Gravitational-like interactions in a cloud of cold atoms?

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Statistical mechanics of self-gravitating systems

- What about lab experiments?

An effective interaction mimicking gravity is needed...

Ultimate goal: a tabletop analog of a galaxy or globular cluster...

More accessible goals: find signatures of the special phase transitions of self gravitating matter, and/or uncover new phenomena with long-range attractive interactions

Experiments?

- Some proposals in the literature (list not exhaustive!):
  - O'Dell et al. (2000): Bose-Einstein Condensate + intense off-resonant laser beams
  - Dominguez et al. (2010): capillary interactions between colloids at a fluid interface $\sim$ 2D gravity.
  - Golestanian (2012): colloids driven by temperature gradients; temperature field induced by the colloids
  - This talk: laser induced interactions in trapped cold atoms $\sim$ 1D and 2D gravity.

Outline

- On 1D and 2D self-gravitating systems
- Trapped cold atoms and long-range laser induced interactions
- Modeling the 1D experiment
- Experimental results
- Towards a 2D experiment?
  - Theory
  - Simulations
  - Experimental challenges

Gravity in 1D

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{Gm^2}{2} \sum_{i \neq j} |x_i - x_j|$$

Two-body potential $V(x) \propto |x|$

-self confining

-not (very) singular

$\rightarrow$ partition function well defined in the scaling limit

- No phase transition; equilibrium density profile

$$\rho(x) = \frac{c}{\cosh^2(x/L)}$$

1D case: fixed temperature dynamics

Dynamical equation with a friction and a white noise (fixed temperature $T = D/\gamma$):

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= -\gamma v_i + \frac{1}{m} \sum_{j \neq i} F_{j \to i} + \sqrt{2D}\eta_i(t) \end{aligned}$$

Associated Non Linear Fokker Planck equation for the one-particle distribution $f(x, p, t)$:

$$\partial_t f + v\partial_x f + C \left(\int f(x', v', t) \mathrm{sgn}(x' - x) dx' dv' \right) \partial_v f = \partial_v \left[\gamma v f + D \partial_v f \right]$$

Asymptotic dynamics: converges to the equilibrium distribution.

Gravity in 2D

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{Gm^2}{2} \sum_{i \neq j} \ln |x_i - x_j|$$

Two-body potential $V(x) \propto \ln |x|$

Short range singularity...

$\rightarrow$ The partition function does not exist if $T < T_c$

The measure concentrates on a Dirac peak containing all the mass.

• The short distance singularity does not create any divergence in the partition function if $T > T_c$.

2D case: fixed temperature dynamics

Dynamical equation with a friction and a white noise (fixed temperature $T = D/\gamma$):

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= -\gamma v_i + \frac{1}{m}\sum_{j\neq i} F_{j\to i} + \sqrt{2D}\eta_i(t) \end{aligned}$$

Associated Non Linear Fokker Planck equation for the one-particle distribution $f(x, p, t)$:

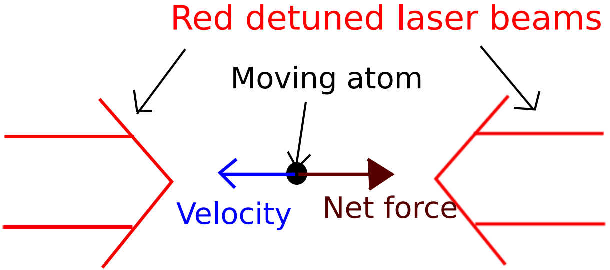
$$\partial_t f + v\nabla_x f + C\left(\int f(x', v', t)\frac{(x'-x)}{|x'-x|^2}dx'dv'\right)\nabla_v f = \nabla_v\cdot[\gamma v f + D\nabla_v f]$$

Related interesting question: dynamics of the collapse for $T < T_c$...

Now: try to show how systems similar to these 1D and 2D self gravitating systems can be engineered with trapped cold atoms.

Trapped cold atoms

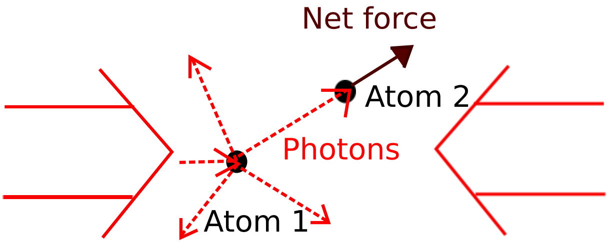
- Techniques developed in the 80's, now routinely used.



Doppler effect → a friction
Spatial trapping: through a magnetic field gradient, or a dipolar trap.

Trapped cold atoms - Multiple diffusion

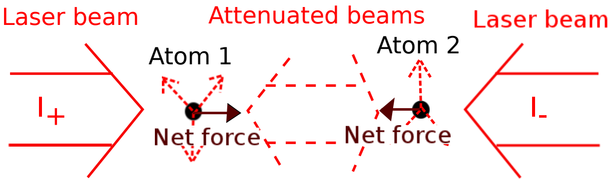
• Multiple diffusion → "Coulomb-like" repulsion (Walker, Sesko, Wieman 90).



→ A research program: instead of considering the repulsion as a limitation, take advantage of it to study "plasma-like" effects in a cloud of cold atoms.

Shadow effect

Laser attenuation $\rightarrow$ laser unbalance $\rightarrow$ effective attraction. This effect has been known since the 80's (Dalibard)



Hypothesis: **small optical thickness** (weak attenuation)

$$I_+(z) = I_0 e^{-\bar{\sigma}\int_{-\infty}^{z}\rho(s)ds} \simeq I_0\left(1 - \bar{\sigma}\int_{-\infty}^{z}\rho(s)ds\right)$$

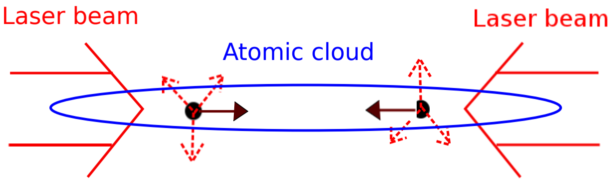
$$\vec{F}_{shadow} \propto I_+ - I_- \;\Rightarrow\; \mathrm{div}(\vec{F}_{shadow}) \propto -\rho$$

$\rightarrow$ a "gravity-like" interaction...

Problem: the repulsive force is stronger...

A quasi 1D or 2D geometry

In a very elongated cloud, most reemitted photons are lost:



$\Rightarrow$ Multiple scattering negligible. Attractive force not modified.

Modeling the 1D experiment

- Radiation pressure force, "full expression"

$$F_{\pm}(z, v_z) = \pm \hbar k \frac{\Gamma}{2} \frac{\Gamma^2}{4(\delta \mp k v_z)^2 + \Gamma^2} \frac{I_{\pm}(z)}{I_s}$$

Coupled to the intensity fields

$$\mathrm{d}I_{\pm} = \mp N \int \sigma_{\pm}(v_z) f(z, v_z, t) dv_z \mathrm{d}z$$

- Photon absorption + random reemission → velocity diffusion
- An external trap (approximately harmonic)

Simplifying assumptions

- Multiple diffusion neglected (OK)
- The radiation force is linearized in v_z (Dangerous!)
  $\rightarrow$ it is decomposed into
  1. A linear friction $\propto -v_z$
  2. The shadow effect
- Small optical width hypothesis ($\pm$ OK)
  $\rightarrow$ the laser intensities disappear, replaced by an effective interaction
- Fast transverse equilibration ($\pm$ OK ?)
  $\rightarrow$ Transverse degrees of freedom integrated out.

Kinetic description $\rightarrow$ A Vlasov Fokker-Planck equation

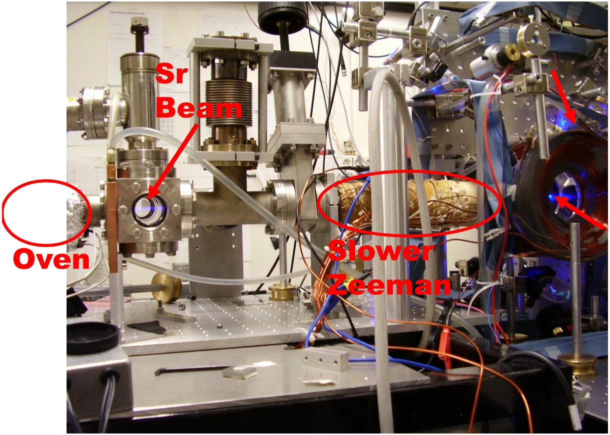
$\rightarrow$ an effective 1D Vlasov Fokker Planck equation
A simplified model for f, the one-particle distribution (contains dipolar trap, friction, "gravity-like" attraction + velocity diffusion):

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} + (-\omega_0^2 z + F_{int}[f](z))\frac{\partial f}{\partial v} = \frac{\partial}{\partial v}\left(\gamma v f + D\frac{\partial f}{\partial v}\right)$$

$\rightarrow$ in principle, equation identical to Vlasov-Fokker-Planck for 1D self gravitating Brownian particles

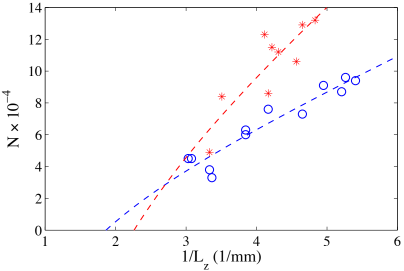
The experiment

- Experiment: Maryvonne Chalony, David Wilkowski (Institut Non Linéaire de Nice)
- Strontium; size $\sim$ 500μm; temperature $\sim 2\mu K$; number of atoms $N \sim 10^5$.



Experimental signatures: cloud's size

Theory, in the self gravitating limit (trap=negligible): $L \propto 1/N$
L = cloud's size; N = number of atoms



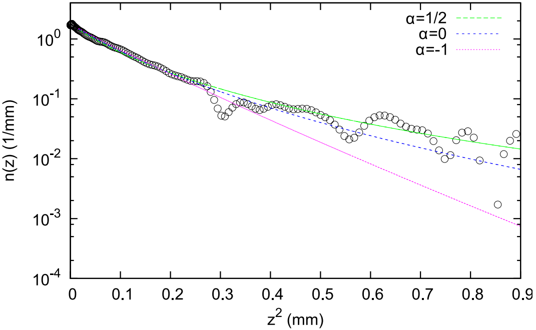
N vs $1/L$. Red: $T \simeq 1.5\mu K$, Blue: $T \simeq 2.1\mu K$; the theory includes the trap.

$\rightarrow$ qualitative agreement; difficult to be more precise...
At least, the size decreases when the number of particles increases.

Experimental signatures: density profile

Theory, in the self-gravitating limit (trap=negligible):

$$\rho(z) = \frac{N}{2L}\frac{1}{\cosh^2(z/L)}$$



Experimental profile vs theory with $1/r^\alpha$ *forces,* $\alpha = -1, 0, 1/2$.

Experimental signatures: breathing frequency

Effective interaction are turned on $\rightarrow$ relaxation to the new "self-gravitating" stationary state through breathing oscillations.

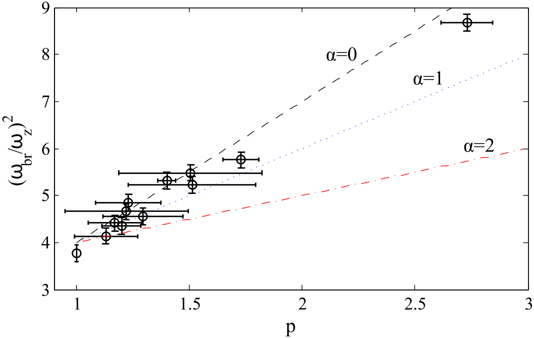
Theory: the breathing frequency depends on the compression factor c and the force exponent α

$$\omega_{br} = \omega_0 \sqrt{(3-\alpha)(c^2-1)+4}$$

c=cloud's size without interaction/ cloud's size with interaction
α= force exponent; $\alpha = 0$ for 1D gravity.

Experiment: the breathing frequency is measured and compared with the theory including the experimentally measured compression factor, varying α.

Frequency vs compression



c^2

Experimental frequency ratio $(\omega/\omega_0)^2$ *vs compression factor* c^2
Theory = dashed line. From top to bottom, force exponent $\alpha = 0, 1, 2$.

Some caveats

- Linearization in velocity
  $\rightarrow$ the force felt by an atom actually depends on its velocity! This is a serious problem: in the experiments that have been performed, the force is "gravitation-like" in a velocity averaged sense...
- The optical thickness is actually $0.2 \leq b \leq 0.6$; this is not very small... The optical thickness is also difficult to measure precisely.
- The theoretical predictions depend very sensitively on the laser detuning, which is difficult to set precisely.

$\rightarrow$ the analogy with a self-gravitating system is only qualitative, and the comparison with the theory cannot be really precise...

Perspectives

A tabletop galaxy: we are not there yet!

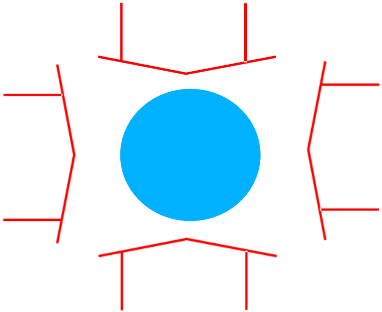
A further step: Study phase transitions with attractive long-range interactions

- 1D self gravitating system: no phase transition.
- 2D self-gravitating system: a collapse at finite temperature T_c

$\rightarrow$ try an experiment with a "pancake shaped" atomic cloud; in this geometry, the attractive interaction is still dominant.

Caution : From here on, work in progress!

Two pairs of contra propagating lasers, a "pancake shaped" cloud:



A Vlasov-Fokker-Planck model (same hypothesis as above...):

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{x}} f - \omega_0^2 \vec{x} \cdot \nabla_{\vec{v}} f + \vec{F}[f](\vec{x}) \cdot \nabla_{\vec{v}} f = \nabla_{\vec{v}} \cdot (\gamma \vec{v} + D \nabla_{\vec{v}} f)$$

where $\vec{F}[f](\vec{x})$ is the effective attraction.

Non potential "gravitation like" interaction

A force which looks like 1D gravity in each direction:

$$F_x(x,y) = -C\int \mathrm{sgn}(x-x')\rho(x',y)dx'$$
$$F_y(x,y) = -C\int \mathrm{sgn}(y-y')\rho(x,y')dy'$$

The divergence is the same as gravitation:

$$\nabla_{\vec{r}}\cdot\vec{F}[f] \propto -\rho\ ,\ \text{but } \vec{F} \text{ is not a gradient}$$

How much does this system look like 2D self gravitating Brownian particles?

Theoretical approaches

- Gradient force $\vec{F} = -\nabla V[\rho]$ $\rightarrow$ we have an implicit equation for the stationary density

$$\rho(\vec{r}) = e^{-\frac{\gamma V[\rho](\vec{r})}{D}}$$

  $+$ The Vlasov-Fokker-Planck equation has a free energy
  $\rightarrow$ our main analytical tools are lost in this case.
- Linearize the dynamics: use periodic boundary conditions so that a uniform density is stationary and study its stability.
  $\rightarrow$ analog of Jeans instability
- Non linear analysis, in a trap: a moment method.
  $\rightarrow$ prove there is a "phase transition"?
- Numerical simulations

Numerical simulations

- The singular interaction must be regularized

1. Particles (Langevin) simulations
Bruno Marcos, Grimaud Pillet (undergraduate student)

2. PDE simulations (overdamped case)
Thanks to Magali Ribot...

1. Does the "gravitational" collapse survives in this setting?
   Open question...
2. If not, is there another type of phase transition?
3. If yes, is it experimentally observable?
   The interesting regime is probably not easy to reach experimentally, but not very far from accessible...