Scattering of seismic waves in the Earth

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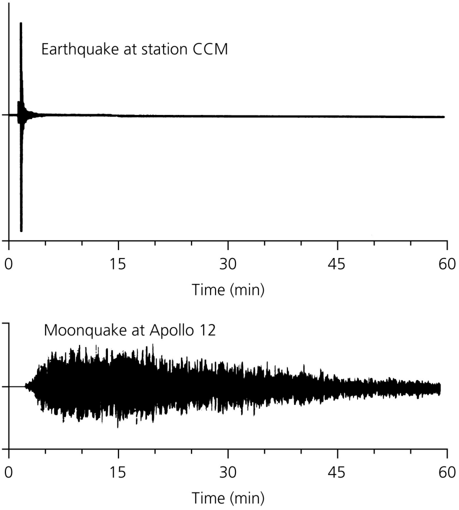
Content

Seismology: A Variety of Propagation Regimes



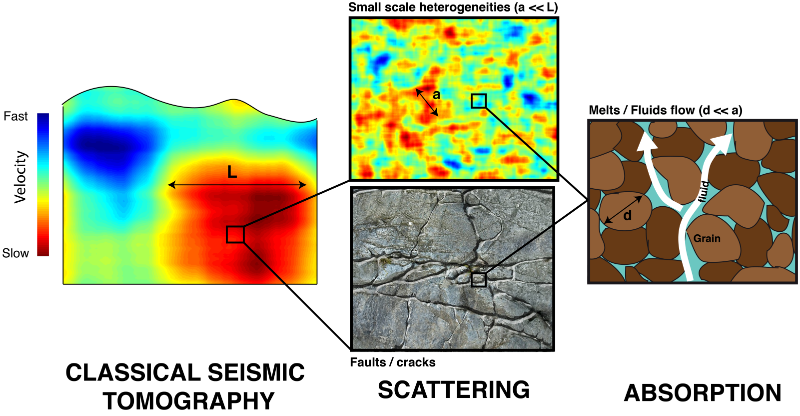
Mitchell, Treatise on Geophysics 2007

Scales

- $f \sim 0.1 - 10$Hz
- $\lambda \sim 0.1 - 10$km

Seismic Attenuation

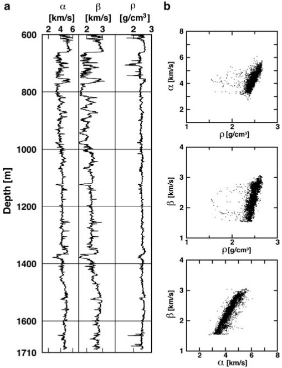
$$\text{Attenuation } Q^{-1} = \quad \text{Scattering } Q_{sc}^{-1} + \text{ Absorption } Q_i^{-1}$$



Goal: Retrieve $Q_{sc}^{-1}(f)$, $Q_i^{-1}(f)$ from energy envelopes characteristics

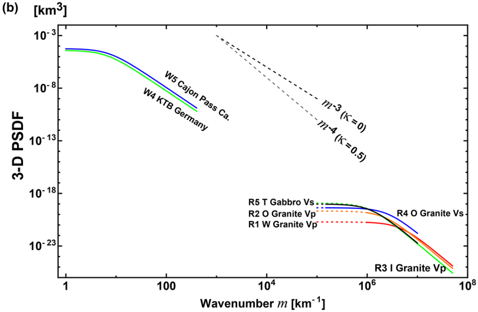
Statistical Description of Heterogeneity

Logging data



Shiomi, 1997

Sato, 2019



Correlation function

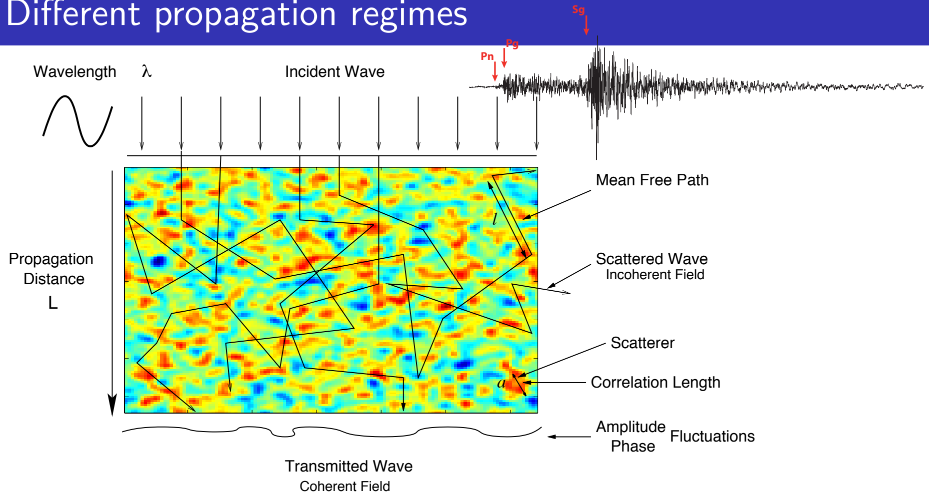
$$C(\mathbf{r}) = \frac{1}{\langle v \rangle^2} \langle \delta v(\mathbf{x} + \mathbf{r}/2) \delta v(\mathbf{x} - \mathbf{r}/2) \rangle$$

Heterogeneity Power Spectrum

$$\Phi(\mathbf{m}) = \int_{\mathbb{R}^3} C(\mathbf{r}) e^{i\mathbf{m}\cdot\mathbf{r}} d\mathbf{r}$$

$$\propto \frac{\epsilon^2 a^3}{(1 + m^2 a^2)^{\kappa+3/2}}$$

Different propagation regimes



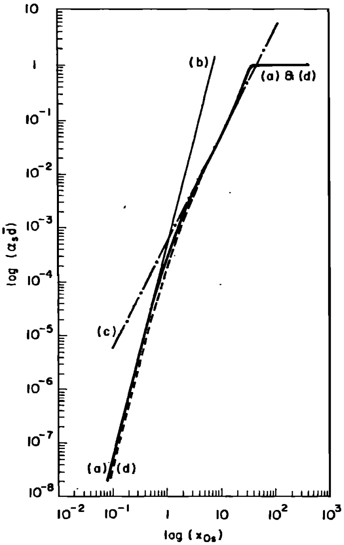
$L \ll l$ $L \gg l$

- Coherent Wave
- Mean Field

- Single $\longrightarrow$ Multiple Scattering
- Transport

- Equipartition
- Diffusion

S-wave attenuation in a polycrystal



Model: exponential correlation function

a: grain size

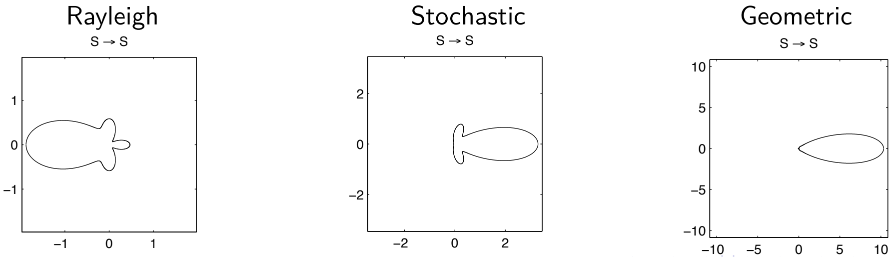
- Rayleigh Regime $ka \ll 1$
  $l \propto (ka)^{-4}$
- Stochastic Regime $ka \sim 1$
  $l \propto (ka)^{-2}$
- Geometric Regime $ka \gg 1$
  $l = a$

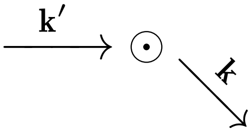
$$Q_{sc} = \omega\tau = \frac{\omega l}{c}$$

Stanke & Kino, 1986

Frequency Regimes: Scattering Anisotropy

$$p(\mathbf{k},\mathbf{k}') \propto \overbrace{F(\delta\alpha,\delta\beta,\delta\rho)}^{\text{Polarization}} \times \Phi(\mathbf{k}-\mathbf{k}')$$

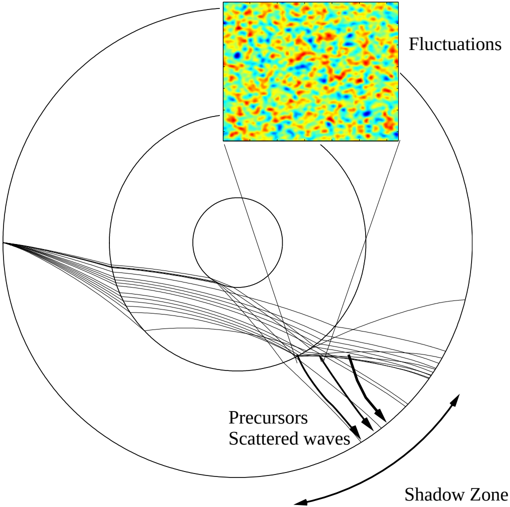


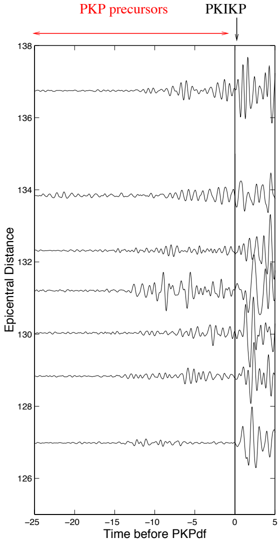


Anisotropy parameter

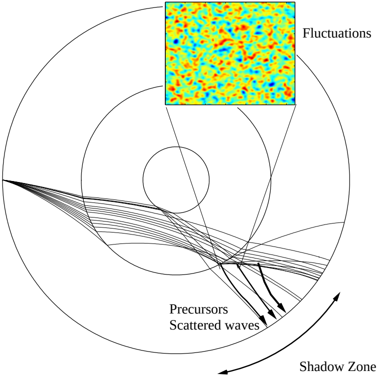
$$g = \int_{4\pi} p(\mathbf{k},\mathbf{k}')\widehat{\mathbf{k}}\cdot\widehat{\mathbf{k}}'\, d\widehat{\mathbf{k}}$$

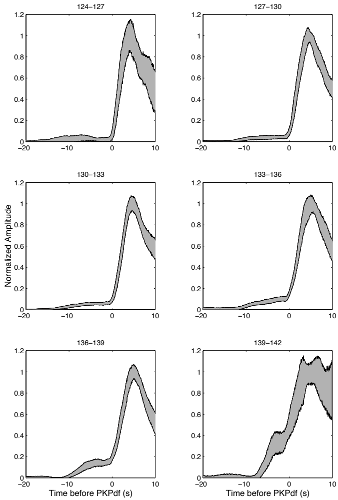
Deep Earth scattering experiment: PKP precursors





Global average observations



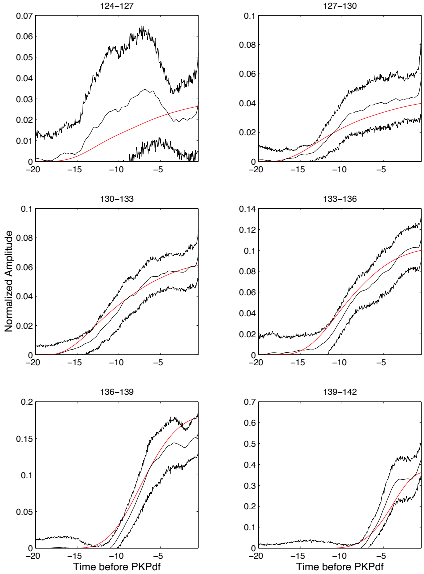


Interpretation:

In a scattering experiment, we scan the power spectrum Φ between 0 and $2k$.

Constraints on the lower mantle fluctuations

Time domain Fits



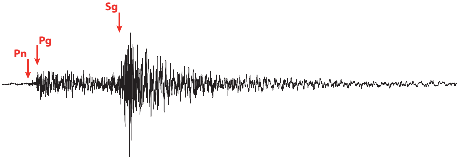
Conclusions

- Preferred model: $\Phi(m) \propto m^{-3}$
- Very weak fluctuations $\propto 0.1\%$
- The correlation length is not resolvable
- Small-scale heterogeneities extend at least 600kms above the CMB

Content

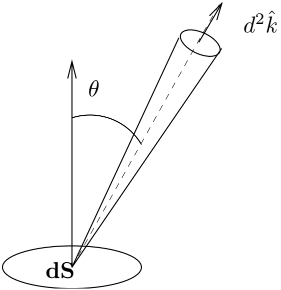
1. Fundamentals: scale lengths, frequency regimes
2. Transport models: separation of absorption and scattering
3. Advanced topic: sensitivity kernels

Equation of radiative transfer



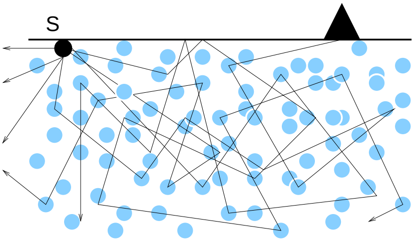
Model: Boltzmann Equation

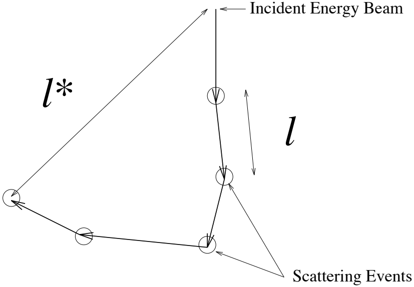
$$\overbrace{\left(\frac{\partial}{\partial t} + c\widehat{\mathbf{k}}\cdot\nabla_{\mathbf{X}} + \omega(Q_{sc}^{-1} + Q_i^{-1})\right)}^{\text{Coherent Propagation}} I(\mathbf{X},\widehat{\mathbf{k}},t) =$$



$$\overbrace{\omega Q_{sc}^{-1}\oint \overbrace{p(\mathbf{k},\mathbf{k}')}^{\text{Scattering anisotropy}} I(\mathbf{X},\widehat{\mathbf{k}}',t)\,d^2\hat{k}'}^{\text{Multiple Scattering Term}} + \overbrace{S(\mathbf{X},\mathbf{k},t)}^{\text{Source term}}$$

Diffusion Limit and Transport Mean Free Path





- Energy Density

$$E(\mathbf{R}, t) = \frac{1}{c} \int_{4\pi} I(\mathbf{R}, \widehat{\mathbf{k}}, t)\, d\widehat{\mathbf{k}}$$

- Random walk with no memory:

$$D = \frac{cl}{d}$$

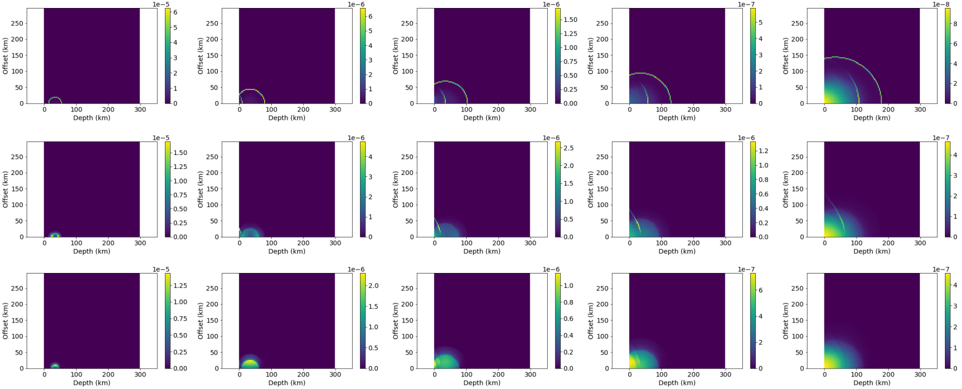
- Persistent random walk:

$$D = \frac{vl}{d(1-g)} = \frac{vl^\star}{d}$$

$$g = \int_{4\pi} p(\mathbf{k}, \mathbf{k}')\widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}}'\, d\widehat{\mathbf{k}}$$

Numerical example for elastic waves in 3-D

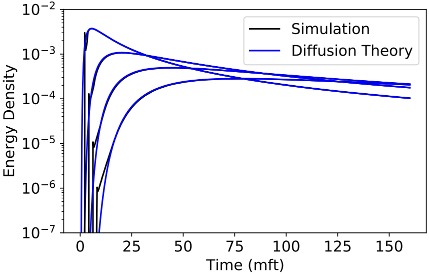
- Source: Explosion at 35kms depth
- Medium: point-like density perturbations in a half-space
- Scattering mean free time: $\tau = 10\text{s}$



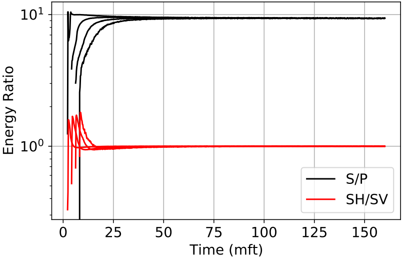
Numerical method: Monte-Carlo simulations

The diffusion/equipartition limit

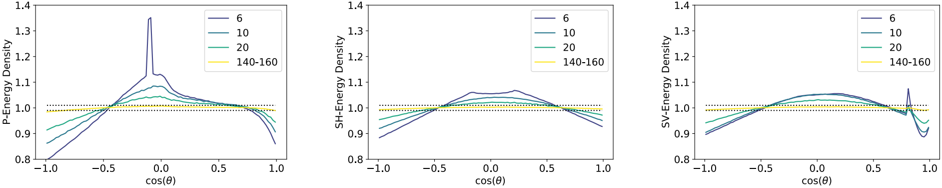
Comparison with Diffusion model



Equipartition

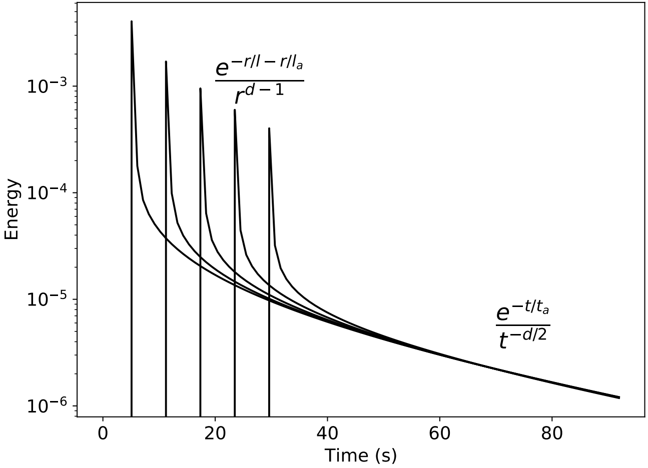


Relaxation towards isotropy



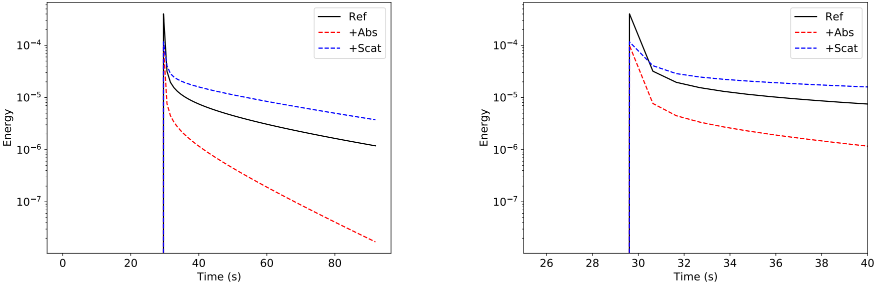
Separation of absorption and scattering

Different scaling for coherent and incoherent intensity



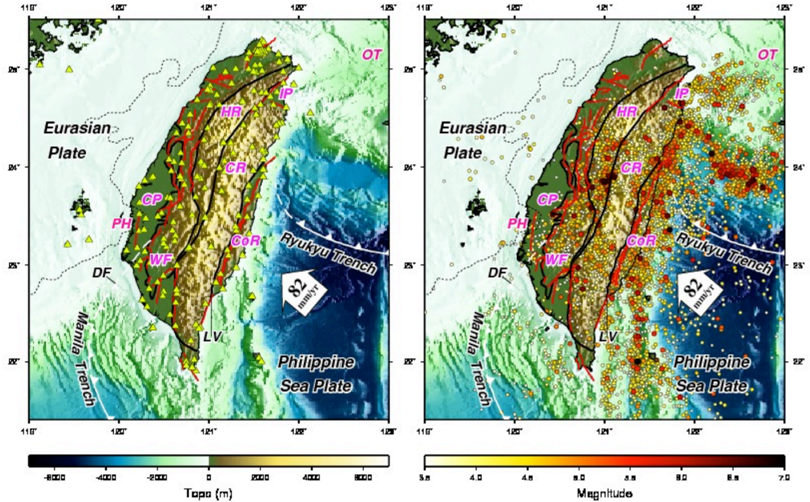
Coherent vs Incoherent energy ratio

Scattering vs Absorption perturbation



- Indeterminacy of scattering/absorption ratio based on ballistic waves alone
- Scattering transfers energy from ballistic waves to the coda
- !!! The coda is mostly sensitive to absorption !!!

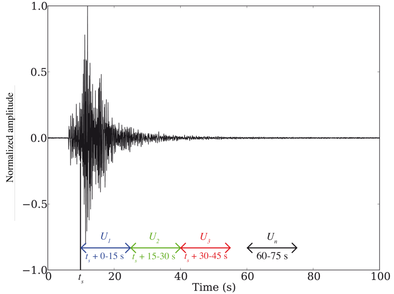
Application to Taiwan

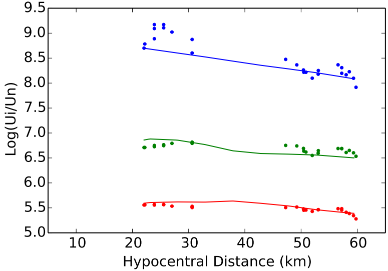


Coastal Plain (CP), Central Range (CR), Coastal Range (CoR), Hueshan Range (HR), Western Foothills (WF)
Longitudinal Valley (LV), Ilan Plain (IP), Peikang Basement High (PH), Deformation Front (DF), Okinawa Trough (OT)

Single-station measurement of crustal attenuation

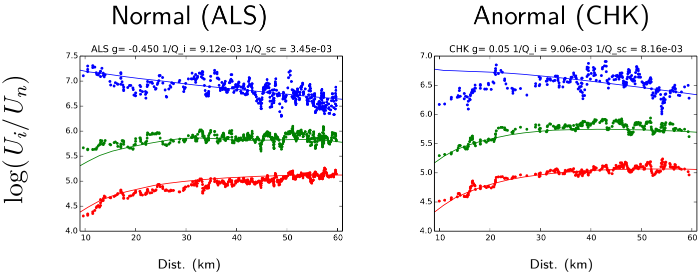
Idea: Measure the energy ratio between ballistic and coda waves

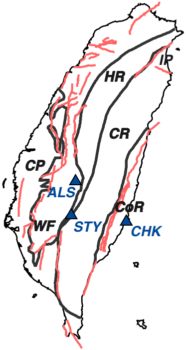




- Multiple Lapse-Time Window Analysis (Sato, Fehler, Hoshiba)
- Model: Boltzmann Equation (isotropic scattering)
- Computation of Energy integrals + partial derivatives wrt Q_{sc}, Q_i
- Optimization with Levenberg-Marquardt algorithm

Anomalous energy distribution in the coda





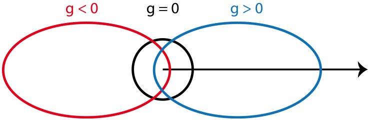
- Clear discrepancy between model and data near the source
- Observed at a number of stations in Taiwan
- Not explained by simple velocity structures

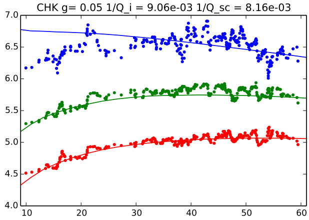
Evidence for scattering non-isotropy in Taiwan

- Henyey-Greenstein phase function:

$$p(\mathbf{k}, \mathbf{k}') = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\mathbf{k} \cdot \mathbf{k}')^{3/2}}$$

g: anisotropy parameter (mean cosine of scattering angle)





$$\boxed{g \approx 0}$$

Evidence for scattering non-isotropy in Taiwan

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$$p(\mathbf{k}, \mathbf{k}') = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\mathbf{k} \cdot \mathbf{k}')^{3/2}}$$

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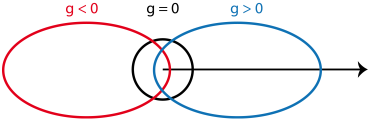
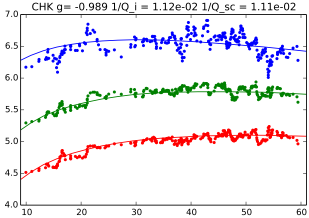
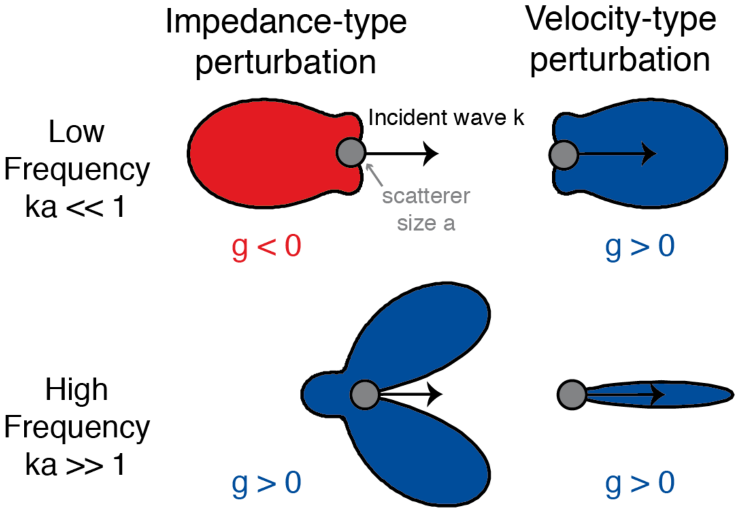


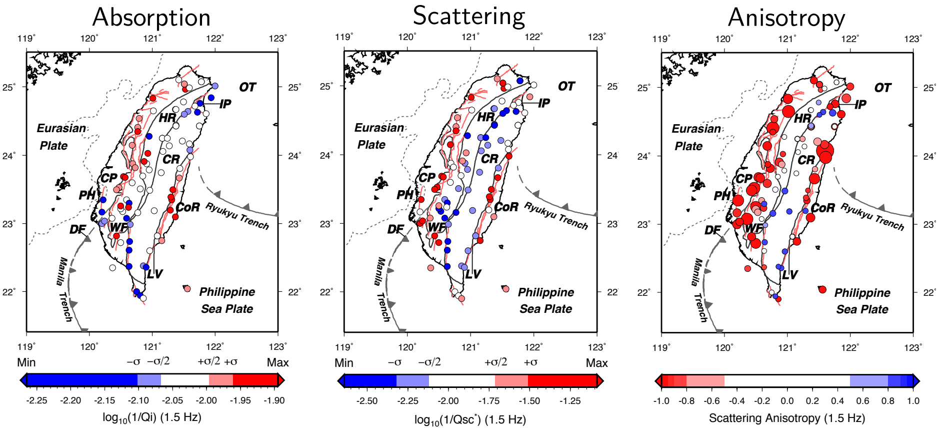


$$\boxed{g \approx -0.95 \;!!}$$

Interpretation of strong backscattering

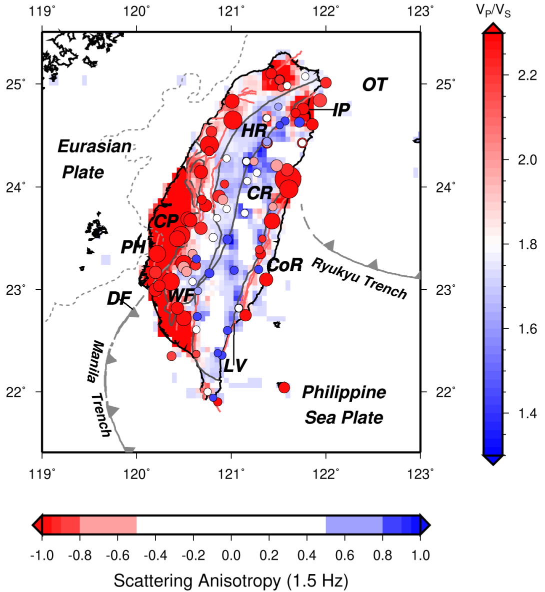


Mapping of attenuation and scattering anisotropy



- Relatively uniform absorption ($Q_i \approx 100$)
- Strong lateral variation of scattering (almost 2 orders of magnitude)
- Scattering anisotropy correlates with Q_{sc}^{-1}

Correlation between V_p/V_s ratio and backscattering



Content

Sensitivity Kernels: Formalism

Transport equation

$$\overbrace{(\partial_t + c\widehat{\mathbf{n}}\cdot\nabla)}^{\text{Variation of intensity along ray}} I(\mathbf{r},\widehat{\mathbf{n}},t) = \overbrace{-\left(\frac{\omega}{Q_{sc}(\mathbf{r})} + \frac{\omega}{Q_i(\mathbf{r})}\right)}^{\text{Loss due to Absorption and Scattering}} I(\mathbf{r},\widehat{\mathbf{n}},t)$$

$$+ \underbrace{\frac{\omega}{Q_{sc}(\mathbf{r})}\oint p(\widehat{\mathbf{n}},\widehat{\mathbf{n}}')I(\mathbf{r},\widehat{\mathbf{n}}',t)\,d\hat{n}'}_{\text{gain through scattering}} + \underbrace{\delta(\mathbf{r}-\mathbf{s})\delta(t)}_{\text{Source term}}$$

1. Perturbation of a reference medium (Q^0_{sc}, Q^0_i)

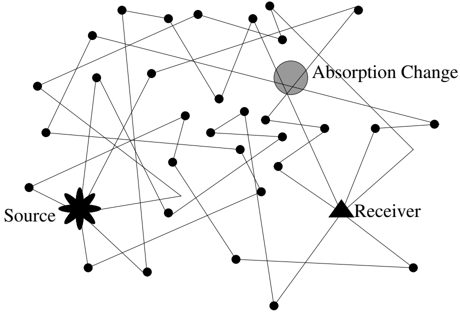
$$\frac{1}{Q_{sc}(\mathbf{r})} = \frac{1}{Q^0_{sc}} + \delta\frac{1}{Q_{sc}(\mathbf{r})} \qquad \frac{1}{Q_i(\mathbf{r})} = \frac{1}{Q^0_i} + \delta\frac{1}{Q_i(\mathbf{r})}$$

2. Linearization:

$$\frac{\delta I(\mathbf{r};\mathbf{s};t)}{I(\mathbf{r};\mathbf{s};t)} = \int \underbrace{K^a(\mathbf{r};\mathbf{x};\mathbf{s};t)}_{\text{Absorption sensitivity kernel}}\,\delta\left(\frac{\omega}{Q_i(\mathbf{x})}\right)dx + \int \underbrace{K^{sc}(\mathbf{r};\mathbf{x};\mathbf{s};t)}_{\text{Scattering sensitivity kernel}}\,\delta\left(\frac{\omega}{Q_{sc}(\mathbf{x})}\right)dx$$

Probabilistic approach to sensitivity functions: absorption

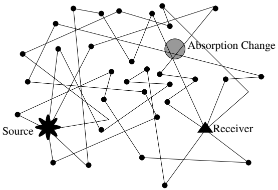
Idea: the sensitivity at time t is proportional to the time spent by the waves in the volume $\delta V(\mathbf{x})$ where the change occurs. (Pacheco & Snieder, 2005)



$$\underbrace{\frac{\delta I}{I}}_{\text{Intensity P}^\circ} = -\underbrace{\omega}_{\text{Freq.}}\,\overbrace{\delta Q_i^{-1}(\omega,\mathbf{x})}^{\text{Absorption perturbation}}\,T(\delta V(\mathbf{x}),t)$$

$$T(\delta V(\mathbf{x}),t) = \underbrace{K^a(\mathbf{x},t)}_{\text{Sensitivity Kernel}} \times \delta V(\mathbf{x})$$

Interpretation of the absorption sensitivity kernel

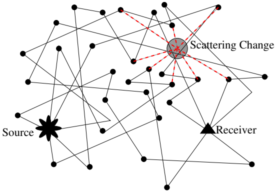


Probability that a random walker arriving at R at time t visited $\delta V(\mathbf{x})$ a time t'?
Application of Bayes Formula:

$$K^a(\mathbf{x}, t) = \int_0^t \oint \frac{\overbrace{I(\mathbf{x}, -\widehat{\mathbf{k}}, \mathbf{r}, t-t')}^{\text{Probability to go from Change to Receiver}} \quad \overbrace{I(\mathbf{x}, \widehat{\mathbf{k}}, \mathbf{s}, t')}^{\text{Probability to go from Source to Change}}}{\underbrace{\oint I(\mathbf{r}, \widehat{\mathbf{k}}, \mathbf{s}, t)\, d\hat{k}}_{\text{Probability to go from Source to Receiver}}} d^2\hat{k}dt'$$

Fundamental Property of the Kernel: $\displaystyle\int_{\text{Full Space}} K_{tt}(\mathbf{x}, t)\, dV(\mathbf{x}) = t$

Interpretation of the scattering sensitivity kernel



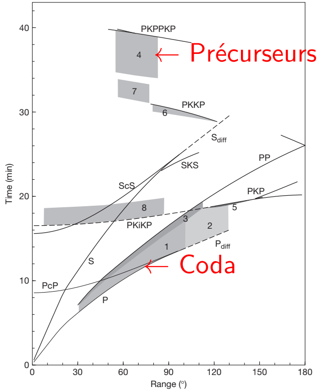
Additional effect: extra probability to go from S to R

$$K^{sc}(\mathbf{r},\mathbf{x},\mathbf{s},t) \approx d(1-g)\frac{\int_0^t \mathbf{J}(\mathbf{x},\mathbf{r},t-t')\cdot\mathbf{J}(\mathbf{x},\mathbf{s},t')\,dt'}{I(\mathbf{r},\mathbf{s},t)}$$

Fundamental property : $$\int_{\text{Full Space}} K^{sc}(\mathbf{r},\mathbf{x},\mathbf{s},t)\,dV(\mathbf{r}) = 0$$

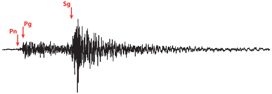
Outlook: a wealth of data to analyze

Observations d'ondes diffusées

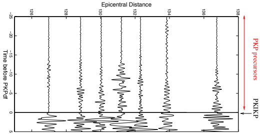


(Shearer, Treatise on Geophysics, 2015)

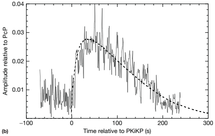
Croûte



Manteau



Noyau



Lune