropic viscosity

+ melt content

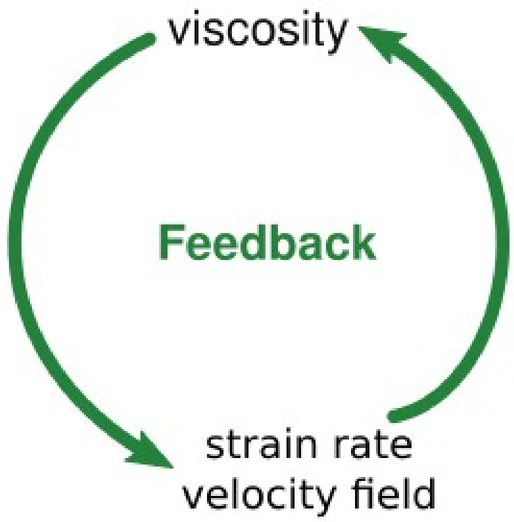
Deformation partitioning (minimal viscosity)



Kohlstedt & Hansen, Treatise on Geophysics, 2015

… but probably <u>coexisting mechanisms</u> rather than bimodal partitioning

Feedbacks between deformation and dynamics



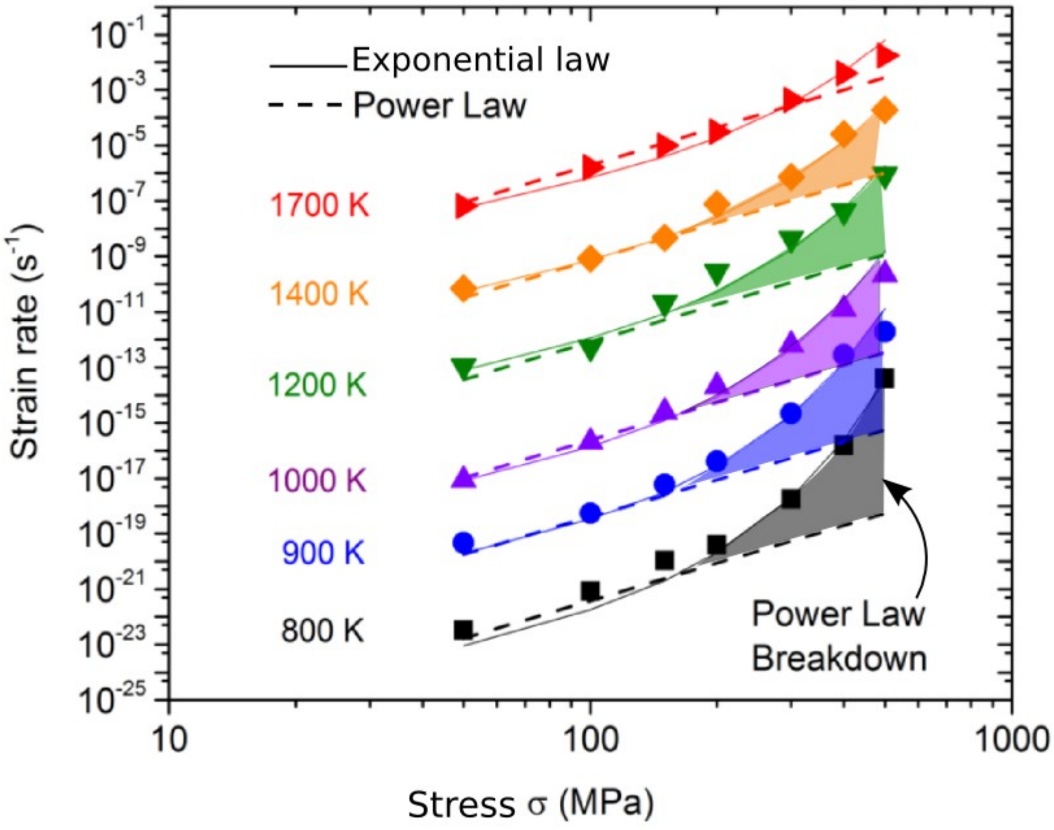
- Dislocation creep

- pseudo-brittle / **yield stress** rheology

with strain-rate dependent viscosity

Reconciling experimental micro-scale rheology and large-scale mantle viscosity

(through numerical modelling)



→ semi-empirical exponential fit

$$\dot{\varepsilon} = A\sigma^{\mathbf{n}} \exp\left[-\frac{E_a}{RT}\left(1-\left(\frac{\sigma}{\sigma_P}\right)^p\right)^q\right]$$

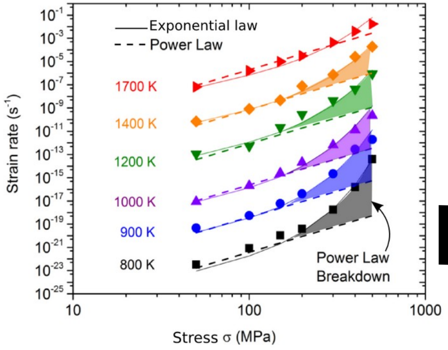
Frost and Ashby, 1982

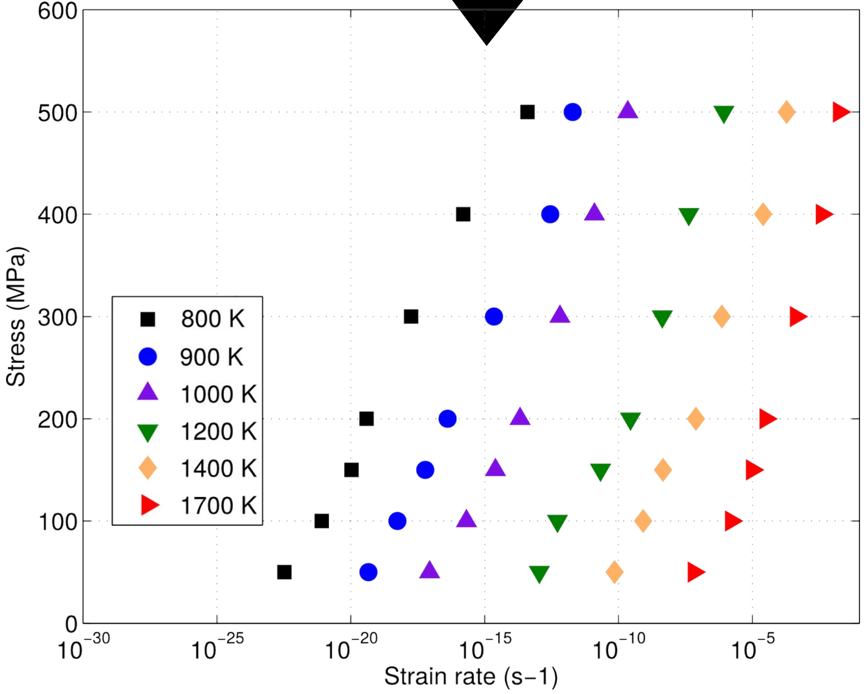
not modeler-friendly...

→ need to express stress as a function of strain rate for **viscosity calculation**

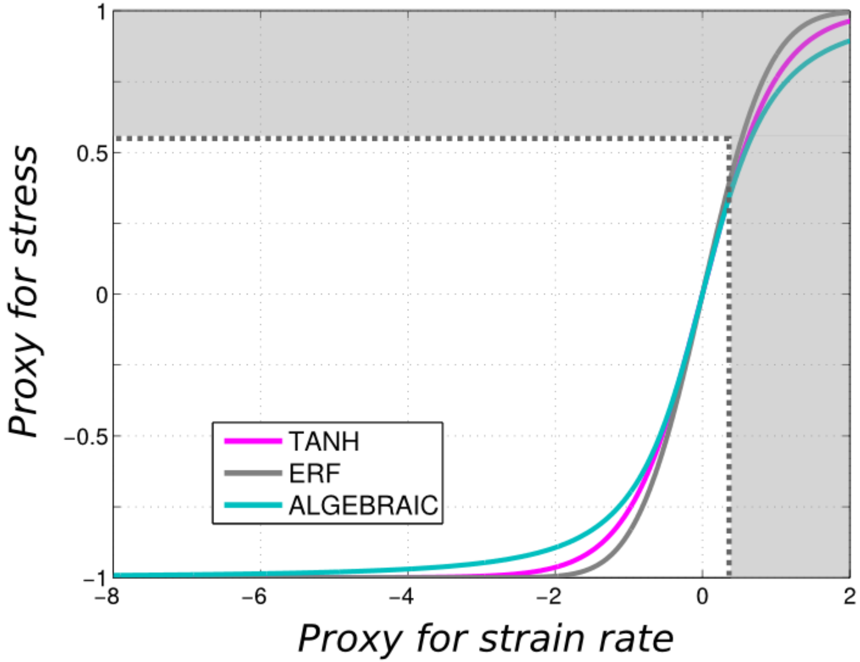
Gouriet et al., EPSL, 2019

Reconciling experimental micro-scale rheology and large-scale mantle viscosity
(through numerical modelling)

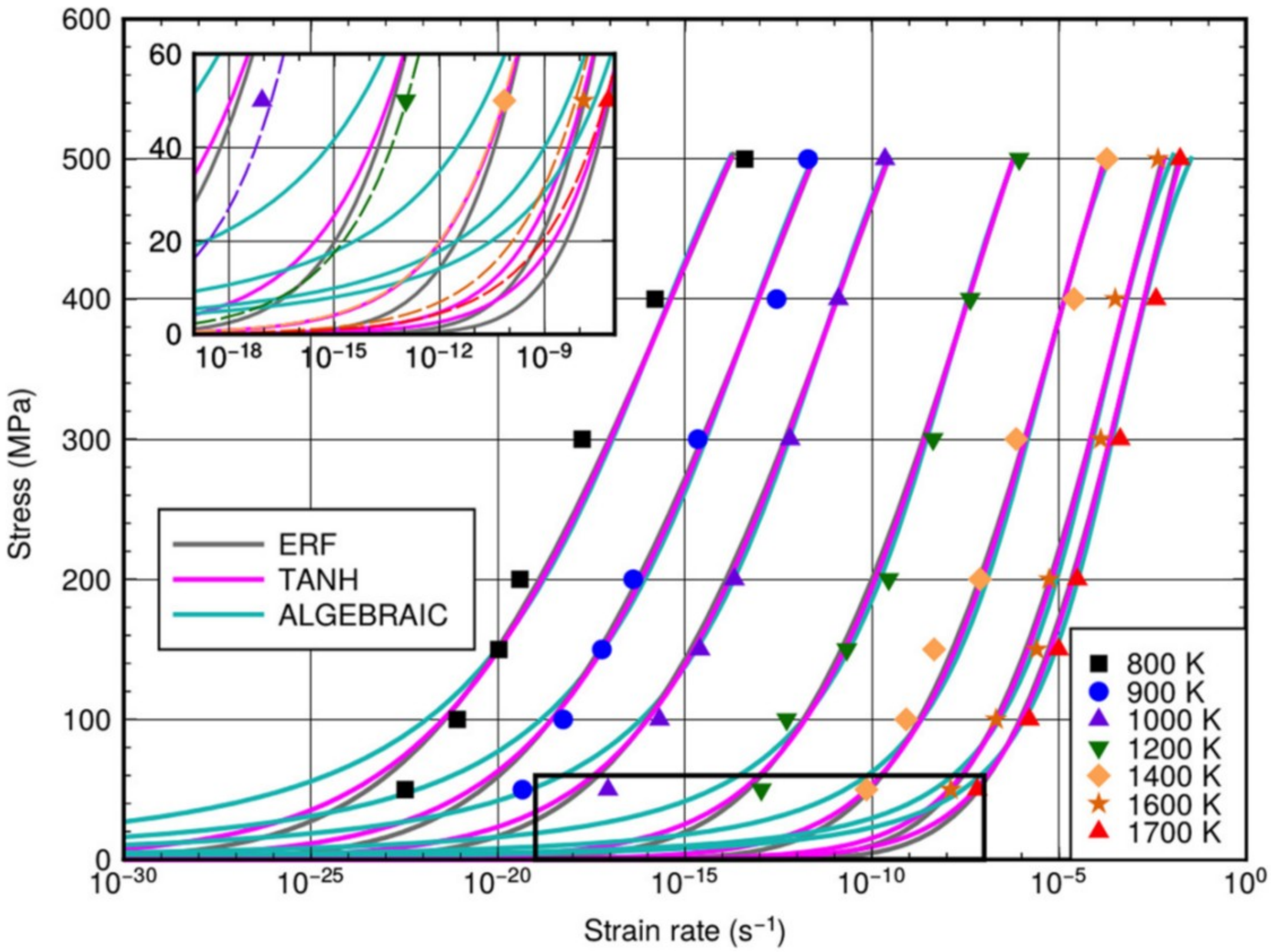




3 ≠ sigmoid functions to fit both high-T and low-T data



3 parameterizations for dislocation creep rheology...
fitting equally well the data !



Garel et al., EPSL, 2020

3 parameterizations for dislocation creep rheology...
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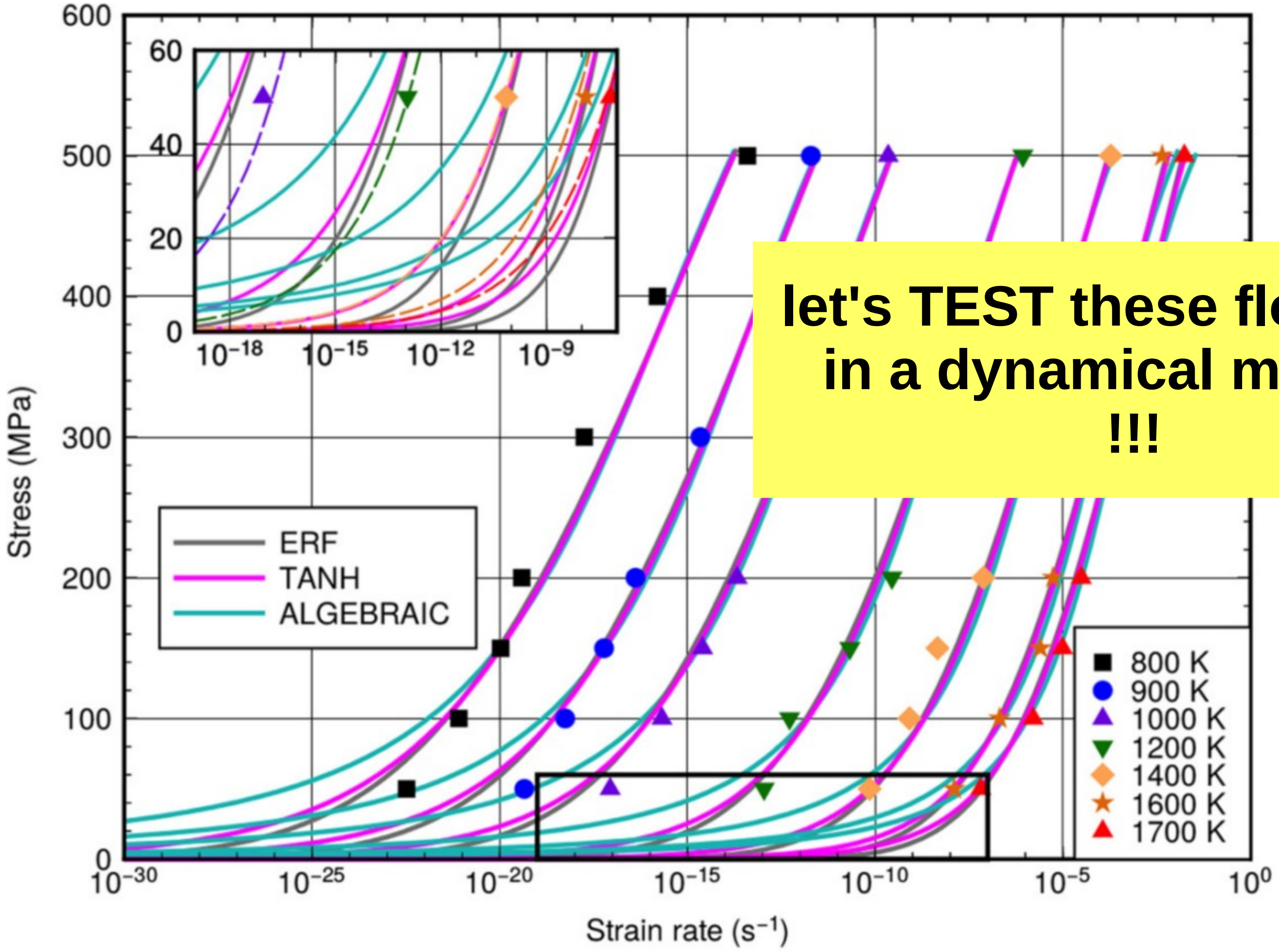
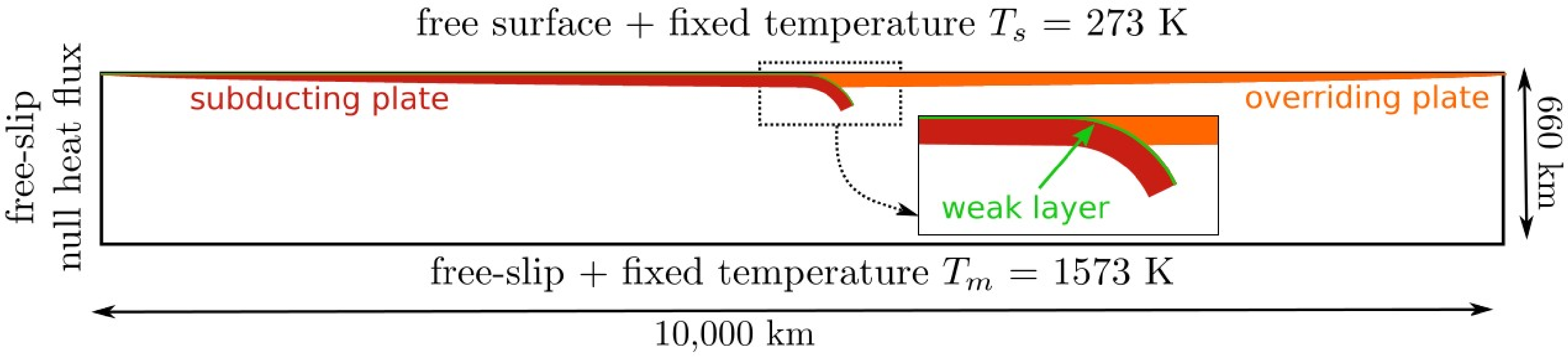


let's TEST these flow laws in a dynamical models !!!

Garel et al., EPSL, 2020

Free subduction model



Composite rheology (addition of strain rates)

$$\boxed{\frac{1}{\mu} = \frac{1}{\mu_{diff}} + \frac{1}{\mu_{disl}} + \frac{1}{\mu_{br}}}$$

- pseudo-brittle / yield stress
- diffusion creep
- **1 single parameterization for dislocation creep** (continuous from low to high temperature)

~~- BEFORE : 2 dislocation creeps (high-T and Peierls low-T)~~

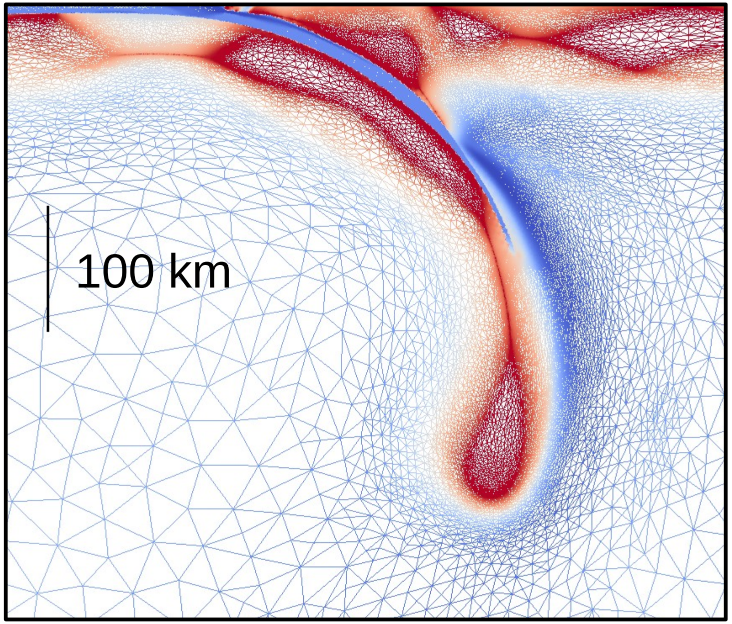
Garel et al., EPSL, 2020

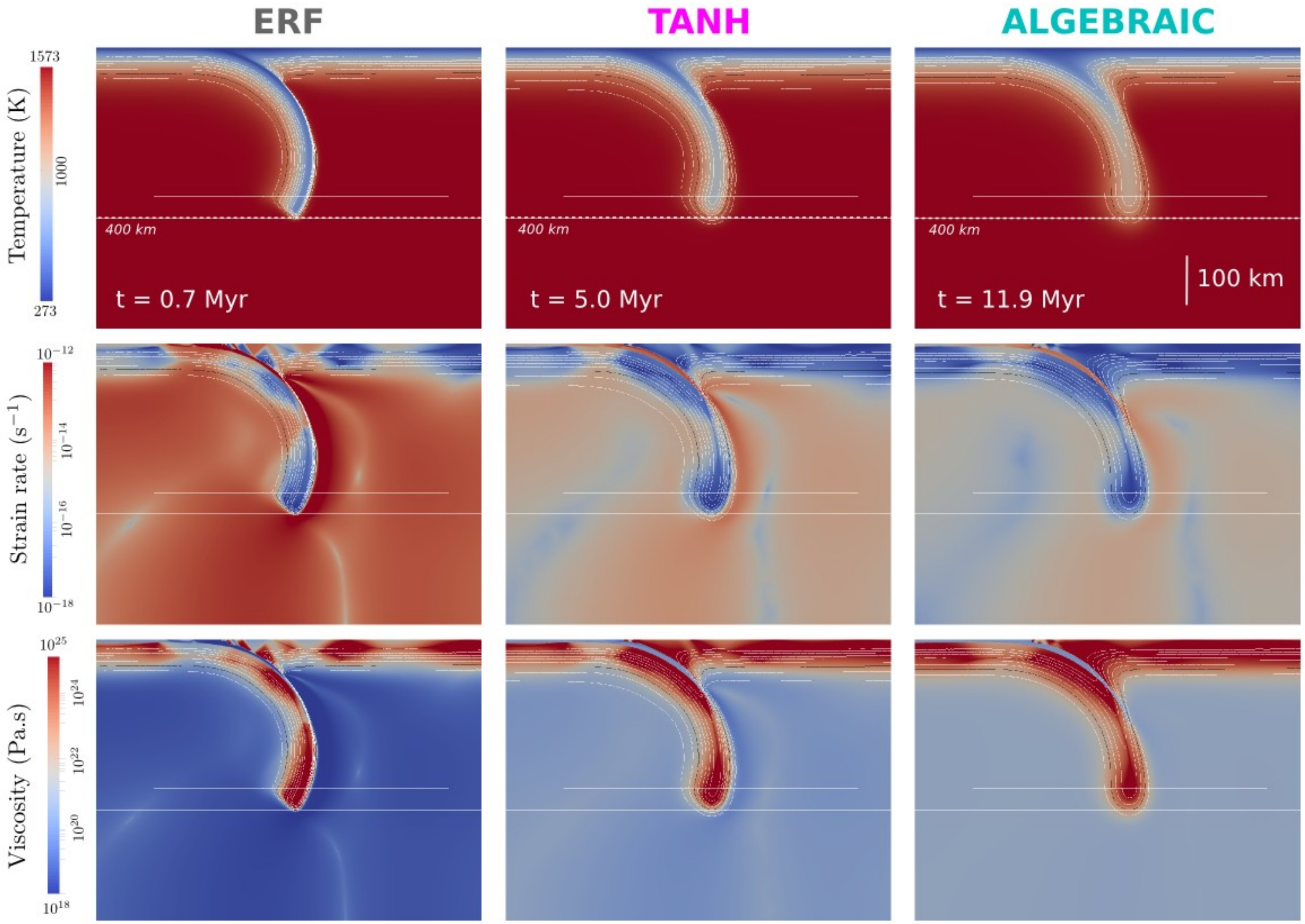
fluidity

Davies et al., G3, 2011
Kramer et al., PEPI, 2012

- **finite-element**, parallel-running code **solving conservation equations**
- developed by the AMCG group (Imperial College London)

- automatic adaptive meshing depending on spatial variations of temperature, velocity, viscosity...

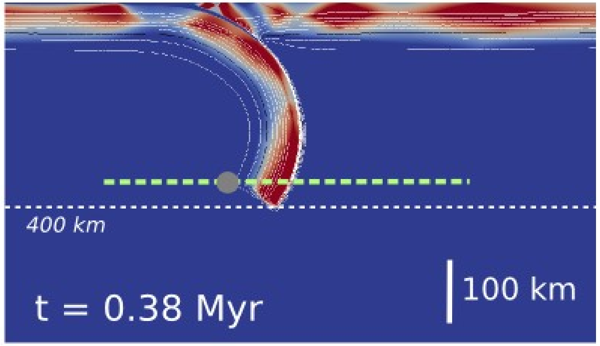




<u>VERY</u> contrasted subduction dynamics

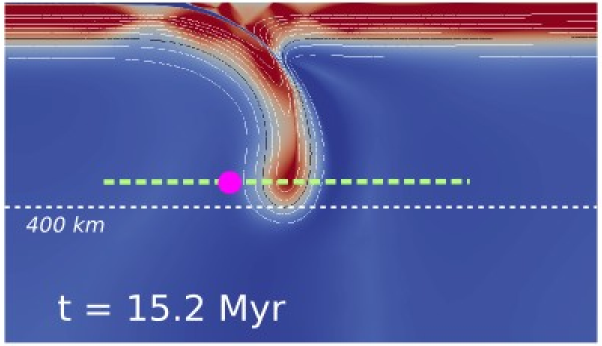
Garel et al., EPSL, 2020

ERF



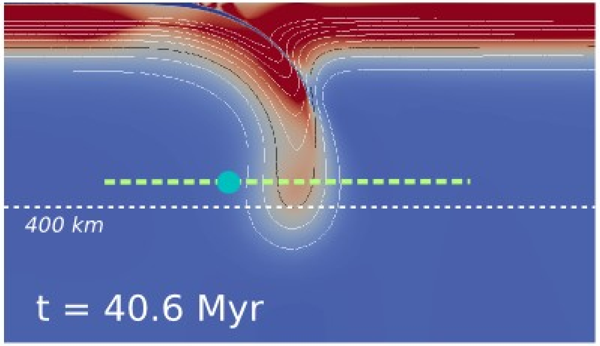
Sinking ++
- very weak asthenosphere
- high asth. strain rates
- very fast subduction
- cold, thin slab

TANH



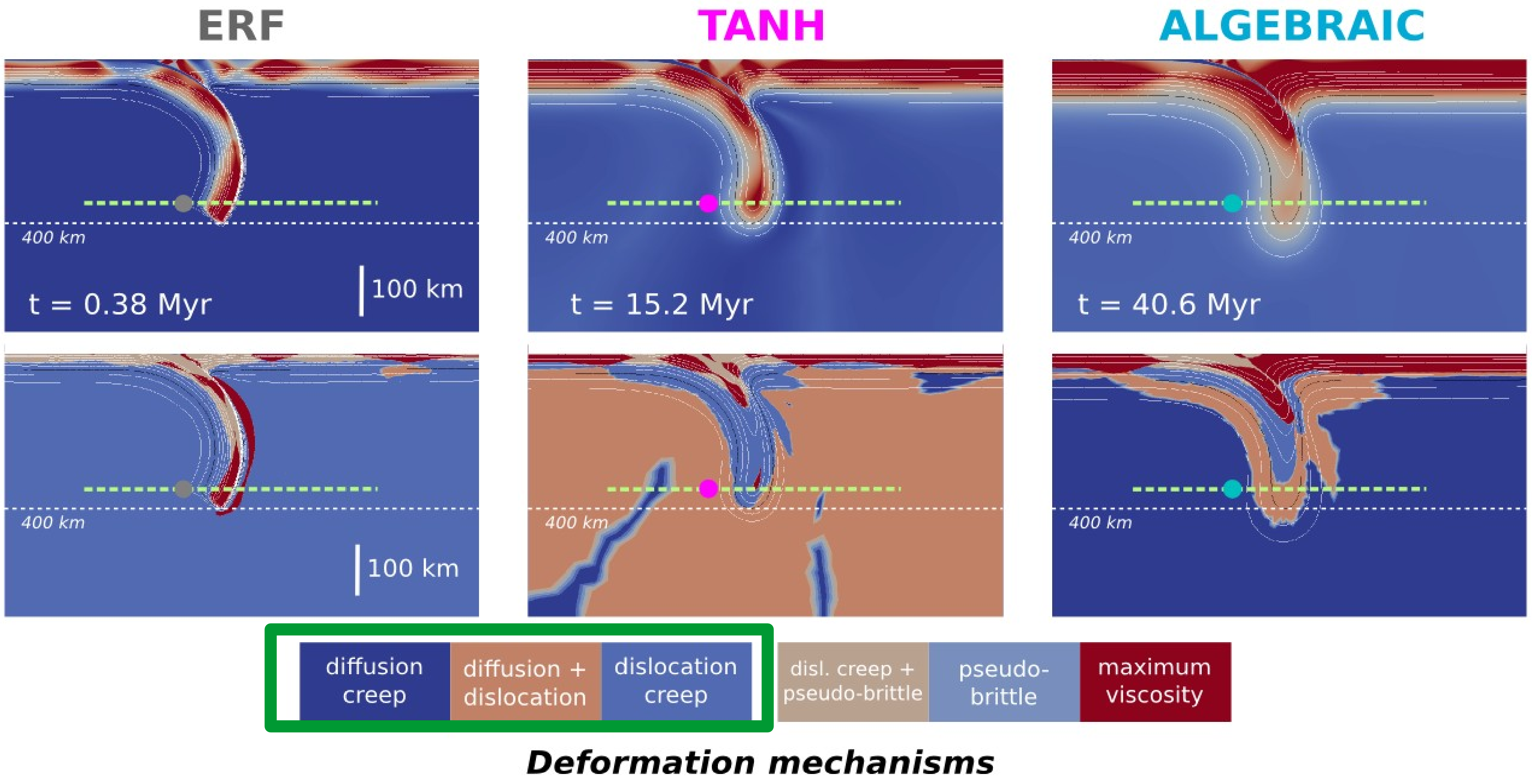
Diffusive AND sinking

ALGEBRAIC



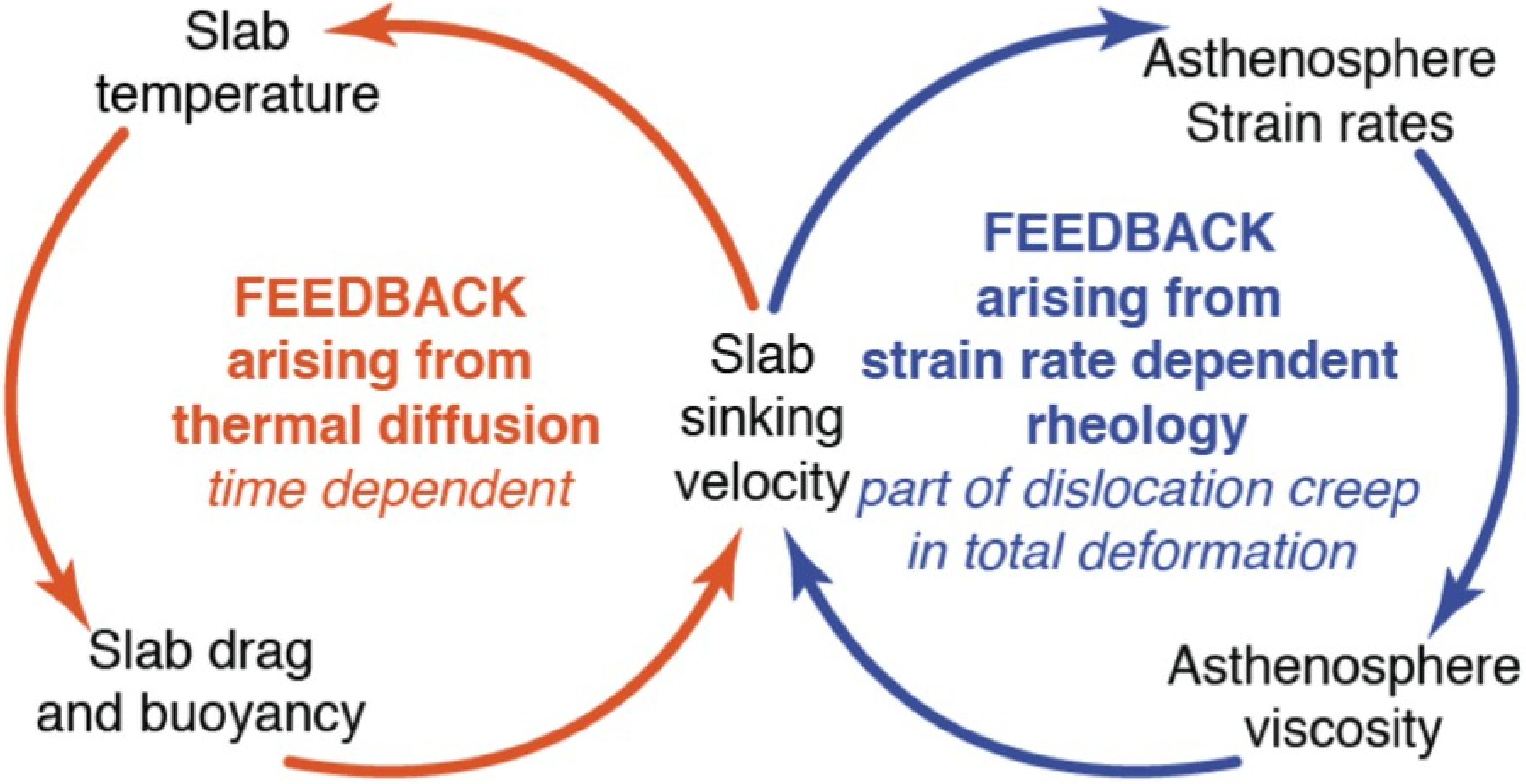
Thermal diffusion++
- high-viscosity asth.
- low asth. strain rates
- very slow subduction
- hot and wide slab

Garel et al., EPSL, 2020



partitioning btn viscosity depending on strain rate (disl. creep, n> 1) **or not** (diff. creep, n=1)

Garel et al., EPSL, 2020

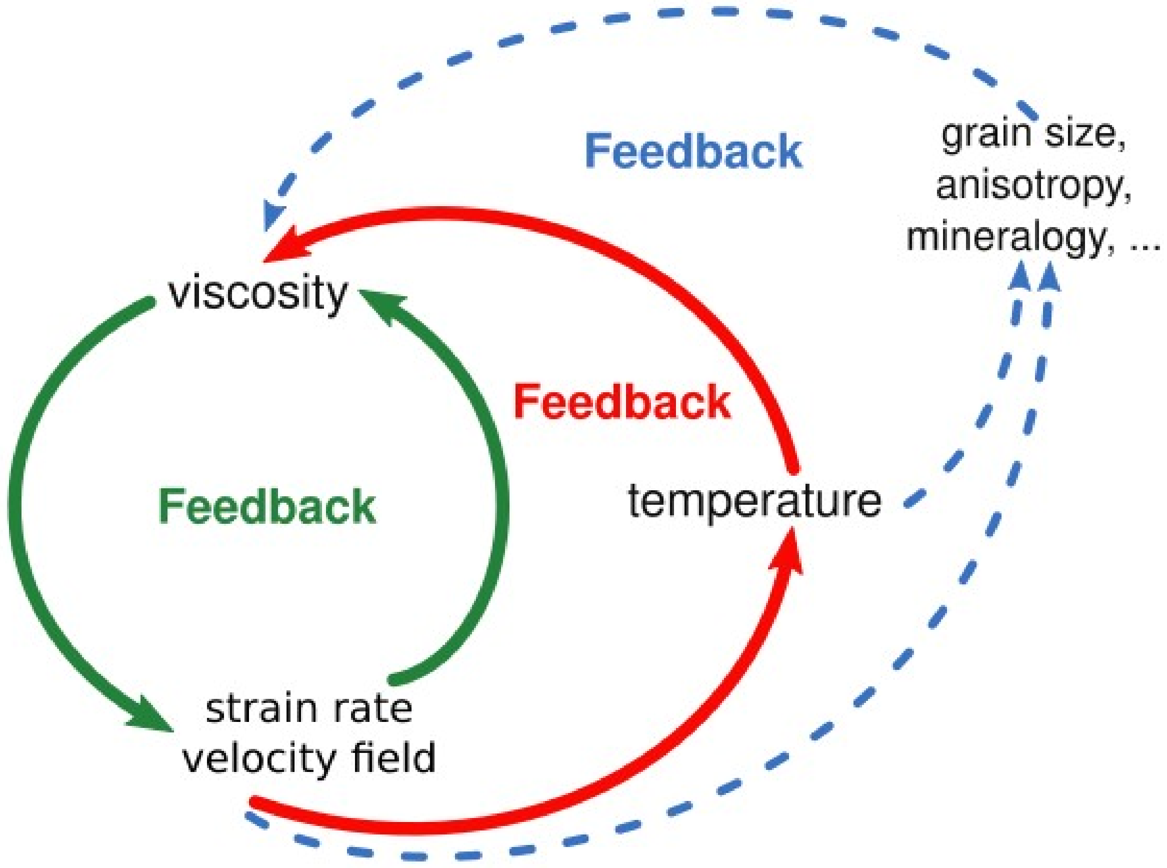


Garel et al., EPSL, 2020

Comparison with geological time-scales observables

Dislocation creep parameterization	ERF	TANH	ALGEBRAIC
Subduction dynamics	Thin, cold slab in free fall in low-viscosity upper mantle No time for thermal diffusion		Hot, diffusive slab in high-viscosity asthenosphere No strain-rate weakening
Time for the slab to reach 400-km	~0.5 Myr	~15 Myr	~40 Myr
Mean asthenosphere viscosity *post-glacial rebound and geoid modelling (~$10^{20\text{-}21}$ Pa.s)*	$10^{18\text{-}20}$ Pa.s	$10^{20\text{-}21}$Pa.s	$10^{20\text{-}21}$Pa.s
surface velocity of the subducting plate *Paleomagnetism and GPS data (1-10 cm/yr)*	~200 cm/yr	~1.5 cm/yr	~0.7 cm/yr
Slab morphology *tomographic imaging*			
Dislocation creep in the upper mantle *Seismic anisotropy*			

Feedbacks between deformation and dynamics



cf Andrea Tommasi and Yanick Ricard's lectures

Feedbacks between deformation and dynamics

Positive feedback between

- shear-induced grain size reduction

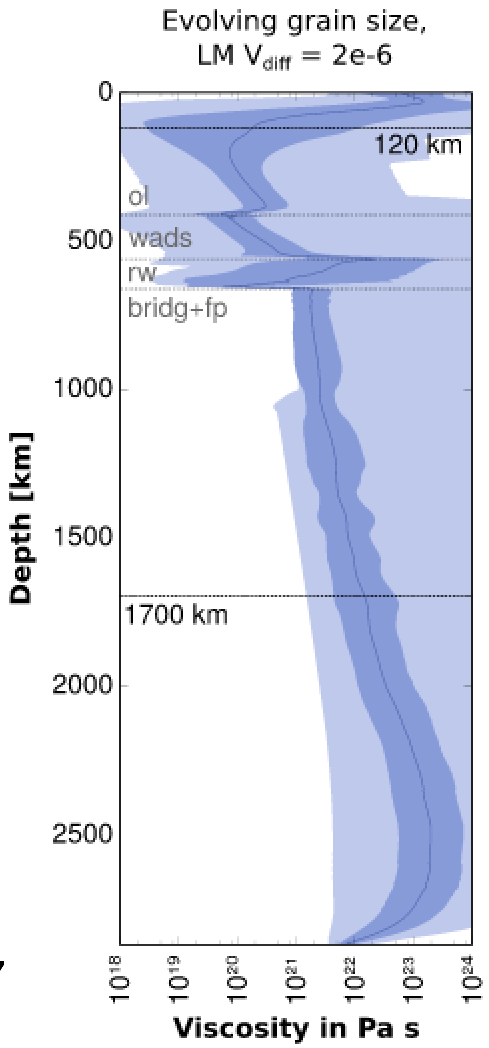
and

- grain-size-controlled viscosity reduction

(**diffusion creep**)

→ deformation localization + low viscosity layer

Dannberg et al., 2017



Introduction to rheology

1. Introduction → definition → why does rheology matter for the Earth ?

→ why is difficult to characterize?

2. Investigating and representing rheology for Earth materials

- stress, strain, strain rate
- lab and numerical experiments
- (micro-)mechanical processes
- flow "laws" and dependencies

3. Contemplating mantle rheology at long geological time-scales

- from flow laws to "effective" viscosity
- feedbacks between deformation and dynamics
- independent constraints on Earth's upper mantle viscosity

4. (open) Conclusion : complexify or simplify ?

+ round-table on upper mantle rheology
Tuesday 13th July

<< 1 µm — 1 cm — ~ 100 km — ~ 1000 km — *length scale*

> 10^{-6} s^{-1} (lab exp)
10^{-23} - 10^{-2} s^{-1} (num exp)

10^{-18} - 10^{-12} s^{-1})

deformation scale

Small-scale data

> acquiring

> upscaling (mineral > rock, strain rates)

INPUT

Parameterizing

Numerical models

- designed to science question
- self-consistent **feedbacks**

PREDICTIONS
SYNTHETICS

COMPARISON

Geological and geophysical data