



dépasser les frontières

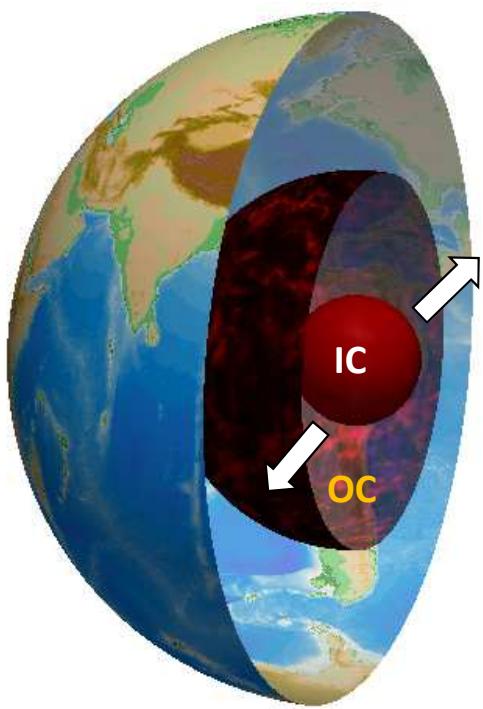
Institut **Terre & Environnement**  
de Strasbourg | ITES | UMR 7063  
de l'Université de Strasbourg  
& CNRS & ENGEES



# Inner-core free oscillations: an observational challenge

Séverine Rosat

# Normal modes related to the inner core



- Slichter mode: *not yet detected...*
- Free Inner Core Nutation: *not really detected...*
- Inner Core Wobble: *not yet detected...*

# History

## POWER SPECTRA ANALYSIS - UCLA EARTH TIDE GRAVIMETER CHILEAN EARTHQUAKE - MAY 22, 1960

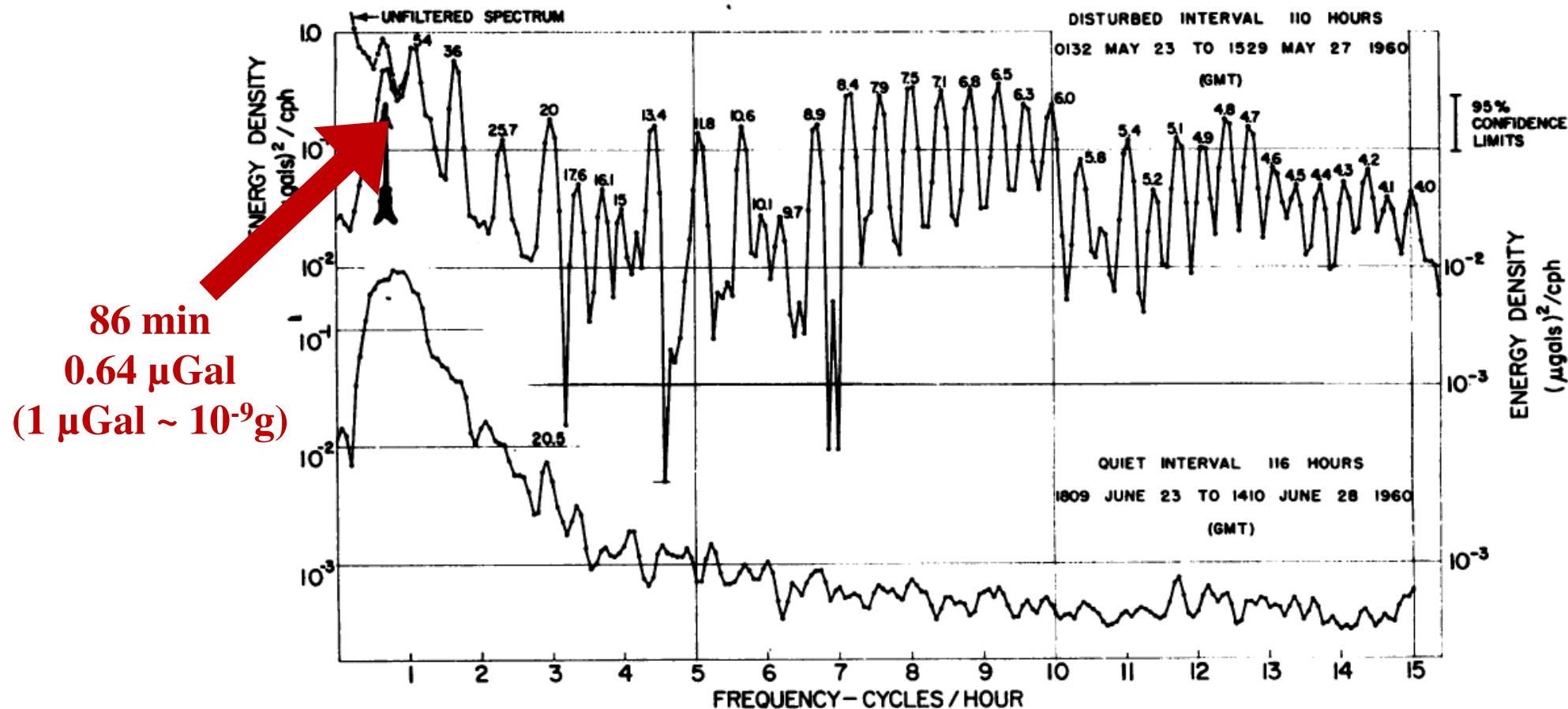


FIG. 1.—Spectral peak (shown by arrow) with period 86 minutes. (Portion of Fig. 4, reference 1, reproduced by courtesy of the authors. The numbers above the peaks indicate the period, in minutes.)

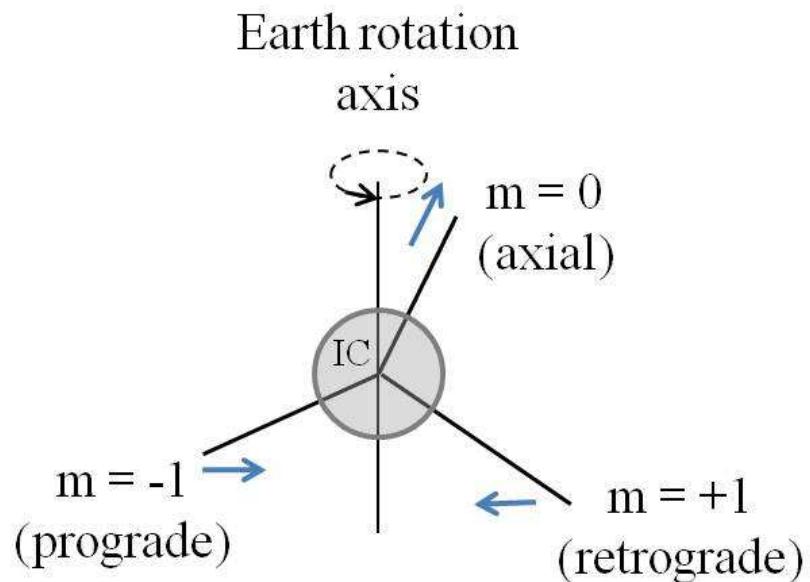
Fig. from Ness et al. (1961) reproduced by Louis B. Slichter (1961)

## Slichter triplet

$^1S_1$  degree-1 mode split by rotation into 3 eigenmodes:

A polar mode (or *axial*) and 2 equatorial modes:

- Positive circularly polarized mode or *prograde*
- Negative circularly polarized mode or *retrograde*



## Theoretical studies

Alsop [1963]: first numerical evidence of the existence of this mode. He noted that this mode was actually a **degenerate spheroidal triplet** of surface spherical harmonic degree one.

Busse [1974]: **rigid** sphere oscillating in a rotating incompressible homogeneous inviscid fluid itself contained within a concentric **rigid** spherical boundary (axisymmetric 'polar' motion of the inner core). He suggested that oscillations of the inner core of detectable amplitude **may be excited by fluid motions in the outer core**.

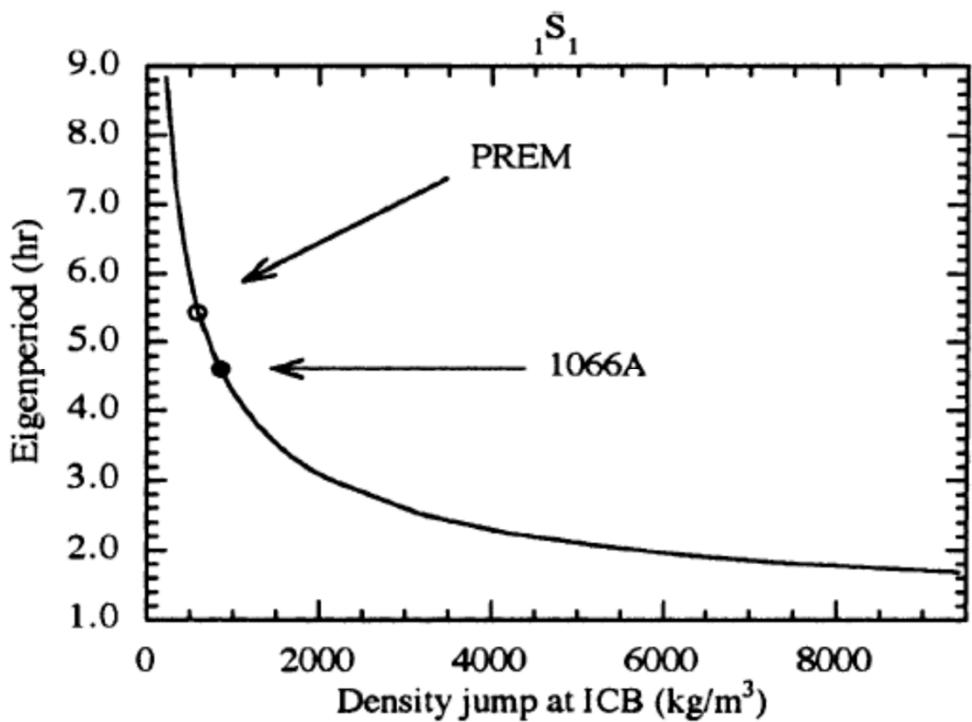
Smith [1976]: normal mode eigensolutions for a **rotating, slightly elliptical Earth**

Dahlen and Sailor (1979), Crossley (1992) etc. periods 4 h - 6 h  
( $T_0 = \mathbf{5.42 \text{ h}}$  for PREM)

## Theoretical studies

- Denis et al. [1997]: eigenperiod of 5.42 h for a non-rotating PREM model  
Earth's hydrostatic flattening  $\rightarrow \Delta\rho$  at ICB should be less than PREM value  
 $\rightarrow$  Slichter period should be  $> 5.42$  h.
- Peng [1997]: **mushy zone** at ICB has a **small but substantial** influence compared with elasticity of the mantle, non-neutral stratification of OC and ellipticity and centrifugal potential  $\rightarrow T > 5.3$  h for PREM model  
(increments in eigenperiods are about 0.6 % for a PREM model, with a mushy zone **5 km in thickness and 50 per cent** in fluid content)
- Rogister [2003]: period decreases with **compressibility of OC**
- Grinfeld and Wisdom [2010]: period could be much shorter than 5 h because of the kinetics of **phase transformations** at the ICB

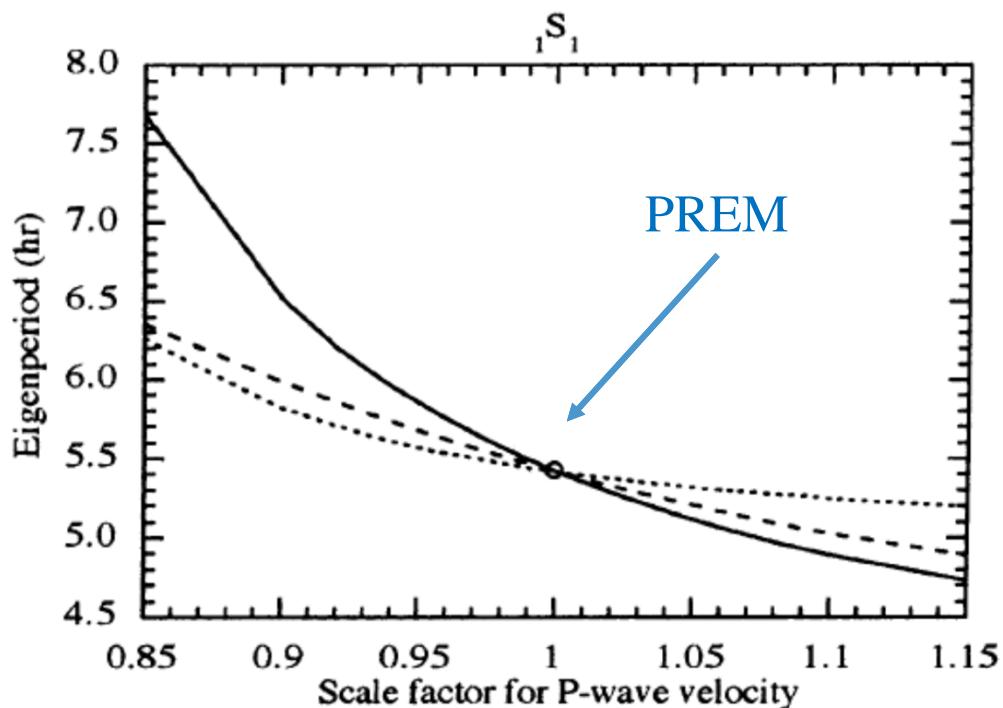
# Theoretical studies



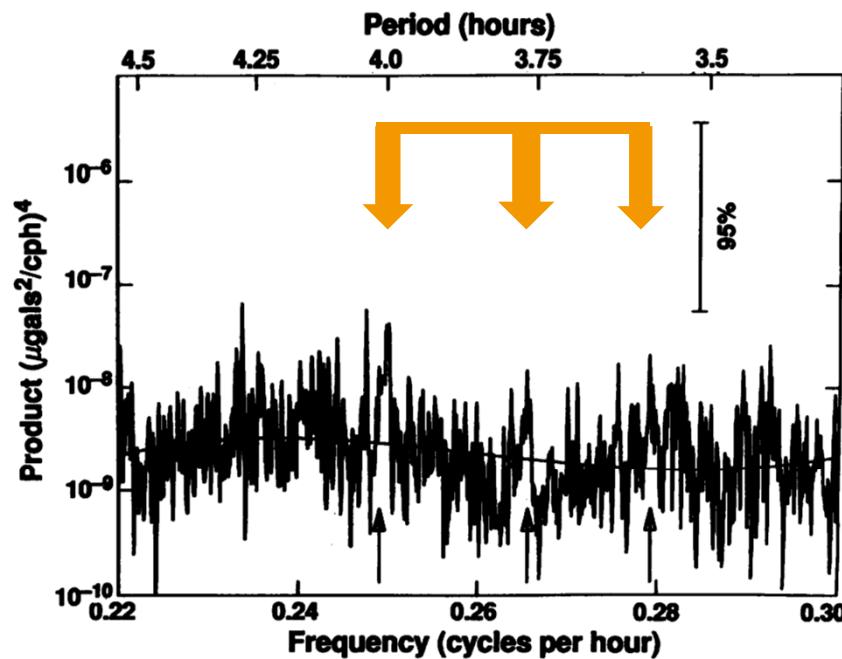
## Influence of compressibility

- Vp multiplied in whole core
- in OC
- ... in IC

[Rogister (2003)]



# A controversial detection



**Fig. 1.** Product spectrum of the four superconducting gravimeter records described in the text. A sinusoid is shown fitted across the whole spectrum to provide a reference noise level. The locations of the resonances identified by their rotational splitting as the triplet of inner core translational oscillations are shown by the arrows. Statistically, the spectrum represents the equivalent of 24.3 years of independent hourly samples. Vertical bar shows 95 percent confidence interval (CI).

Smylie [1992]:  $T = 3.6, 3.8, 4.0$  h

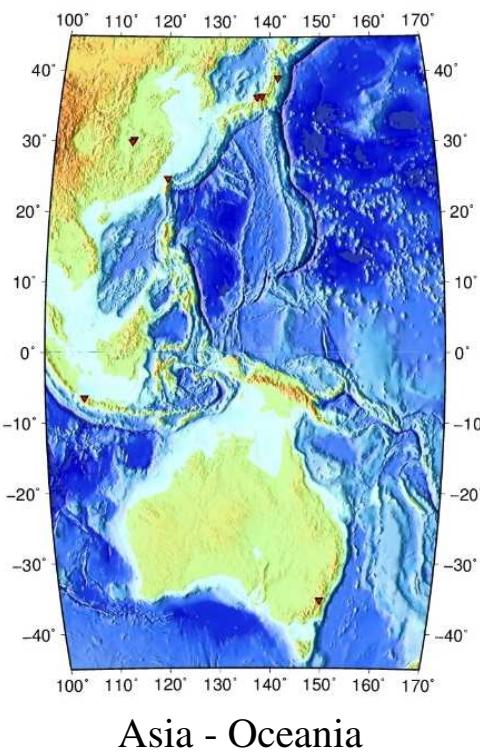
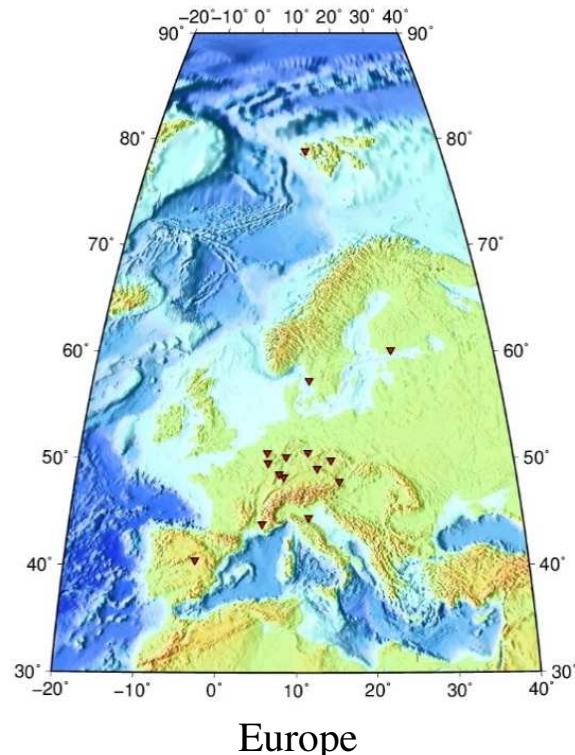
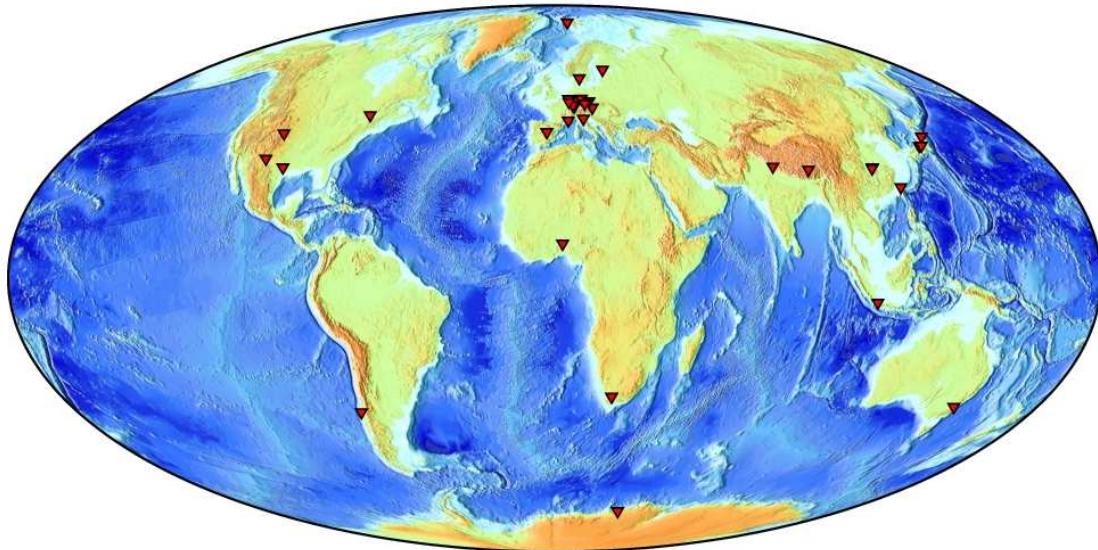
Smylie (1999), Smylie & McMillan (2000): splitting  $\rightarrow$  dynamic viscosity of OC at ICB:  $1.22 \cdot 10^{11}$  Pa.s

$\mu = 1.6 \cdot 10^{-2}$  Pa.s using laboratory experiments (Rutter et al. 2002)

Mathews and Guo (2005) have proposed  $\mu \leq 1.7 \cdot 10^5$  Pa.s using nutation data

## *Effective viscosity*

# Superconducting Gravimeter Observations



## International Gravity and Earth Tides Service

(IAG Service under the umbrella of the International Gravity Field Service, since 2015)

<http://igets.u-strasbg.fr>

former Global Geodynamics Project

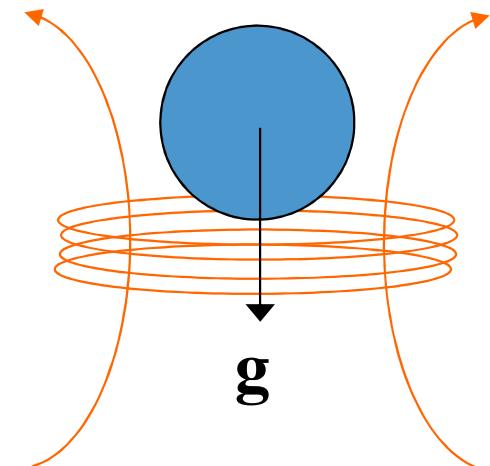
~30 Superconducting Gravimeter stations

One of the Scientific Objectives

→ Earth's **normal modes**

by combination of data to reduce local gravimetric effects

# Superconducting Gravimeter Observations



Magnetic levitation of a Niobium sphere

Sensitivity  $\sim 1 \text{ nGal} = 10^{-12} \text{ g}$  ( $g = 9.81 \text{ m/s}^2$ )

Weak instrumental drift (few  $\mu\text{Gal/year}$ )

→ Suitable instruments for the study of a wide range of geophysical phenomena from seismic modes to long-period processes  
(Crossley et al. 1999; Hinderer et al. 2007)

**At Slichter period (~ 5 h) SGs are instruments with the lowest noise levels**  
→ Search for  $^1\text{S}_1$  signal in SG time-varying gravity data from the SG network

[Crossley et al. 1999; Rosat et al. 2004; Rosat & Hinderer 2011]

# A controversial detection

**Never undoubtedly detected...** despite many attempts for > 25 years

→ Search for  ${}_1S_1$  signal in surface time-varying gravity data

## Controversial detection:

Smylie (1992), Courtier et al. (2000),  
Pagiatakis et al. (2007)

Rieutord (2000): such an observation  
incompatible with theory

## No detection:

Hinderer et al. (1995), Jensen et al. (1995),  
Rosat et al. (2003, 2006, 2007, 2008), Guo et  
al. (2006, 2007), Abd El-Gelil and Pagiatakis  
(2009), Ding and Shen (2013), Ding and  
Chao (2015), Luan et al. (2019)

Combination of more and less noisy data

$T > 4 \text{ h}$

**NEVER DETECTED → What is the expected  
Slichter mode surface amplitude?**

Depends on the excitation process and damping mechanism

## « Slichter mode »: damping mechanisms?

Seismic anelasticity:

Crossley et al. (1991)  $\rightarrow Q \sim 5000$

Outer-core viscosity:

Mathews and Guo (2005)  $\rightarrow Q \sim 5000$

Magnetic damping:

Buffett and Goertz (1995)  $\rightarrow 2200 < Q < 5.8 \cdot 10^5$

$\rightarrow$  Should be revised (D. Jault pers. com.)

$Q \geq 2000$

Damping time of 144 days

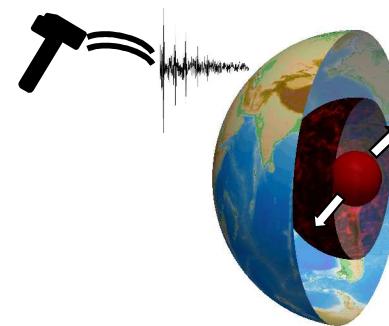
→ **Weak damping**

Which excitation processes?

# « Slichter mode »: Excitation mechanisms?

- Smith (1976)
- Crossley (1987; 1992)
- Rosat (2007)
- Greff-Lefftz and Legros (2007)
- Rosat and Rogister (2012)
- Rosat et al. (2014)

} Seismic excitation

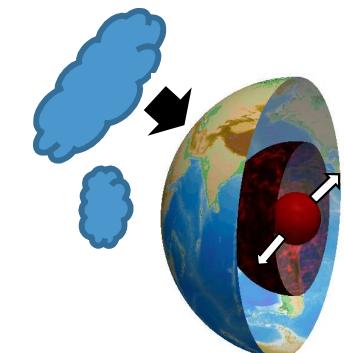


} Pressure flow at core boundaries



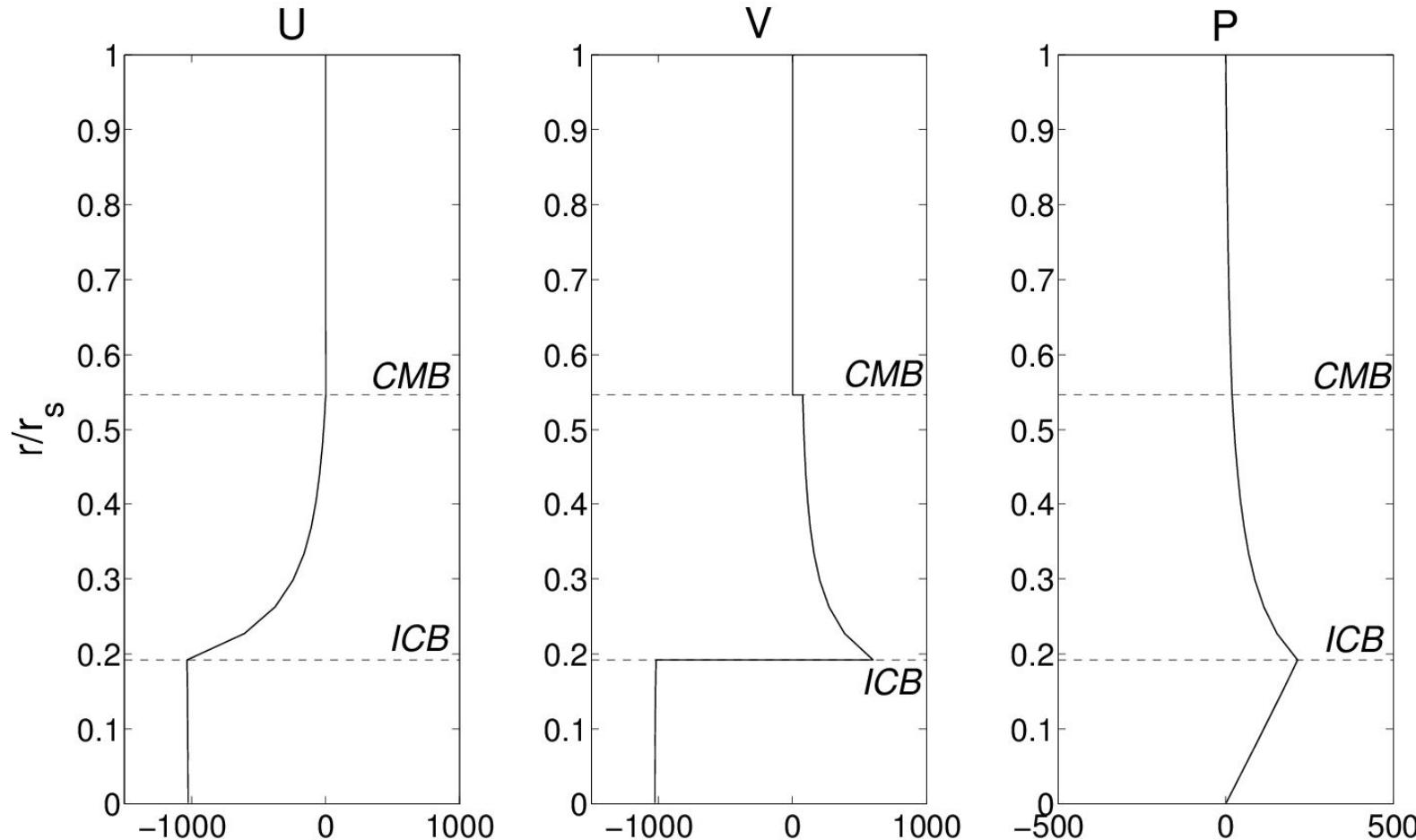
} Meteoroid impact, surface pressure load

} Surface atmospheric pressure load  
from meteorological center data  
(ECMWF and NCEP/CFSR )



## « Slichter mode »: Excitation mechanisms?

perturbs surface gravity: inertial 3% + **free-air displacement** 96% + potential perturbation 1% of the total effect (Dahlen and Tromp, 1998)



[Rosat & Rogister 2012]

→ small excitation amplitude expected

## « Slichter mode »: Excitation mechanisms?

$$\begin{aligned}\mathbf{s}(\mathbf{x}, t) = & \int_{-\infty}^t \int_V \mathbf{G}(\mathbf{x}, \mathbf{x}'; t - t') \cdot \mathbf{f}(\mathbf{x}', t') dV' dt' \\ & + \int_{-\infty}^t \int_S \mathbf{G}(\mathbf{r}, \mathbf{x}'; t - t') \cdot \mathbf{t}(\mathbf{x}', t') d\Sigma' dt'\end{aligned}$$

$$\mathbf{G}(\mathbf{x}, \mathbf{x}'; t) = \Re \sum_k (i\omega_k)^{-1} \mathbf{s}_k(\mathbf{x}) \mathbf{s}_k(\mathbf{x}') e^{i\omega_k t}, \text{ for } t \geq 0$$

# « Slichter mode »: Excitation mechanisms?

[Rosat 2007]

Event	Chile 1	Chile 2	Chile 1+2	Alaska	Bolivia	Peru	Andaman-Sumatra	Maule-Chile	Tohoku
Date	1960	1960	1960	1964	1994	2001	<b>2004</b>	2010	2011
$M_w$	9.5	9.6	9.8	9.2	8.2	8.4	<b>9.3<sup>1</sup></b>	8.8	9.1
<i>Reference for the source model</i>	<i>Kanamori and Cipar (1974)</i>			<i>Kanamori (1970)</i>					<i>Global CMT*</i>
Surface gravity effect in <b>nGal</b> ( $= 10^{-2} \text{ nm/s}^2$ )									
Smith (1976)	0.94	1.2	-	0.58	-	-	-		
Crossley (1992)	0.724	0.835	<b>1.52</b>	0.34	0.02 <sup>2</sup>	-	-		
Rosat (2007)	0.656	0.853	<b>1.51</b>	0.19	0.007	0.010	<b>0.29</b>	0.095	0.145

<sup>1</sup> Stein and Okal (2005)

<sup>2</sup> personal communication

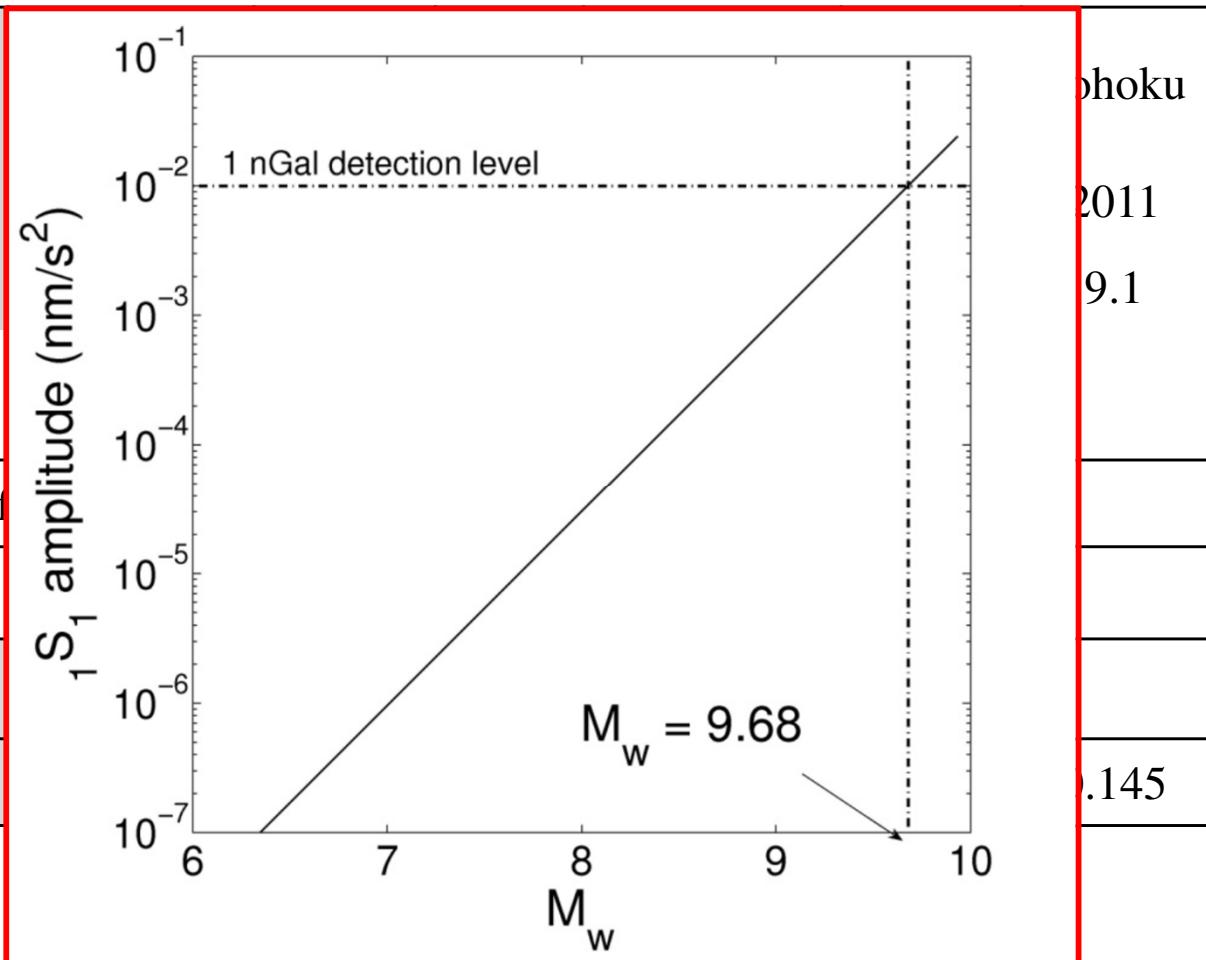
( $\leftrightarrow$ surface  
displ.  $\sim 1 \mu\text{m}$ )

# « Slichter mode »: Excitation mechanisms?

Event	Chile 1	Chile 2	Chile 1+2		phoku
Date	1960	1960	1960		2011
$M_w$	9.5	9.6	9.8		9.1
<i>Reference for the source model</i>	<i>Kanamori and Cipar (1974)</i>				
	Surf				
Smith (1976)	0.94	1.2	-		
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Vertical dip-slip  $M_w > 9.7 \rightarrow A > 1 \text{ nGal}$

[Rosat 2007]

## « Slichter mode »: Excitation mechanisms?

### Meteoroid impact

Location	Date	Diameter (m)	Density (kg/m <sup>3</sup> )	$M_w$	$\Delta g$ (nm/s <sup>2</sup> )
Ries Crater Germany	$15.1 \pm 0.1$ My BP	1500	2700 (rock)	7.4	$3.9 \cdot 10^{-6}$
Rochechouart France	$214 \pm 8$ My BP	1500	3350 (stony-iron)	7.5	$4.9 \cdot 10^{-6}$
Chesapeake Bay USA	$35.5 \pm 0.3$ My BP	2300	2700 (rock)	7.8	$1.4 \cdot 10^{-5}$
Chicxulub Mexico	$65 \pm 0.05$ My BP	17500	2700 (rock)	9.6	$6.7 \cdot 10^{-3}$

[Rosat & Rogister 2012]

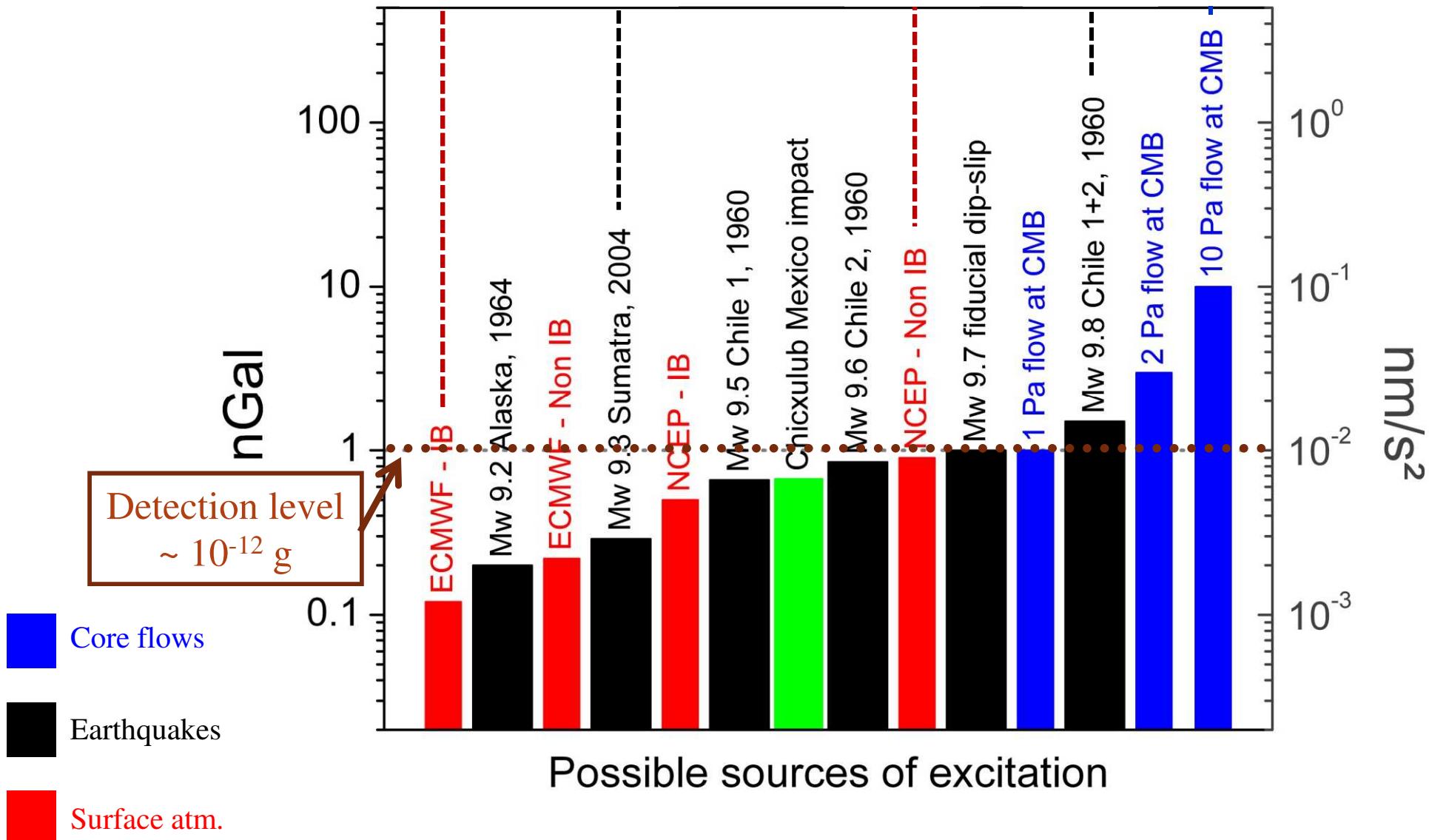
# Collision with a meteoroid (seismic impact)

Location	Date	Diameter (m)	Seismic efficiency $10^{-5} < k_s < 10^{-3}$ <i>(Schultz and Gault, 1975)</i>	$\sigma^2$
Ries Crater Germany	$15.1 \pm 0.1$ My BP	1500	$10^{-6}$	
Rochechouart France	$214 \pm 8$ My BP	1500	$10^{-6}$	
Chesapeake Bay USA	$35.5 \pm 0.3$ My BP	2300	$2700$ (rock) <u><math>M_w = 10.2</math></u> $= 6.7 \cdot 10^{-2} \text{ nm/s}^2$	$7.8 \cdot 10^{-5}$
Chicxulub Mexico	$65 \pm 0.05$ My BP	17500	$2700$ (rock) <u><math>9.6 \cdot 10^{-3}</math></u>	$1.4 \cdot 10^{-5}$

[Rosat & Rogister 2012]

# « Slichter mode »: Maximum surface amplitudes

surface displacement: 0.5  $\mu\text{m}$       1  $\mu\text{m}$       3  $\mu\text{m}$       5  $\mu\text{m}$       0.05 mm  
IC displacement: 0.5 mm      1 mm      3 mm      5 mm      50 mm



# « Slichter mode »: Maximum surface amplitudes

### surface displacement:

0.5 μm

1 μm

3 μm

5 μm

0.05 mm

## IC displacement:

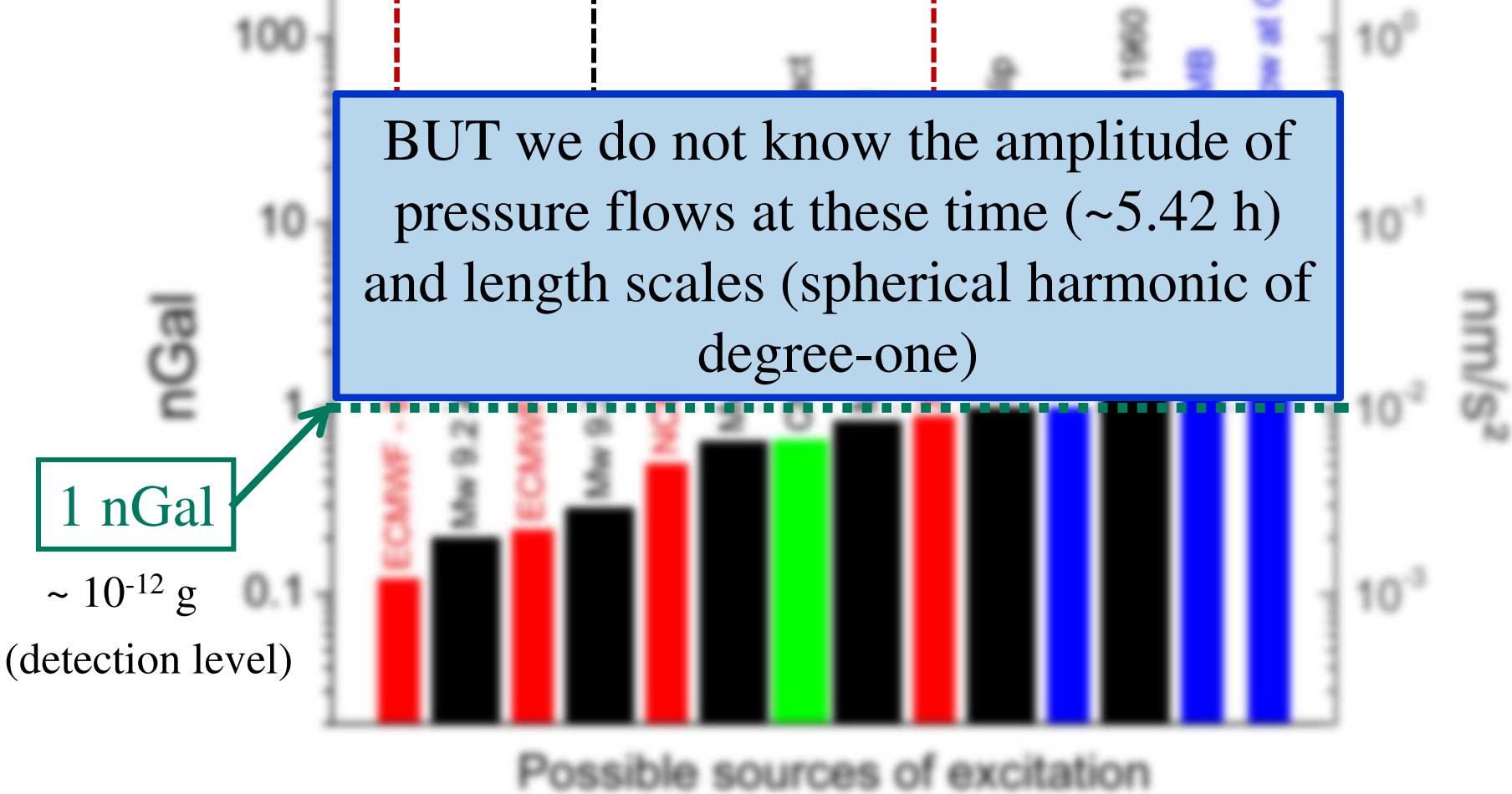
0.5 mm

1 mm

3 mm

5 mm

50 mm



→ Maximum surface gravity effect for known sources < 1 nGal

## « Slichter mode »: summary

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- Maximum surface gravity effect for known sources < 1 nGal, the limit of detection by current instruments (Superconducting Gravimeters)
- Largest excitation amplitudes are reached for pressure flow acting at the core boundaries but actual flow amplitudes at such time-scales are unknown
- **Effect of phase transformations** needs to be revisited
- Coupling of Slichter modes with gravito-inertial modes for given Brunt-Väisälä frequency values (Rogister & Valette (2009))
- Hope: new instruments (gradiometers) developed to detect GWs...

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Slichter modes → Still a challenge!!!



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# Back up slides

# The Brunt-Väisälä frequency

Describes the role of **Archimedean force** in a stratified fluid.

$$N^2(r) = -g(r) \left[ \frac{\dot{\rho}(r)}{\rho(r)} + \frac{\rho(r)g(r)}{\kappa(r)} \right]$$

$N$  = angular frequency to which a particle will oscillate (or not) (adiabatically) when displaced from equilibrium.

- $N^2 > 0$  : **stable stratification**  $\rightarrow$  **gravity wave** (the larger  $N^2$ , the larger the feedback force, the faster the oscillation)
- $N^2 = 0$  : **neutral** stratification  $\rightarrow$  a displaced particle stays at its position ( $N^2=0$  for seismic and inertial modes)
- $N^2 < 0$  : **unstable** stratification  $\rightarrow$  a displaced particle will go away  $\rightarrow$  no oscillation but convection is possible

# The core, a multi-disciplinary approach

## Geomagnetic field

(secular variations)

(space/ground observatories)

*Crustal  
magnetic field*

## Geodesy/Gravimetry

(Earth's rotation by VLBI, GNSS surface deformation, space/ground gravity changes)

## Seismology

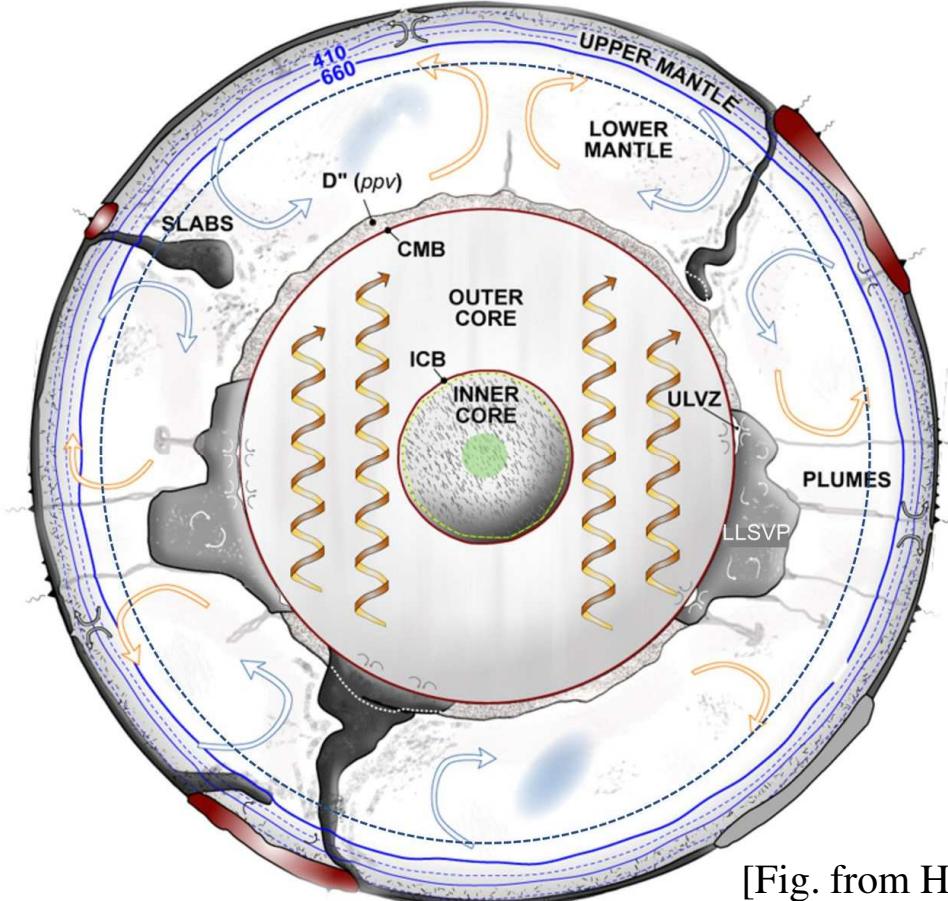
(seismic waves, seismic modes)

*Surficial layer processes  
(atmosphere, oceans, hydrosphere)*

*Crust, mantle  
attenuation*

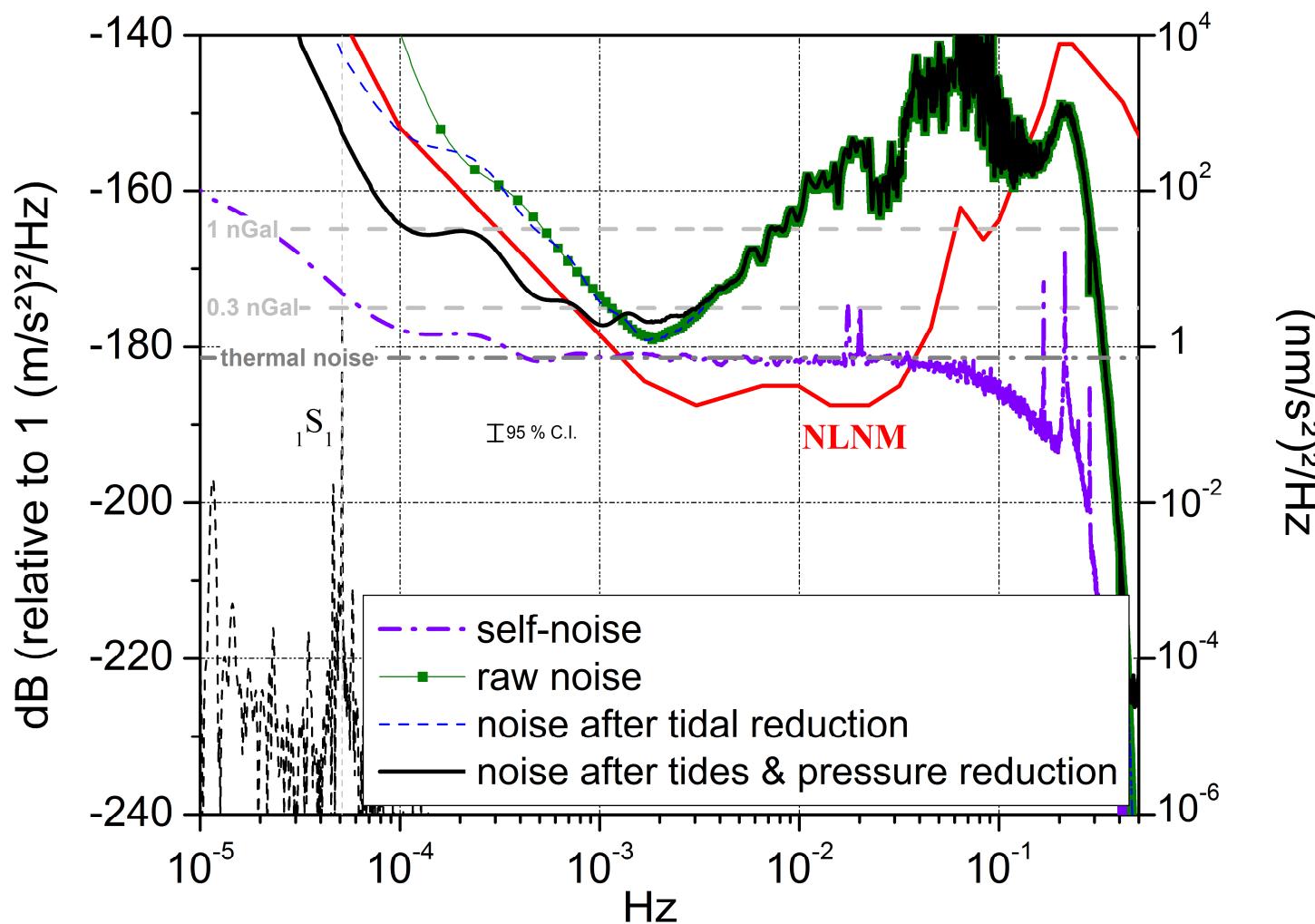
Lab. experiment  
Mineral physics  
Geochemistry

Geodynamo /  
convection / MHD  
numerical modeling



[Fig. from Heron 2018; Garnero et al. 2005]

## « Slichter mode »: limit of detection of gravimetric signal on Earth



Stacking of 10 SGs with similar low noise level would increase the SNR by a factor 3 in amplitude (10 dB)...

# Surface atmospheric load

$$\Delta g(r, \theta, \phi; t) = \frac{r_s^2 U(r)}{i\nu} [U(r_s)g_0 + P(r_s)]$$

$$[\int_{-\infty}^t e^{i\nu t'} (\sigma_{10}(t') \cos \theta + \sigma_{11}^c(t') \sin \theta \cos \phi + \sigma_{11}^s(t') \sin \theta \sin \phi) dt']$$

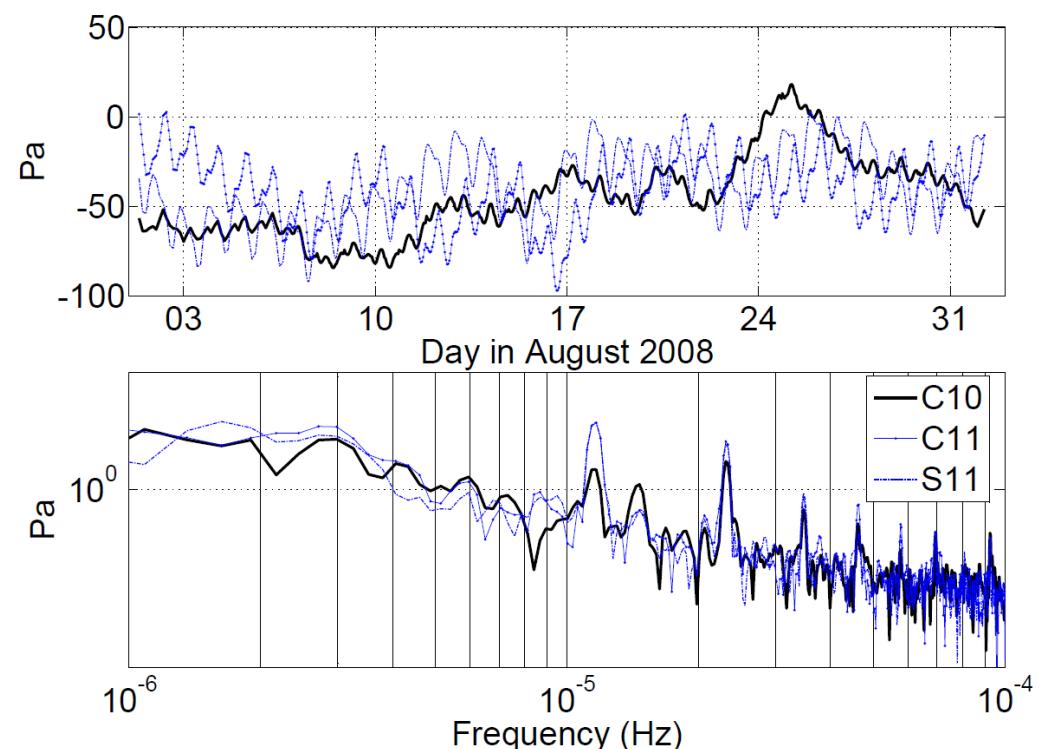
$$[-\omega^2 U(r_s) + \frac{2}{r_s} g_0 U(r_s) + \frac{2}{r_s} P(r_s)].$$

inertial
free-air
potential

August 2008: hourly ECMWF (European Centre for Medium-Range Weather Forecasts) data available in the frame of the CONT08 intensive VLBI measurements (usually 3 h temporal resolution)

[Rosat et al. (2014) PEPI]

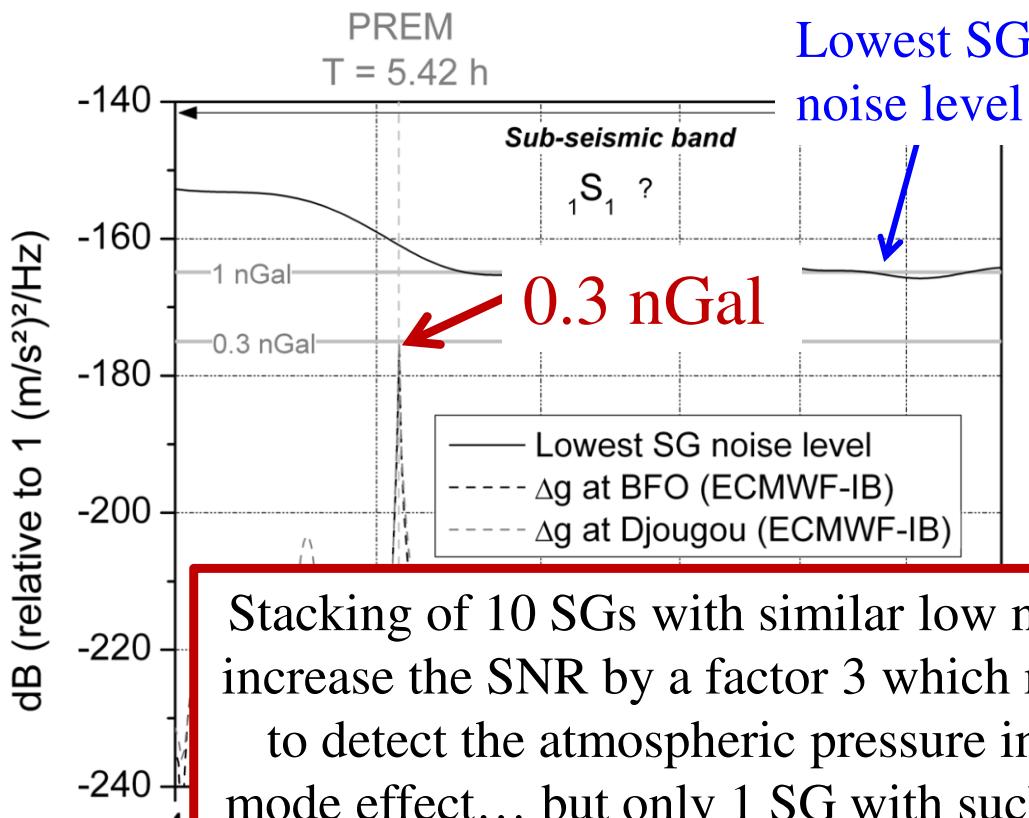
Degree-one surface load (from international meteorological center)



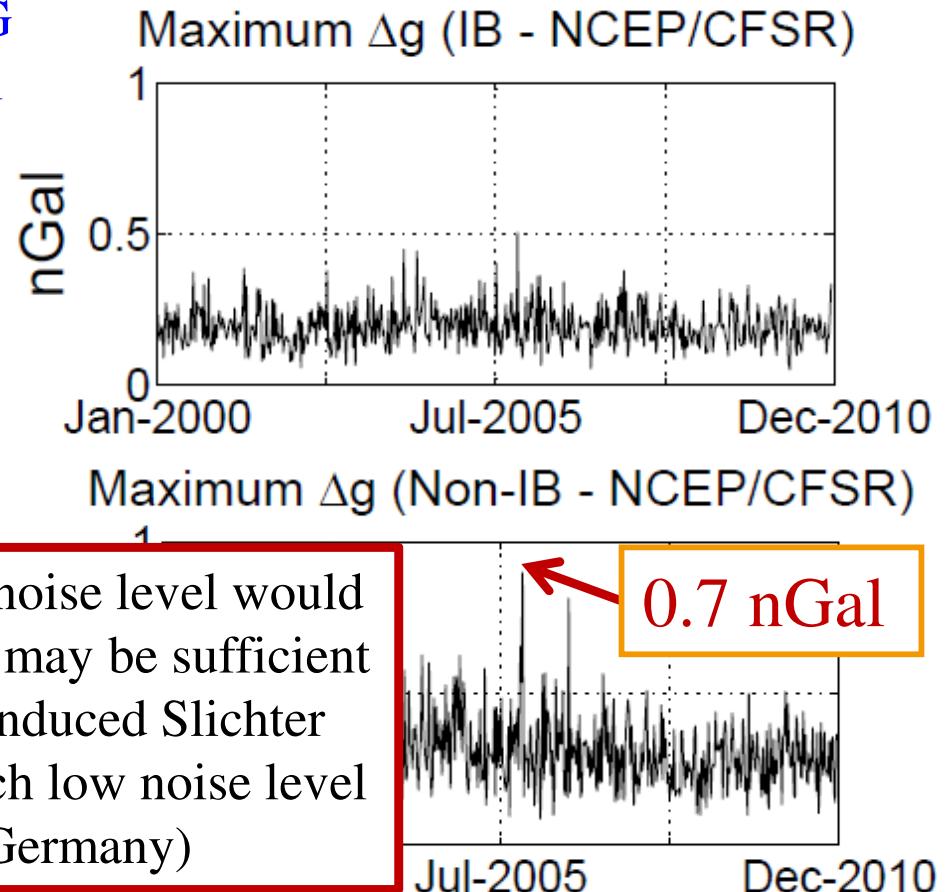
Time-varying degree-one ECMWF pressure field for an IB ocean response during August 2008

# Surface atmospheric load

Ex: ECMWF



Ex: NCEP/CFSR



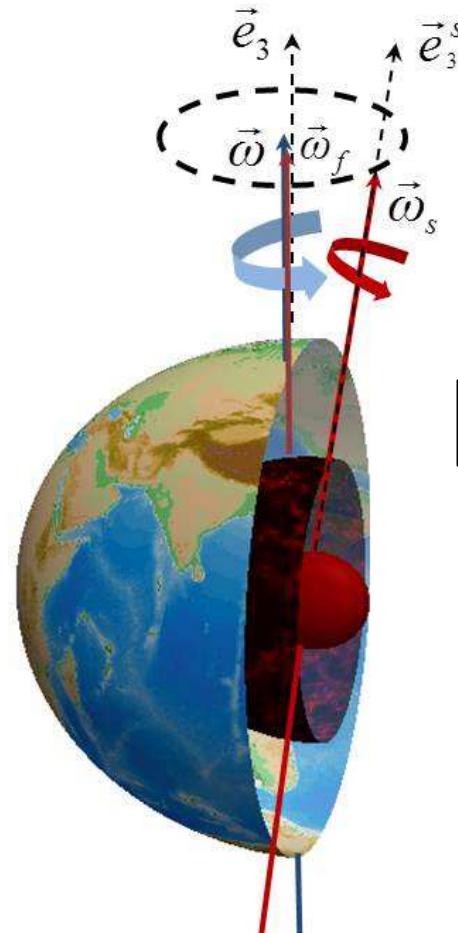
## Forcing:

- Hourly degree-one ECMWF atmospheric pressure field during August 2008;
- Hourly degree-one NCEP/CFSR from 2000 until 2011.

Response of the oceans: inverted (IB) and a non-inverted barometer (NIB).

[Rosat et al. (2014) PEPI]

# Free Inner Core Nutation (FICN)



IC angular momentum  $\sim 1/1400$  global Earth

**Never clearly detected**

Neither the mode nor its resonance (by tidal forcing)

$\sim$  a few tens of  $\mu\text{as}$  ( $\sim 0.3 \text{ mm}$ )  $\rightarrow \Delta g \sim 0.1 \text{ nGal}$  ( $\sim 10^{-13} \text{ g}$ )

## Motivation

- Ellipticity of ICB?
- Density jump at ICB?
- Magnetic field at ICB?
- Viscosity at ICB?
- Topographic coupling effects at ICB?

## Free Inner Core Nutation

$T \approx [400 - 1000]$  sid. days?

$Q \approx [400 - 500]?$

Prograde in space



# Free Core Nutation (FCN) & Free Inner Core Nutation (FICN)

FICN  
resonance?

Frequency of forcing  
(diurnal tide)

$$T_g(\sigma) = \delta_{ref} + \frac{N_{2,g}}{\sigma' - S_2} + \frac{N_{3,g}}{\sigma' - S_3}$$

FCN  
parameters

FICN  
parameters



Bayesian inversion

