Experimental Thermal and Fluid Science 20 (1999) 19–24

Experimental Thermal and Fluid Science

www.elsevier.nl/locate/etfs

Quasi-2D MHD turbulent shear layers ☆

T. Alboussière [a], V. Uspenski [b], R. Moreau [c,*]

[a] Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK

[b] Institute of Mechanics, University of Moscow, Michurinskii Pr. 1, 119899 Moscow, Russian Federation

[c] Laboratory EPM-MADYLAM, ENSHM de Grenoble, BP 95, 38402 St-Martin d'Hères, France

Received 12 July 1998; received in revised form 12 January 1999; accepted 14 June 1999

Abstract

An experiment has been carried out to investigate the properties of 2D turbulence in a mercury layer in the presence of a steady magnetic field. Visualizations of the vortices were possible via the observation of the image of a regular grid mesh reflected on the free surface. In parallel, quantitative measurements of the velocity were recorded with the use of potential probes located at the bottom of the fluid layer. The basic flow consists in a shear flow. The mean part of the turbulent flow and the spectrum of its fluctuations are characterized. The global influence of turbulence on both momentum and heat transfers is examined. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: MHD turbulence; Quasi-2D MHD; Shear layers

1. Introduction

MHD turbulence exhibits quite specific properties associated with its tendency to become 2D [1,2]. But, so far, most of the experiments performed with electrically conducting fluids in the presence of a uniform magnetic field were devoted to homogeneous turbulence (except within the thin Hartmann layers), so that the need of knowledge and understanding of the behaviour of quasi-2D turbulence in MHD shear layers is still enormous. As a matter of fact, those layers, which develop along the walls parallel to the magnetic field or from any kind of singularity (either geometrical, such as a sharp angle, or electrical, such as a jump in the electrical conductivity) in the direction of the magnetic field, are of special interest because all sorts of transport phenomena take place there. The first observation, purely qualitative, of the turbulent character of these layers is due to Lehnert [3] and the first quantitative description of their properties is due to Kljukin and Kolesnikov [4]. However, to our knowledge, nothing is known on the transport of a scalar quantity, such as heat, by this kind of turbulent flow, and its dynamics is still far from being well understood. Of course, none of the available models for ordinary turbulent shear flows can be applied to this quasi-2D turbulence. The purpose of this paper is therefore to present new experimental results which might be used as a reference to be compared with predictions of adequate modeling. Another paper in this conference (Dumont et al. [5]) presents a attempt to model this 2D turbulence by using a maximum entropy theory, initially proposed by Robert and Sommeria [6]. The equipment has been realized with a special care to insure the stability and uniformity of all the control parameters and to select a combination of them which guarantees the 2D character of this turbulence. And a special effort has been made to improve the accuracy of the diagnostic techniques, based on the measurements of temperatures and electric potential differences.

2. Apparatus and measurement techniques

The working fluid is a layer of mercury. It is located in a horizontal circular cell (internal diameter 22 cm) above an electrically insulating bottom plate. The upper boundary may be either a free surface, allowing to observe the deformation produced by the turbulent eddies, or an insulating horizontal plate, to privilege the 2D [1] when the quantitative measurements are performed. This cell is located in a vertical coil which generates a uniform vertical magnetic field. Both the

☆ Presented at the 11th Symposium on Turbulent Shear Flows.

* Corresponding author. Tel.: +33-476-82-52-06; fax: +33-476-82-52-49.

E-mail address: rene.moreau@polycnrs-gre.fr (R. Moreau)

PII: S0894-1777(99)00023-0

depth of the fluid layer and the strength of the magnetic field are adjusted in such a way that the typical time scale to establish a good 2D, $\tau_{2D} = (\rho H^2)/(\sigma B_0^2 l^2)$ (ρ stands for density, σ for the electrical conductivity, B_0 for the magnetic field, H for the fluid depth and l for the typical horizontal length scale of a turbulent eddy) is much smaller than the typical turnover time $\tau_{tu} = l/u$ (u stands for a typical velocity scale). Typically, with $H = 9.4$ mm and $B_0 = 0.17$ T, which are the values selected for all the results presented in this paper, $\tau_{2D} \sim 10^{-2}$ s whereas $\tau_{tu} \sim 1$ s, so that 2D is well achieved. At this stage, it may be noticed that, because the energy transfer is directed towards the large scales in 2D turbulence, this condition is well satisfied as soon as it is satisfied for the length scale of the forcing mechanism.

On the bottom plate a set of 292 small electrodes are located on a circle of radius 9.3 cm and make an anode from which the electric current I arrives, whereas the electrically conducting external vertical plate collects this electric current. Because the Hartmann number Ha is significantly larger than one (42 with the chosen values), this radial electric current is passing through the Hartmann layers which behave like an electric motor and drive the fluid in rotation. From the theory of the Hartmann layer (see [2, p. 124–131]), if the flow were laminar, the core velocity between two Hartmann layers should be $V_\theta = I/(2\pi r\sqrt{\sigma\rho\nu})$, where r denotes the distance from the center of the cell and ν the kinematic viscosity. This means that a shear is necessarily established between the rotating external annulus and the central core region which stays at rest in a laminar regime. The typical maximum velocity which is expected is of the order of 40 cm/s. And, if the flow was laminar, the typical thickness of the circular shear layer above the ring of electrodes, would be of the order of 0.15 cm. But as expected, this circular shear (or mixing) layer is unstable. 2D columnar eddies aligned with the applied magnetic field develop, with an initial wave length which corresponds to the most linearly unstable disturbances (close to 2 cm), and interact. These structures remain 2D since any 3D disturbance induces an electric current and a damping Lorentz force (2D eddies do not induce any current). Therefore, an energy transfer towards the large scales feeds larger and larger vortices, until the largest allowed scale which is the cells radius (11 cm). This arrangement and the 2D shear flow are sketched in Fig. 1.

Besides, a central part of the bottom plate (radius 3.65 cm) made of a thick copper piece (but electrically insulated from the mercury) may be heated from below at a given power (0 or 8 W, in this paper) whereas the external vertical wall is maintained at a given and precisely controlled temperature (with fluctuations less than 0.02°C). This allows to study how a 2D turbulent shear flow transports a scalar quantity such as heat, as well as angular momentum (Fig. 1). On the contrary, if the central part of the bottom plate was heated without passing any electric current, the apparatus would give the possibility to study an axisymmetric convective flow such as the one sketched on Fig. 2. This is not the case in this paper which focuses on the behaviour of the 2D shear flow with or without heating. It has been checked that the convective flow is always masked by the electrically driven shear flow.

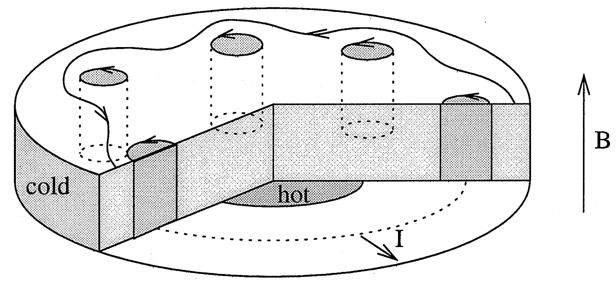

Fig. 1. Qualitative pattern of the electrically driven 2D turbulent shear flow (the control parameter I is the intensity of the electric current passing from the ring of electrodes to the external vertical wall).

The temperature measurements are easy to perform with thermocouples. To measure the two velocity components in the radial and azimuthal directions, electric potential sensors were used. Between two such sensors close enough to each other, the electric potential difference is equal to the product of the velocity component in the direction perpendicular to the sensors line times the magnetic field B_0 and the distance between these sensors. We have nevertheless developed a significant originality in these techniques, since our sensors are in the same time thermocouples and velocity probes. Each sensor is made of two small wires, one made of platinum and the other of constantan, both of diameter 0.2 mm. A cylinder of alumina whose external diameter is 0.9 mm containing two holes of diameter 0.2 mm is used to electrically insulate the two wires and to maintain them in place. Each probe, located at the bottom boundary of the fluid, behaves like a Pt–Cst thermocouple and provides the local temperature. Measuring the electric potential between the Pt surfaces of two neighbouring probes gives the local and instantaneous velocity. Platinum was chosen as one of these materials precisely because it has almost the same absolute thermoelectric power as mercury, in order to minimize to error due to the addition of a thermoelectric Pt–Hg contribution to the MHD electrical potential difference. All the results presented in this paper have been obtained with such

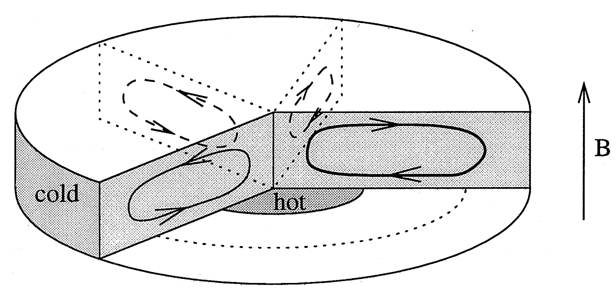

Fig. 2. Qualitative pattern of the buoyant axisymmetric flow generated by heating the center of the bottom plate.

sensors inserted in the insulating bottom plate. Their total number is 88. They are located, either on some radial lines (to provide the angular velocity component), or on some circles (to provide the radial velocity component).

A flow visualization may also be realized when a free surface is present, either using tracers moving with the free surface of mercury, looking at the image of a regular grid reflected by the mercury free surface. Each eddy capable to locally depress the plane surface yields a disturbance of the image. It can be proved for small deformations, that a linear dependency of the area of the mesh elements exists with respect to the Laplacian of the surface height: the area decreases for a depression (light rays are focussed) and increases for a hump. On the video shown during the oral presentation of this paper, it is clear that the initial instability of the mixing layer (made of about 30 periodic eddies of typical length scale of the order of 2 cm) rapidly feeds an inverse cascade. At high values of the electric current (of the order of 20 or 30 A) only a small number of large coherent structures (2 or 3 structures with length scales of the order of 10 cm) moving around the cell at a velocity smaller than the maximum fluid velocity by about one order of magnitude are present.

3. Results and interpretation

The mean velocity profiles are shown in Fig. 3. It is remarkable that, whereas the wall shear layer thickness is almost the same as in a laminar regime (estimated at about 2 mm), the free shear layer thickness is increased by about one order of magnitude. It also noticeable that the maximum angular velocity is nearly half that predicted for the laminar regime. For instance, the expression $I/(4\pi r\sqrt{\sigma\rho\nu})$ predicts a velocity of 43 cm/s for $I = 20$ A, but the measured value is close to 23 cm/s. And, consistently, a significant angular momentum is present in the central part of the cell where no electric current is flowing within the Hartmann layers. This illustrates the turbulent transport of angular momentum by the turbulence. If one would wish to derive a thickness for the turbulent free shear layer from the maximum slope of the mean velocity profile, one would get values of the order of 3 cm (instead of 0.15 cm in a laminar regime). Nevertheless, the total angular momentum present in this flow remains significantly smaller than the value predicted for a purely 2D core flow bounded by classical Hartmann layers. The deficit may be explained by the influence of some Ekman pumping, which drives an inward radial flow within the Hartmann layers. By continuity, this yields an outward radial flow within the core. This 3D recirculation transports a fraction of the angular momentum toward the wall shear layer where it is dissipated. Very recent measurements performed under high magnetic field (up to 6 Tesla, not reported in this paper), demonstrate that, when this 3D recirculation is damped out, the predicted velocity within the rotating annulus $I/(4\pi r\sqrt{\sigma\rho\nu})$ is recovered.

In exactly the same conditions, the mean temperature profiles are shown in Fig. 4. At low electric current (0 or 1 A), the angular velocity of the rotating annulus is quite small. Similarly, the buoyant flow due to the heating is efficiently damped by the applied magnetic field. One may therefore consider that the temperature profiles measured at $I = 0$ and 1 A, which are indeed very close to each other, correspond to almost pure thermal conduction. This means that the observed temperature difference between the central part and the external wall (14.5 K) may be associated with a Nusselt number close to unity. When the electric current (and the velocity) increases to 10 A, the maximum temperature difference reduces to 5.2 K, which is the minimum value ever observed and corresponds to a Nusselt

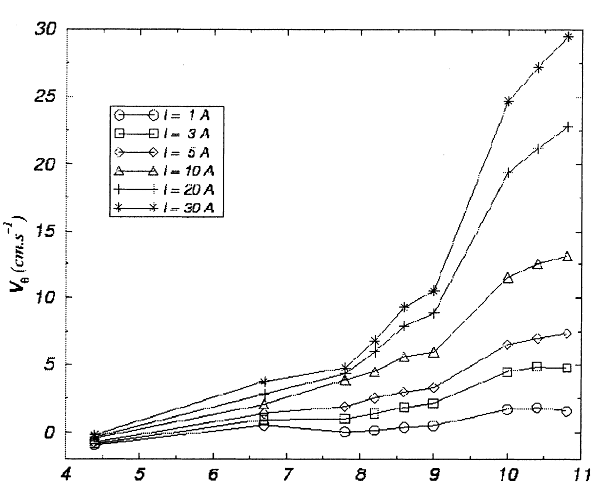

Fig. 3. Profiles of the mean angular velocity component for different values of the control parameter I ($B_0 = 0.17$ T).

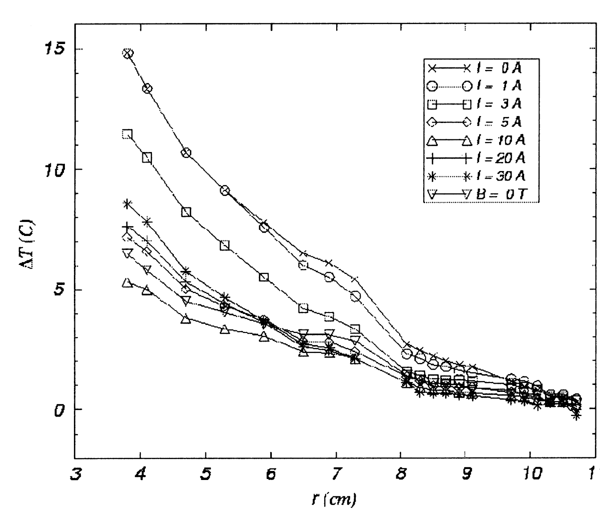

Fig. 4. Mean temperature profiles for different values of the control parameter I ($B_0 = 0.17$ T).

number of nearly 2.8. Then, if the electric current still increases to 30 A, the maximum temperature difference increases again to about 8.6 K, which corresponds to a Nusselt number of 1.7. This result, which would imply a non-monotonic variation of the heat flux across the shear layer versus the maximum velocity, has still to be checked and confirmed. Indeed, the thermal inertia of the system (30–50 min, due to pure conduction in the central part) is much higher than its mechanical inertia (a few minutes). It is therefore difficult to guarantee that the mean temperature profile is well established. Now, if we look more locally at the slopes of the mean temperature profiles, it seems clear that the slope remains significantly smaller in the free shear layer (between $r = 8$ cm and $r = 11$ cm) than in the central part of the cell where the mean velocity is small (but the turbulence intensity high). Precisely, Fig. 5 shows the profiles of the rms values of the fluctuations of the two velocity components V'_θ and V'_r. The first point to notice is the importance of the turbulence level: the ratio V'_θ/V_θ is of the order of 10–12% in the free shear layer and goes to values of the order of 35% in the central part. Indeed, the rms of the azimuthal velocity fluctuation is almost uniform all over the cell, whatever the value of the mean velocity. One may also notice that the rms of the radial velocity fluctuation is about half that of the angular fluctuation. And, finally, the almost linear increase of V'_r with the distance from the wall when we move from the external wall towards the centre, seems to be quite consistent with continuity.

Two typical energy spectra are shown in Fig. 6 (Fig. 6(a) for a moderate electric current $I = 3$ A and Fig. 6(b) for a high value of the electric current $I = 20$ A). Both figures show nice signatures of the large coherent structures illustrated on the video, in the forms of peaks in a range of small wave numbers (the tail of the spectra for k smaller than 0.3 cm^{-1}, only due to the calculation of the power spectral density, has no significance). The first peak corresponds to the largest length scale, namely the diameter of the cell. For $I = 3$ A, other individual peaks can be identified, corresponding to smaller structures (vortices). The spectra were produced from temporal records of the velocity and transformed in spatial spectra using the Taylor hypothesis. The peaks correspond to the distance between a vortex and itself (after one turn), to a vortex and the one before it, etc. About seven peaks can be seen. The smallest well-defined structure (distance between two neighbouring vortices) is about 3 cm. For $I = 20$ A, only three peaks can be identified. The smallest well-defined structure is about 7 cm. This shows clearly the increase of the size of the large coherent structures (or the reduction of their number) when the kinetic energy of the turbulence increases.

At larger wave numbers, but before those dominated by the noise, an inertial range seems to be well-defined, although it starts with a clear reminiscence of the peaks. In both cases ($I = 3$ A and $I = 20$ A) a power law fits well with the measurements, but the exponent should be close to $-5/3$ at low electric current (or low kinetic energy) and close to -3 at high electric current (or high kinetic energy). These results are in agreement with previous measurements. Sommeria et al. [7] had also measured $k^{-5/3}$ spectra in a 2D homogeneous turbulence dominated by an inverse energy cascade. And many authors, in Riga (see Lielausis, [8]), in Purdue (Brouillette and Lykoudis, [9]; Gardner and Lykoudis, [10]) and in Grenoble (Alemany et al. [11]) had previously reported k^{-3} energy spectra in MHD turbulence, but in quite different conditions (see a discussion of these conditions in Moreau, [2]). It is important to discuss the origin of the k^{-3} slope for the energy spectrum. In ordinary 2D turbulence, with a well characterized forcing at a wave number $k*$, such a spectrum for $k > k*$ results from the enstrophy cascade, whereas an inverse energy cascade yields a $k^{-5/3}$ spectrum for $k < k*$. But it should be noticed that MHD flows have their own reason to display a k^{-3} spectrum, even in the range characterized by an inverse energy cascade. This is due to the energy dissipation within the Hartmann layers. It appears

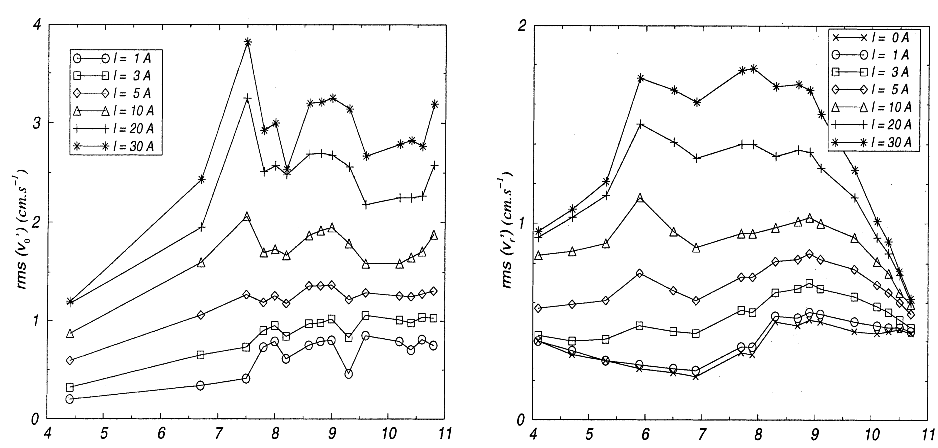

Fig. 5. Profiles of the rms of the velocity fluctuations for different values of the control parameter $I(B_0 = 0.17$ T); (a) angular velocity component, (b) radial velocity component.

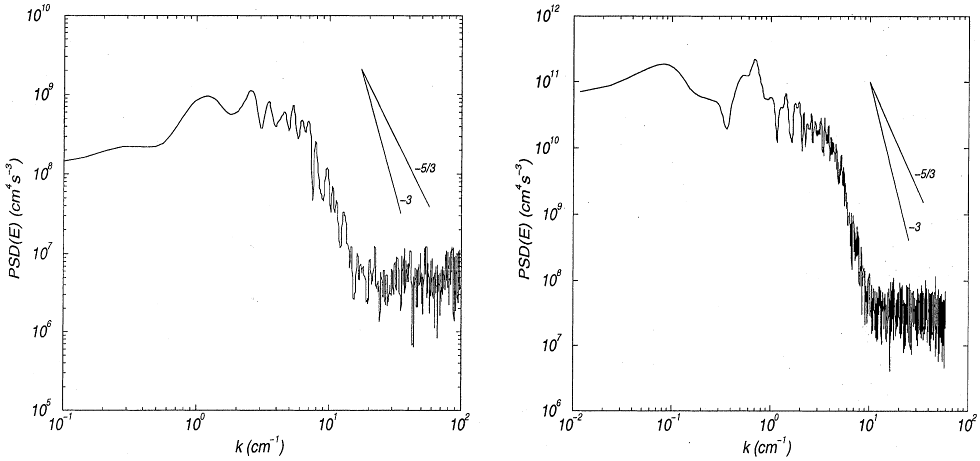

Fig. 6. One-dimensional energy spectra of angular velocity at $r = 96$ mm; (a) for $I = 3$ Amp, (b) for $I = 20$ Amp.

possible to get a quasi-steady equilibrium at each k between the local inertial transfer of energy and the local Hartmann damping. The first one has a typical time scale $1/(uk)$, whereas the second one has a typical time scale $(H\sqrt{\rho})/(B_0\sqrt{\sigma v})$, which is k independent (Sommeria and Moreau, [1]). This damping which results from both viscous and ohmic effects (it results from the Hartmann layer theory that they are equal to each other) is the only dissipation suffered by this 2D turbulence.

The transition between the $k^{-5/3}$ and the k^{-3} energy spectral slope can also be believed as dependent on the relative importance of energy dissipation versus enstrophy dissipation. When the turnover time scale is longer than the damping time due to the Hartmann layer, energy dissipation is important and leads to its $k^{-5/3}$ spectrum. When the turnover time scale is much less than the dissipation time, energy can be considered as conserved, whereas enstrophy is still dissipated. The enstrophy cascade towards the sink at small scales leads to a k^{-3} spectrum for energy and feeds also large scales, by conservation of energy.

4. Concluding remarks

A mercury cell located in a vertical magnetic field is used to observe the singular properties of quasi-2D turbulent shear layers. A number of striking results have been obtained, among which the most noticeable seem to be the following:

1. The velocity field is dominated by a number of large coherent structures fed from the instability of the shear layer via a energy transfer towards the large scales.
2. This kind of turbulence is, as expected, a good way to efficiently transport the momentum, but it seems to have a much lower efficiency to transport a scalar quantity such as heat. This point is particularly noticeable since it suggests that the turbulent Prandtl number should not tend to unity.
3. The energy spectrum exhibits peaks (at low wave numbers) corresponding to the large structures and an inertial range (at higher wave numbers) whose typical law is $k^{-5/3}$ or k^{-3} depending on the time scale of energy transfer compared to the time scale of dissipation in the Hartmann layer. Two possible mechanisms are a quasi-steady equilibrium of energy transfer and dissipation in the Hartmann layer at all scales, or an enstrophy cascade.

Acknowledgements

The present work was supported by the European contract No. CIPA-CP-0080.

References

[1] J. Sommeria, R. Moreau, Why, how and when MHD turbulence becomes two-dimensional, J. Fluid Mech. 118 (1982) 507–518.
[2] R. Moreau, Magnetohydrodynamics, Kluwer Academic Publishers, Dordrecht, 1990.
[3] B. Lehnert, Instability of laminar flow of mercury caused by an external magnetic field, Proc. Royal Soc. London A 233 (1955) 299.
[4] A.A. Kljukin, Yu.B. Kolesnikov, MHD instabilities and turbulence in liquid metal shear flows, in: J. Lielpeteris, R. Moreau (Eds.), Liquid Metal Magnetohydrodynamics, Kluwer Academic Publishers, Dordrecht, 1989, pp. 449–454.
[5] J. Sommeria, T. Dumont, R. Robert, Model of two-dimensional turbulence for liquid metal flows in a magnetic field, paper presented to the 11th Symposium on Turbulent Shear Flows, Grenoble, Sept. 1997.
[6] R. Robert, J. Sommeria, Statistical equilibrium state for two-dimensional flows, J. Fluid Mech. 229 (1991) 291–310.
[7] J. Sommeria, J.M. Nguyen Duc, P. Caperan, Two-dimensional MHD turbulence, in: J. Lielpeteris, R. Moreau (Eds.), Liquid Metal Magnetohydrodynamics, Kluwer Academic Publishers, 1989, pp. 441–448.
[8] O.A. Lielausis, Liquid metal magnetohydrodynamics, Atomic Energy Review 13 (1975) 527–581.

[9] E.C. Brouillette, P.S. Lykoudis, Magneto-fluid-mechanic channel flow I experiments, Phys. Fluids 10 (1967) 995–1001.
[10] R.A. Gardner, P.S. Lykoudis, Magneto-fluid-mechanic pipe flow in a transverse magnetic field. Part I. Isothermal flow, J. Fluid Mech. 47 (1971) 737–764.
[11] A. Alemany, R. Moreau, P. Sulem, U. Frisch, Influence of an external magnetic field on homogeneous MHD turbulence, J. de Méca. Théor. et Appl. 18 (1979) 277–313.