Introduction to core convection

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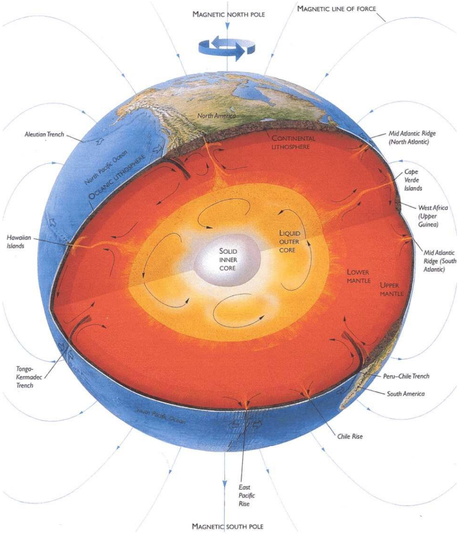
Overview

1. Evidence for convection in the core
2. Driving forces
3. Physical properties of the core
4. Governing equations for convection
5. So many waves... geostrophy and turbulence
6. Convection in the inner core

1. Evidence for convection in the core

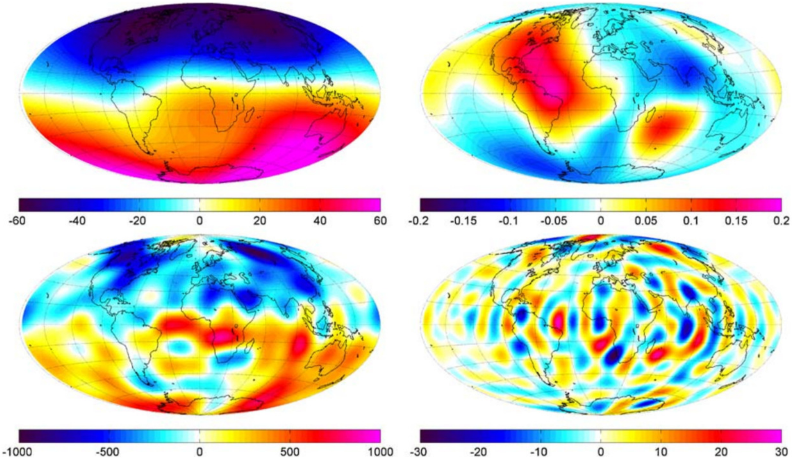


- 1906, Richard Oldham, existence of the core
- 1913, Beno Gutenberg, CMB within 1%
- 1919, Joseph Larmor, "dynamo effect" for the Sun (and Earth)
- 1926, Harold Jeffreys, liquid core
- 1946, Walter Elsasser, Earth's dynamo

Geomagnetism

- Back in 1633, Henry Gellibrand discovers magnetic variation
- 1906, Bernard Brunhes discovers magnetic reversals
- 1949, Louis Néel discovers (anti)ferromagnetism
- 1600 – present days, measurements of declination (naval traffic, magnetic observatories, satellite data)

Magnetic data

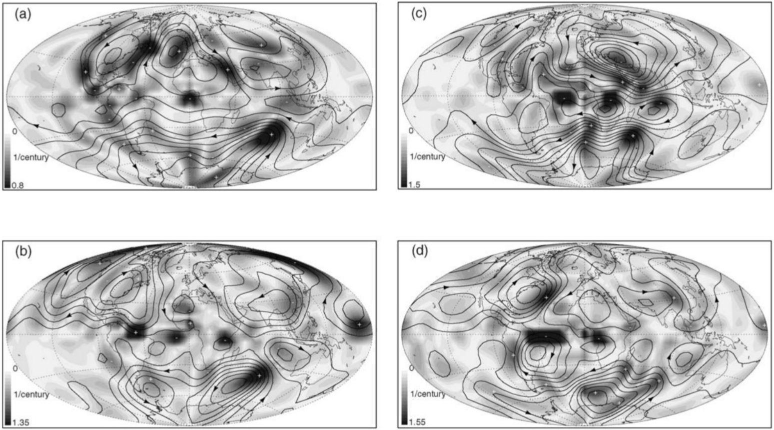


CHAOS-2s, Olsen *et al.*, **179**:3, JGI, 2009

Velocity at the top of the core

$$\frac{\partial B_r}{\partial t} = -\nabla_s \cdot (B_r \mathrm{u}_s)$$

~ 20 km/year



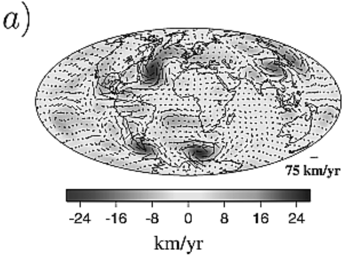
a) 1840 b) 1890 c) 1940 d) 1990

Amit and Olson, PEPI, **155**, 2006

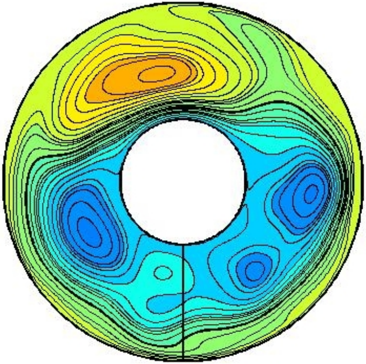
(Tangential)-Geostrophic inversion

A. Pais and D. Jault, GJI 2008

N. Gillet, A. Pais and D. Jault, GJI 2009



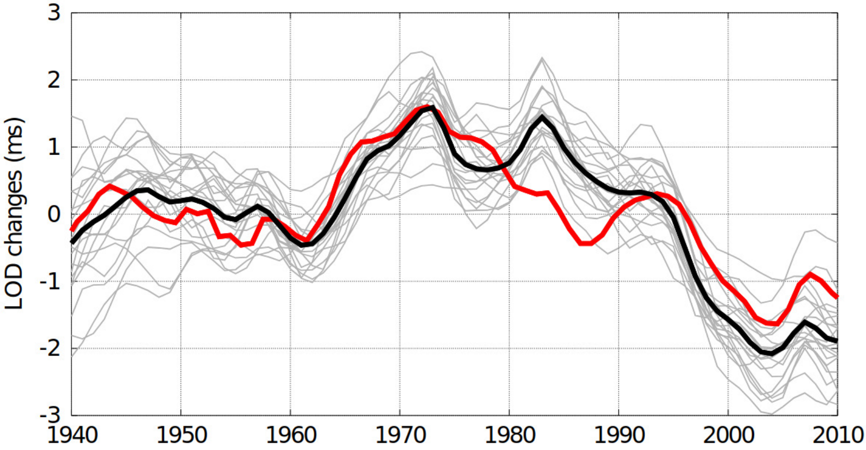
Pais *et al.*, JGR, **109**, 2004
Epoch 1980



Asymmetric gyre

Length Of Day

Average geostrophic (geomagnetic) velocity variations versus LOD



Gillet *et al.*, JGR, **120**:6, 2015

Seismic radial profiles

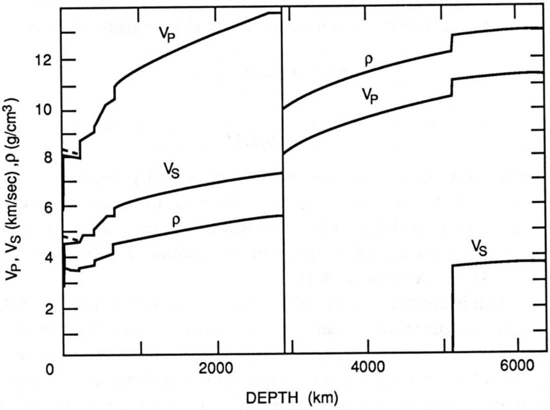
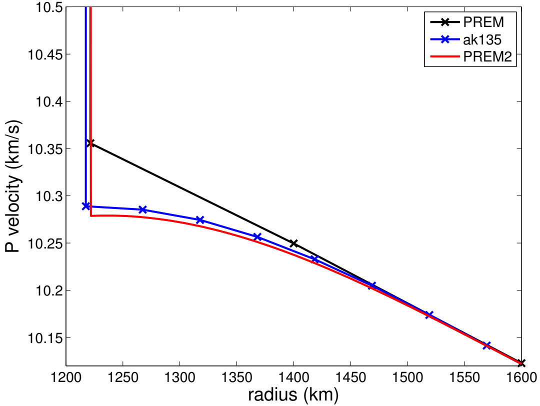


Figure 7.2. PREM model: Seismic velocities and density profile (after Dziewonski and Anderson 1981).

PREM radial model: isentropic outer core by construction

Seismic radial profiles: bottom dense layer

Other models, not bounded by the isentropic hypothesis



Kennett *et al.*, *GJI*, **122**, 1995, ak135

Song and Helmberger, *JGR*, **100**, 1995, PREM2

And at the top of the core?

No definite seismic observation, but a few reasons in favour of its existence

- light elements from the inner core crystallization might end up there
- light elements from exsolution might end up there
- diffusion of light elements from the mantle

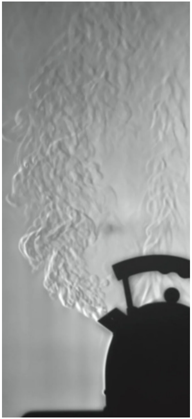
and a few against its existence

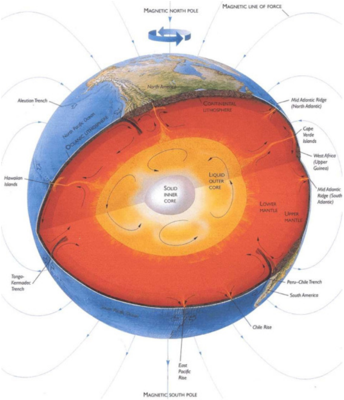
- existence of 'fast' geomagnetic variations (jerks)
- upwellings/downwellings in core velocity models

2. Driving forces

- Thermal cooling from the top
- Composition buoyancy at the ICB
- Precessional forcing
- Compositional buoyancy from exsolution
- Radiogenic heating?

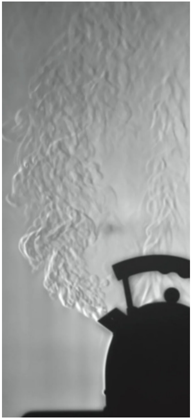
Thermal convection

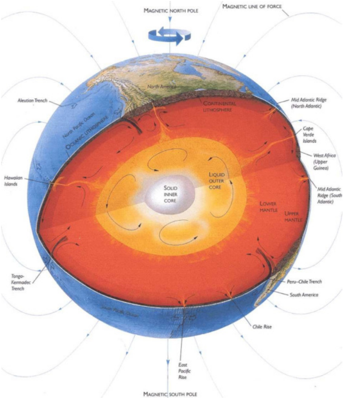




- Flux imposed by the mantle
- Geothermal flux 47 TW ± 2
- of which 5 to 15 TW extracted from the core

Thermal convection





- Flux imposed by the mantle
- Geothermal flux 47 TW ± 2
- of which 5 to 15 TW extracted from the core: start a debate here...

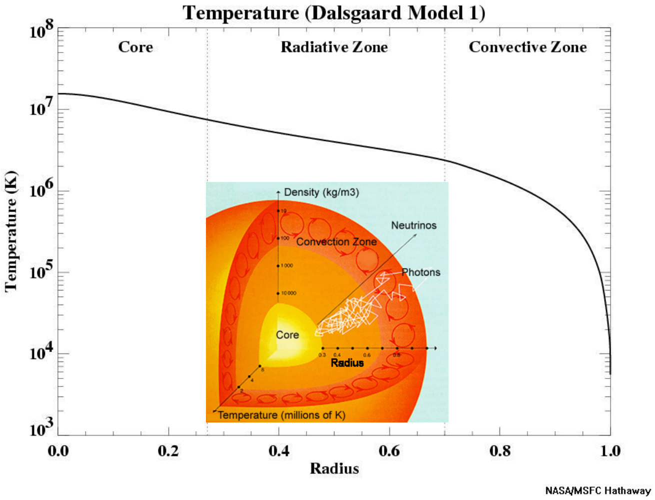
Compressible convection

- Entropy mixing
- In a hydrostatic pressure gradient
- Produces an adiabatic (isentropic) temperature gradient
- Because iron is a good thermal conductor
- Generates a significant conduction flux

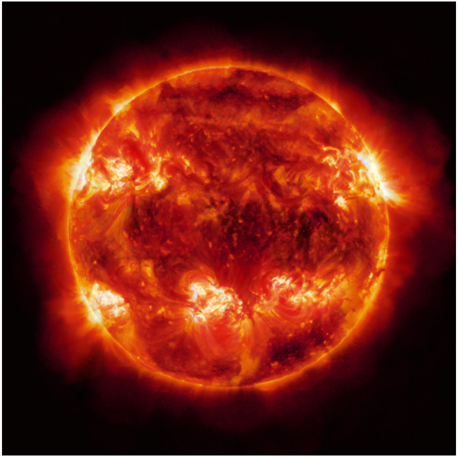
Compressible convection

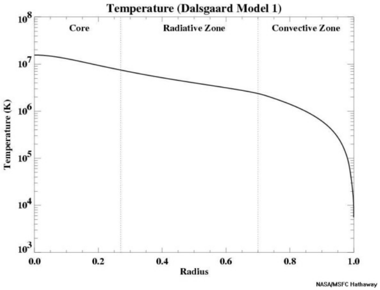
- Only a heat flux exceeding the flux conducted along the adiabat can generate convection (Schwarzschild 1906, Adams-Williamson 1923, Jeffreys 1930)
- A smaller flux is conducted along a stable temperature gradient: no motion
- That condition may depend on depth and epoch

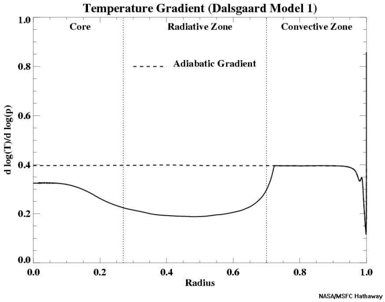
Profil de température à l'intérieur du soleil



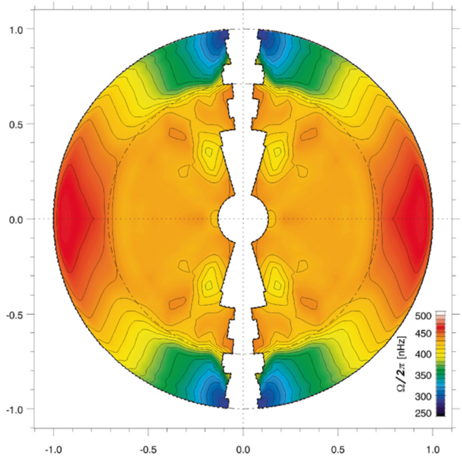
Convection dans le soleil







Rotation différentielle du soleil



Compositional convection

The density jump across the ICB (590 kg m^{-3} in PREM, 850 kg m^{-3} for normal modes and ICB reflexion) is due to

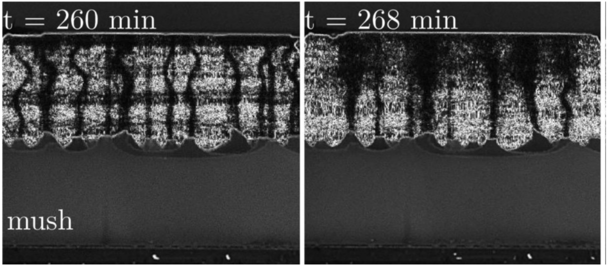
- phase change (latent heat) 200 to 240 kg m^{-3}
- fractionation of light elements (H,Si,0,S,C,...)

Generally considered as the strongest source of convection...

liquid core: 6 to 10% less dense than liquid iron
inner core: 2 to 3% less dense than solid

Compositional convection in experiments

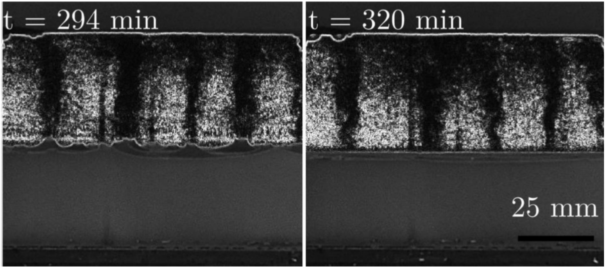
Crystallization of an ammonium chloride solution NH_4Cl



Huguet *et al.*, **204**, GJI, 2016

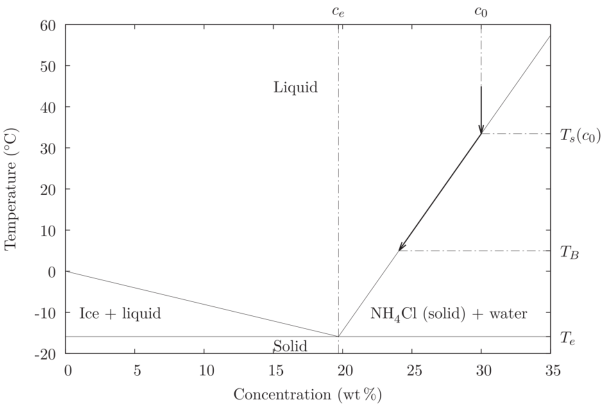
Compositional convection in experiments

And melting...



Huguet *et al.*, **204**, GJI, 2016

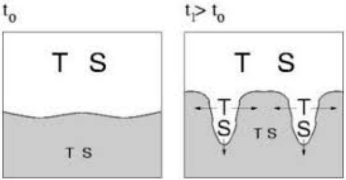
Phase diagram NH_4Cl in water

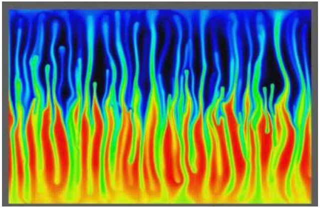


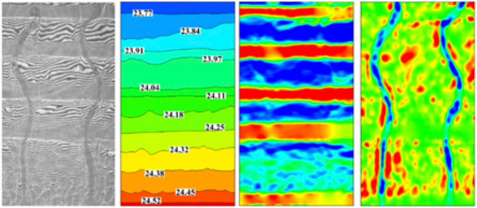
Specificity of compositional convection

- Its strength is directly linked to that of thermal convection: because thermal cooling is directly responsible for inner core crystallization
- However, molecular diffusivity of species is much smaller than thermal diffusivity, by three to four orders of magnitude $Le = \kappa/D \sim 10^4$
- Origin of double-diffusive effects

Double diffusion



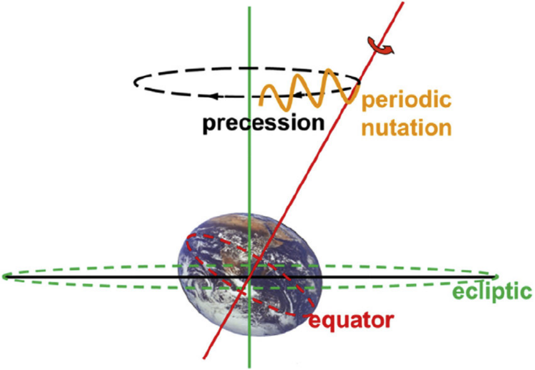




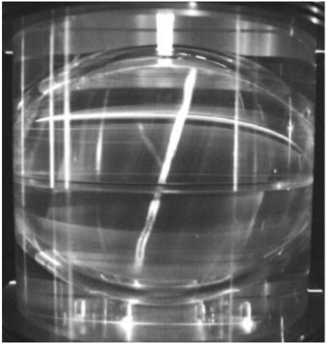
Precession

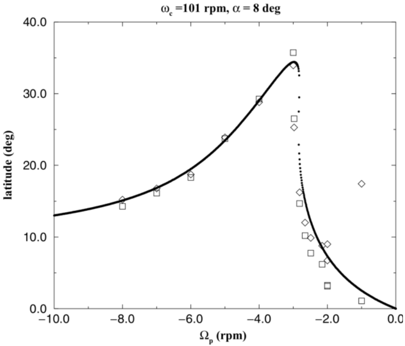
Equatorial buldge leads to precession: 25772 years, 23.5 °

Precession of Moon's orbit generates nutations: 18.6 years, 9.2 ″



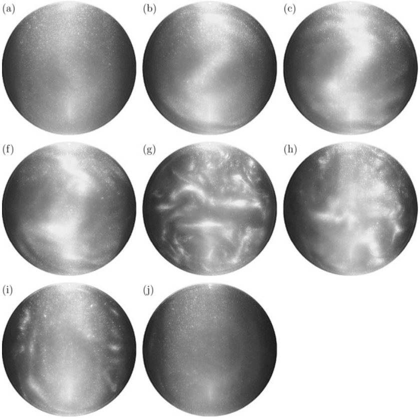
Effect of precession





Noir *et al.*, **154**, GJI, 2003

Precession and mode interactions

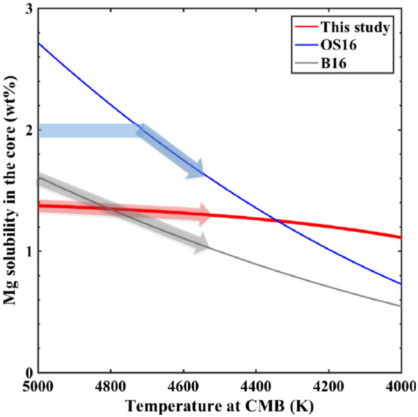


Goto *et al.*, Phys. Fluids, **26**, 2014

Precession

- Effect of (small) viscosity
- Many studies in the last 10-15 years
- Potential to dissipate energy
- Limited amount of rotational energy, $2\ 10^{29}$ J, *i.e.* 7 TW for 1 Ga
- most of it goes probably in ocean tidal dissipation
- but may be enough to power the geodynamo

Exsolution



O'Rourke and Stevenson, Nature, **529**, 2016
Du *et al.*, GRL, **44**:22, 2017

3. Physical properties of the liquid core

- density
- heat capacity
- thermal expansion
- viscosity
- thermal conductivity
- electrical conductivity

and what is the temperature of the core?

Density

PREM is enough

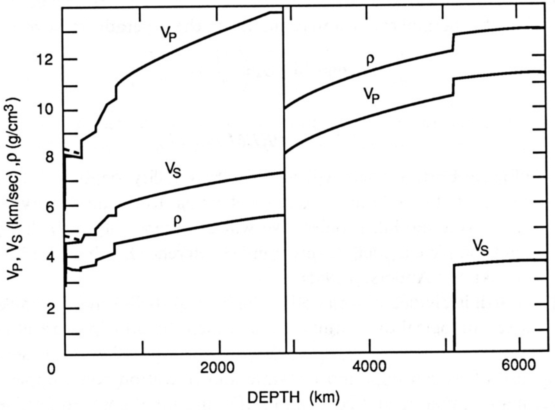
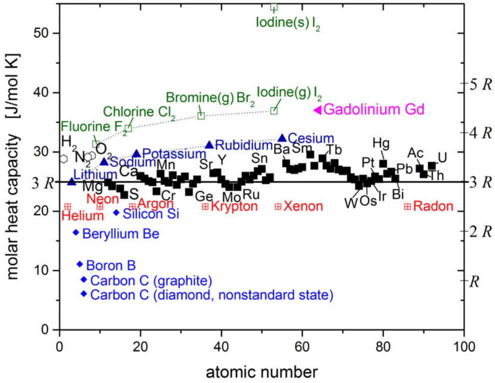


Figure 7.2. PREM model: Seismic velocities and density profile (after Dziewonski and Anderson 1981).

Heat capacity

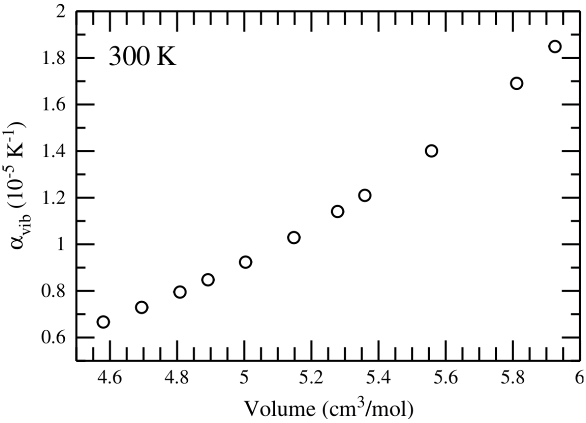
The law of Dulong and Petit is enough: "six calories per gram atom", *i.e.* $C_p = 3R$ with $R = 8.314$ J $K^{-1}mol^{-1}$ the ideal gas constant



Moreover, c_p and c_v are very similar in condensed matter.

Coefficient of thermal expansion

$$\alpha = \frac{1}{v} \frac{\partial v}{\partial T}\bigg|_P$$



Murphy *et al.*, JGR, **118**:5, 2013

$$\alpha \sim \rho^{-n}$$

with $n = 3$ to $n = 4$ (Vinet or Murnaghan)

Viscosity

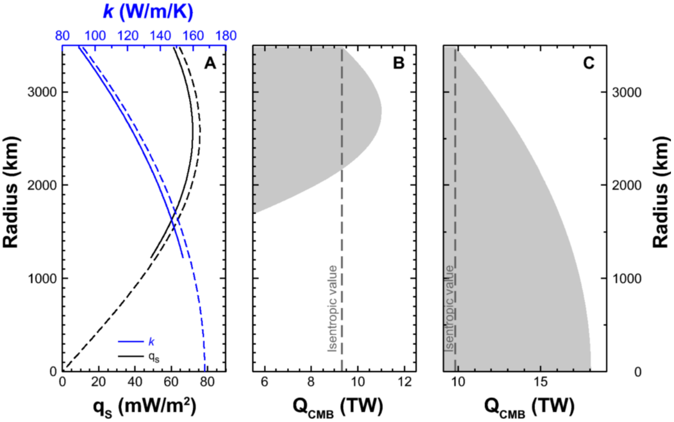
General agreement for small values $\eta \simeq 10^{-3}$ Pa s
Except Smylie in a few papers on Slichter modes...

Thermal conductivity

Its value was considered to be around 30 W $m^{-1}K^{-1}$ from the 70's (Matassov PhD thesis, shock experiments, 1977; Stacey and Davis, *Physics of the Earth*, CUP, 2008), but re-evaluated in the last 10 years.

Thermal conductivity

A crucial, determining, parameter for convection, because thermal conduction along the adiabatic gradient is a significant part of the total radial heat flux.

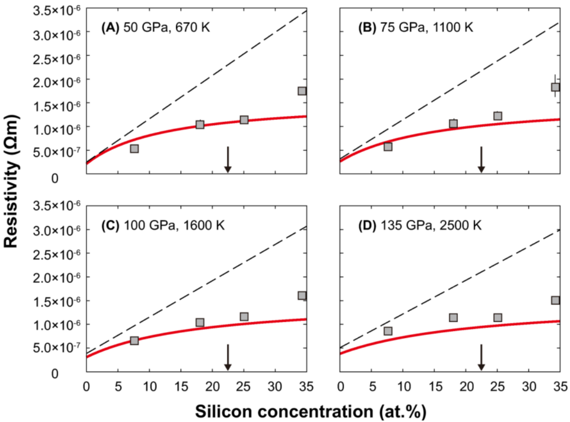


Gomi *et al.*, PEPI, **224**, 2013

Electrical conductivity

- Important for dynamo action!
- Wiedemann–Franz law: $\frac{k}{\sigma} = LT$, where $L = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 \simeq 2.44\ 10^{-8}$ W Ω K^{-2} is the Lorenz number
- was hence also revised upward recently

Electrical conductivity



Gomi *et al.*, PEPI, **224**, 2013

$$\sigma \simeq 10^6\ \Omega^{-1}\mathrm{m}^{-1}$$

4. Governing equations for convection

- Convection
- Navier-Stokes with Coriolis and Lorentz forces
- Induction equation
- Adiabatic gradient and superadiabatic forcing

Convection equations

Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{v}) = 0$$

Momentum conservation (Navier-Stokes)

$$\rho \frac{\mathrm{D}\mathrm{v}}{\mathrm{D}t} = -\nabla P + \rho \mathrm{g} + \nabla \cdot \tau$$

τ : deviatoric stress tensor

Energy conservation (or entropy equation)

$$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = \dot{\varepsilon} : \tau + \nabla \cdot (k \nabla T)$$

and an equation of state EoS: $T = T(\rho, s)$, $P = P(\rho, s)$

$$\dot{\varepsilon}_{ij} = 1/2\,(\partial_i v_j + \partial_j v_i)$$

$$\tau_{ij} = 2\eta \left(\dot{\varepsilon}_{ij} - \frac{1}{3}(\nabla \cdot \mathrm{v})\delta_{ij} \right)$$

The 'adiabatic' gradient

In a well-mixed fluid, entropy is uniform. In addition, the fluid is often close to hydrostatic equilibrium.

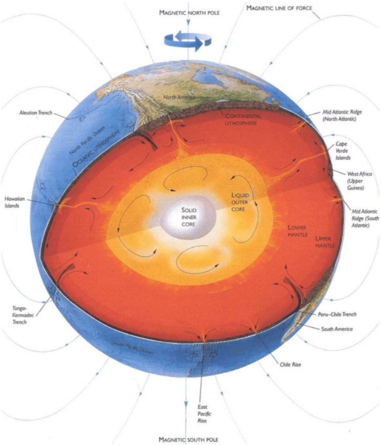
$$\boxed{\mathrm{d}s = \left.\frac{\partial s}{\partial P}\right|_T \mathrm{d}P + \left.\frac{\partial s}{\partial T}\right|_P \mathrm{d}T = 0}$$

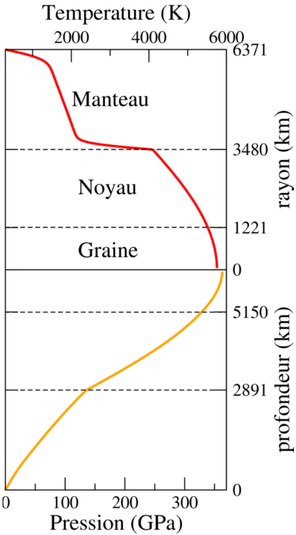
$$\left.\frac{\partial s}{\partial P}\right|_T = -\frac{\alpha}{\rho} \qquad \mathrm{d}G = -s\mathrm{d}T + \frac{1}{\rho}\mathrm{d}P$$

$$\left.\frac{\partial s}{\partial T}\right|_P = \frac{c_p}{T} \qquad \mathrm{d}H = T\mathrm{d}s + \frac{1}{\rho}\mathrm{d}P = c_p\mathrm{d}T + \left.\frac{\partial H}{\partial P}\right|_T \mathrm{d}P$$

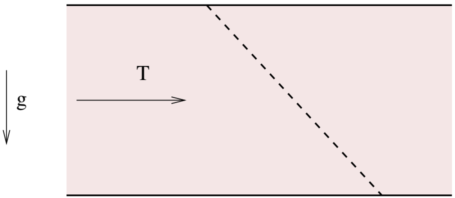
$$\text{Hence} \quad \left.\frac{\partial T}{\partial P}\right|_s = \frac{\alpha T}{\rho c_p} \quad \longrightarrow \quad \boxed{\frac{\mathrm{d}T}{\mathrm{d}z} = -\frac{\alpha g T}{c_p}}$$

Earth's convection



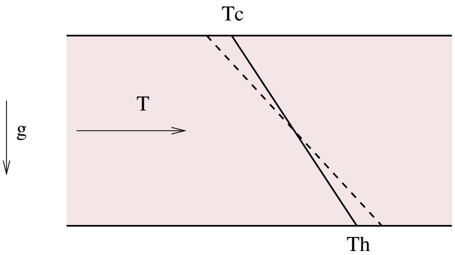


Adiabatic (isentropic) temperature profile



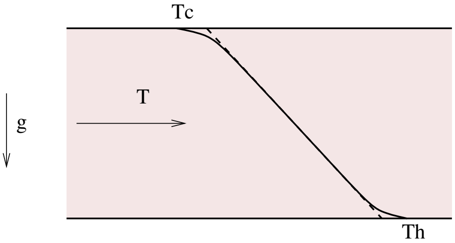
T_a (dashed line) is the 'adiabatic' temperature profile. We also define ρ_a, P_a, c_{pa}, α_a...

Stable temperature profile



$\Delta T \leq \Delta T_a \quad \longrightarrow \quad$ no convection

Unstable temperature profile



$\Delta T > \Delta T_a \quad \longrightarrow \quad$ convection

Anelastic (liquid) approximation

continuity $$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathsf{v})$$ $$\boxed{\nabla \cdot (\rho_a \mathsf{v}) = 0}$$

Navier-Stokes $$\rho \frac{\mathrm{D}\mathsf{v}}{\mathrm{D}t} = -\nabla P + \rho \mathsf{g} + \nabla \cdot \tau$$

$$\boxed{\rho_a \frac{\mathrm{D}\mathsf{v}}{\mathrm{D}t} = -\rho_a \nabla \left(\frac{P}{\rho_a} \right) + \rho_a \alpha_a T' g \hat{\mathsf{e}}_z + \nabla \cdot \tau}$$

entropy $$\rho T \frac{\mathrm{D}s}{\mathrm{D}t} = \dot{\epsilon} : \tau - \nabla \cdot \phi$$

$$\boxed{\rho_a \frac{\mathrm{D}(c_{pa} T')}{\mathrm{D}t} = -\rho_a \alpha_a g \, T' v_z + \dot{\epsilon} : \tau + \nabla \cdot (k \nabla (T' + T_a))}$$

Heat flux

The heat flux is the sum of the conduction flux along the adiabat and the extra flux driven by the superadiabatic temperature difference ΔT_{sa}

$$\Phi = k\frac{\Delta T_a}{H} + Nu\ k\frac{\Delta T_{sa}}{H}$$

where Nu is the resulting dimensionless heat flux, that must be a function of the dimensionless parameters defining the problem

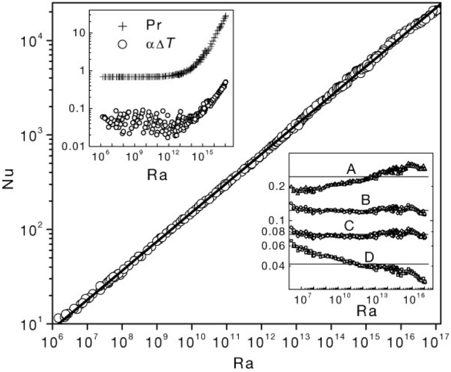
$$Ra = \frac{\alpha\, g\, \Delta T_{sa} H^3}{\nu \kappa}$$

$$Pr = \frac{\nu}{\kappa}$$

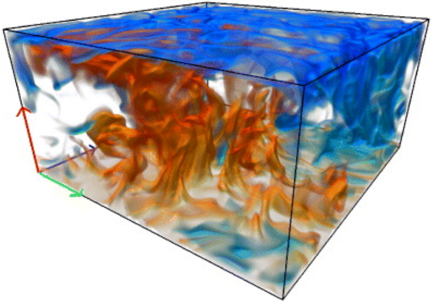
$$\mathcal{D} = \frac{\alpha\, g\, H}{c_p}$$

Rayleigh, Prandtl and Dissipation number

Heat flux



Niemela *et al.*, Nature, vol. 404, 2000



Lülff *et al.*, New J. of Phys., vol. 13, 2011

Heat flux

Assuming

- a heat flux conducted along the adiabat of 3 to 10 TW
- a superadiabatic flux of 1 to 10 TW
- vertical velocities of 10^{-4} m s^{-1}

Hence the convective heat flux $\rho c_p v \, \Delta T_{sa}$ can transfer easily 10 TW with

$$\Delta T_{sa} \sim 10^{-4} \ \mathrm{K}$$

Superpowers of core convection!

Then the Rayleigh number is

$$Ra = \frac{\alpha \, g \, \Delta T_{sa} H^3}{\nu \kappa} \sim 10^{23}$$

Coriolis forces

Navier-Stokes has an extra force term in a rotating frame of reference

$$\rho_a \frac{\mathrm{D}\mathsf{v}}{\mathrm{D}t} = -2\,\rho_a\,\Omega \times \mathsf{v} - \rho_a \nabla \left(\frac{P}{\rho_a} \right) + \rho_a \alpha_a T' g \hat{\mathsf{e}}_z + \nabla \cdot \tau$$

Electromagnetic coupling

$$\rho_a \frac{\mathrm{D}\mathrm{v}}{\mathrm{D}t} = -2\,\rho_a\,\Omega \times \mathrm{v} + \mathrm{j} \times \mathrm{B} - \rho_a \nabla \left(\frac{P}{\rho_a} \right) + \rho_a \alpha_a T' g \hat{\mathrm{e}}_z + \nabla \cdot \tau$$

where $\mathrm{j} = \nabla \times \mathrm{B}/\mu_0$ and the magnetic field obeys the induction equation

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times (\mathrm{v} \times \mathrm{B}) + \nabla \times \left(\frac{1}{\mu_0 \sigma} \nabla \times \mathrm{B} \right)$$

and $\nabla \cdot \mathrm{B} = 0$

5. So many waves...geostrophy and turbulence

- inertial waves
- geostrophy
- Alfvén waves
- torsional waves
- big gyre, LOD variations
- turbulence

Inertial waves (general method)

Ignore viscosity, non-linear inertia, Lorentz forces, buoyancy, density variations...

$$\frac{\partial \mathrm{v}}{\partial t} = -2\,\Omega \times \mathrm{v} - \nabla P$$

We can take the curl to remove pressure

$$\frac{\partial (\nabla \times \mathrm{v})}{\partial t} = -2\,\nabla \times (\Omega \times \mathrm{v})$$

Expand the possible velocity fields as planar waves

$$\mathrm{v}(\mathrm{x}, t) = \hat{\mathrm{v}}(\mathrm{k}, t) e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)}$$

Inertial waves (general method)

After substitution in Navier-Stokes

$$\omega \mathrm{k} \times \hat{\mathrm{v}} = 2\, i\, (\mathrm{k} \cdot \Omega)\hat{\mathrm{v}}$$

Taking another $\mathrm{k}\times$ product leads to

$$\omega \mathrm{k} \times \mathrm{k} \times \hat{\mathrm{v}} = -4(\mathrm{k} \cdot \Omega)^2 \hat{\mathrm{v}}$$

We do not forget the continuity equation, $\nabla \cdot \mathrm{v} = 0$

$$\mathrm{k} \cdot \hat{\mathrm{v}} = 0$$

Then the wave equation becomes

$$-\omega^2 k^2 \hat{\mathrm{v}} = -4(\mathrm{k} \cdot \Omega)^2 \hat{\mathrm{v}}$$

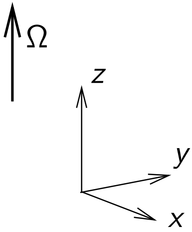
Dispersion relation

$$\omega^2 k^2 = 4(\mathrm{k} \cdot \Omega)^2 \qquad \text{hence} \qquad \boxed{\omega = \pm 2 \frac{\mathrm{k} \cdot \Omega}{k}}$$

Phase and group velocities

Phase velocity ω/k, or in 3D $\omega/k\,\hat{\mathbf{e}}_k$

$$\left(2\Omega\frac{k_x k_z}{k^3}, 2\Omega\frac{k_y k_z}{k^3}, 2\Omega\frac{k_z^2}{k^3}\right)$$



Group velocity $\partial\omega/\partial k$, or in 3D $\partial\omega/\partial k_i$

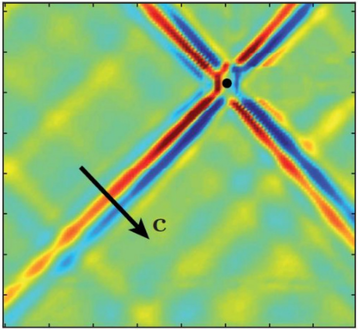
$$\left(-2\Omega\frac{k_x k_z}{k^3}, -2\Omega\frac{k_x k_z}{k^3}, 2\Omega\frac{k^2 - k_z^2}{k^3}\right)$$

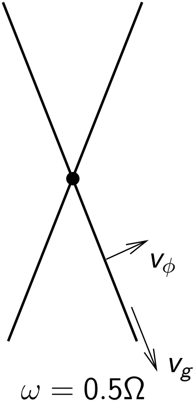
Properties

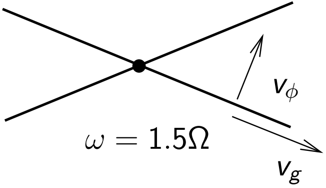
- phase and group velocities are orthogonal
- maximum phase velocity in the direction Ω
- minimum in perpendicular directions, but max group velocity along Ω

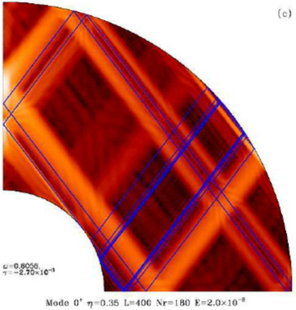
Visualization

Cortet *et al.*, Phys. Fluids, **22**, 2010









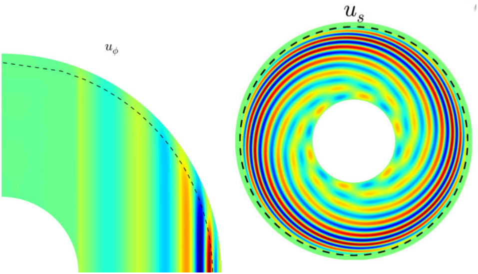
Rieutord *et al.*, JFM, **435**, 2001

Geostrophy

When a flow is indepedent of z, it escapes inertial waves and there can be an equilibrium between Coriolis and pressure gradient. This is

geostrophic balance

- atmosphere
- oceans
- Jupiter, Saturn ?
- Earth's core ?



Vidal and Schaeffer, GJI, **202**:3, 2015

Alfvén waves

In a uniform magnetic field B_0 along z, the linear coupled equations of Navier-Stokes and induction lead to

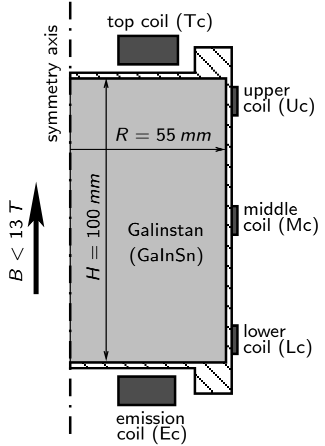
$$\frac{\partial^2 v}{\partial t^2} = \frac{B_0^2}{\rho\mu_0}\frac{\partial^2 v}{\partial z^2}$$

Alfvén waves propagate in the direction of B_0 and opposite direction, with a (phase) velocity

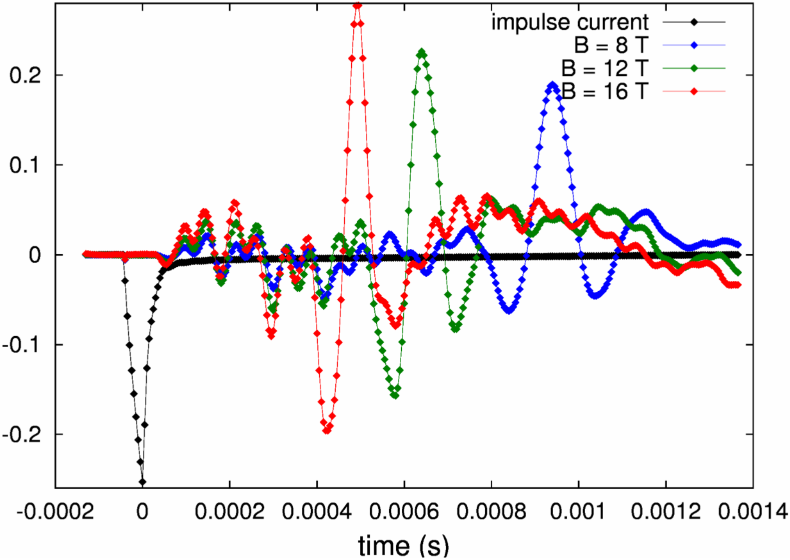
$$V_A = \frac{B_0}{\rho\mu_0}$$

which also the group velocity (no dispersion)
In the core, inertial and Alfvén waves are present!

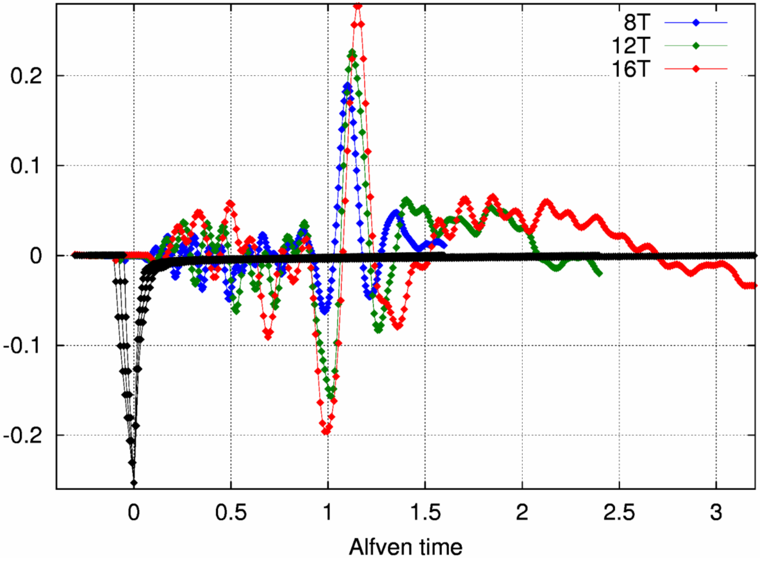
Experiments with gallium



Experiments with liquid sodium



Experiments with liquid sodium



Torsional waves, LOD

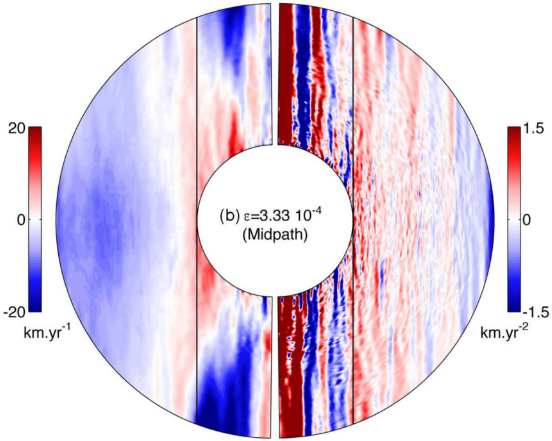
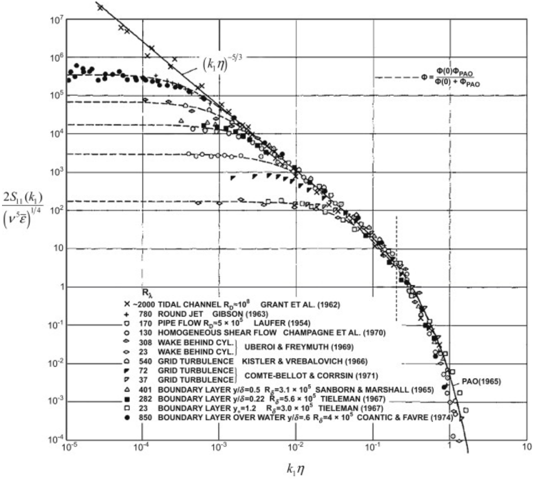


Figure 9. Meridional cuts of axisymmetric azimuthal (or zonal) flow snapshots (left, blue is westwards) and zonal flow acceleration snapshots (right), in the start-of-path (a) and Midpath model (b). The vertical black lines mark the axial cylinder tangent to the inner core (the tangent cylinder).

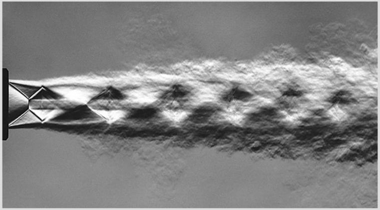
Aubert, GJI, **214**, 2018

Turbulence



$(k_1\eta)^{-5/3}$
$\Phi = \frac{\Phi(0)\Phi_{PAO}}{\Phi(0)+\Phi_{PAO}}$
$\frac{2S_{11}(k_1)}{(\nu^5\bar{\varepsilon})^{1/4}}$
$k_1\eta$
R_λ
~2000 TIDAL CHANNEL $R_D=10^8$ GRANT ET AL. (1962)
780 ROUND JET GIBSON (1963)
170 PIPE FLOW $R_D=5\times10^5$ LAUFER (1954)
130 HOMOGENEOUS SHEAR FLOW CHAMPAGNE ET AL. (1970)
308 WAKE BEHIND CYL.
23 WAKE BEHIND CYL.
UBEROI & FREYMUTH (1969)
540 GRID TURBULENCE KISTLER & VREBALOVICH (1966)
72 GRID TURBULENCE
37 GRID TURBULENCE
COMTE-BELLOT & CORRSIN (1971)
401 BOUNDARY LAYER $y/\delta=0.5$ $R_\delta=3.1\times10^5$ SANBORN & MARSHALL (1965)
282 BOUNDARY LAYER $y/\delta=0.22$ $R_\delta=5.6\times10^5$ TIELEMAN (1967)
23 BOUNDARY LAYER $y_*=1.2$ $R_\delta=3.0\times10^5$ TIELEMAN (1967)
850 BOUNDARY LAYER OVER WATER $y/\delta=.6$ $R_\delta=4\times10^5$ COANTIC & FAVRE (1974)
PAO(1965)

Weak turbulence



The complex interactions between shock waves and expansion waves in an "overexpanded" supersonic jet. The flow is visualized by a schlierenlike differential interferogram.

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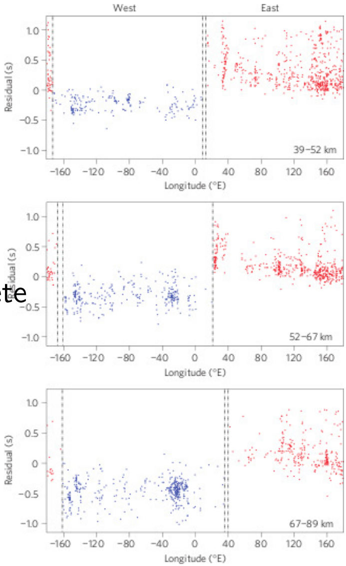
6. Convection in the inner core

- evidence for inner core dynamics
- convection model
- top/down mantle effect

Another seismic observation

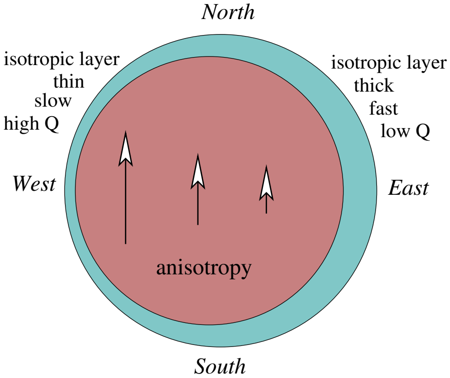


Waszek *et al.*,
Nature Geoscience **4**:4, 2011

There exists a sharp, discrete difference between eastern and western hemispheres

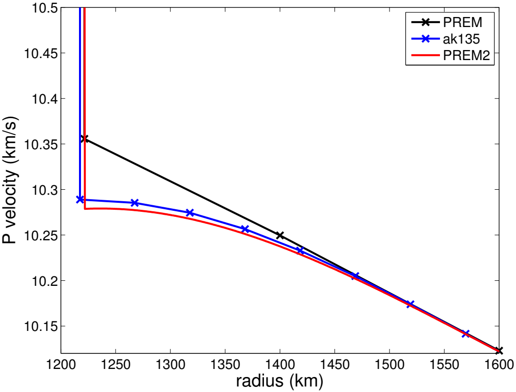
Hemispheric asymmetry

Eastern and Western hemispheres have different surface properties
S. Tanaka and H. Hamaguchi, JGR 1997

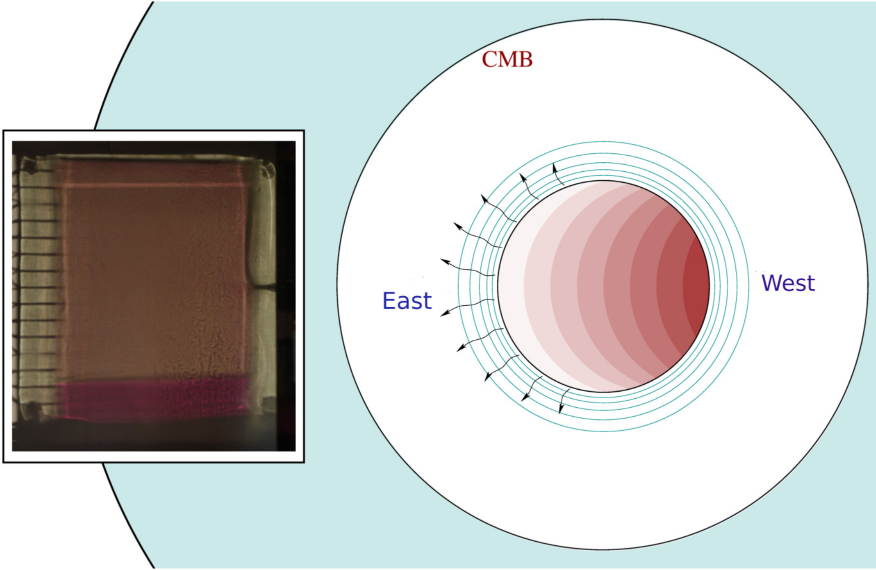


A slow layer above the ICB

- Adiabatic gradient throughout A.M. Dziewonski and D.L. Anderson, *PEPI*, **25**, 1981 PREM
- First observation A. Souriau and G. Poupinet, *GRL*, **18**, 1991
- Global P-velocity model B.L.N. Kennett, E.R. Engdahl and R. Buland, *GJI*, **122**, 1995 ak135
- Earth's core P-velocity model X. Song and D.V.A. Helmberger, *JGR*, **100**, 1995 PREM2



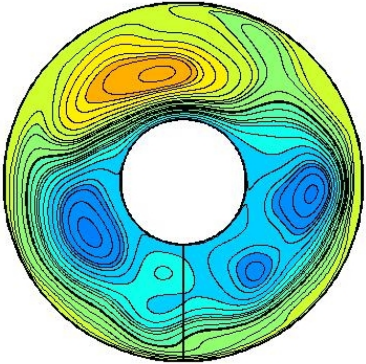
Asymmetric forcing for the outer core



Asymmetric gyre in the outer core

A. Pais and D. Jault, GJI 2008

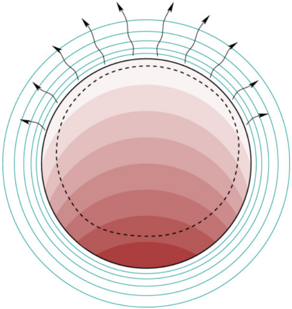
N. Gillet, A. Pais and D. Jault, GJI 2009

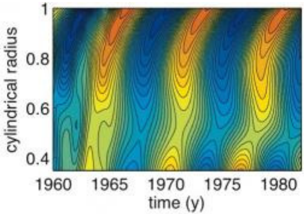


Buoyancy

Coriolis

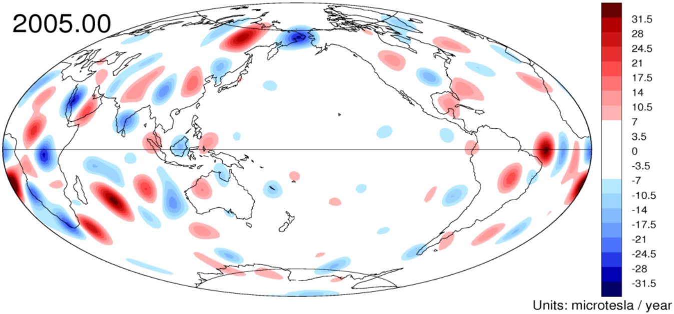
Lorentz





Torsional oscillations Gillet *et al.*, Nature, **465**, 2010

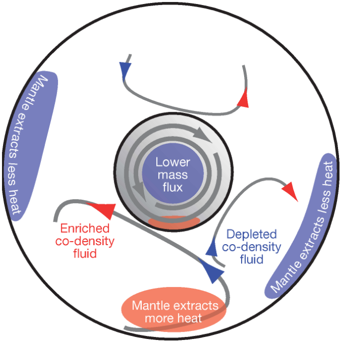
Heterogeneity of magnetic secular variations



from Andy Jackson

Alternative view: role of the mantle

Aubert *et al.*, Nature **454**, 2008
Control of the outer core dynamics
by mantle heterogeneity



Gubbins *et al.*, Nature **473**, 2011
suggest convection can locally
melt the ICB

A few thoughts

- we expect progress from computer power: large *Ra*, very different diffusivities
- we still need some experiments
- we need more thoughts on convection models: compressibility, (weak) turbulence models
- we need better knowledge of α, σ, k
- debate between internal or external origin of forces driving inner core convection