# Experiments on reflection of internal gravity waves: without and with rotation

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**Abstract**

When internal waves are impinging onto a sloping bed, striking phenomena are expected to occur close to the slope due to the reflection properties. We present several laboratory experiments at low and moderately large Reynolds numbers. Reflection with or without rotation is addressed in details. We have in particular devised several internal wave generators producing wide beams almost similar to a plane wave. Using a temporal filtering technique, we carefully study the generation of harmonic frequencies due to nonlinear effects in the impact zone. It allows comparisons with previous theoretical predictions. Firstly, the selection criteria of the reflected wavelengths are discussed. Secondly, evanescent waves whose frequencies are larger than the Brunt-Väisälä frequency are observed. The amplitudes of the different harmonics after reflection are then obtained. Finally, we also discuss preliminary results on the consequences for the advection of particles close to the slope.

## 1. Introduction

Linear and nonlinear dynamics underlying the reflection of internal waves impinging on a sloping topography induce intriguing features such as energy focusing of the reflected beam and generation of harmonics. Of special interest is the case of critical incidence (see Philipps (1977)) $\theta = \alpha$, with $\theta$ the angle of propagation of the incident beam and $\alpha$ the angle of the slope, with respect to the horizontal. Recently, Dauxois and Young (1999) have derived a weakly nonlinear solution using a matched asymptotic expansion in the critical case. Subsequently, Tabaei et al. (2005) have nicely studied the reflection properties of a beam with a finite width, but for $\theta \neq \alpha$. In this paper, we briefly review the experimental set-ups we devised to test these analytical predictions.

## 2. Preliminary small scale experiments

Using classical Schlieren visualizations, we had previously considered a perspex $40 \times 30 \times 10$ cm tank, filled with a linearly stratified fluid leading to a Brunt-Väisälä frequency $N = 3.1 \pm 0.1$ rad/s (see Dauxois et al. (2004)). Internal waves were generated in the shape of the usual Saint Andrew cross by a cylinder of radius $R = 1.5$ cm vertically oscillating. We studied the beam hitting a slope inclined at an angle $\alpha = 35°$. Figure 2 of Dauxois et al. (2004) presents successive snapshots obtained during one excitation period for $\theta - \alpha = -8°$. The evolution of the waves was emphasized by spontaneously generated intrusive layers (see also McPhee-Shaw and Kunze (2002)). Corresponding to the usual isopycnal lines, they agree *qualitatively* with theoretical predictions derived by Dauxois and Young (1999). In particular, it was possible to exhibit wave overturns and the amplification of the wave amplitude close to the slope.

Building on this previous work, we improved the experimental set-up by measuring the density gradients with the synthetic Schlieren visualization to get *quantitative* results (Dalziel et al. (2000)). This technique reveals different features of the internal waves reflection (see Fig. 1). One might in particular evaluate the amplitude of the beams. The reflected one follows the linear reflection law, while the color scale emphasizes the energy density focusing. The analysis is however perturbed by the unavoidable reflections on the boundaries of the three other emitted beams.

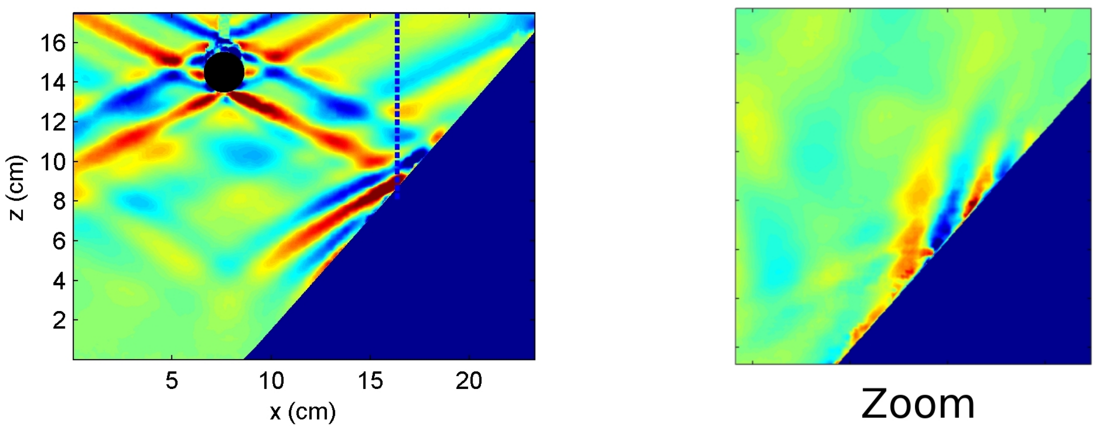


Figure 1: Left panel presents the vertical density gradient obtained experimentally with the synthetic Schlieren technique for a sub-critical incidence. The excitation frequency is $f = 0.2$ Hz while the slope angle is $\alpha = 48°$. The black disk corresponds to the vertically oscillating cylinder. Right panel shows a zoom on the impact zone after filtering at twice the excitation frequency. Note that the reflection of the incident wave is downslope, while the second harmonics is upslope.

In non critical conditions, this experimental set-up is particularly appropriate. For example, right panel of Fig. 1 presents the zoom on the impact zone of the vertical density gradient filtered at twice the excitation frequency. This method, described in details in Gostiaux et al. (2006a), allows to precisely distinguish the harmonics from the incident signal. Figures 1 correspond to a sub-critical incidence $\theta < \alpha$. Note that the beam reflected at the excitation frequency propagates downslope whereas the second harmonic propagates upslope. Even away from the critical conditions, the reflection is already strongly nonlinear since the harmonics were absent from the incident beam. This picture attests that the transfer between the incident frequency and the second harmonic is important.

Nevertheless, in critical conditions, the incident beam seems to disappear close to the slope as shown by Fig. 2. This is presumably due to dissipative effects. A careful inspection of the measurements also reveals discontinuities in the density gradients. The synthetic Schlieren technique is presumably not appropriate to study these properties, since the astigmatism conditions are not satisfied any longer. In addition, the small scale of the experiments does not allow to study the reflection for small $\alpha$ ($\sim 10°$), usually encountered in oceans. Larger scale experiments are thus required.

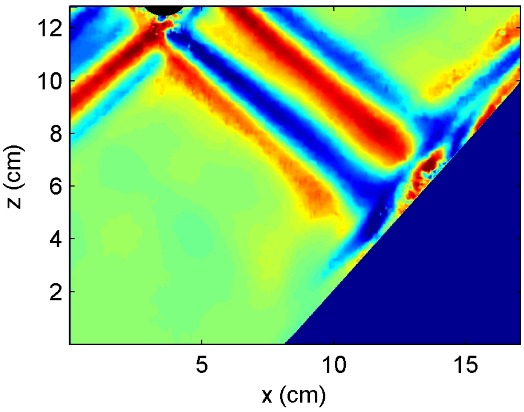


Figure 2: Horizontal density gradients visualized by synthetic Schlieren in critical conditions $\theta = \alpha = 48°$. The excitation frequency is $f = 0.3$ Hz.

## 3. Experiments at larger scale

### 3.1. First set of experiments

Two different series of experiments were performed at the Coriolis platform in Grenoble together with H. Didelle, S. Mercier, J. Sommeria and S. Viboud. Thanks to the world-unique size of the facility (13m diameter large tank), the disturbing reflections on the boundaries of the small tank can be drastically reduced. The high quality Particle Image Velocimetry (PIV) technique developed in this laboratory was of crucial importance to improve the quantitative study of internal waves.

During the first experiments 2005 campaign, we used a paddle-like internal wave generator inspired by Teoh et al. (1997) and De Silva et al. (1997) (see Gostiaux et al. (2006a) for details). The tank was filled with 1 m of salted water, stratified so as to present a Brunt-Väisälä frequency $N = 0.53$ rad/s. Visualizations were performed in the vertical median plane by illuminating small beads with a laser sheet followed by a digital camera. Recording at 60 frames per period, we used again the filtering technique to get the averaged velocity field and associated information on the phase. Tuning the excitation frequency, it was again possible to study sub- and super-critical cases, as shown in Figs. 3: downslope reflection for the left panel and upslope reflection for the right panel. The energy does not seem to be conserved, since the colors do not suggest that the energy of the reflected beam is increased while the width of the beam has been drastically reduced. The explanation lies beyond the scope of the *linear* approach since harmonics have now to be taken into account.

Figure 4 presents the second and third harmonics in the super-critical case $\theta - \alpha = 1.8°$. The vertical velocity field is plotted since the propagation is steeper for these harmonics. It is obvious that, although the amplitudes are one order of magnitude lower than the fundamental one, both of them have unexpectedly approximately the same value. A careful study of different experiments suggests a different mechanism between the sub- and super-critical case: nonlinearity being of higher importance in the first situation (see Gostiaux et al. (2006a)). Note also that the third harmonic is evanescent (non propagative) since its frequency is larger than the Brunt-Väisälä frequency.

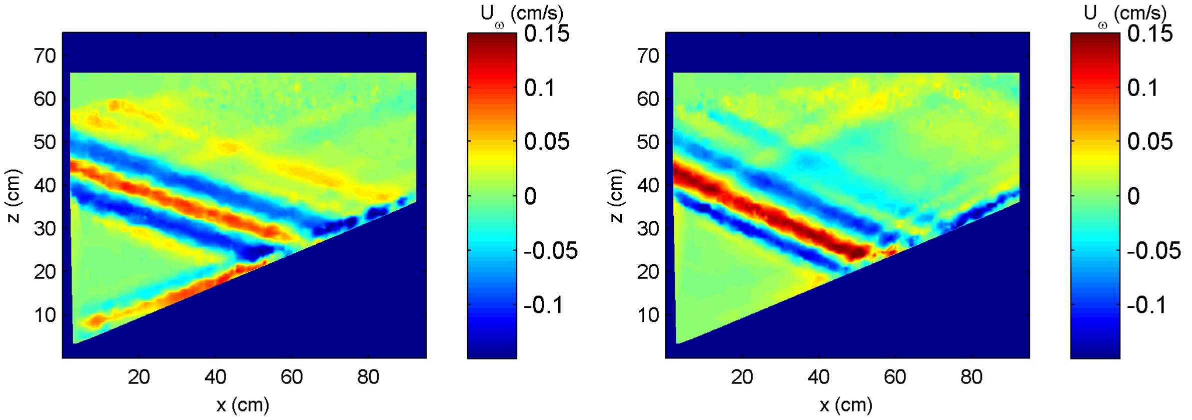


Figure 3: Horizontal velocity field, filtered at the excitation frequency when $\theta - \alpha = -3°$ (left panel) and $1.8°$ (right panel).

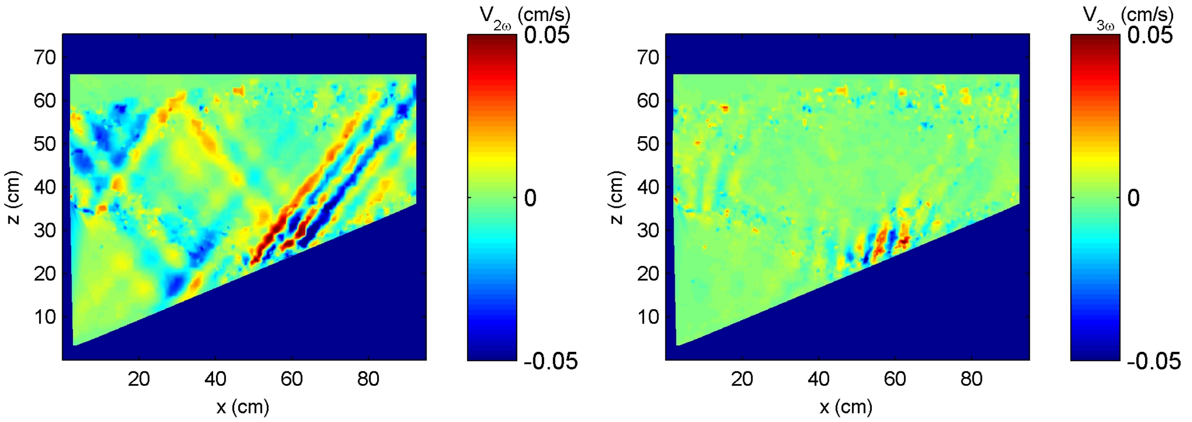


Figure 4: Vertical velocity field filtered at the second harmonic (left panel) and third harmonic (right panel) in a super-critical case ($\theta - \alpha = 1.8°$).

### 3.2. A novel internal wave generator

We performed a second set of experiments in 2006, in order to get better estimates of the amplitudes of the reflected beams. Firstly, we devised a new internal wave generator emitting a plane wave of much higher spatial and temporal monochromaticity. All details of this generator are given in Gostiaux et al. (2006b). Secondly, we considered more realistic cases with a less steep slope ($\alpha = 9.5°$). It ensures that all the observed harmonics are no longer evanescent but propagative.

Figure 5 presents the horizontal velocity field filtered at the fundamental frequency in a quasi-critical case. Note that the width of the incident beam covers more than 4 wavelengths. The reflected beam, theoretically concentrated along the slope, is not visible in the impact zone. We will present during the conference quantitative results on this reflection process *with* and *without* rotation of the Coriolis table.

## 4. Advection

Finally, the reflection of internal waves is believed to be a pivotal point to understand and quantify mixing properties close to the oceanographic topography. Therefore, we study *analytically* the advection of particles initially located close to the slope.

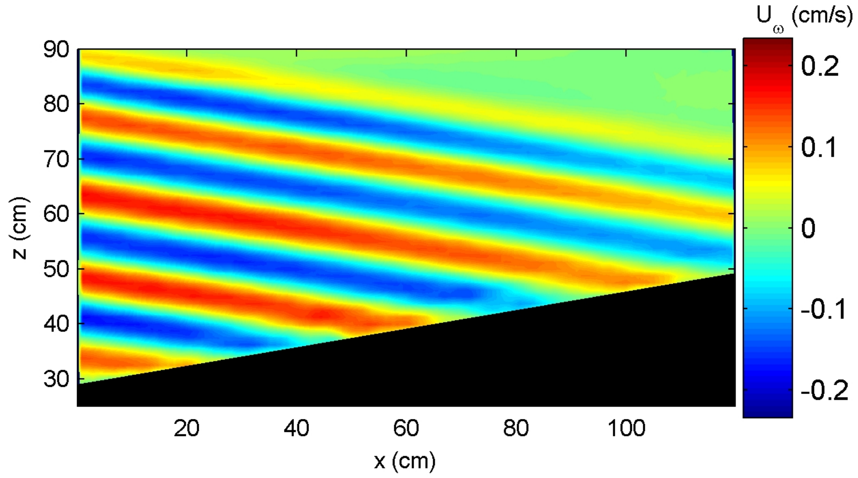


Figure 5: Horizontal velocity field filtered at the fundamental frequency for $\theta = 8°$ and $\alpha = 9.5°$.

This has been done by using the analytical solution of the weakly nonlinear problem solved by Dauxois and Young (1999). In this case, the advection of a passive tracer is accounted for by the Stokes drift due to the leading order of the velocity field (the superposed incident and reflected waves) on the one hand, and the contribution of the next order of the velocity field (the non-linear interaction of the incident and reflected waves) on the other hand. For non-critical incidence ($\alpha \neq \theta$), the two contributions balance and the fluid particles oscillate about their initial position. For critical incidence ($\alpha = \theta$), the singular solution takes the form of rolls propagating upslope. Figure 6 shows that the fluid particles close to the slope are advected by these propagative rolls and move upslope as well, at the phase speed of the rolls.

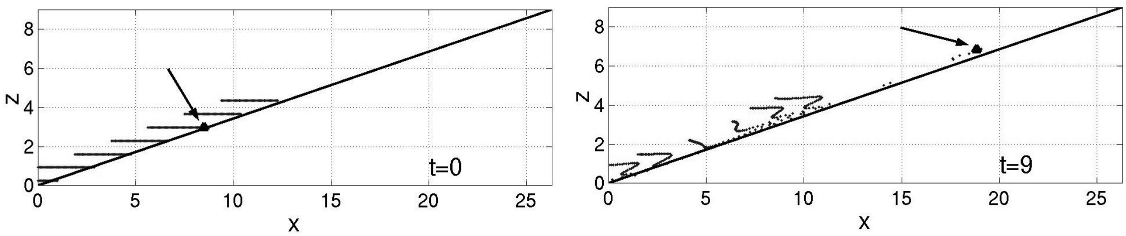


Figure 6: Passive tracers initially located on iso-density lines and advected by the velocity evaluated for critical incidence.

## 5. Main conclusions

The experimental study of the reflection of internal waves has emphasized the following characteristics:

- The reflection of internal waves unambiguously generates nonlinearities, induced by the interaction between the incident and the reflected beams. The existence of the second and third harmonics agrees well with theoretical predictions.
- We showed for the first time the existence of *evanescent* waves trapped close to the slope for harmonics whose frequencies are larger than the Brunt-Väisälä frequency.

- The measured *amplitudes* for the second and third harmonics do not agree with analytical predictions. The third harmonic is unexpectedly intense.
- We do confirm that the wavelength selection during the reflection process is governed by the projection of the wavevectors onto the slope.
- Experiments reveal an unexpected difference between *sub* and *super-critical* conditions, that we are not yet able to explain.

**Acknowledgements**

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