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Preface

Serge Aubry

Photograph by Claude Baesens

This special issue of Physica D is dedicated to Serge Aubry's influence and achievements, for his 60th birthday. It is an outgrowth from a conference on "Nonlinear Physics: Condensed Matter, Dynamical Systems and Biophysics" which was held in his honour at the Institut Henri Poincaré, Paris, 30–31 May 2005.

Serge Aubry is a remarkably creative and inspiring physicist. He has made great contributions to nonlinear phenomena in solid-state physics, dynamical systems theory and biological physics. His approach combines attention to experimental results, deep physical insight, formulation of key problems, incisive numerical simulation, innovative use of mathematics, unconventional interpretations and imaginative predictions.

Let us survey his main achievements.

- **Solitons and chaos in ferroelectrics**. Linear theory was incapable of explaining a central peak observed in the dynamic response of ferroelectric materials near their structural transition. Aubry proposed that it is due to the displacement of domain walls in a chain of double well potentials, thus bringing solitons into solid-state physics (simultaneously with Krumhansl and Schrieffer). He also realised that if one moves away from the continuum limit, the equilibrium states of a one-dimensional nearest neighbour chain are in one-to-one correspondence with the orbits of an associated area-preserving map, allowing him to deduce much about the possible equilibrium states, in particular the existence of quasiperiodic sliding states and chaotic pinned states.
- **Ground-states of Frenkel–Kontorova models**. The Frenkel–Kontorova model is a simple model for one-dimensional structures with competing periodicities, e.g. epitaxial layers. Most authors treated it via the continuum limit, but Aubry realised that he could develop the mathematics of the discrete case and discovered fundamental differences, in particular the "transition by breaking of analyticity" for incommensurate structures. His key observation is that the graphs of two minimum energy sequences

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can cross at most once. From this, he proved a complete classification of the minimum energy states. The high points are that, for each irrational ω, there is always a set of minimum energy states of mean spacing ω, which forms either a circle or a Cantor set (corresponding to sliding and pinned incommensurate structures, respectively), and the dependence of physical quantities on parameters is typically a "devil's staircase". In addition to their intrinsic interest for structure theory in the solid state, his results have profound implications for Hamiltonian dynamics, via the correspondence he found to an associated area-preserving map. Thus Aubry showed that the fate of its KAM circles as a deviation from integrability is increased is that they break into invariant Cantor sets with the same irrational rotation number (proved independently by Mather).

- **Weak periodicity of ground-states**. For a very general class of structural models, Aubry proved that the ground-states must exhibit a weak form of periodicity, which includes periodic and quasiperiodic but is more general, like the mathematical notion of almost periodicity. He gave an example to show that the ground-state could fail to be quasiperiodic. This analysis and example play a fundamental role in theory that is still being developed by others.
- **Localisation in quasiperiodic potentials**. Aubry showed that the phenomenon of Anderson localisation, where the wave functions in a random potential are localised, can also occur in strong enough quasiperiodic potentials. This work initiated a line of deep research by others.
- **Anti-integrable limits**. In Hamiltonian dynamics, integrable systems play a central role, firstly because they can be "solved" and secondly because many results hold for near-integrable systems, e.g. KAM theory. Aubry invented an opposite limit, which he named "anti-integrable". It is most clearly explained in the context of the standard map

$$\begin{aligned} y' &= y - \frac{k}{2\pi}\sin 2\pi x \\ x' &= x + y' \end{aligned} \tag{1}$$

which has an integrable limit $k = 0$ and an anti-integrable limit $k = \infty$. Now $k = \infty$ does not define a map, but if one makes the correspondence to a Frenkel–Kontorova model

$$W = \sum_{n\in\mathbb{Z}} \frac{1}{2}(x_{n+1} - x_n)^2 + \frac{k}{4\pi^2}\cos 2\pi x_n \tag{2}$$

then one could scale the energy W by $\frac{k}{4\pi^2}$ and obtain an equivalent variational problem

$$W' = \sum_{n\in\mathbb{Z}} \frac{\lambda}{2}(x_{n+1} - x_n)^2 + \cos 2\pi x_n \tag{3}$$

in which $\lambda = \frac{4\pi^2}{k}$. Then the variational problem with $k = \infty$ makes sense as $\lambda = 0$, for which it is immediate that the equilibrium states are all the sequences with each $x_n \in \mathbb{Z}/2$. Aubry realised that perturbation theory of the variational problem about this limit gives many interesting orbits of the standard map for large k. This approach has been very fruitful.
- **Bipolarons**. Aubry applied his concept of anti-integrability to models of electron–phonon systems, such as the Holstein model. For the adiabatic case in which the phonons are treated as classical, the uncoupled case (atomic limit) provides a solvable case about which perturbation theory can be performed. This results in equilibrium states that can be described as arbitrary configurations of polarons and bipolarons. He has proposed that these are relevant to charge density wave materials, giving an insulating regime, in contrast to the Peierls–Fröhlich conducting regime, and hence a new metal-insulator transition. Generalised to models including a Hubbard repulsion, he has also proposed that bipolarons are relevant to the cuprate superconductors. In particular, he found "spin-resonant" bipolarons and, in a special parameter regime, "quadrisinglets", which are relatively easy to displace and which he proposed as candidates for the charge carriers in the cuprates.
- **Discrete breathers**. An analogous perturbation theory from the uncoupled limit showed that, in Hamiltonian networks of anharmonic oscillators, there can occur spatially localised time-periodic oscillations named "discrete breathers". Aubry developed proofs of existence and stability, and numerical methods for their computation, and studied their interactions with phonons and each other and their mobility. His insights led him to propose that discrete breathers can be left behind on cooling a lattice, leading to stretched exponential relaxation, and that materials that are insulating at the linearised level can become conducting above a threshold forcing amplitude, by formation of "rivers" of discrete breathers.
- **Targeted transfer**. A development from the concept of discrete breathers that deserves special discussion is Aubry's theory of targeted transfer of excitation from one spatial location to another. These excitations can be vibrational or electronic. The important idea is that it can occur in a very efficient and directional fashion if a nonlinear resonance condition is satisfied: the frequencies of vibration should remain in resonance at all stages of the transfer. Aubry proposes that this mechanism may be relevant to many situations in biophysics, in particular photosynthesis.

Many of the above results were obtained in collaboration with students or others, but we think all would agree that Aubry was the "motor" and would wish to express their gratitude to him for his stimulation.

The special issue presents a mix of valuable surveys and original research papers on topics relating to Aubry's interests.

The editors join with the authors in wishing Serge continuing success in innovating research and inspiring ideas.

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