EXPLOSIONS
et
TRANSITION FLAMME-DETONATION

CAS DES SUPERNOVAE ?

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Fluides réactifs

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla.\mathbf{u} \qquad \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}.\nabla$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p \boxed{-\frac{Gm}{r^2}\mathbf{e}_r, \quad r \equiv |\mathbf{r}| \quad m \equiv 4\pi\int_0^r \rho r^2 dr}$$

Eq. d'état: $p(\rho, T)$

chaleur de réaction

$$\rho c_v \frac{DT}{Dt} = \nabla.(\lambda\nabla T) - p\nabla.\mathbf{u} + \sum_j Q^{(j)}\dot{W}^{(j)},$$

Vitesses de réaction: $\dot{W}^{(j)}(T, \rho, ..Y_i..)$

$$\rho\frac{DY_i}{Dt} = \nabla.(\rho D_i \nabla Y_i) + \sum_j \vartheta_i^{(j)}\mathcal{M}_i\dot{W}^{(j)},$$

Conditions initiales + aux limites

Combustion classique dans les gaz (réactions moléculaires)

$e.V.$

$T_b : 1000 - 3500K$

$U : 10^{-1} - 3.10^3 m/s$

$\Delta T/T : 3 - 12$

$\Delta p/p : 10^{-2} - 25$

complexité: 300 réactions - 50 espèces

Combustion nucléaire (fusion)

MeV

$T_b : 10^7 - 10^{10} K$

$\rho : 1 - 10^{14} g/cm^3$

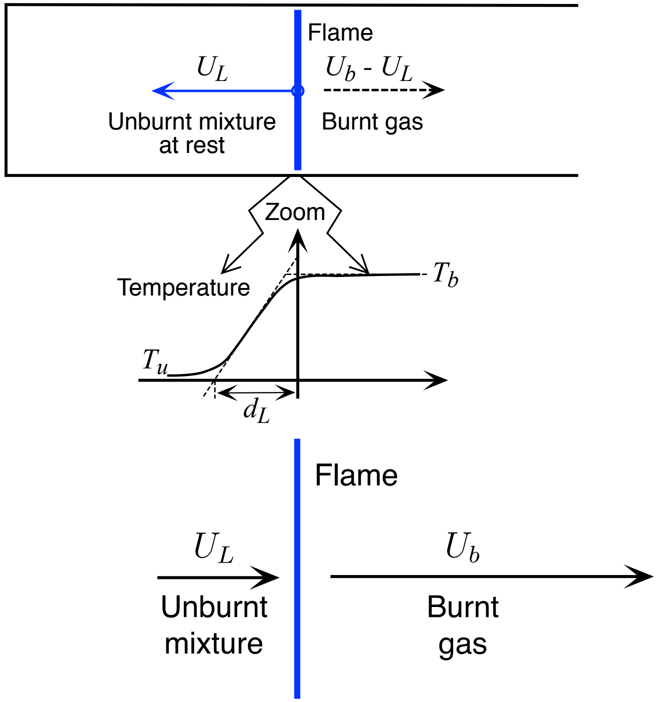
$U : 10^2 - 10^4 km/s$

complexité des schémas réactionnels

Flammes: ondes subsoniques

Davy 1810 $U_L/a_u \ll 1$

Ondes planes quasi-isobare



U_L?

Ondes de réaction-diffusion

$$R \rightarrow P + Q$$

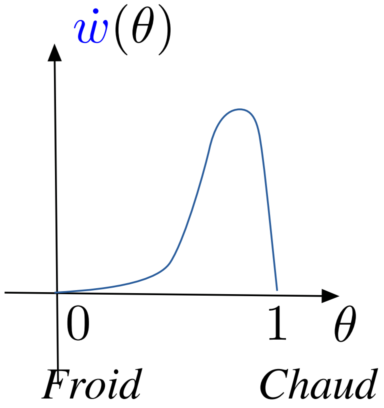
vitesse de flamme

$$\frac{\partial\theta}{\partial t}$$

$$\mu\frac{d\theta}{dx} - D\frac{d^2\theta}{dx^2} = \frac{\dot{w}(\theta)}{\tau_r}, \qquad \dot{w} > 0,$$

$$x = -\infty : \theta = 0, \;\; \dot{w} = 0,$$

$$x = +\infty : \theta = 1, \;\; \dot{w} = 0,$$

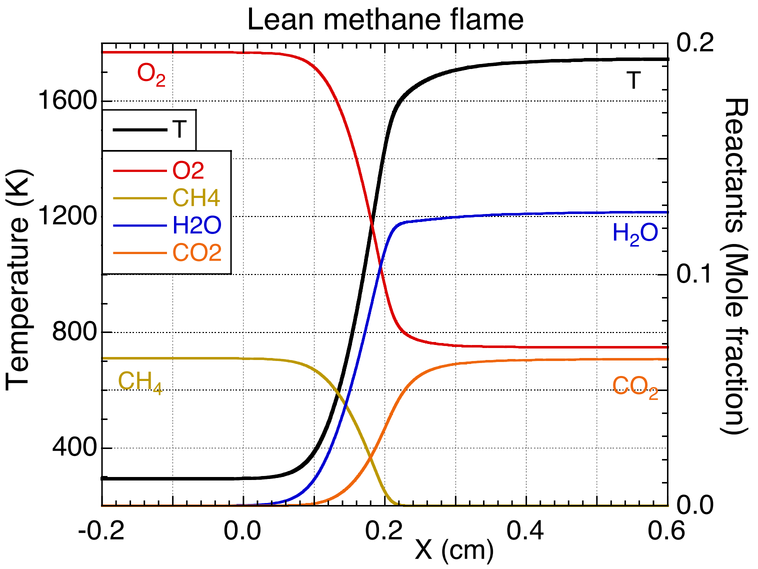


$$\frac{\mu}{\sqrt{D/\tau_r}}? \qquad \tau_r \gg \tau_{coll} \Rightarrow \mu/a_u \ll 1$$

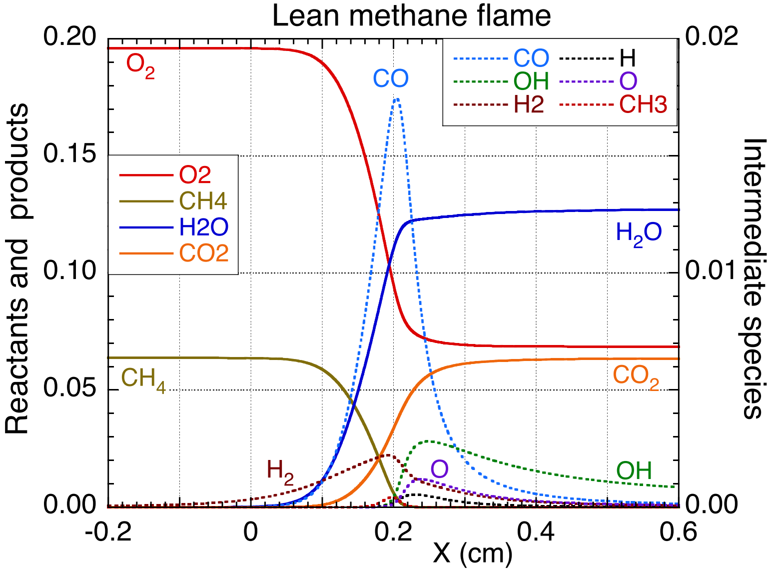
ZFK 1938 *KPP 1938*

$$D \approx 0.15cm^2/s. \quad \tau_r \approx 10^{-6}s. \quad \boxed{\times 10^2!} \qquad \mu \approx 40\,cm/s \qquad d \approx 5.10^{-2}cm$$

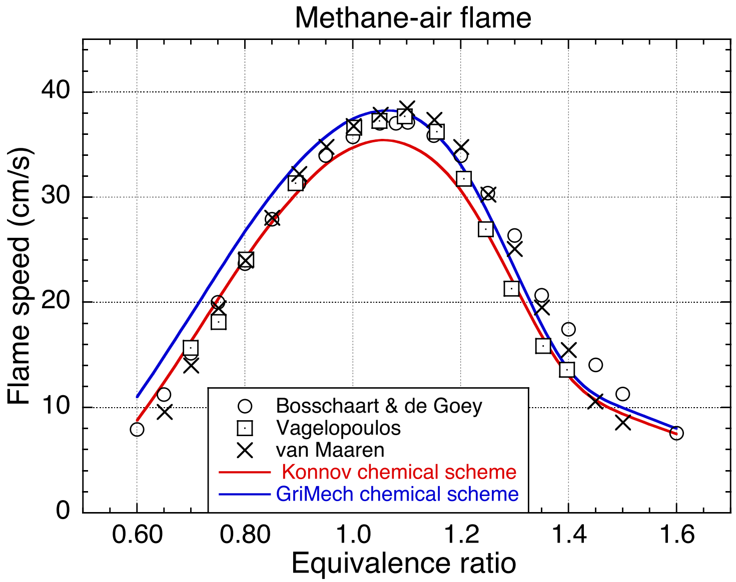
flammes usuelles



Lean methane flame

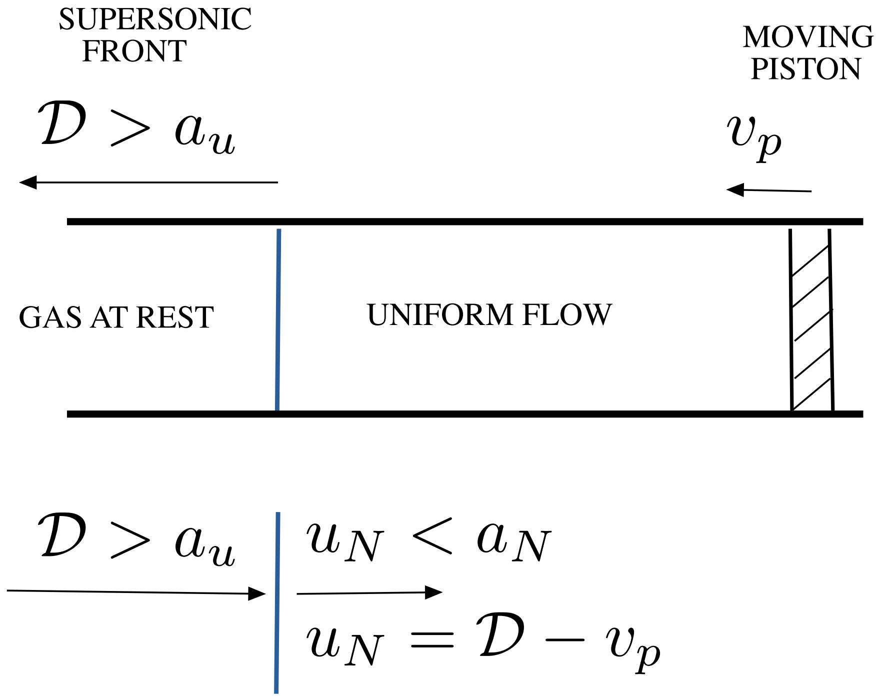


Lean methane flame



Methane-air flame

PLANAR SHOCK WAVE
INERT GAS



SUPERSONIC FRONT

$\mathcal{D} > a_u$

MOVING PISTON

v_p

GAS AT REST

UNIFORM FLOW

$\mathcal{D} > a_u$

$u_N < a_N$

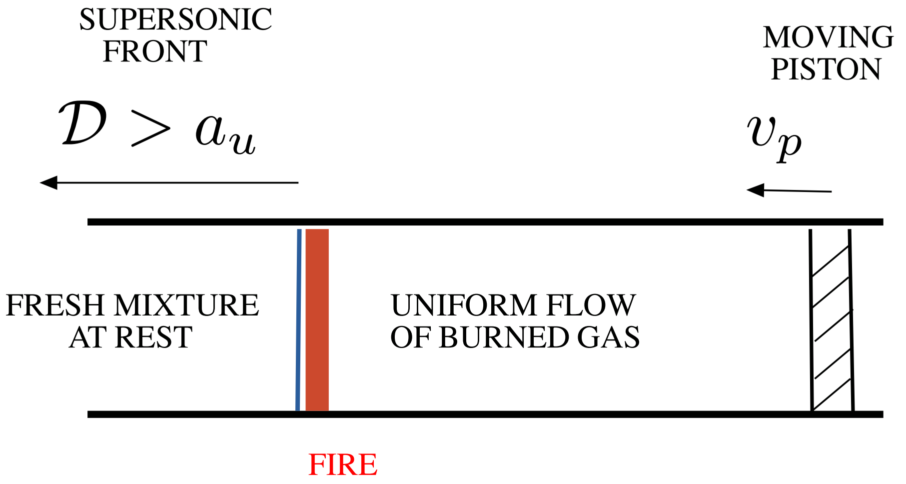
$u_N = \mathcal{D} - v_p$

Supersonic *Subsonic*

Poisson 1808, Stokes 1848, **Riemann** 1860, Rankine 1869, **Hugoniot** 1889, Rayleigh 1910

OVERDRIVEN DETONATION
REACTING GAS

PISTON SUPORTED SUPERSONIC WAVE



SUPERSONIC
FRONT

$\mathcal{D} > a_u$

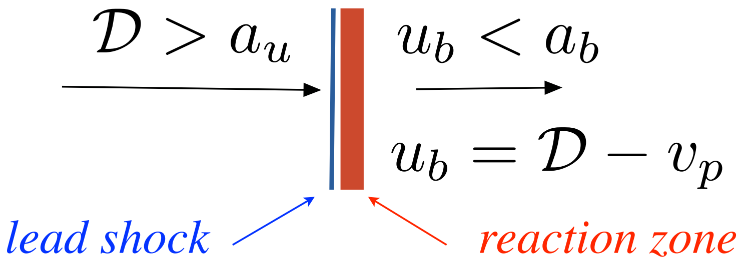
MOVING
PISTON

v_p

FRESH MIXTURE
AT REST

UNIFORM FLOW
OF BURNED GAS

FIRE



$\mathcal{D} > a_u$

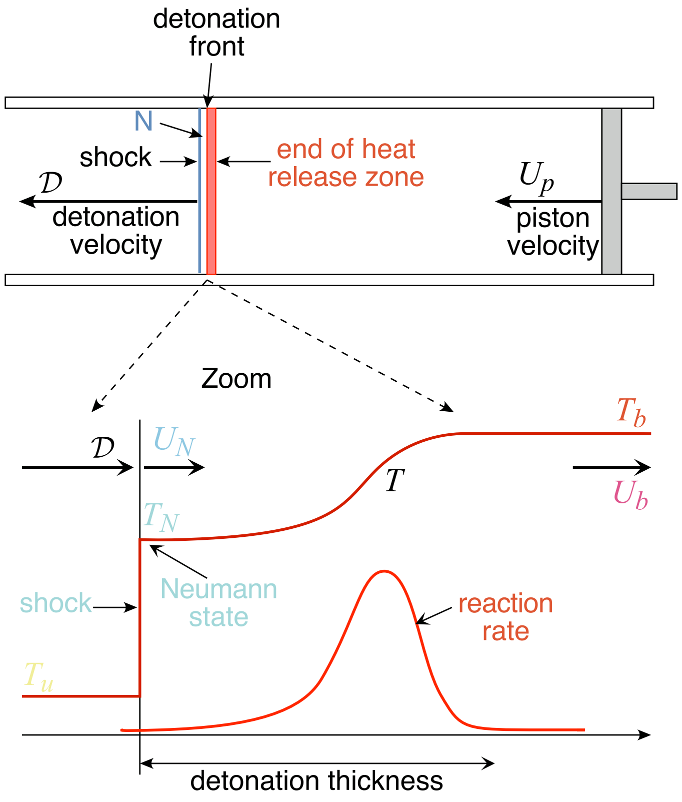
$u_b < a_b$

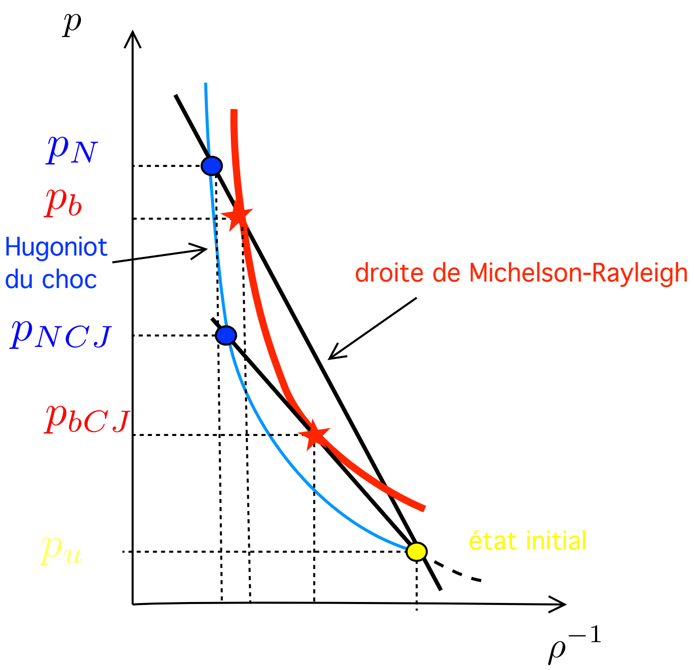
$u_b = \mathcal{D} - v_p$

lead shock

reaction zone

Abel 1870, Berthelot et Vielle 1881, Mallard et Le Chatelier 1881, **Mikhel'son** 1893, Chapman 1899, Jouguet 1904, Vielle 1900, **Zel'dovich** 1940, von Neumann1942, Döring 1943,





Mikhel'son 1893,

$$\mathcal{D} \approx \sqrt{q} \approx 1.5\,10^3 - 2.8\,10^3 m/s$$

détonations gazeuses usuelles

$$T_N \approx 1100 - 1600K$$

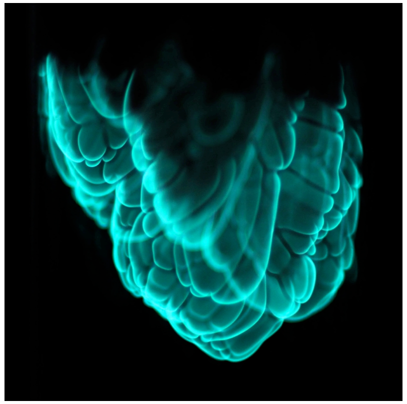
$$d = U_N \tau_r \approx 10^{-1} - 1\,cm/s$$

$$p_N/p_u \approx 10 - 30$$

Stabilité de ces ondes et leur dynamique

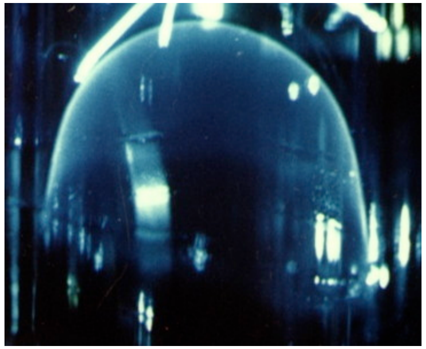
Flammes cellulaires

DL instab
hydro

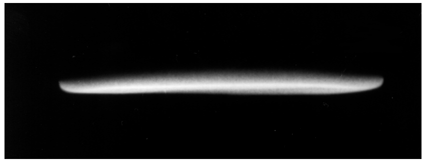


“Turing” instab
thermo diff

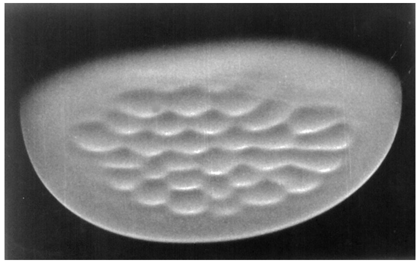
L. Boyer, G. Quinard et J. Quinard, IRPHE 1980-2000



RT instab



stabilisé par anti RT



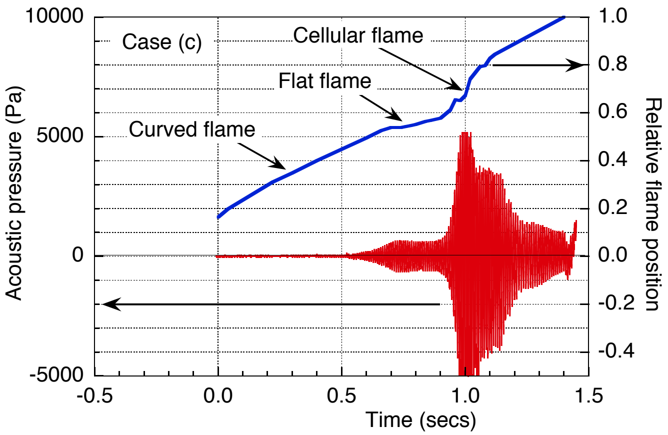
Insta thermo-acoustique

Rayleigh 1878

Feedback

$$\partial^2 p / \partial^2 t - a^2 \Delta p = \boxed{\partial \dot{q} / \partial t}$$

Sound generation



Acoustic instability in Premixed Flames

© IRPHE G. Searby

G. Searby & D. Rochewerger *JFM* (1991)

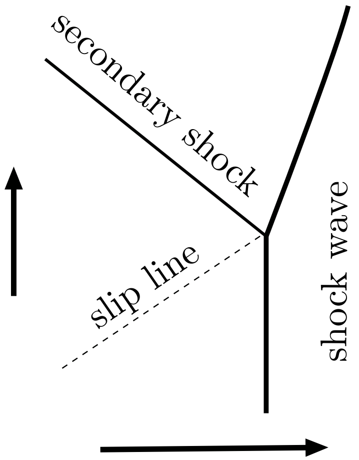
C.Clanet, G. Searby & P.C. *JFM* (1999)

INTERACTION
ONDE DE CHOC - VORTEX
ONDE DE CHOC - TURBULENCE

Formation de singularités en temps fini

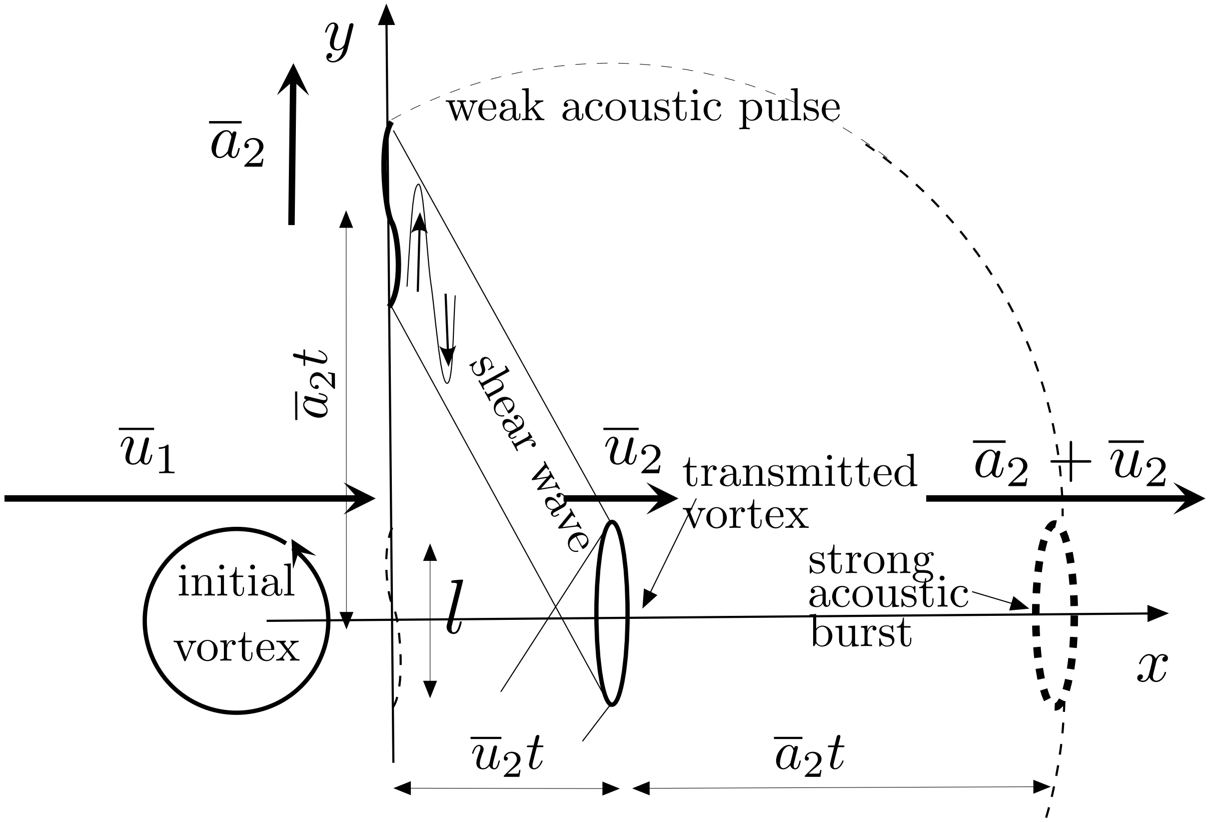
P. C. 2012

FORMATION DE POINT TRIPLE (MACH-STEM)



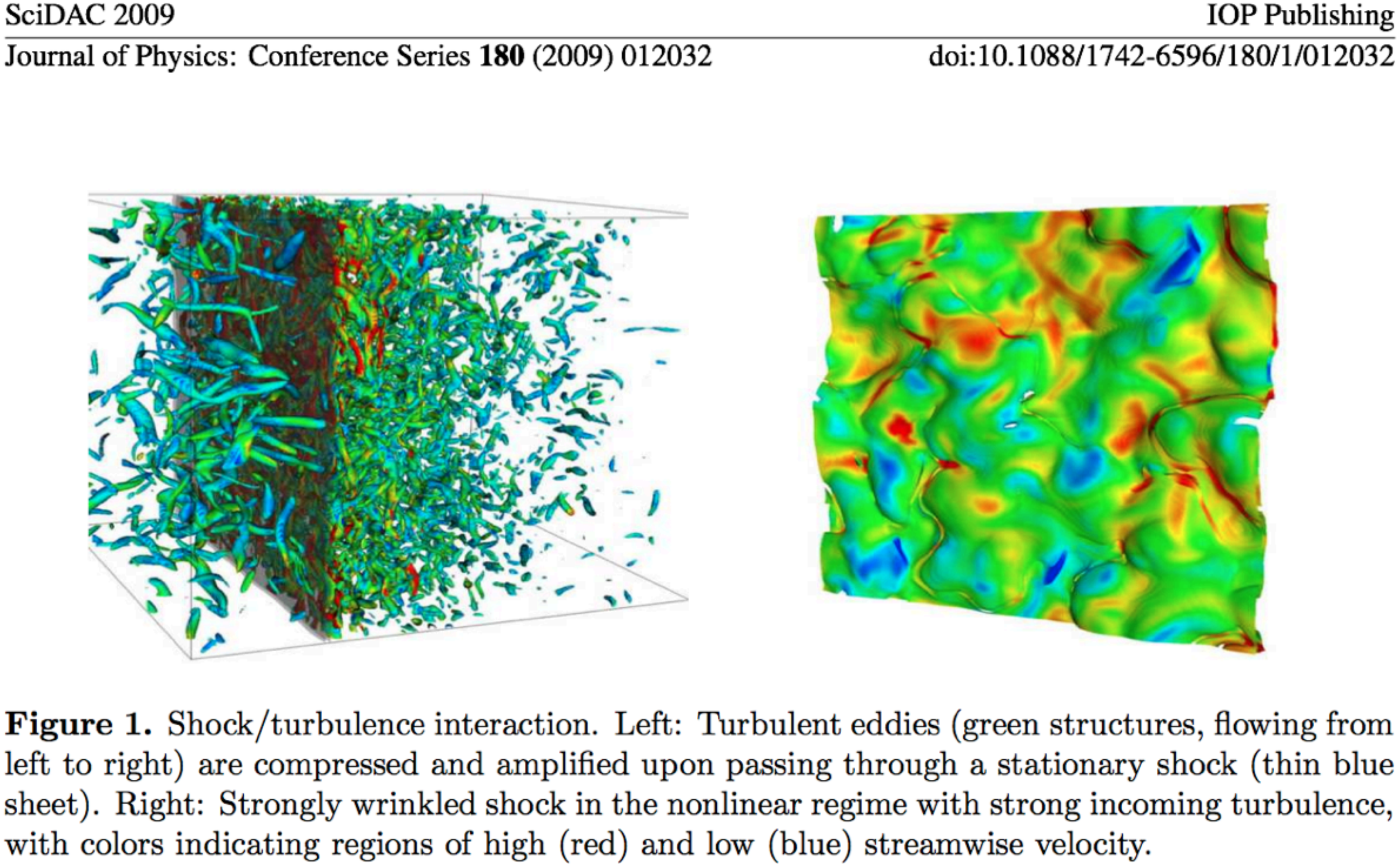
Stabilité et dynamique des fronts de choc.

Ondes transverses



S.K. Lele and J. Larsson
Standford University

Shock-turbulence interaction: What we know and what we can learn from simulations



SciDAC 2009 IOP Publishing
Journal of Physics: Conference Series **180** (2009) 012032 doi:10.1088/1742-6596/180/1/012032

Figure 1. Shock/turbulence interaction. Left: Turbulent eddies (green structures, flowing from left to right) are compressed and amplified upon passing through a stationary shock (thin blue sheet). Right: Strongly wrinkled shock in the nonlinear regime with strong incoming turbulence, with colors indicating regions of high (red) and low (blue) streamwise velocity.

Analyse faiblement linéaire pour les chocs forts dans la limite Newtonienne

Reduced equation for the shock front $\tilde{\mathcal{A}}(\eta, \zeta, \tau)$

$$\frac{\partial^2 \tilde{\mathcal{A}}}{\partial \tau^2} - \bigtriangledown^2 \tilde{\mathcal{A}} + \frac{\partial | \bigtriangledown \tilde{\mathcal{A}}|^2}{\partial \tau} = O$$

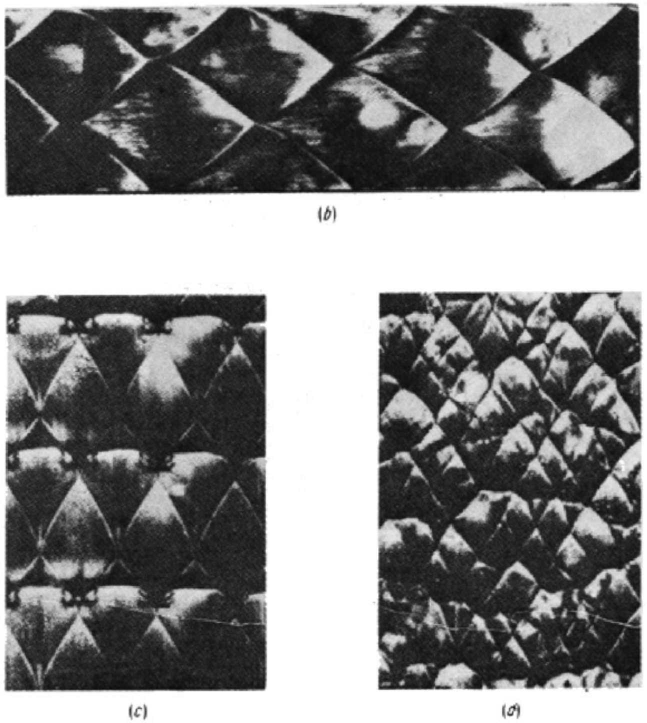
effet d'une perturbation amont (vortex, turbulence)

$$\frac{\partial^2 \tilde{\mathcal{A}}}{\partial \tau^2} - \bigtriangledown^2 \tilde{\mathcal{A}} + \frac{\partial | \bigtriangledown \tilde{\mathcal{A}}|^2}{\partial \tau} = \frac{\mathcal{H}(\eta, \zeta, \tau/\epsilon)}{\epsilon^2}$$

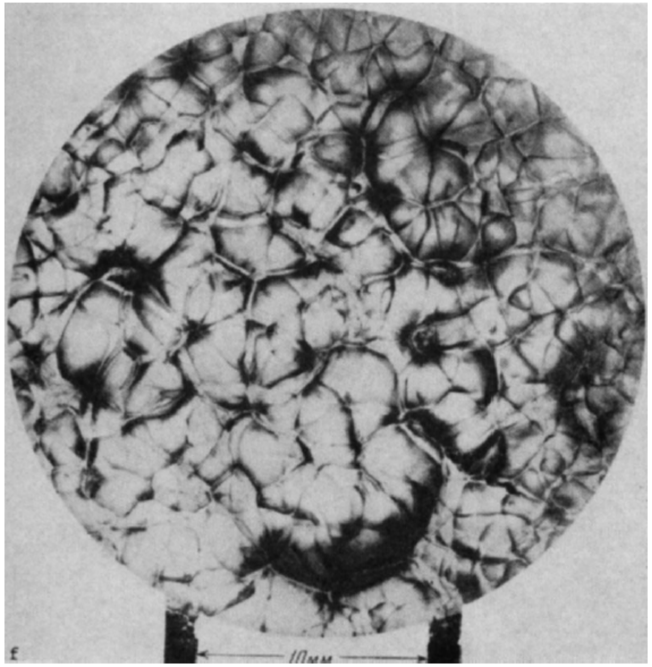
$\mathcal{H}(\eta, \zeta, .)$ function of order unity

effet de l'intermittence

Spinning, 1926
Cellular, 1950-1960



Markings left on sooted-coated foils at the walls



Front view
Shchelkin&Troshin 1965

Cellular detonations are instructve examples of Mach stem formation in shock waves

Shchelkin & Troshin 1965

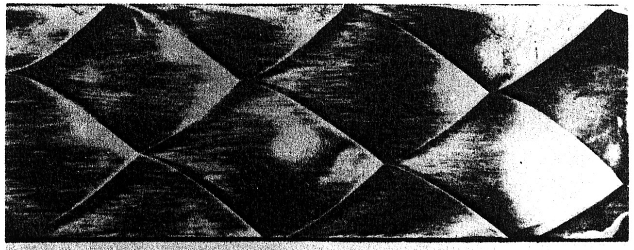
Strehlow 1984

Transverse structures

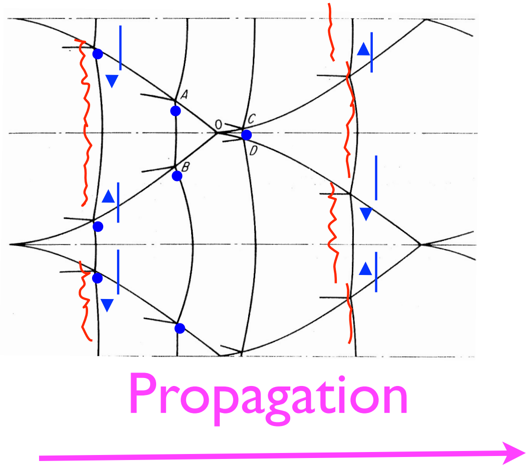
DNS:

Oran & Boris (1987)
Bourlioux & Majda (1992)

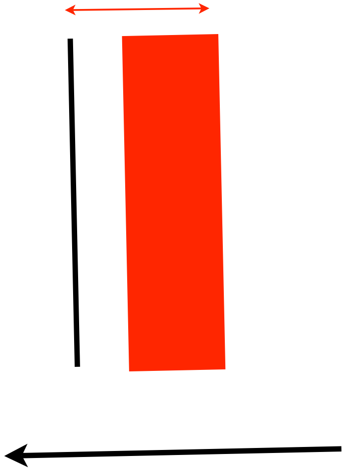
Sec. 7A OVERVIEW



PLANAR MODE $2H_2 + O_2$ + 70% Ar 70 TORR



INSTABILITÉ 1-D des détos



oscillations

gaz frais

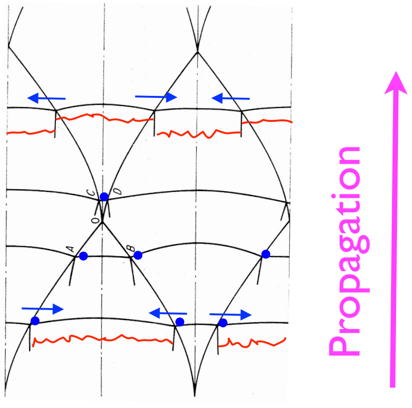
gaz brûlés

vitesse de propagation

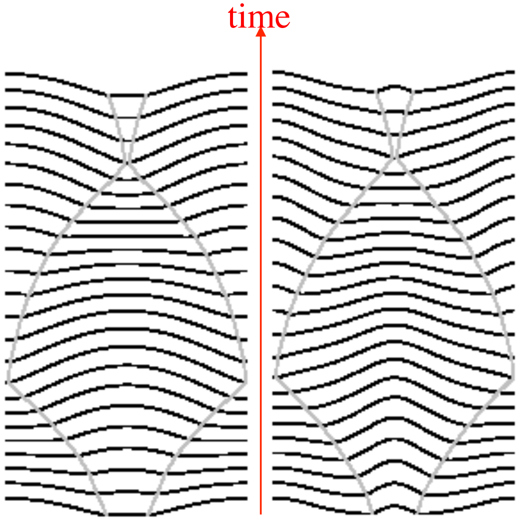
P.C. & L.He *JFM* 1996

Bifurcation analysis: model equation for the detonation front

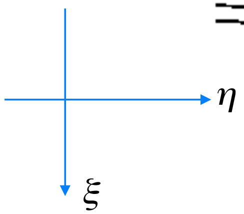
P.C. & B. Denet, *Phys. Rev. Lett. 2002*



Simple waves



General case



Mean shape of the front
$\xi = \overline{\alpha}(\eta)$

Pbs d'initiation directe en espace libre:

Pas trop difficile pour les flammes

Difficile pour les détos $p_N/p_u \approx 10 - 30$

L.He & P.C. *JFM* 1994

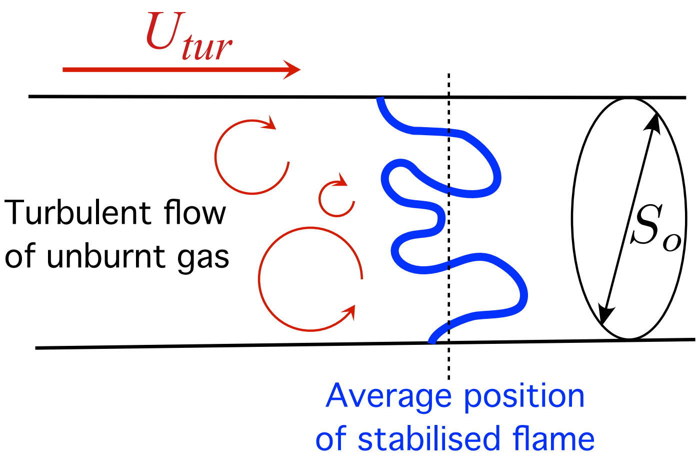
Transition déflagration détonation DDT

Flamme = piston semi-imperméable

Pas facile même en espace confiné.

Non observé en espace libre.

Flamme turbulente

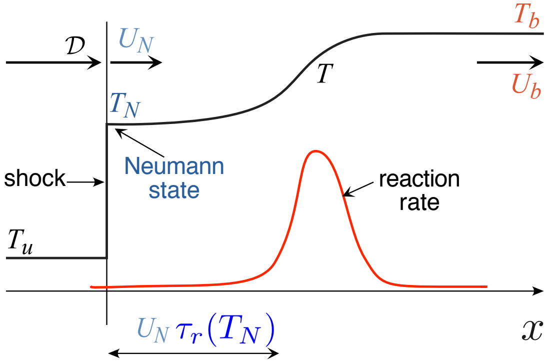


vitesse de flamme turbulente

surface de flamme

$$\frac{U_{tur}}{U_L} = \frac{\Sigma}{S_o}$$

DDT observé dans les tubes pour $U_{tur}/U_L \approx 10^2$

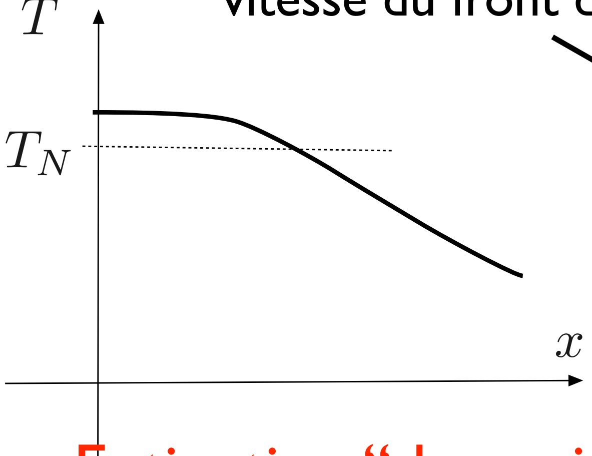


$$\boxed{\frac{d\tau_r}{dT} < 0}$$

Allumage spontané des détonations

Zeldovich et al. 1970, Zeldovich 1980

Gradient de température $T > T_N$ établie sur des temps courts $t < \tau_r(T)$



vitesse du front d'allumage

$$\boxed{\left|\frac{d\tau_r}{dT}\,\frac{dT}{dx}\right|^{-1} = a(T)}$$

vitesse du son

Extinction "dynamique" à plus basse température

point de retournement

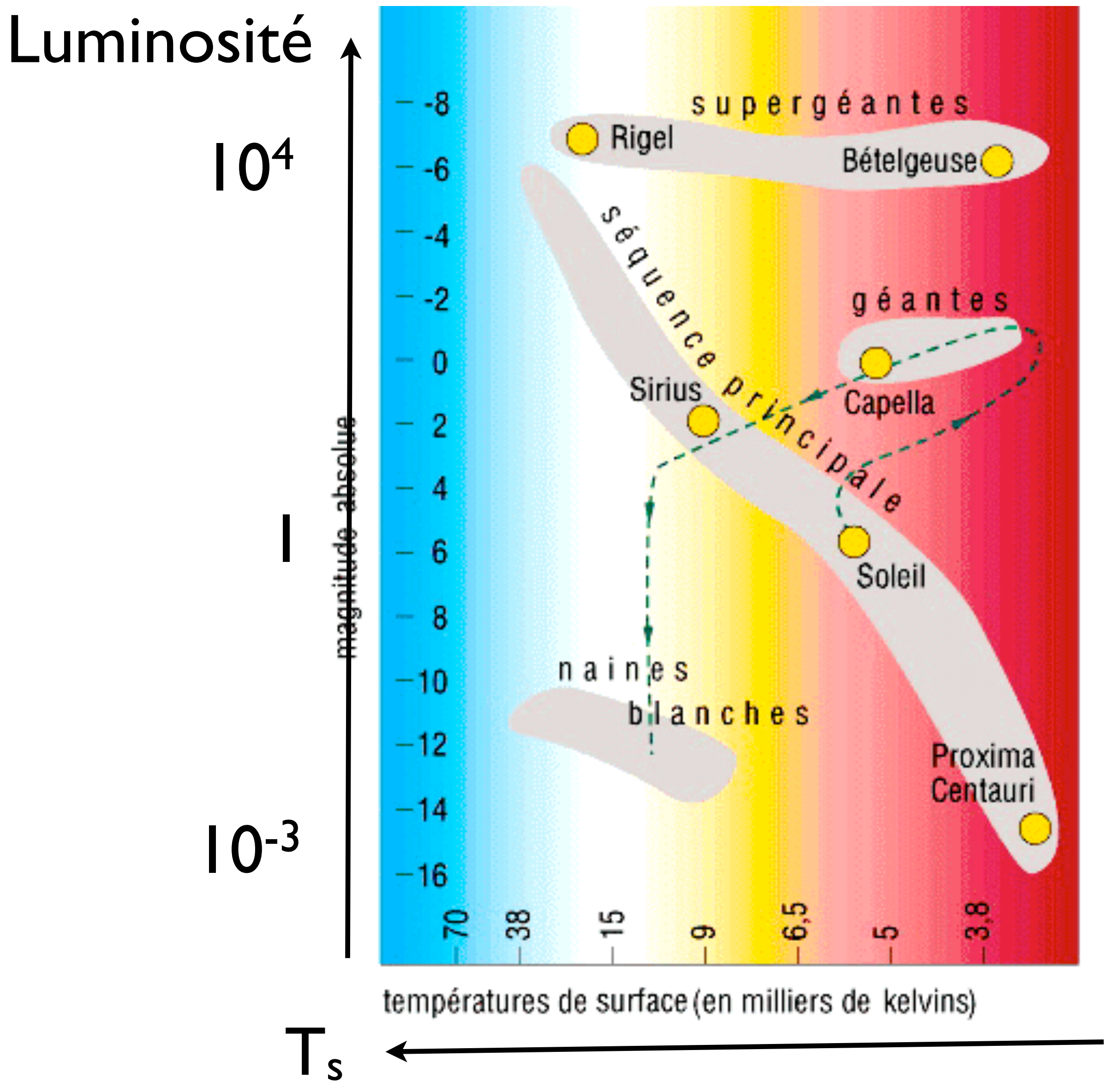
L. He & P.C. 1992, 1994

LES ÉTOILES

Zel’dovich & Novikov *“Stars and relativity”* Dover 1970

-1) États quasi-stationnaires

Étoiles en régime quasistationnaire



A. Bouquet 1998

Équilibre hydrostatique
sans réaction nucléaire

Hypothèses: *sphérique, pas de convection, "eq. d'état":* $\boxed{p = K\rho^\gamma}$

$$m(r) = 4\pi \int_0^r \rho(r')r'^2 dr' \qquad -\frac{1}{\rho(r)}\frac{dp}{dr} = \frac{Gm(r)}{r^2}$$

Données: *masse de l'étoile* M, ***Paramètres***: γ, K, G

Conditions aux limites: $r = 0 : d\rho/dr = 0, \quad r = R? : \rho = 0$

Méthode du tir corrigé

$$r = 0 : d\rho/dr = 0, \rho = \rho_c \Rightarrow R \text{ et } M \Rightarrow R(M)$$

Équation de Lane-Emden (1907)

$$n \equiv 1/(\gamma - 1) \quad l^2 \equiv [(n+1)K/(4\pi G)]\rho_c^{1/n-1} \quad \xi \equiv r/l \quad \theta \equiv \rho/\rho_c$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2 \frac{d\theta}{d\xi}\right] = -\theta^n$$

$$\xi = 0 : \quad \theta = 1, \quad d\theta/d\xi = 0$$

$$\gamma = 1 + \frac{1}{n} \qquad p = K\rho^{\gamma}$$

$$M = \frac{b(n)}{G^{3/2}} \frac{p_c^{3/2}}{\rho_c^2} \qquad \rho_c \propto M^{2n/(3-n)}$$

$n < 3, \quad (\gamma > 4/3):$ **Stable**

$n > 3, \quad (\gamma < 4/3):$ **Instable**

Naines blanches

$$\rho_c \ll 10^6 g/cm^3 \Rightarrow \gamma = 5/3$$

$$\rho_c \gg 10^6 g/cm^3 \Rightarrow \gamma = 4/3 \qquad p = K^* \rho^{4/3}$$

Masse critique

$$M^* \approx 1.4 M_{\odot}$$

Pertes thermiques radiatives

Compensées / réactions nucléaires

Combustible épuisé ?

$$U = -4\pi G \int_0^R \rho(r) \frac{m(r)}{r} r^2 dr \qquad E_T = 4\pi \int_0^R e(r) r^2 dr \qquad E = E_T + U$$

$$4\pi \int_0^R r^3 dr \left[-\frac{dp}{dr} = \rho(r) \frac{Gm(r)}{r^2} \right] \quad \Rightarrow 4\pi \int_0^R 3p(r) r^2 dr = -U$$

$$\text{gaz parfait :} \qquad 3p = 2\rho e$$

$$2E_T = -U \Rightarrow E = -E_T$$

$$\boxed{E \searrow \qquad T \nearrow}$$

de 3 x 10 ^{15} g/cm ^{3}, ce qui explique pourquoi il ne disparait pas dans un trou noir.

Réactions thermonucléaires majeures au sein des étoiles

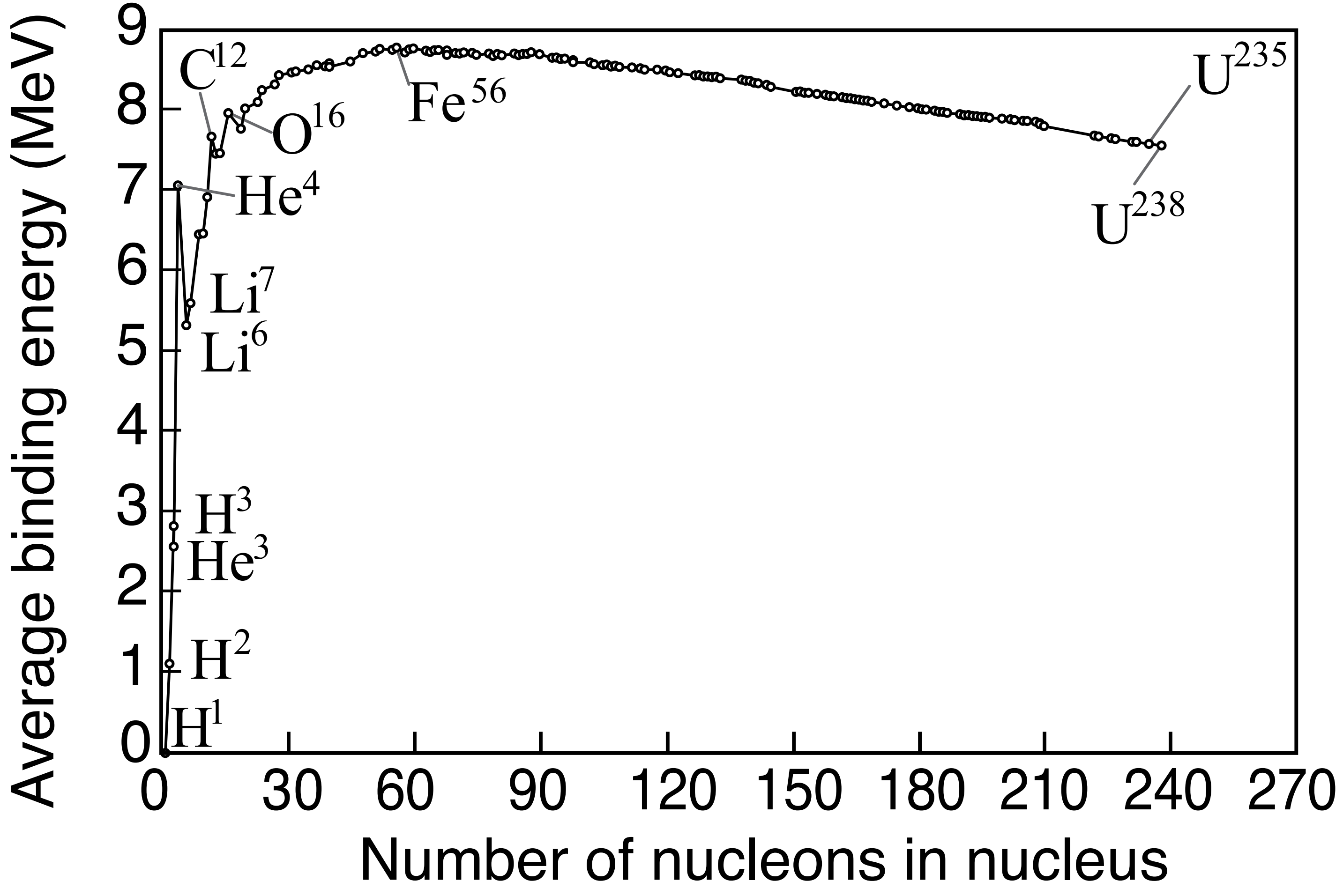
F.Terrin

Nature des réactions	Température d'ignition (en millions de degrés K)
Combustion de l'hydrogène $4\ ^{1}H \dashrightarrow\ ^{4}He$ (réaction proton-proton)	10
Combustion de l'hélium $3\ ^{4}He \dashrightarrow\ ^{8}Be + ^{4}He \dashrightarrow\ ^{12}C$ $^{12}C + ^{4}He \dashrightarrow\ ^{16}O$	100
Combustion du carbone $2\ ^{12}C \dashrightarrow\ ^{4}He + ^{20}Ne$ $^{20}Ne + ^{4}He \dashrightarrow\ n + ^{23}Mg$	600
Combustion de l'oxygène $2\ ^{16}O \dashrightarrow\ ^{4}He + ^{28}Si$ $2\ ^{16}O \dashrightarrow\ 2\ ^{4}He + ^{24}Mg$	1.500
Combustion du silicium $2\ ^{28}Si \dashrightarrow\ ^{56}Fe$	4.000
Photodissiociation du fer $^{56}Fe \dashrightarrow\ 13\ ^{4}He + 4\ n$	6000

Ces réactions sont classées par température d'ignition croissante, depuis 10 millions de degrés pour l'hydrogène jusque 6 milliards de degrés pour le fer. Seule

	T (K)	$\rho\,(\mathrm{g.cm}^{-3})$	ans
H → He	$6.\,10^7$	5	$7.\,10^6$
He → C, O	$2.\,10^8$	$7.\,10^2$	$5.\,10^5$
C → O, Ne, Mg	$9.\,10^8$	$2.\,10^5$	$6.\,10^2$
Ne → O, Mg, Si	$1.7\,10^9$	$4.\,10^5$	1
O → Si, S	$2.3\,10^9$	10^7	0.5
Si → Fe	$4.\,10^9$	$3.\,10^7$	1 jour

Nucléosynthèse calme $M = 25\,M_\odot$


Average binding energy (MeV)
0
1
2
3
4
5
6
7
8
9
0
30
60
90
120
150
180
210
240
270
Number of nucleons in nucleus
H1
H2
He3
H3
Li6
Li7
He4
C12
O16
Fe56
U235
U238

Que se passe-t-il quand les réactifs sont épuisés ?

Fin de combustion

étoiles de petite masse $\neq$ étoiles de grande masse

$M < 0.7\,M^*$ Naine blanche d'He

$M \approx 0.7\ M^*$ Géante Rouge, Naine blanche C et O

$M \leqslant 10 M_\odot$ Naine blanche C et O après expulsion de l'enveloppe !? $0.5M_\odot < M_{C-0} < 0.8M_\odot$

$10\,M_\odot < M < 12\,M_\odot$ Naine blanche Ne et O après expulsion de l'enveloppe !? $0.8M_\odot < M_{Ne-0} < 1.4M$

$12\,M_\odot < M$ $T_c \nearrow 5\,10^8\,K$ Combustion $\nearrow$ Fe étoile à neutrons $M_{Ne-O} < M^*$

SN II

Bethe *Rev. Mod. Phys. 1990*

Étoile de grosse masse

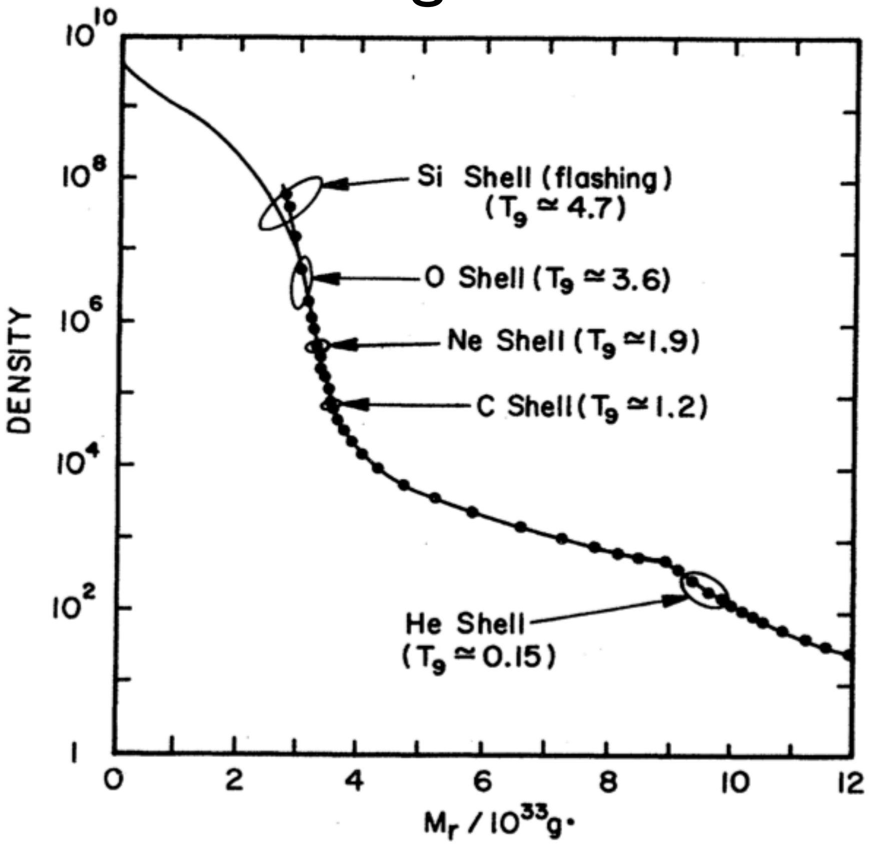
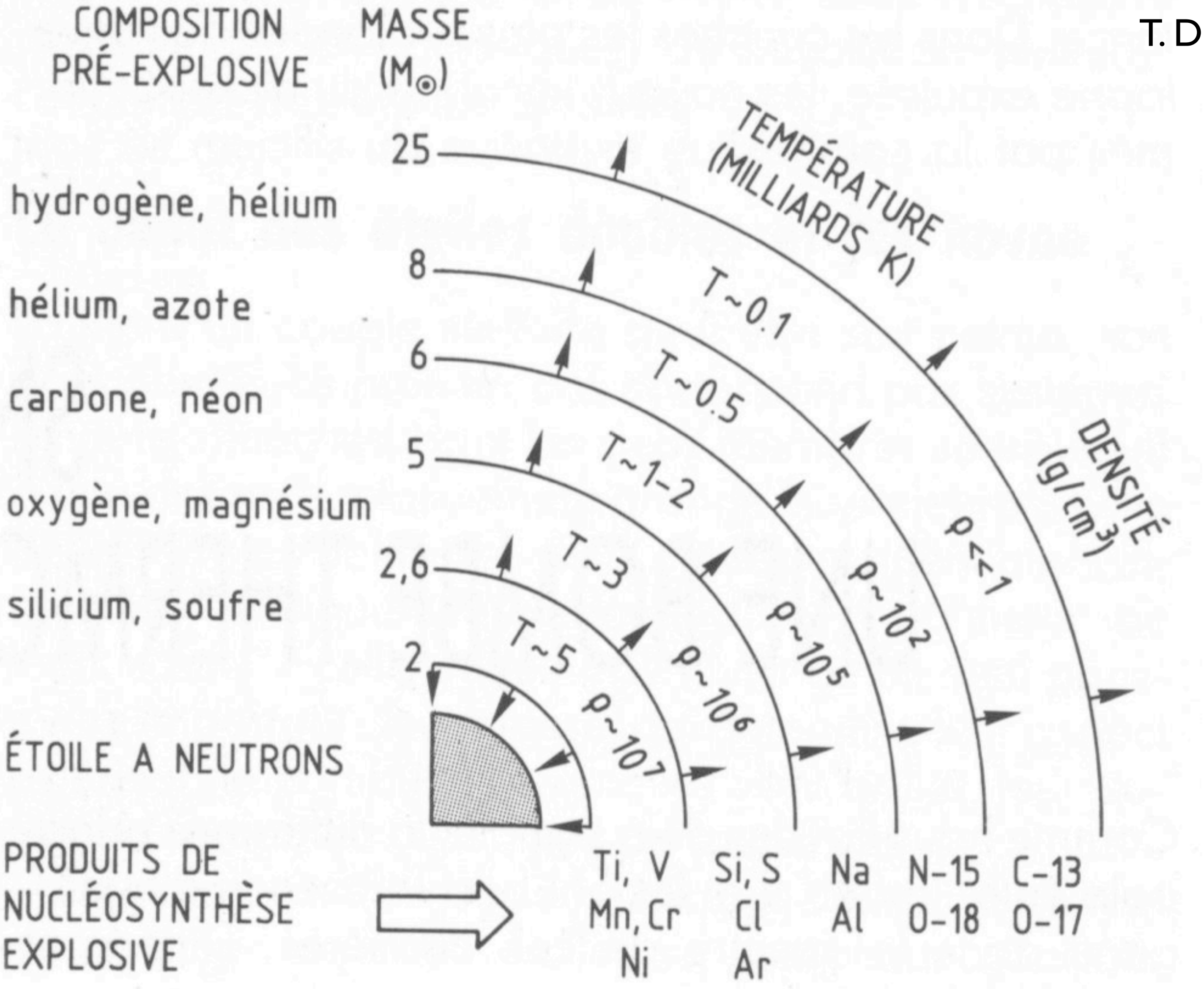


FIG. 1. Density distribution of a star before supernova collapse, according to Arnett (1977a). The enclosed mass is given in units of 10^{33} grams; the Sun's mass is 2×10^{33} g. The location of shells is indicated in which various nuclear reactions take place, together with the temperature in units of 10^9 K.

T. Douvion 1999



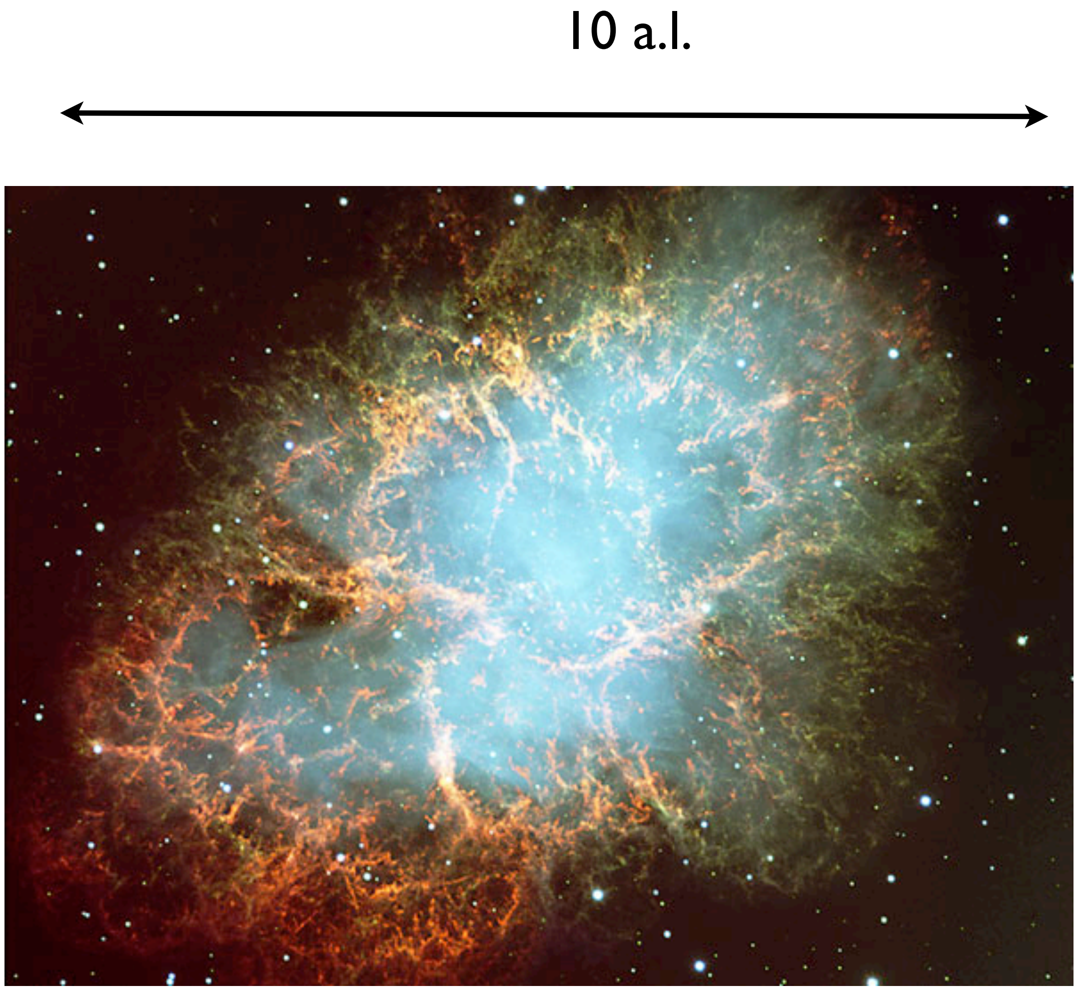
Supernovae: évènements courts, violents extraordinaires et exceptionnels

brillance = 10^9 soleil, durée: plusieurs semaines à 1 an,
éjection d'une énorme quantité de matière à très grande distance,
production des éléments lourds

185 bc, 1006, 1054 Chine, 1572 Tycho Brahe, 1650 Kepler, ...1987,..
une centaine répertoriée à ce jour

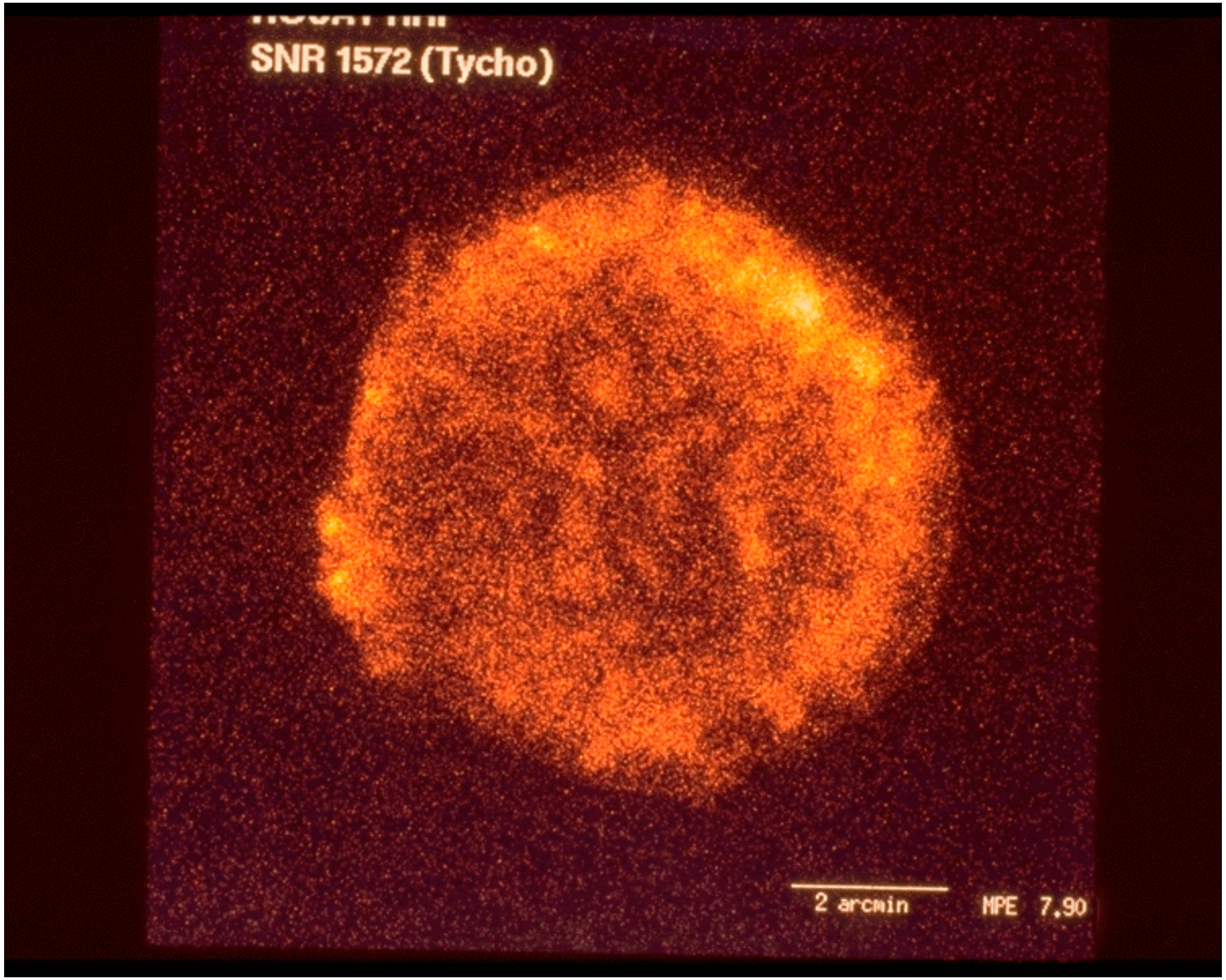
courbe de lumière + spectre: classification.......

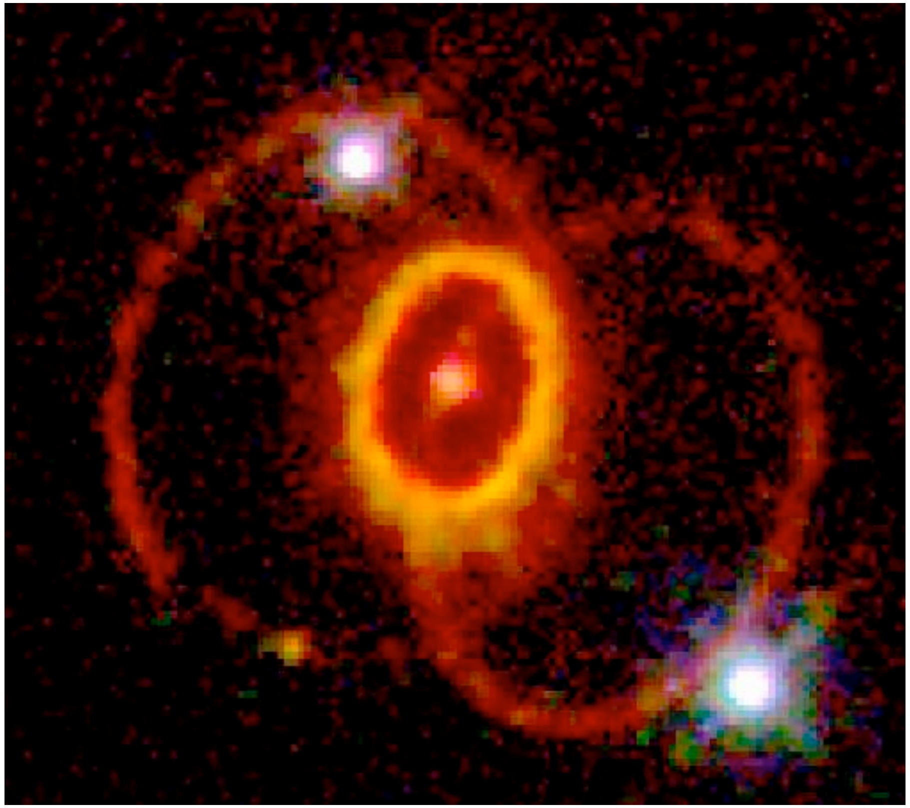
traces visibles + longtemps



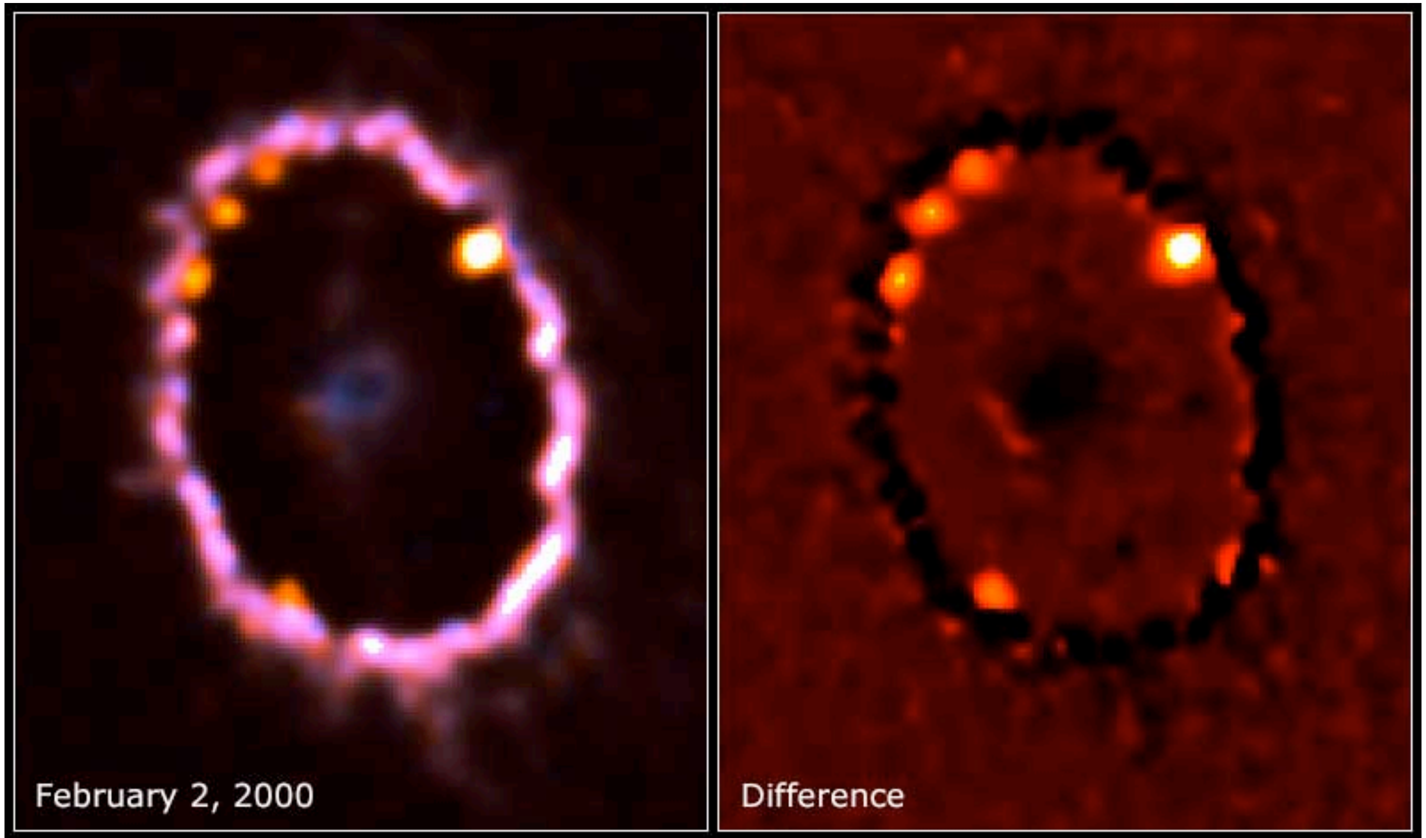
vitess d'expansion
qq 1500 km/s
densité des filaments
10^3 part/cm^3

SN 1054 Nébuleuse du Crabe ds constellation du Taureau (voie lactée)
distance: 6 520 a.l.
au centre: étoile à neutron, pulsar rotation 33/s


SNR 1572 (Tycho)
2 arcmin
MPE 7.90



SN 1987: Grand nuage de Magellan17 10^4 a.l.

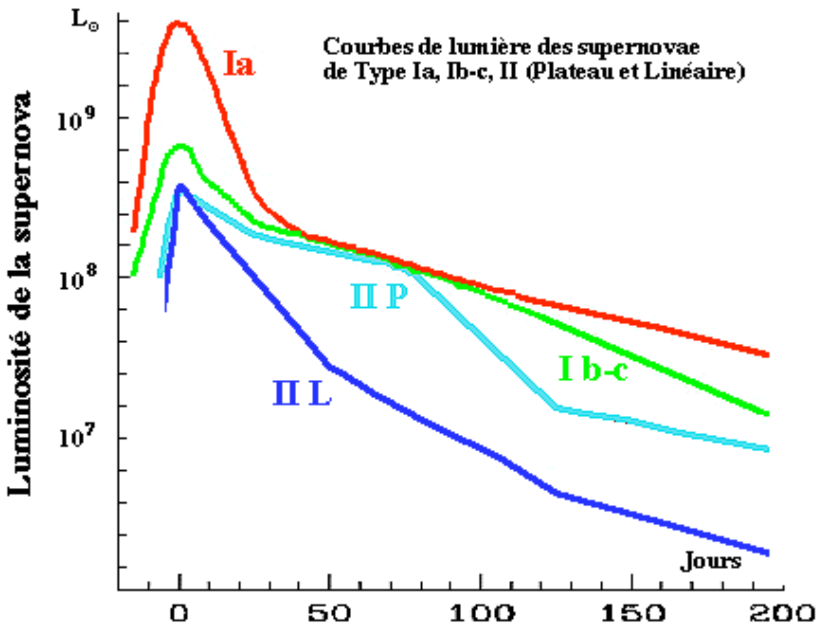


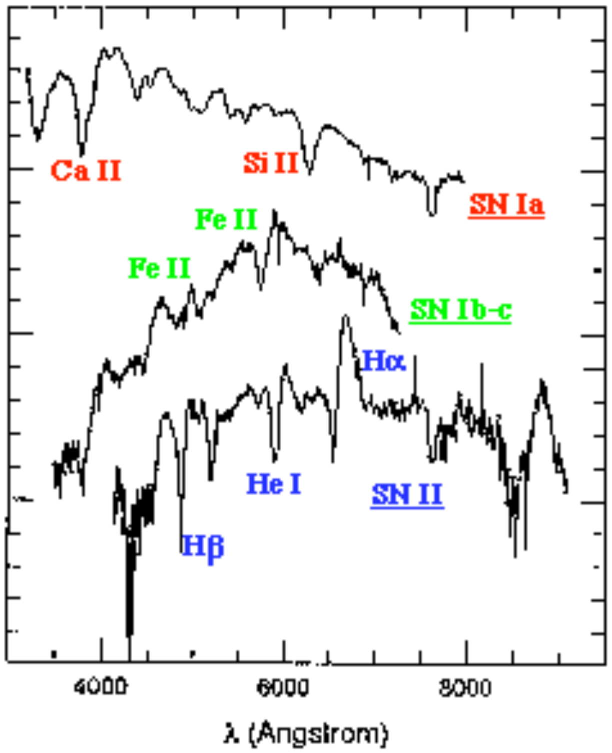
Supernova 1987A in the Large Magellanic Cloud **HST** • WFPC2

NASA, P. Challis and R. Kirshner (CfA), P. Garnavich (University of Notre Dame)
and The SINS Collaboration • STScI-PRC00-11

SN I: pas de raies H

SN II: raies H





A. Bouquet 1998

SN Ia: Combustion explosive

...Wheeler & Harkens *Rep. Prog. Phys* (1990), Khokhlov et al. *A&A* (1992), ... Baron et al. *ApJ* (2012)

Petite étoile ancienne: naine blanche

M $\nearrow$ par interaction binaire $10^{-10} - 10^{-5} M_{\odot}/an$

$\rho : 10^6 - 10^7 g/cm^3 \quad T \approx 10^8 K$

Simulations numériques:

Allumage d'une flamme turbulente
transition vers une détonation

Mes critiques sont nombreuses

SN II: collapse

Yahil *ApJ* (1983), Bethe *Rev. Mod. Phys* (1990), Woosley Heger *Rev. Mod. Phys* (2002),
Janka et al. *Phys. Rep.* (2007), Keshet & Balberg *P. R.L* (2012),

Grosse étoile jeune en pelure d'oignon

Analytique + simulations numériques:

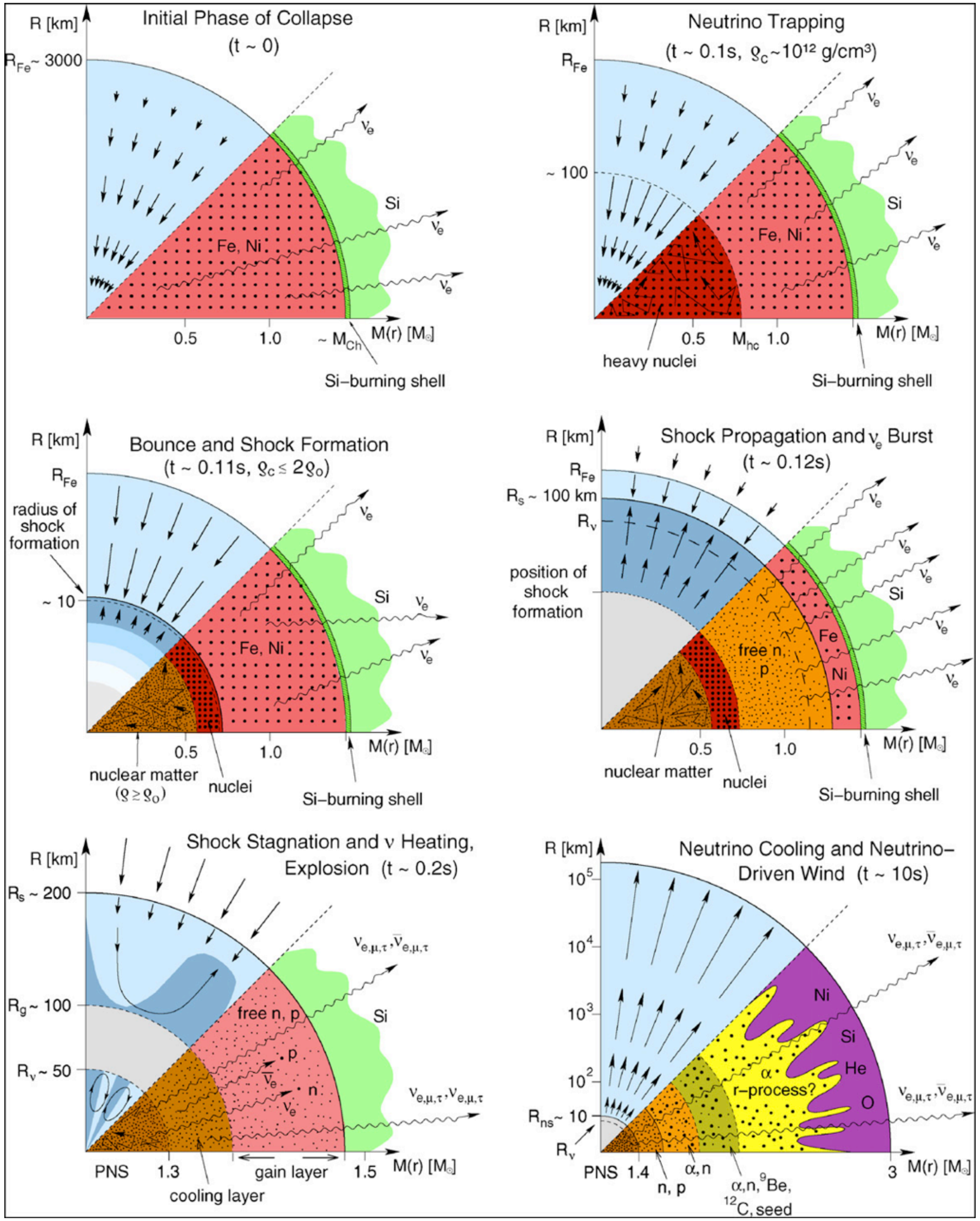
Effondrement brutal de l'enveloppe
sur le noyau incompressible.

$$\rho : 10^{12} - 10^{14} g/cm^3$$

Formation d'un choc se propageant vers l'extérieur

Accélération et renforcement du choc par les neutrinos

Janka et al. *Phys. Report 2007*



NUMERICS:

HOMOLOGOUS COLLAPSE NEAR THE CORE

QUASY-STEADY OUTER CORE

CRUCIAL ROLE OF NEUTRINOS
FOR A STRONG SHOCK

Self similar solution for the collapse in spherical geometry for a steady outer core

$$p = K\rho^{\gamma} \qquad 6/5 < \gamma \leqslant 4/3$$

$$r^2 \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u r^2)}{\partial r} = 0 \qquad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{Gm}{r^2}, \qquad m \equiv 4\pi \int_0^r \rho r^2 dr$$

dimensional parameters : $G,\ K,\ \cancel{M}$

$$X = K^{-1/2} G^{\frac{\gamma-1}{2}} \frac{r}{(-t)^{2-\gamma}}$$

$$\rho = \frac{1}{G}\frac{1}{(-t)^2} D(X) \qquad X \to \infty : \quad D(X) \propto 1/X^{\frac{2}{2-\gamma}}$$

$$u = \frac{K^{1/2}}{G^{(\gamma-1)/2}} \frac{1}{(-t)^{\gamma-1}} V(X) \qquad X \to \infty : \quad V(X) \propto 1/X^{\frac{\gamma-1}{2-\gamma}}$$

$$[V + (2-\gamma)X]D'/D + V' = -2 - 2V/X$$

$$\gamma D'/D^{2-\gamma} + [V + (2-\gamma)X]V' = -(4\pi/X^2)\int_0^X D(X)X^2 dX - (\gamma - 1)V$$

$$[V + (2-\gamma)X]DX^2 = (4 - 3\gamma)\int_0^X D(X)X^2 dX \qquad \Rightarrow X \to 0 : D \to D_c?,\ V \approx -2X/3$$

sonic point !

Self-similar stellar collapse

Yahil. *ApJ 1983*

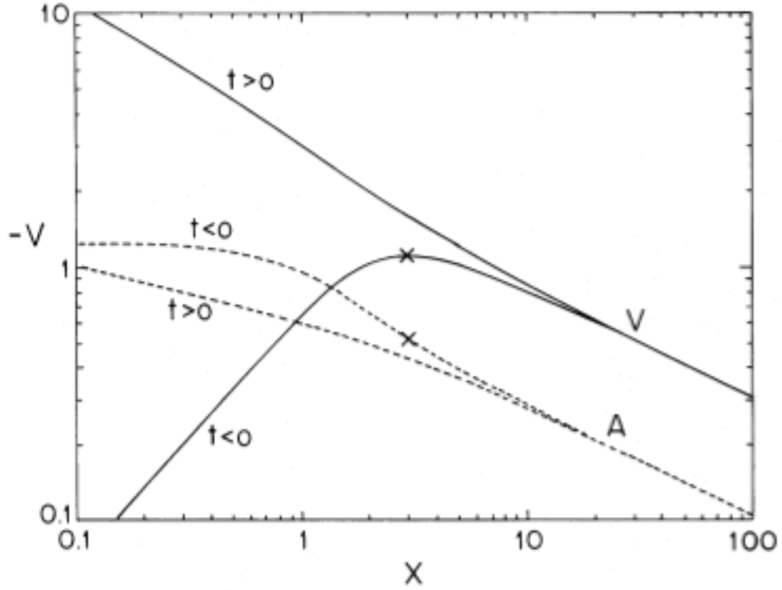


FIG. 1.—Dimensionless infall velocity $-V$ and sound speed A for $\gamma = 1.30$. Both precatastrophe $(t < 0)$ and postcatastrophe $(t > 0)$ solutions are shown. A cross at the point of maximum infall velocity marks the edge of the inner core. The same notation holds in Figs. 2–5.

Critical conditions for core-collapse supernovae

Keshet & Balberg *P. R.L* (2012),

Analyse quasi-statique au moment du ralentissement critique

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{Gm}{r^2}, \qquad 4\pi\rho u r^2 = \dot{m}$$

$$u\left(\frac{de}{dr} - \frac{p}{\rho^2}\frac{\partial \rho}{\partial r}\right) = -\frac{L}{r^2} + kT^6$$

heating by the outgoing neutrino flux

cooling by the neutrino-emission

$$n + \nu_e \rightarrow p + e^- + q$$

neutrino-emission by electron capture $p + e^- \rightarrow n + \nu_e - q$

Conditions limites: sur le coeur et au choc

Résultat: point de retournement avec des valeurs “pertinentes” des paramètres !