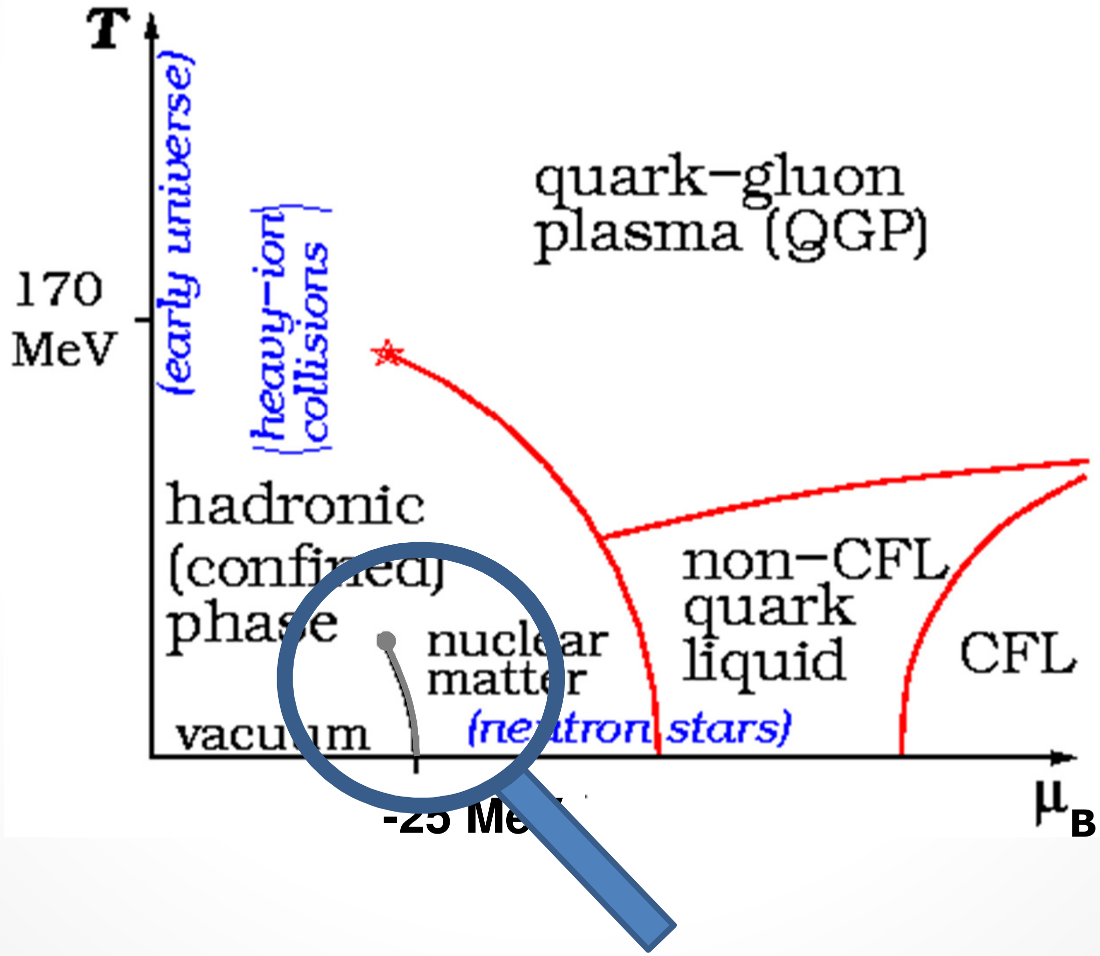
Coulomb effects in the phase diagram of neutron stars

F.Gulminelli – LPC and University of Caen

The QCD phase diagram of dense matter



Source: WIKIPEDIA

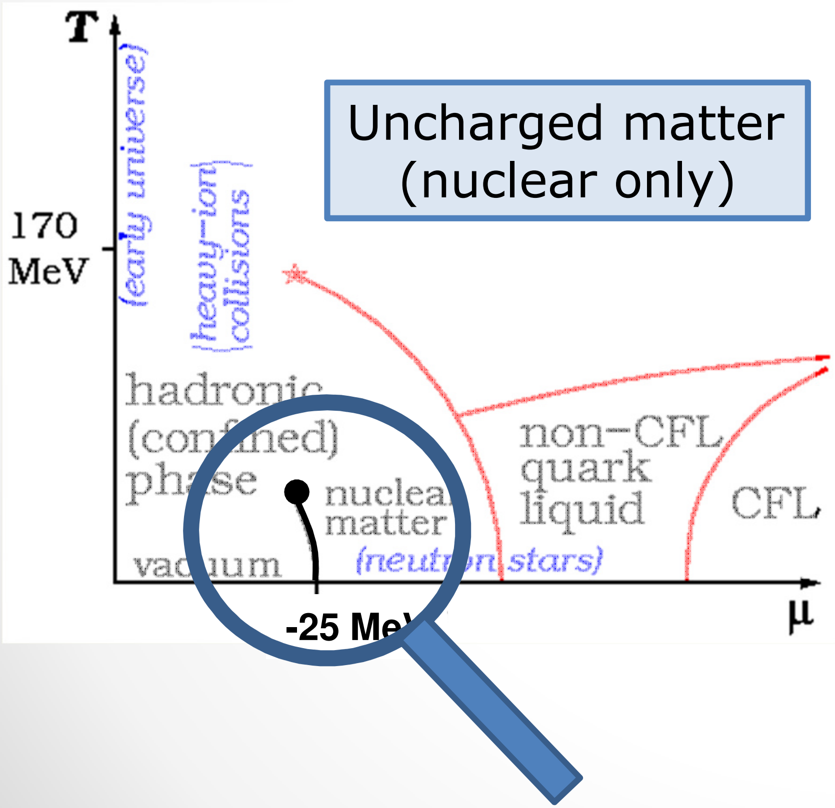
(T,μ_B) description of dense matter in Neutron Stars

- **Is a schematic nuclear-only phase diagram relevant?**
- 3 good quantum numbers $(\rho_B,\ \rho_Q, \rho_L) \leftrightarrow (\mu_B,\ \mu_Q, \mu_L)$
- Charge neutrality: 2 dof + 1 constraint
  $(\rho_B, \rho_Q = 0, \rho_L) \leftrightarrow (\mu_B, \mu_L)$

- **Is a grandcanonical description adequate?**
- Attractive&repulsive, short&long range interactions
- Gravity imposes a density profile => a canonical description

Collaboration (ANR SN2NS)

Adriana Raduta, IFIN Bucarest
Micaela Oertel LUTH Meudon
Panagiota Papakonstantinou, IPN Orsay
Jerome Margueron, IPN Orsay

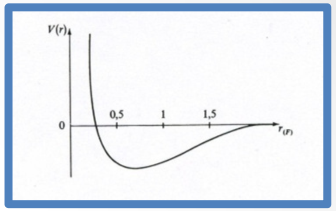


- No first order phase transition
- No critical point
- Instability to finite size fluctuation
- **Ensemble inequivalence**

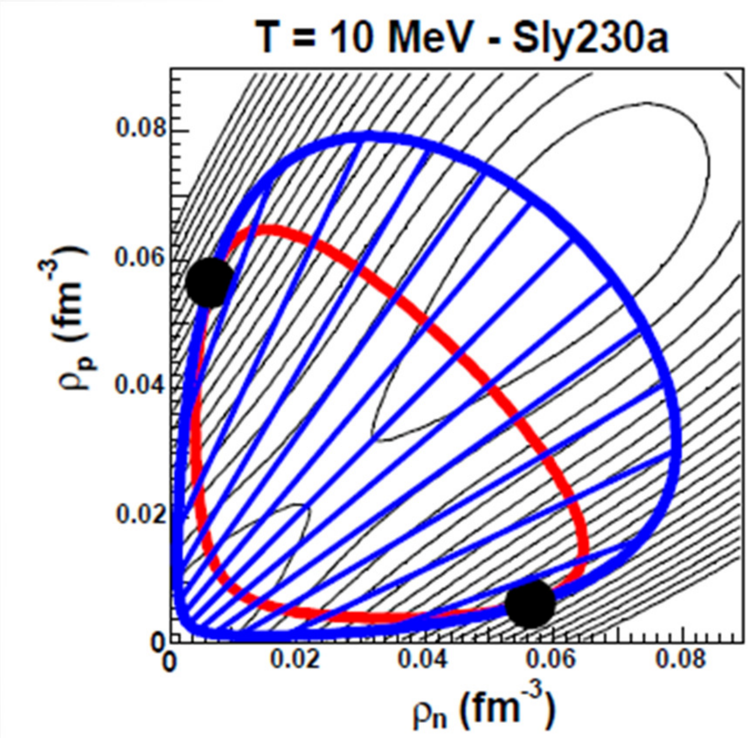
Because of Coulomb correlations
Neutral ≠ Uncharged

Phase Diagram of « nuclear matter »

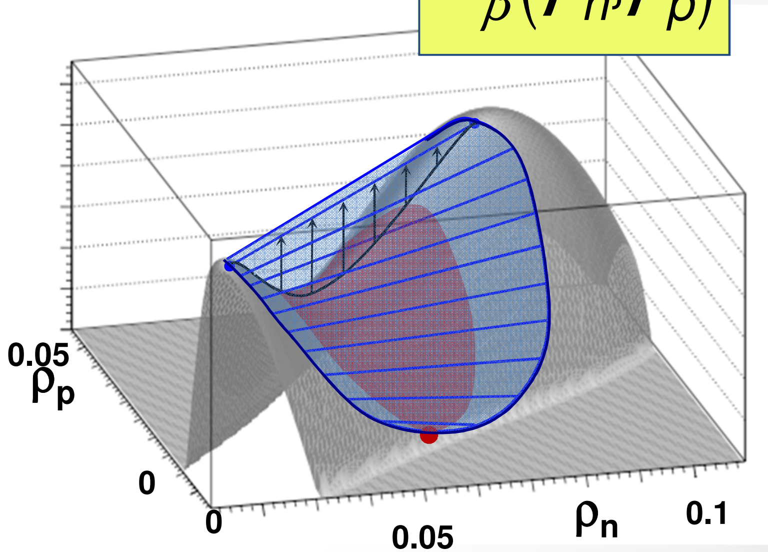
= n,p (short-range nuclear only)



$$-\beta f_\beta(\rho_n,\rho_p) = s^{HF}(\rho_n,\rho_p) - \beta e(\rho_n,\rho_p)$$



$$-f_\beta(\rho_n,\rho_p)$$



1st&2nd order PT; $\rho = \rho_n + \rho_p$ order parameter
=> »LG »

Phase Diagram of « nuclear matter »

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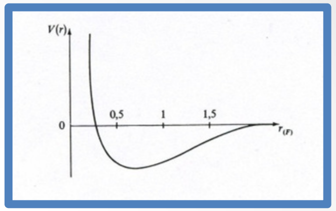
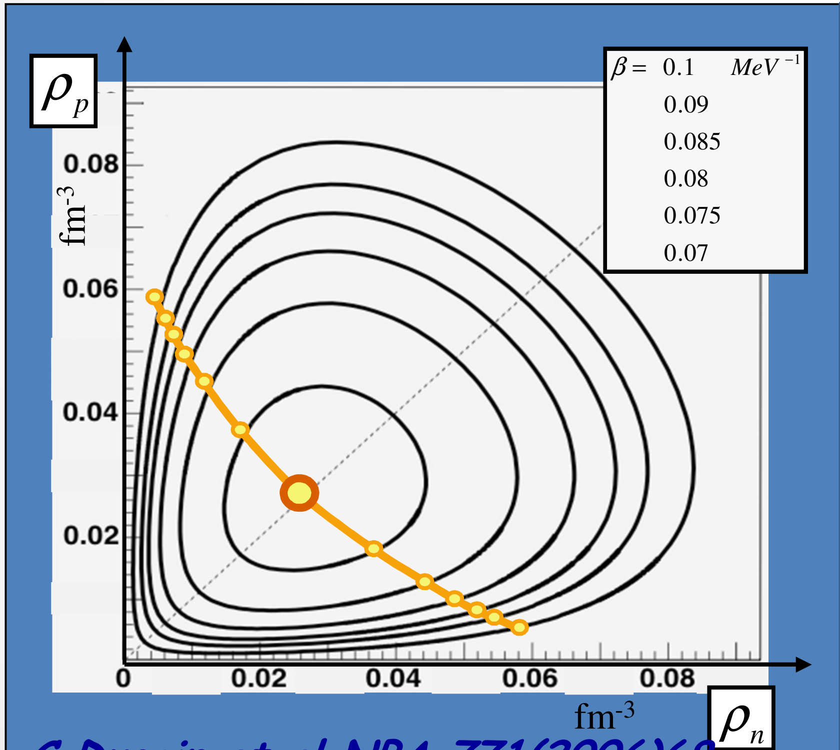
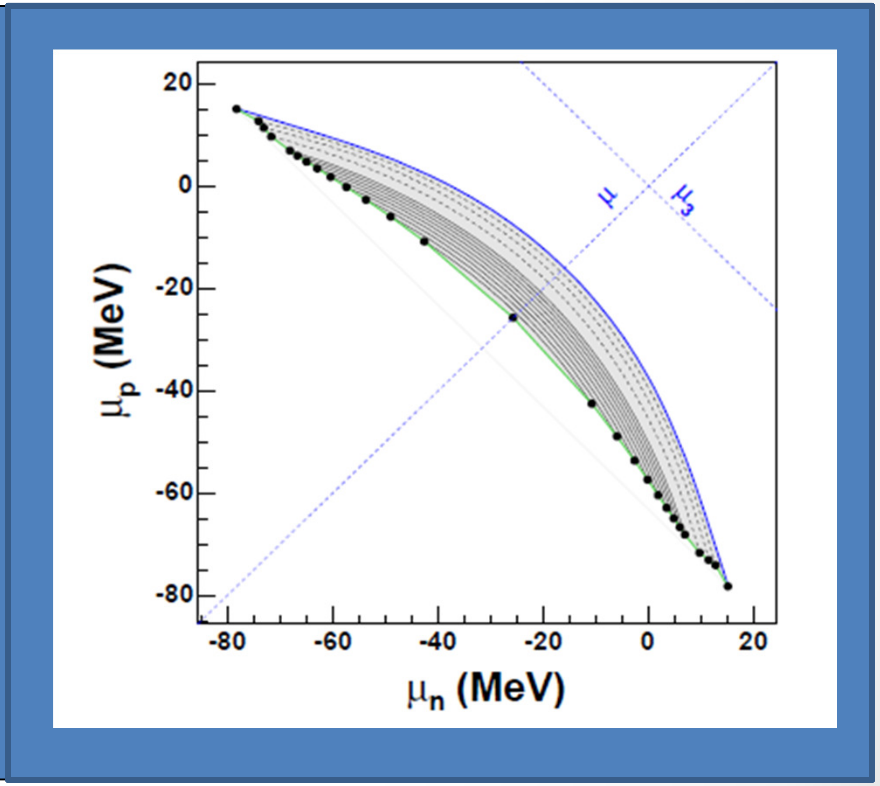




C.Ducoin et al NPA 771(2006)68



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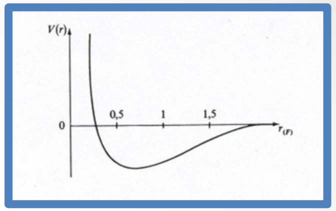
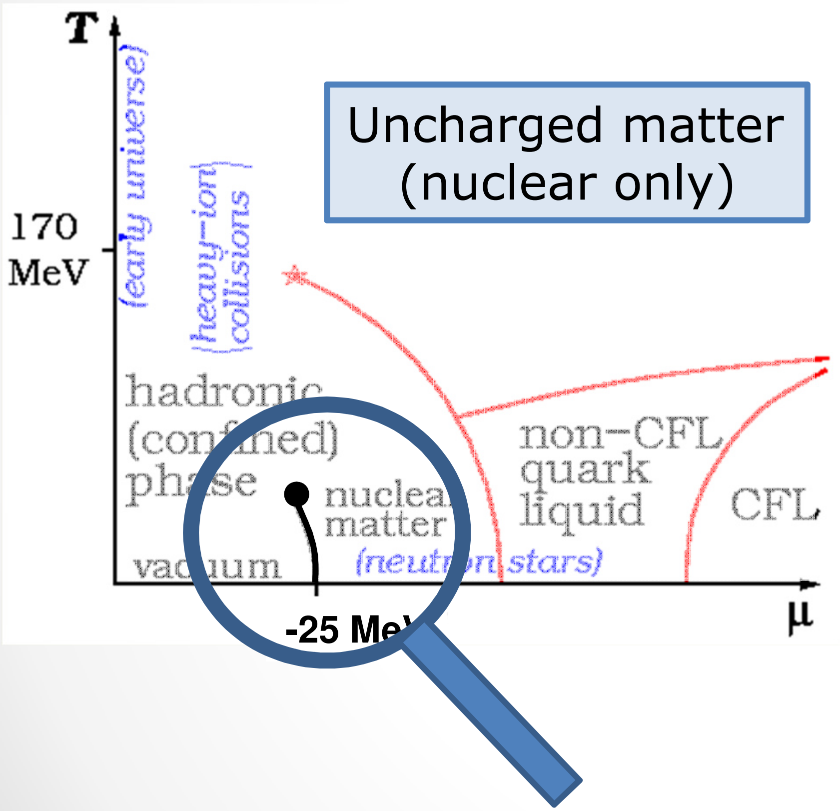
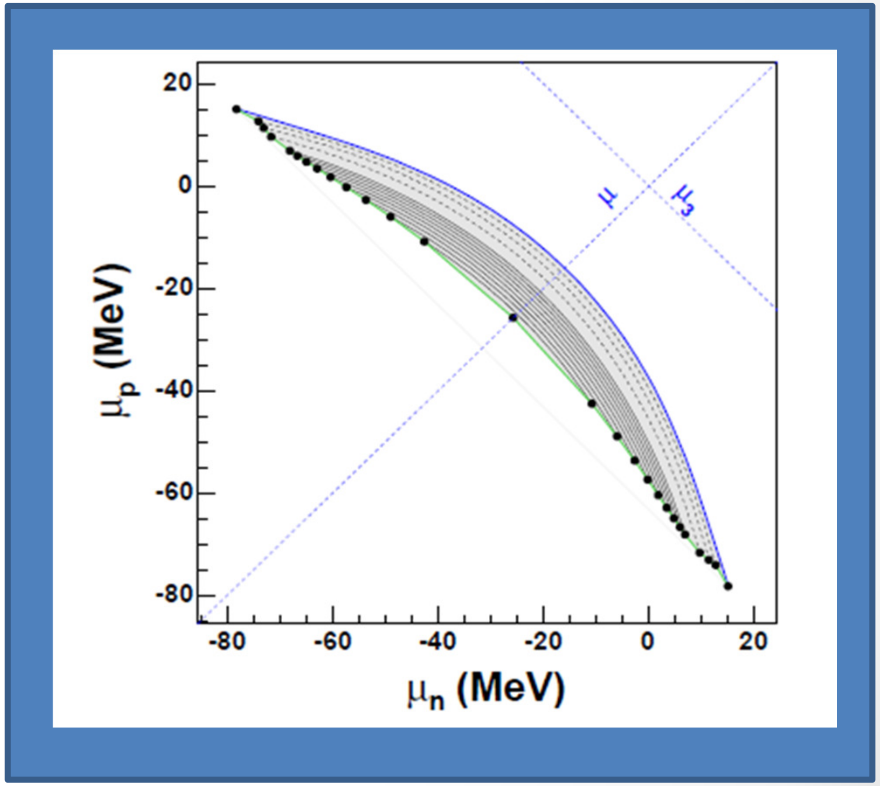
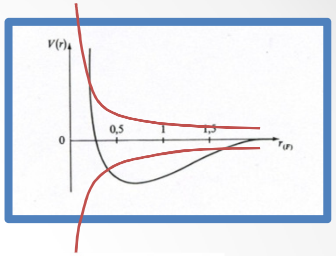






1st&2nd order PT; $\rho=\rho_n+\rho_p$ order parameter
=> »LG »

Stellar Matter =n,p (short-range nuclear) +e (long range Coulomb)



- $\widehat{H} = \widehat{H}_{np} + \widehat{K}_e + \sum_{ij=p,e} \widehat{V}_{ij}$

$$\hat{V}_{ii'} = \frac{\alpha q_i q_{i'}}{1+\delta_{ii'}} \int \frac{\rho_i(\vec{r})\rho_{i'}(\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r} d\vec{r}'$$

$$\frac{\sum_{ij}\langle \widehat{V}_{ij}\rangle}{\Omega} \propto {\rho_Q}^2 \Omega^{2/3}$$

- Strict charge neutrality required at the thermo limit
- Multipole expansion: $E_{Coul} = \sum_{l,l',m,m'} Y_{LM}^{ll'mm'}(\theta,\varphi) \frac{Q_l^m Q_{l'}^{m'}}{r^{L+1}}$ => the longest range interaction among neutral subsystems is the D-D interaction $E_{coul} \propto D \cdot D'/R^3$
- Effective short range BUT Coulomb correlation imposes a constraint on the order parameter

$$(\rho_n, \rho_p, \rho_e) \leftrightarrow \left(\rho_n, \rho_0 = \frac{\rho_p + \rho_e}{2}, \rho_q = \rho_p - \rho_e = 0\right) \Rightarrow (\rho_n, \rho_0)$$

-

Ensemble inequivalence

with a background of neutralizing charge

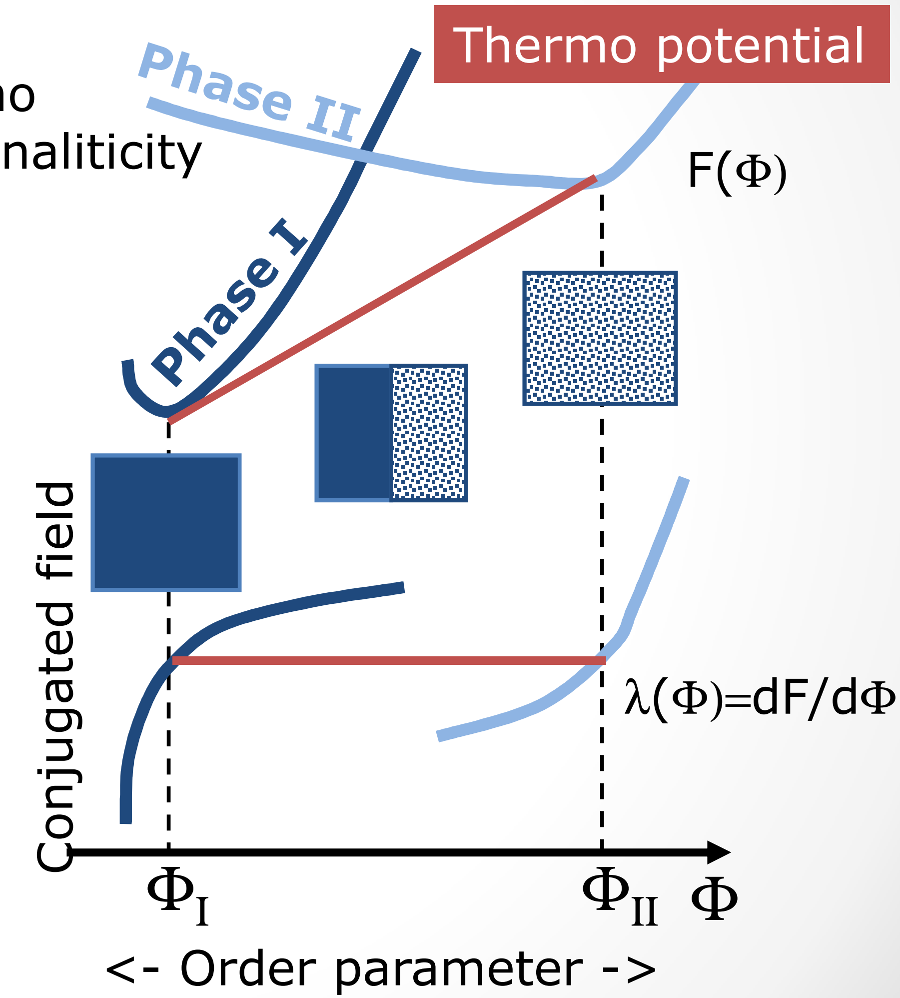
⇔ constraint on the order parameter $\rho_p = \rho_e$

A.Raduta,F.G.,PRC 82:065801 (2010)
PRC 85:025803 (2012)

Inequivalence and phase transition

- Phase mixing minimizes the thermo potential F and leads to the non-analiticity
- This requires **additivity of F** over macroscopic domains

$$f(\varphi) = \frac{F}{V} = \alpha f_I(\varphi_I) + (1-\alpha) f_{II}(\varphi_{II})$$



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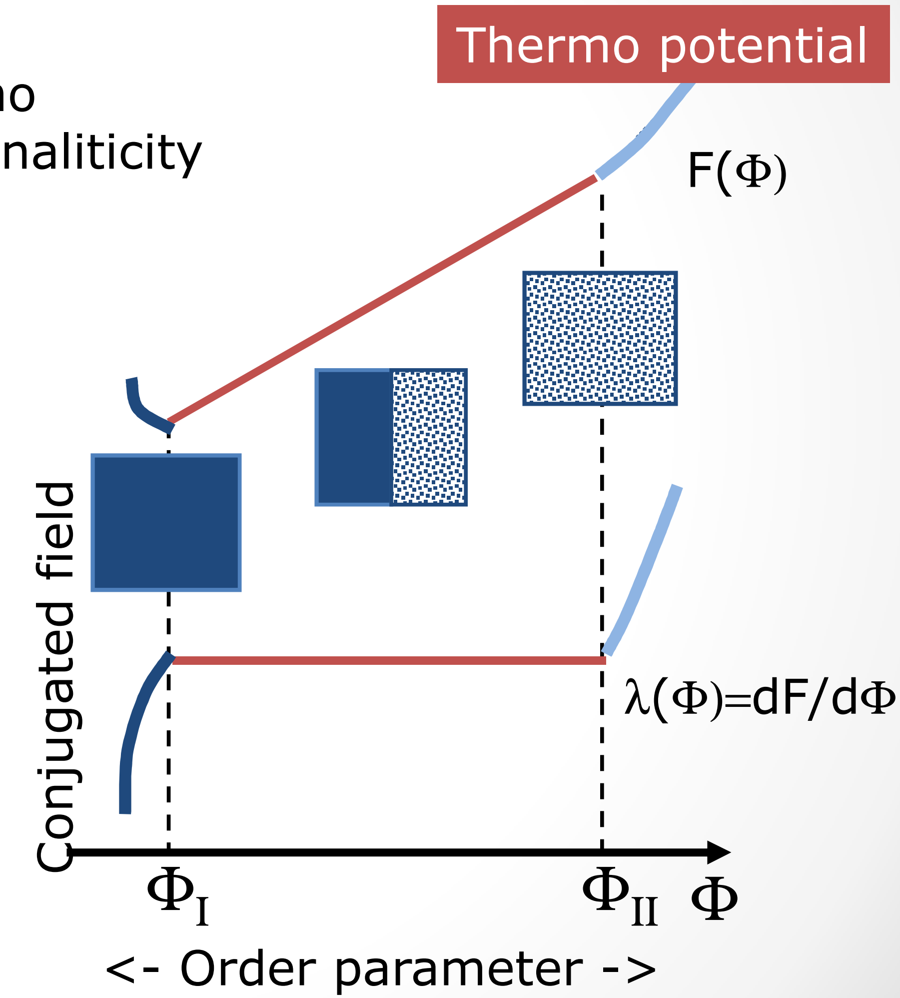

Inequivalence and phase transition

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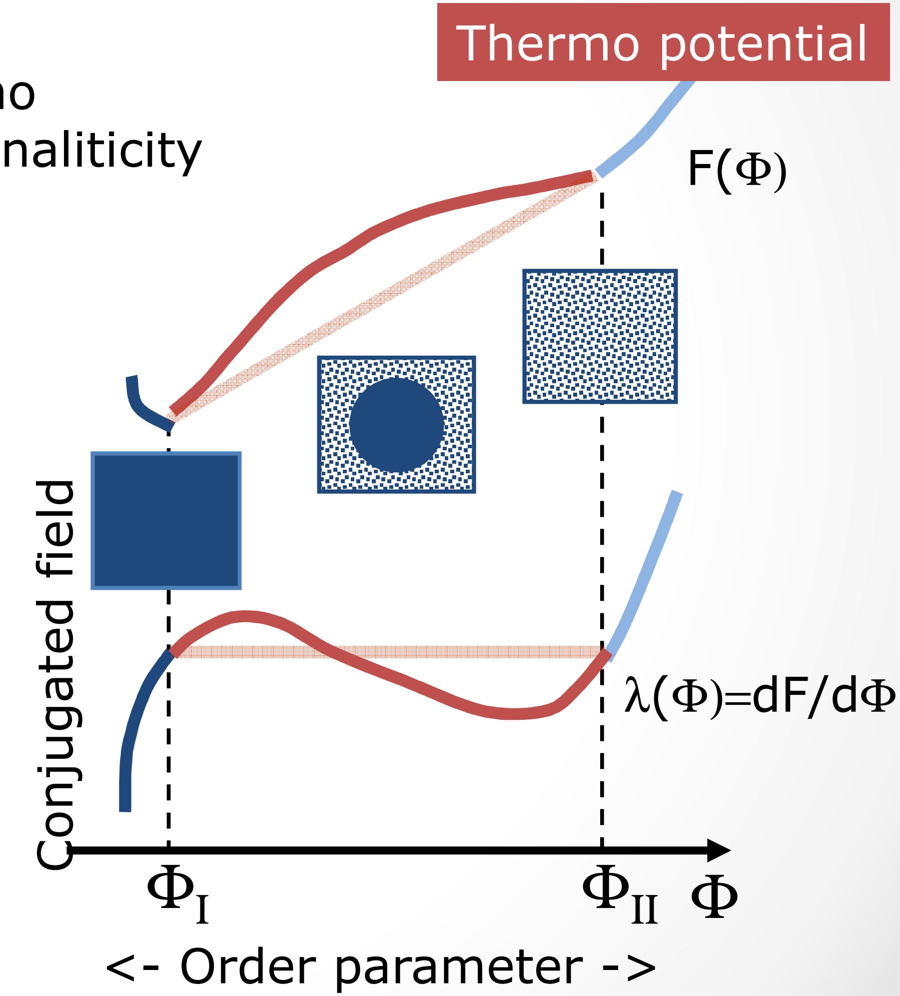
$$f(\varphi) = \frac{F}{V} > \alpha f_I(\varphi_I) + (1-\alpha) f_{II}(\varphi_{II})$$

- The surface entropy variation is not negligible in a finite system

=> Backbending EoS



Inequivalence and phase transition

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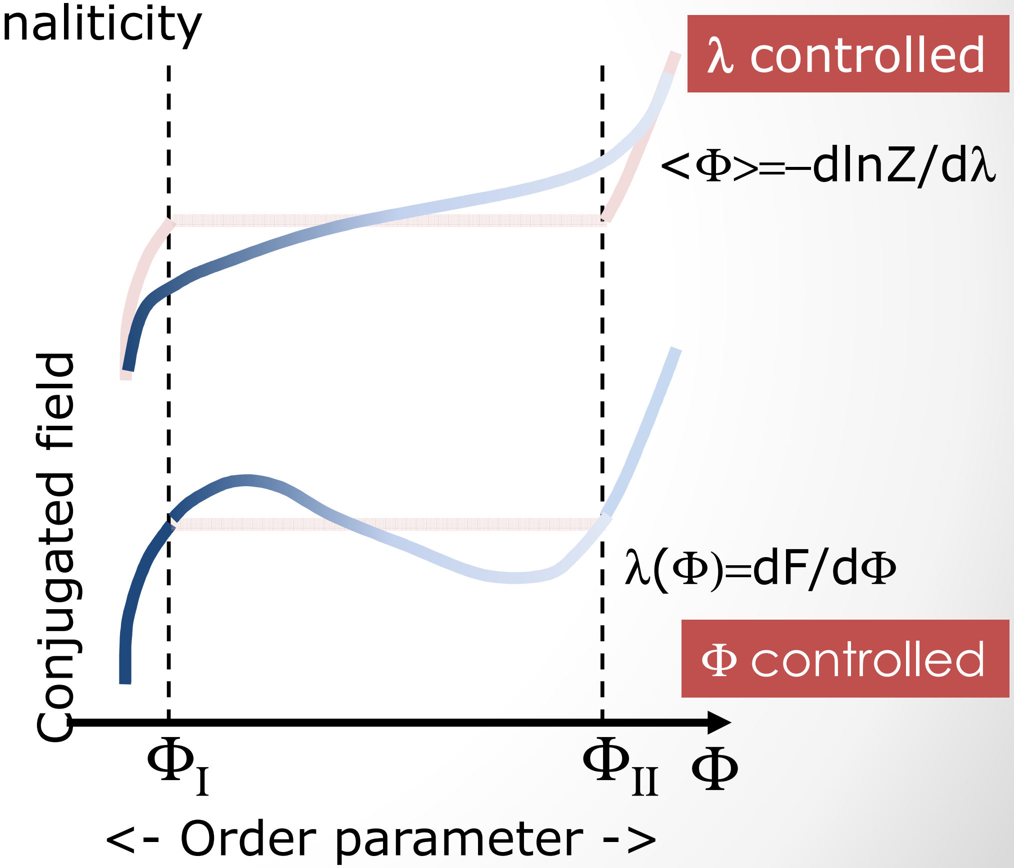
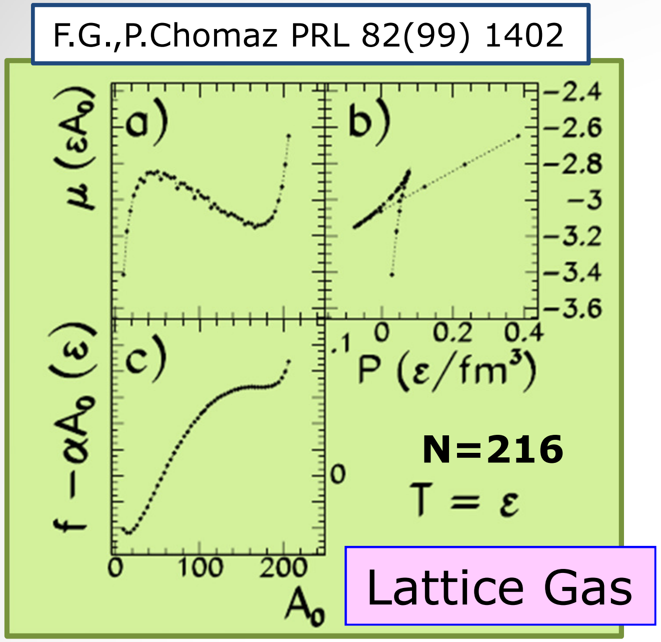
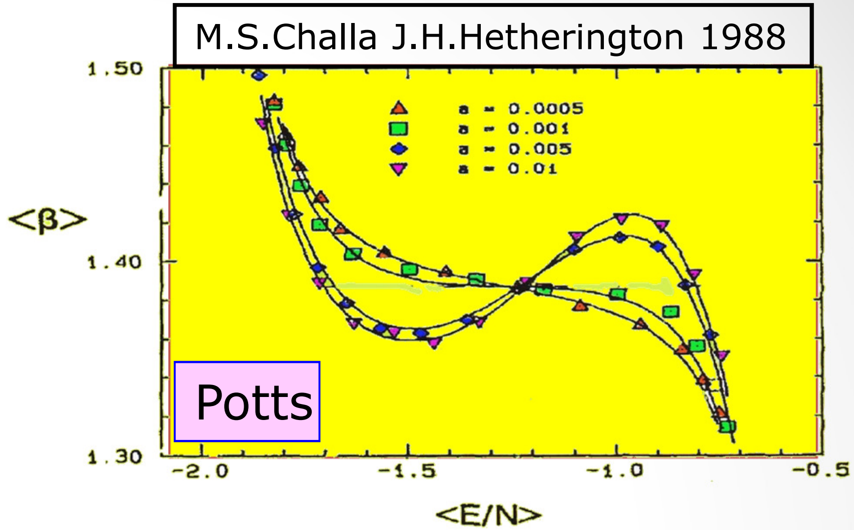
$$f(\varphi) = \frac{F}{V} = \alpha f_I(\varphi_I) + (1-\alpha) f_{II}(\varphi_{II})$$

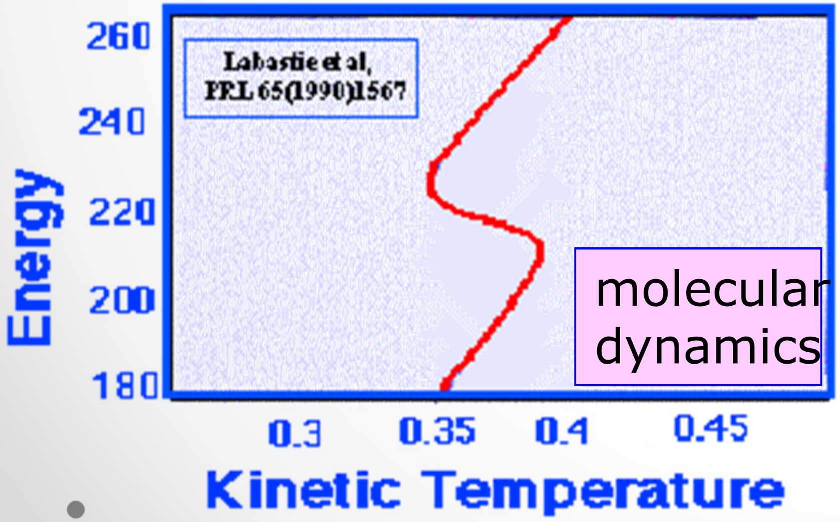
- The surface entropy variation is not negligible in a finite system => Backbending EoS
- This naturally leads to ensemble inequivalence in finite systems. This inequivalence might be kept up to the thermo limit, if interactions are long range.

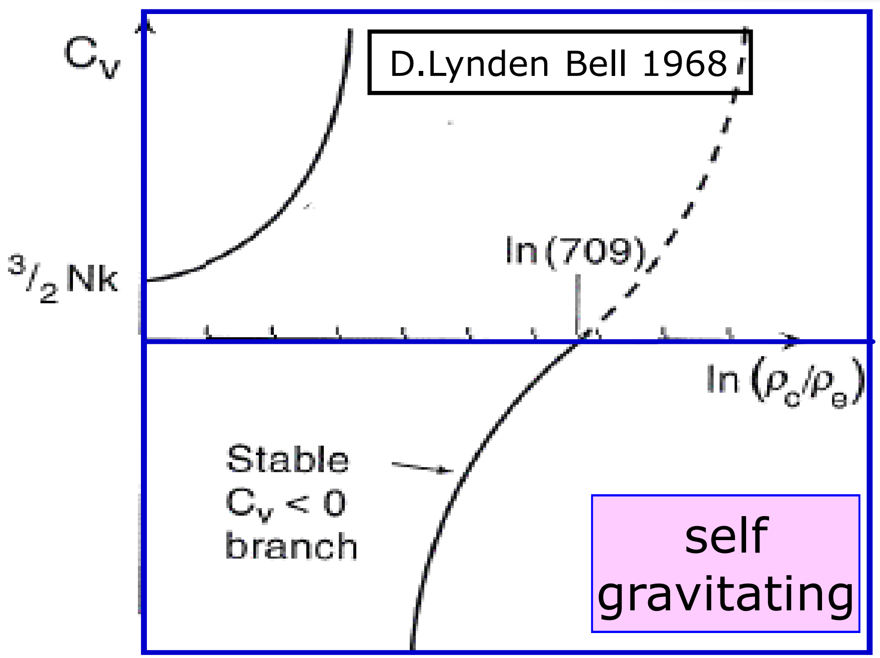


A.Campa, T.Dauxois, S.Ruffo Phys. Rep. 480 (2009) 57

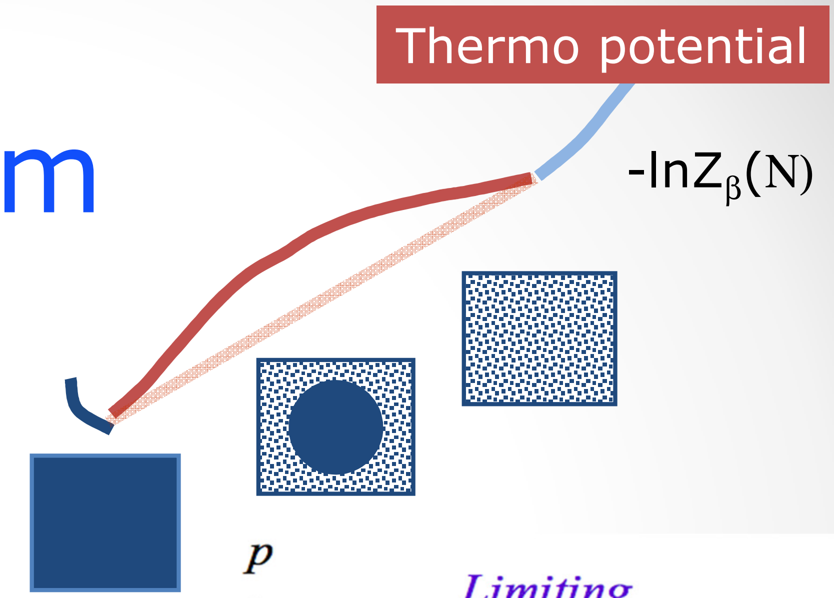

F.G.,P.Chomaz PRL 82(99) 1402
a)
b)
c)
μ (εA0)
f − αA0 (ε)
−2.4
−2.6
−2.8
−3
−3.2
−3.4
−3.6
0 0.2 0.4
P (ε/fm³)
0 100 200
A0
N=216
T = ε
Lattice Gas


M.S.Challa J.H.Hetherington 1988
<β>
1.50
1.40
1.30
a = 0.0005
a = 0.001
a = 0.005
a = 0.01
Potts
-2.0 -1.5 -1.0 -0.5
<E/N>


Labastie et al,
PRL 65(1990)1567
Energy
260
240
220
200
180
molecular
dynamics
0.3 0.35 0.4 0.45
Kinetic Temperature


Cv
D.Lynden Bell 1968
3/2 Nk
ln(709)
ln (ρc/ρe)
Stable
Cv < 0
branch
self
gravitating

Behavior of the finite npe system

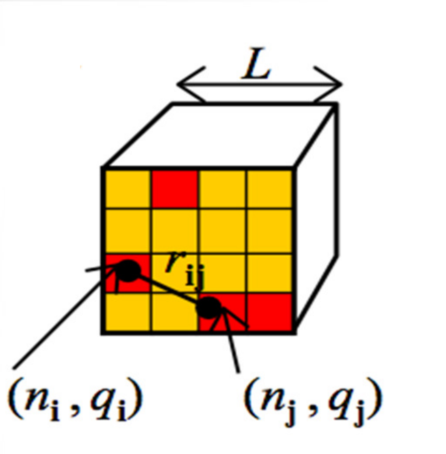


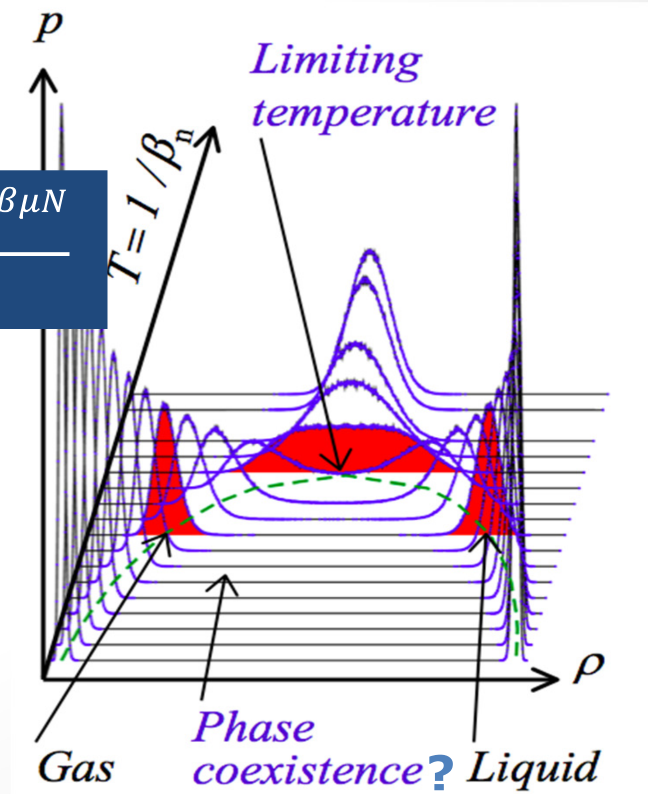
Ising model with Coulomb-like and nuclear-like interactions

$$H^{(\ell)} = -\frac{\varepsilon}{2}\sum_{\langle ij \rangle} n_i n_j + \frac{\chi}{2}\sum_{ij} q_i q_j \frac{1}{r_{ij}}$$

$$q_i = n_i - \frac{1}{N}\sum_{k=1}^{N} n_k$$

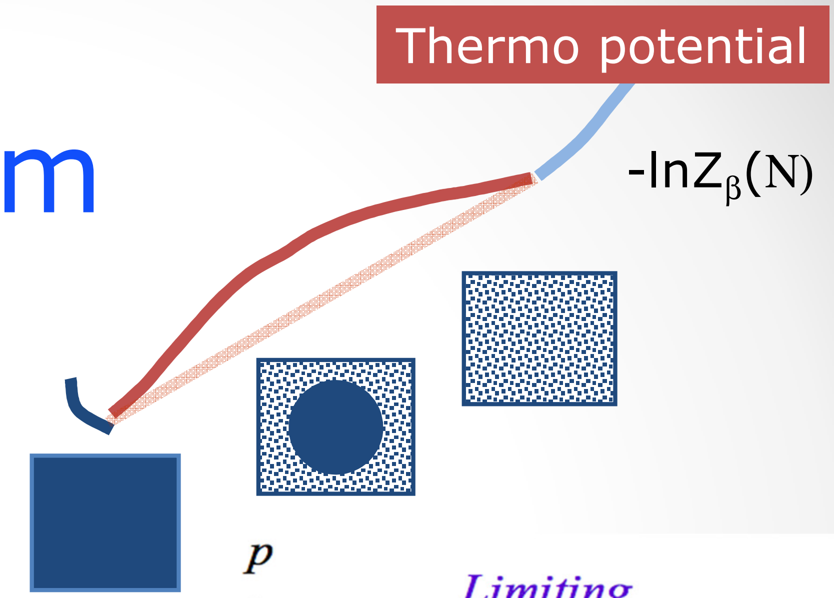
$$P_\mu(N) = \frac{\ln Z_\beta(N) e^{\beta\mu N}}{Z_{\beta\mu}}$$





- *P.Napolitani et al, PRL 98, 131102 (2007)*

Behavior of the finite npe system



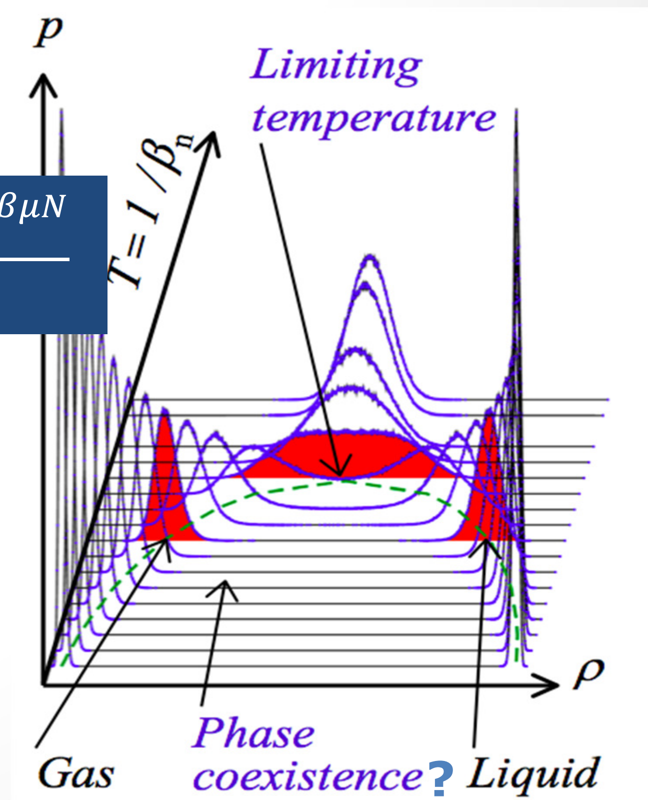
- What is the thermo limit of a concave thermo potential?

(Extensive ensemble)

$$P_{\mu}(N) = \frac{lnZ_{\beta}(N)e^{\beta\mu N}}{Z_{\beta\mu}}$$

- What is the thermo limit of a bimodal distribution?

(Intensive ensemble)

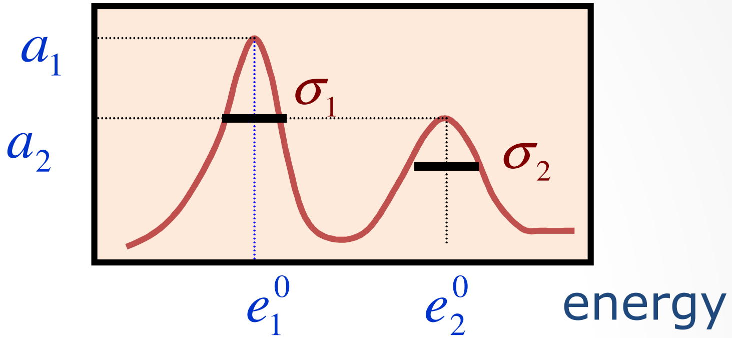


Thermo limit for the intensive ensemble Z_β

$$Z_\beta = \beta_0^{-1} Z_{\beta_0} \int db P_0\left(\frac{e}{\beta_0}\right) e^{-te}$$

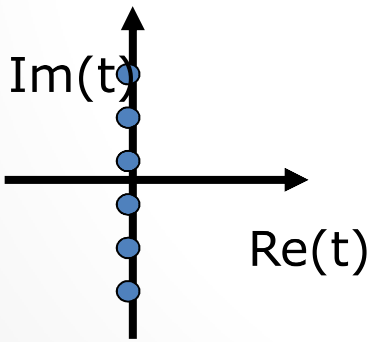
$$P_0 = P_1 + P_2$$

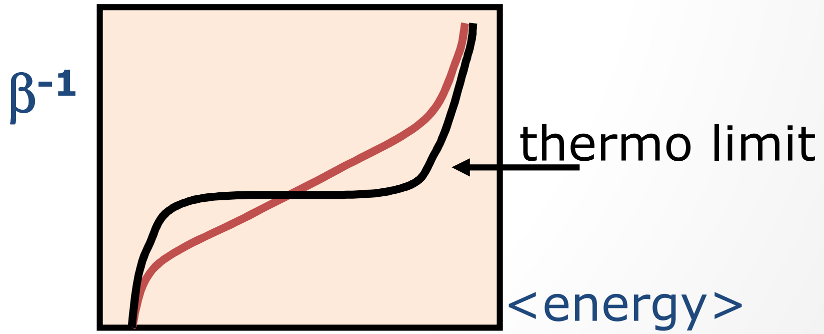
Singularities of LogZ ⟺ Zeros of Z



double saddle point approximation ➡

$$i(2k+1)\frac{\pi}{2} = \frac{1}{2}\log\frac{a_1}{a_2} - t_k\frac{e_2^0 - e_1^0}{2} - t_k^2\frac{\sigma_2^2 - \sigma_1^2}{4}$$



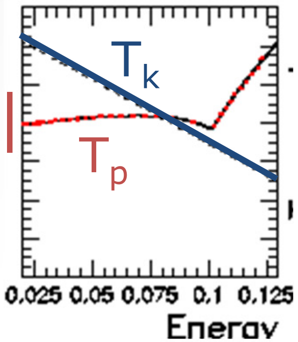


➡ Standard phase transition in the thermo limit (Yang Lee unit circle theorem)

K.C.Lee Phys.Rev.E 53 (1996) 6558
P.Chomaz, F.G. Physica A 330 (2003) 451.

Thermo limit for the extensive ensemble W(E)

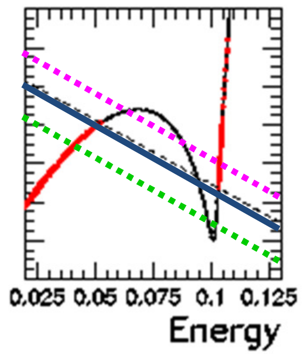
$c_k < |c_p|$



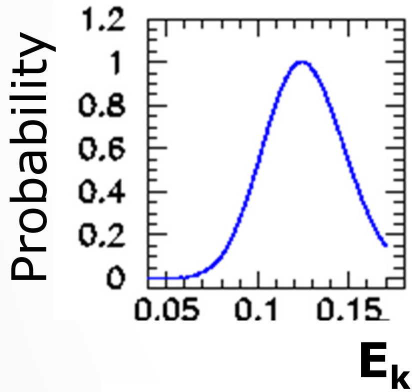
$E = E_k + E_p$

$P_E(E_k) = e^{(Sk+Sp-S)}$

extrema: $T_k = T_p$

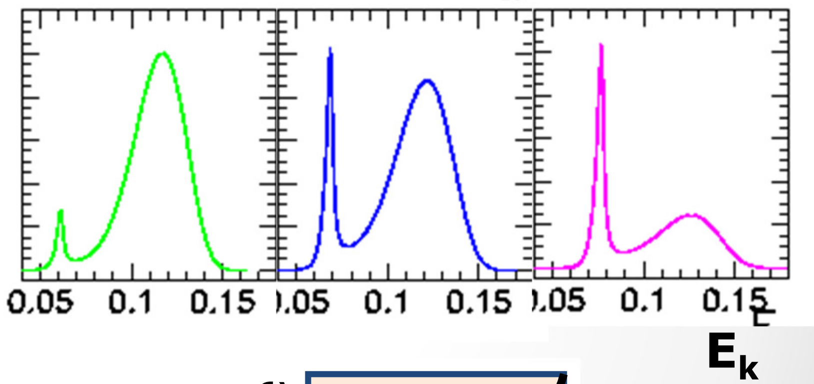


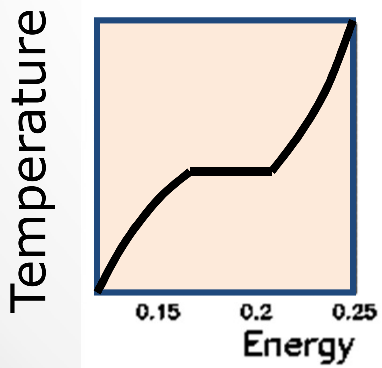
$c_k > |c_p|$



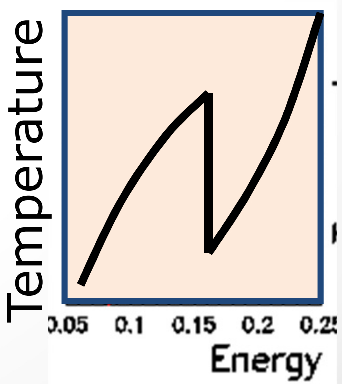
$<E_k>$

temperature estimator





Ensembles do not converge at the thermo limit !



D.Lynden-Bell, Monthly Not.Astr.Soc.138 (1968) 435
F.G, P.Chomaz, Phys. Rev. E 66 (2002) 046108.

Thermo-limit and ensemble (in)equivalence

- Back to neutron star matter: consider a fixed temperature $\beta^{-1}$and chemical potential $\vec{\mu} = (\mu_n, \mu_P, \mu_e)$

- $P_{\beta\,\vec{\mu}}(\rho_n, \rho_e) \propto Z_B(\rho_n)\, Z_L(\rho_e)$

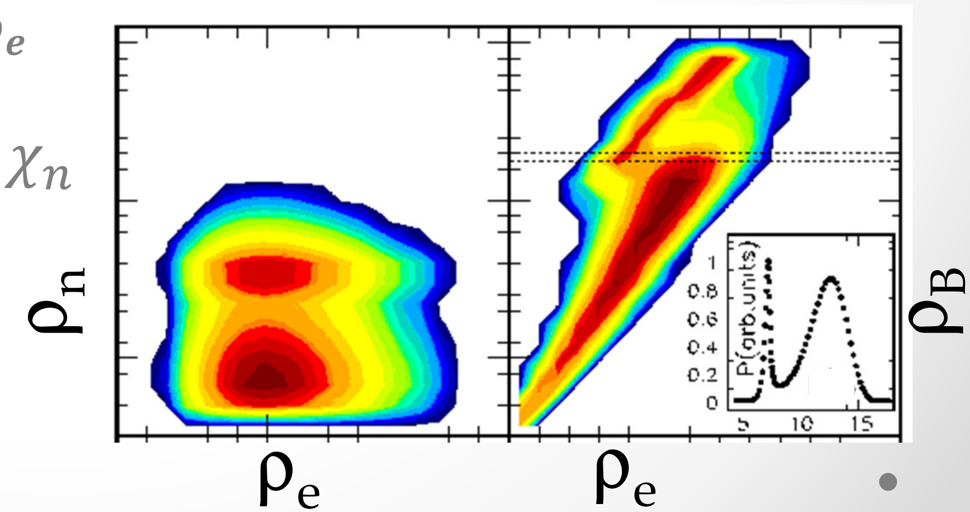
Uncorrelated pdf in the intensive ensemble without the constraint

- $P_{\beta\vec{\mu}}(\rho_B, \rho_p) \propto Z_B(\rho_B - \rho_p)\, Z_L(\rho_p)$

Conservation constraint $\boldsymbol{\rho_p = \rho_e}$

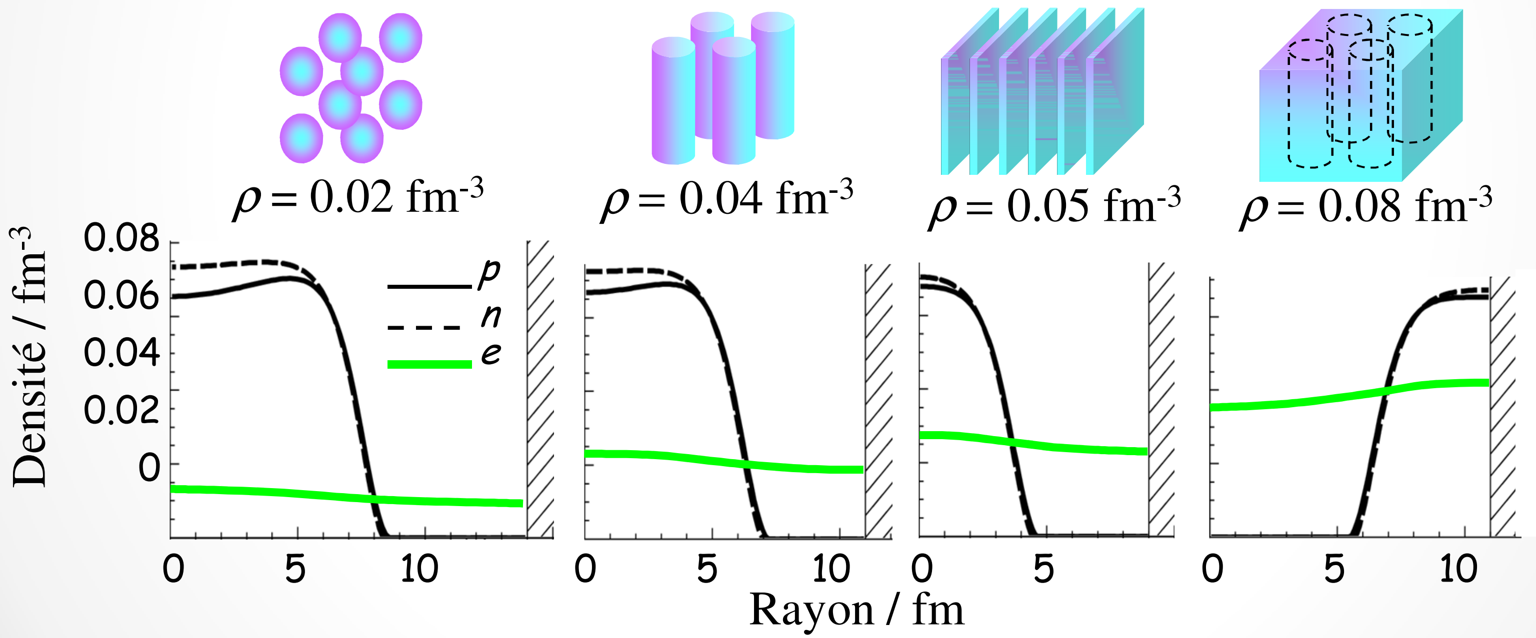
- $\chi_e = \frac{d^2 lnZ_L}{d{\rho_e}^2} > |\frac{\partial^2 lnZ_B}{\partial {\rho_n}^2}| = \chi_n$

Inequivalence condition is typically fulfilled



Quasi-incompressible e- gas

T. Maruyama et al. PRC 72, 015802 (2005)



RMF
T=0
y=0.5

A.Raduta,F.G.,PRC 82:065801 (2010)
PRC 85:025803 (2012)

The extended NSE model

- Mixture of nucleons, clusters of all sizes, photons, electrons, positrons, neutrinos

$$Z = Z_{lep}(\beta,\mu_e) Z_\gamma(\beta) Z_n(\beta,\mu_n,\mu_p) Z_N$$

- Nucleons treated in the Skyrme-HF approximation

$$Z_n = \exp\left(\beta (V - V_N)\left(\frac{\hbar^2\tau_n}{3m_n^*} + \frac{\hbar^2\tau_p}{3m_p^*} + \langle \hat{h}_{sp}\rangle - \langle \hat{h}_{mf}\rangle\right)\right) \cdot \exp\beta(\mu_n\rho_n + \mu_p\rho_p)$$

- Nuclei form a statistical ensemble of excited clusters interacting via Coulomb and excluded volume

$$Z_N(\beta,\mu,\tilde{\mu}) = \prod_{\substack{a>1 \\ i\in(-a,a)}} \exp(\omega_{a,i} + \beta\mu a)$$

$$Z_N(\beta,A,\tilde{\mu}) = \sum_{\{n_a\}} \prod_{a=2}^{\infty} \frac{\omega_{a\tilde{\mu}}^{n_a}}{n_a!} = \frac{1}{A}\sum_{a=2}^{A} a\,\omega_{a\tilde{\mu}} Z_N(\beta, A-a, \tilde{\mu})$$

$$\omega_{a\tilde{\mu}} = (V - V_{N+n}) \sum_{i=-a}^{a} g_{ay}(\beta) \left(\frac{m_{ay}}{2\pi\beta}\right)^{3/2} e^{-\beta(e_{ai}(\rho,\rho_p) - \tilde{\mu} i)}$$

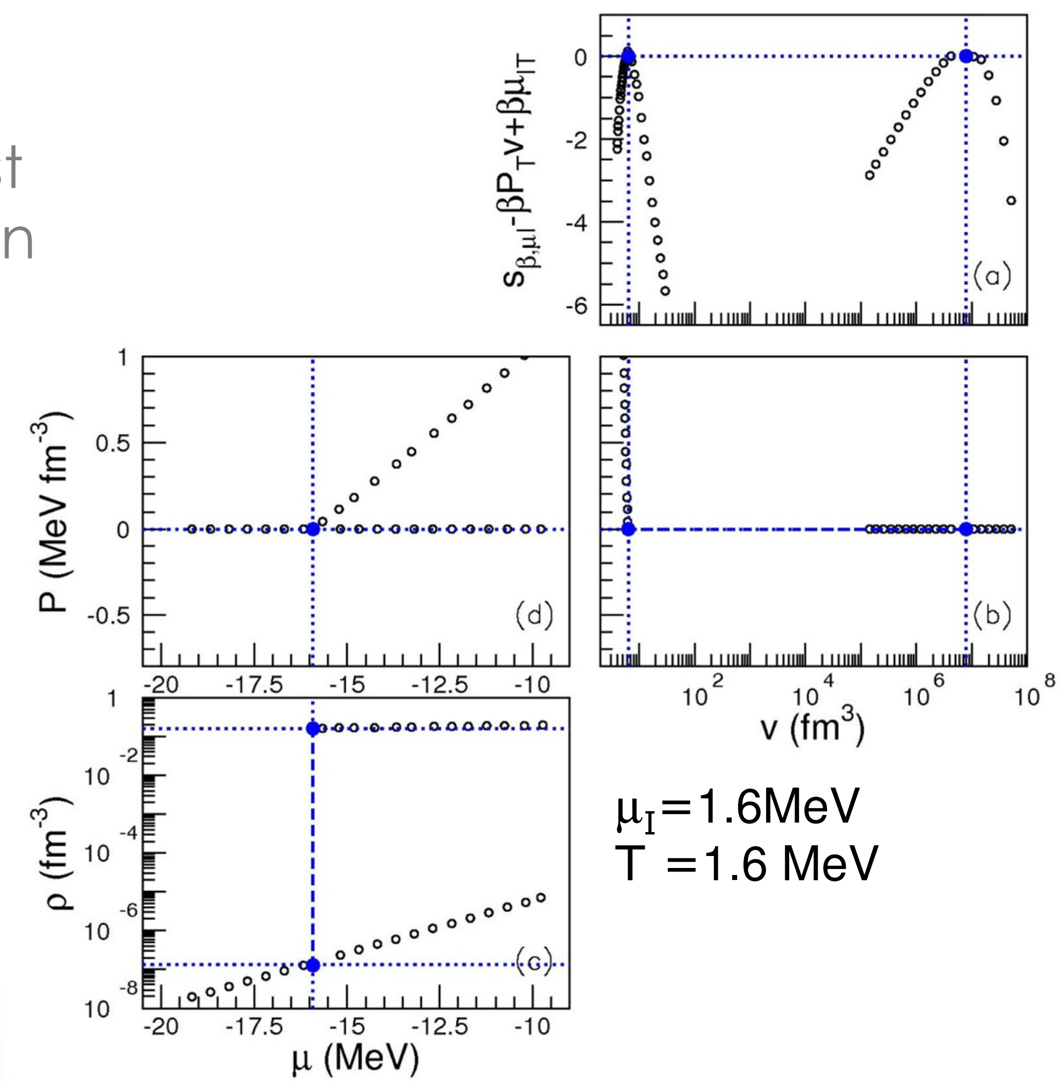
- Thermodynamic consistency between the different components

$$\mu_i^{nucleons} = \mu_i^{clus} \qquad i = n, p$$

$$P = P^{nucleons} + P^{clus} \quad ; \quad \rho_i = \rho_i^{nucleons} + \rho_i^{clus}$$

The extended NSE model

- Grandcanonical thermodynamics: first order phase transition



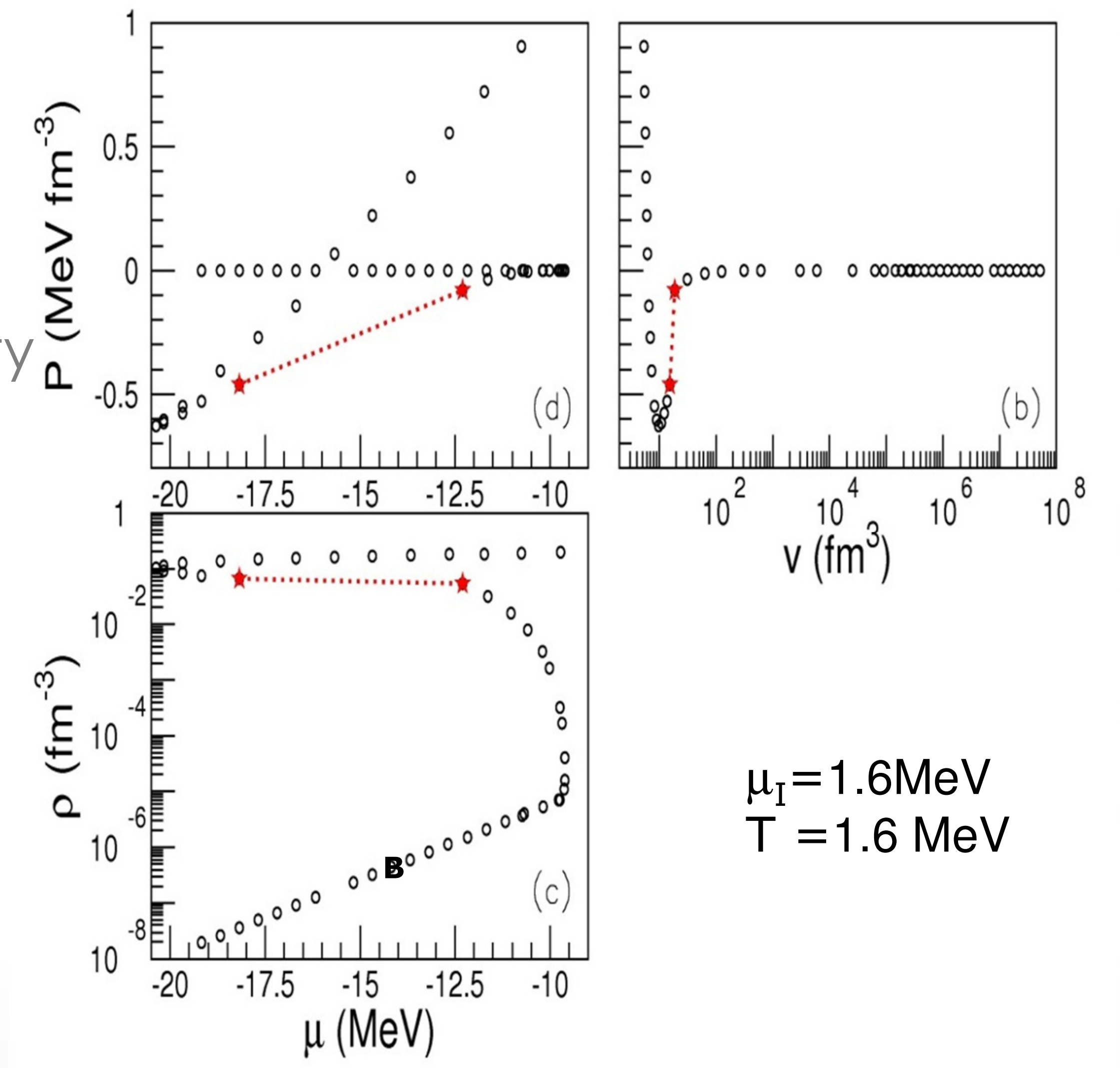
μ_I=1.6MeV
T =1.6 MeV

A.Raduta,F.G.,PRC 82:065801 (2010)

- PRC 85:025803 (2012)

The extended NSE model

- Canonical thermodynamics: backbending EoS and chemical potential discontinuity

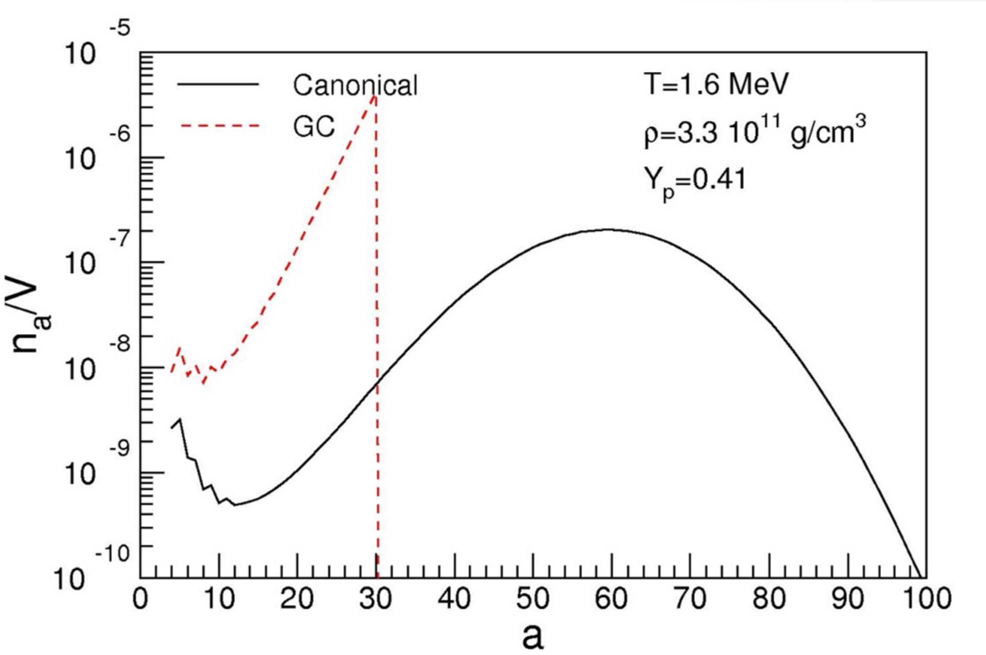


A.Raduta,F.G.,PRC 82:065801 (2010)

- PRC 85:025803 (2012)

The extended NSE model

- Canonical thermodynamics: backbending EoS and chemical potential discontinuity
- Large distribution of cluster sizes in the inequivalence region



A.Raduta,F.G.,PRC 82:065801 (2010)
PRC 85:025803 (2012)

Conclusions

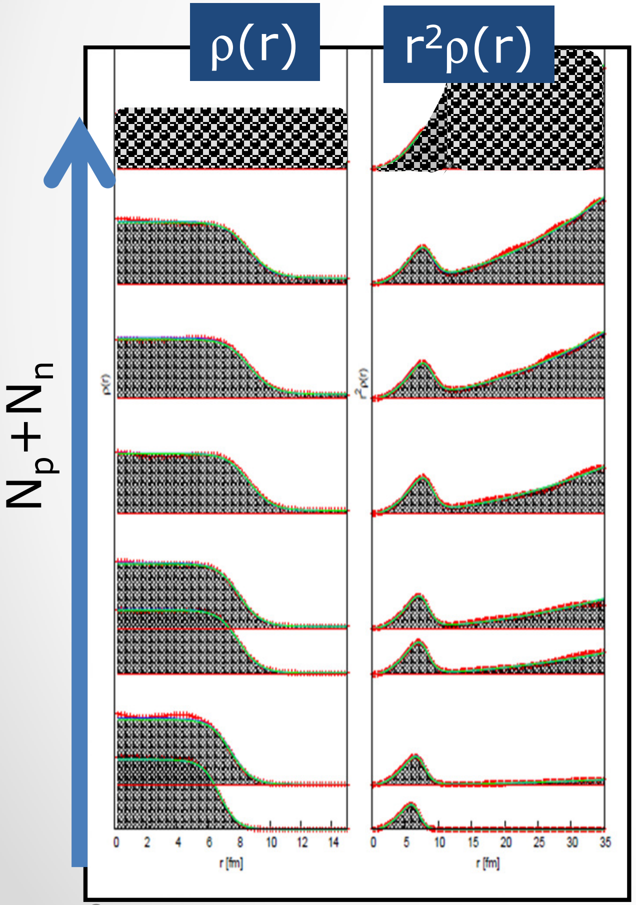
- Neutron star matter: a playing ground for statistical mechanics
- Baryonic matter (n,p) unstable with respect to density fluctuations
- Ultra-relativistic electrons ~ constitute a uniform incompressible ideal gas background
- Coulomb correlations: strict constraint on the order parameter $\rho_p = \rho_e$

$\Rightarrow$ **Quenching of the first order phase transition in the canonical ensemble**

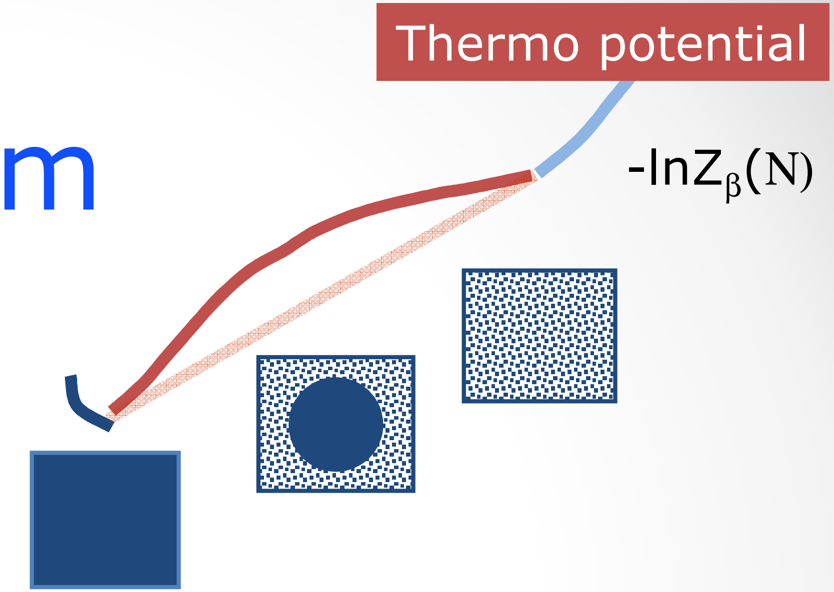
=> Sizeable effects on the composition of stellar matter

Behavior of the finite npe system

T=0 Microscopic HF-BCS calculation

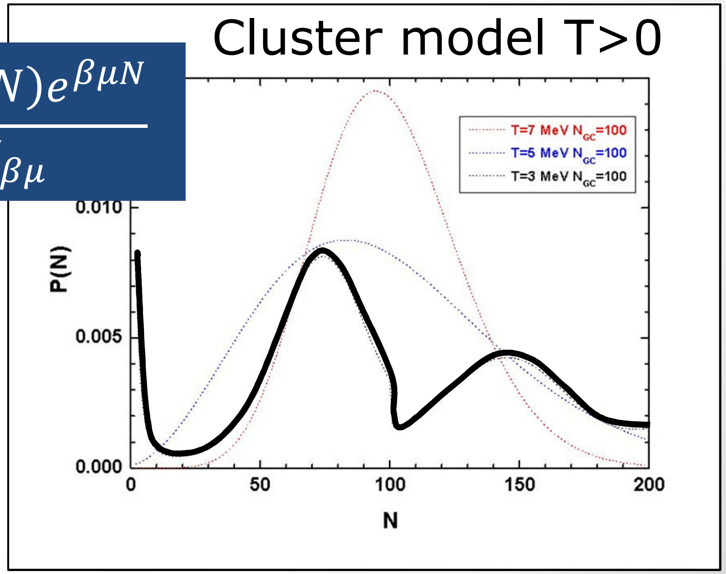


P.Papakonstantinou et al, in preparation



$$P_\mu(N) = \frac{lnZ_\beta(N)e^{\beta\mu N}}{Z_{\beta\mu}}$$

Cluster model T>0



G.Chaudhuri et al, in preparation

The order or the « L-G » transition

• • •

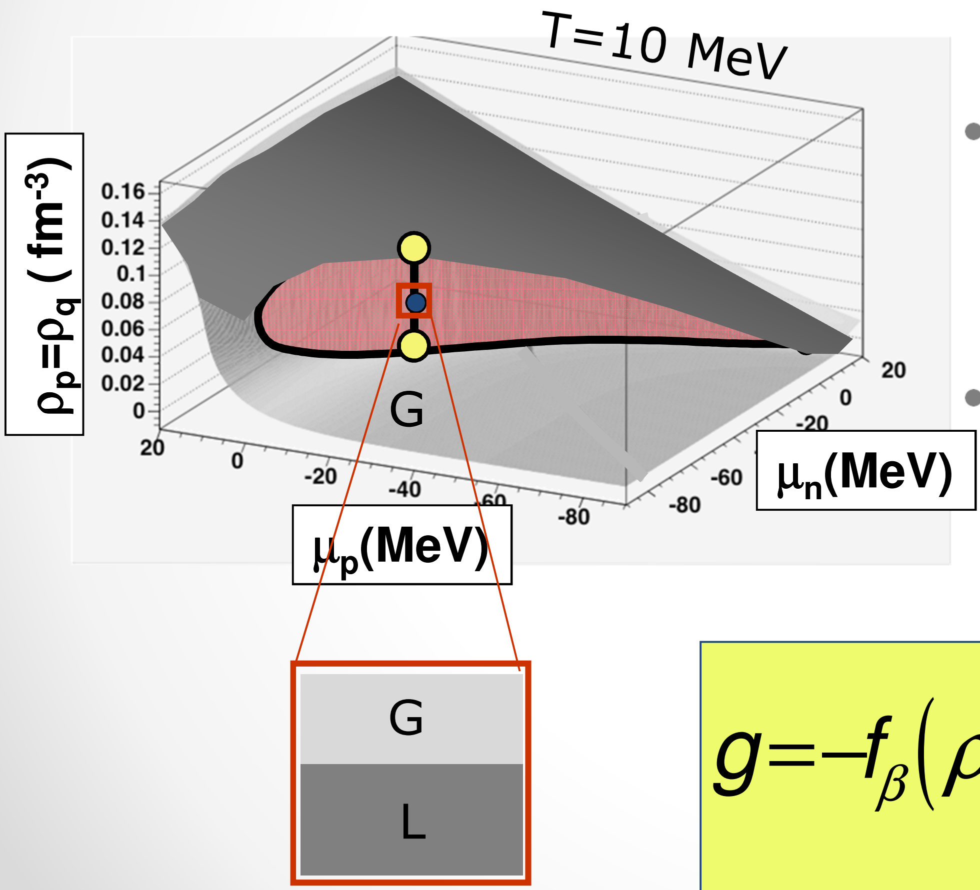
with a background of neutralizing charge

⇔ constraint on the order parameter $\rho_p = \rho_e$

C.Ducoin, F.G. et al NPA 771 (2006) 68,
NPA 789 (2007) 403,
PRC 75 (2007) 065805.

Phase transition in n+p matter

$\rho_q = \rho_p - \rho_e = \rho_p$



- First order phase transition:
- (proton) density jump at constant chemical potential
- Phase mixture at intermediate density

$$g = -f_\beta(\rho_n, \rho_p) + \sum_{i=n,p} \mu_i \rho_i \qquad \rho_q = -\frac{\partial g}{\partial \mu_q}$$

Phase transition in n+p+e matter

- Macroscopic charge neutrality: $\rho_q = 0$
- Mean field:
  $f_\beta(\rho_n,\rho_p)=f_{np}(\rho_n,\rho_p)+f_e(\rho_e=\rho_p)$

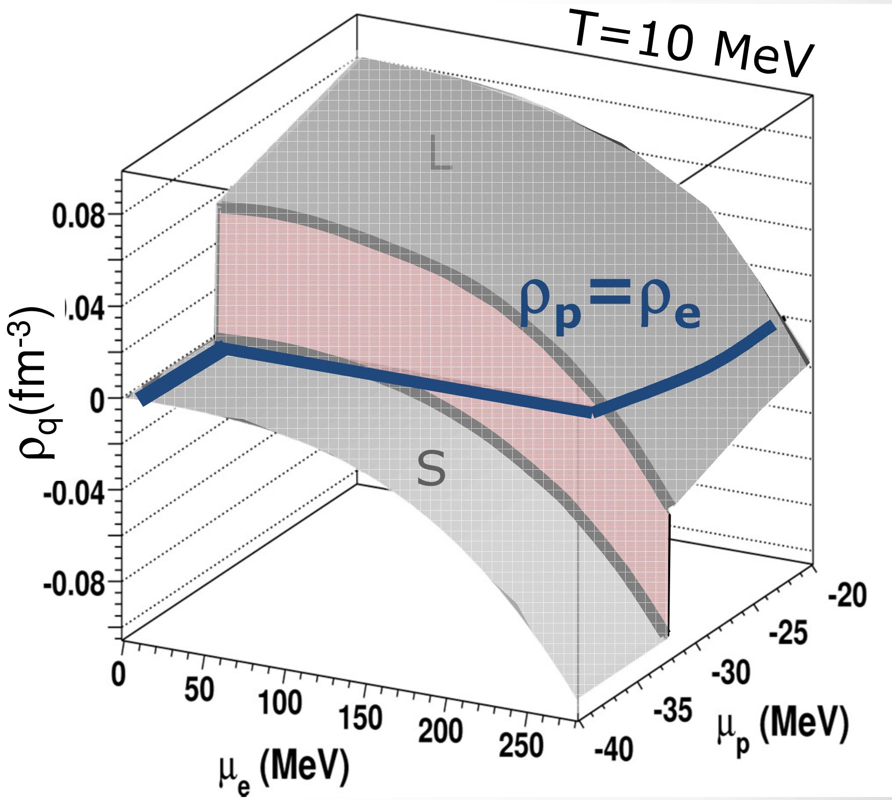
- Model case: charged particles independently neutralized by an homogeneous background of opposite charge $\delta\rho_i(\vec{r}) = \rho_i(\vec{r}) - \langle\rho_i\rangle$

$$\hat{V}'_{ii'} = \frac{\alpha q_i q_{i'}}{1+\delta_{ii'}} \int \frac{\delta\rho_i(\vec{r})\delta\rho_{i'}(\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r} d\vec{r}'$$

- $f_\beta(\rho_n,\rho_p)=f'_\beta(\rho_n,\rho_0,\rho_q=0)$ $\begin{cases} \rho_q = \rho_p - \rho_e \\ \rho_0 = (\rho_p + \rho_e)/2 \end{cases}$

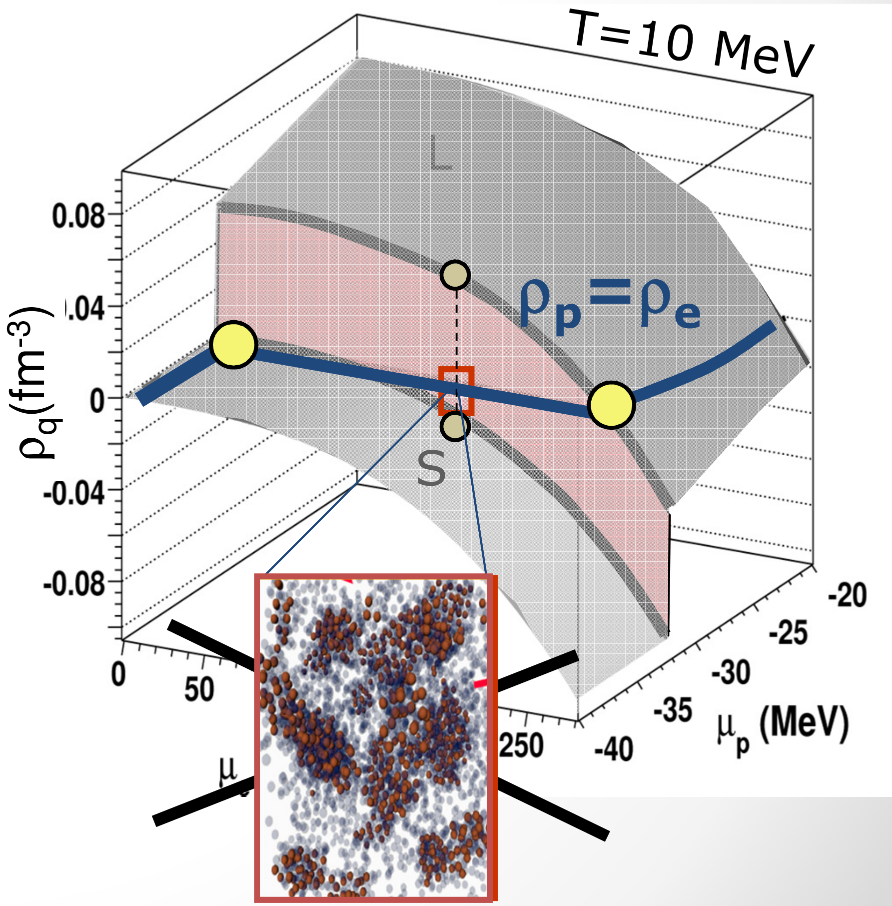
$$g_{\mu_n,\mu_0,\mu_c} = -f_\beta'(\rho_n,\rho_0,\rho_q) + \sum_{i=n,0,q} \mu_i \rho_i$$

$$\rho_q = -\frac{\partial g}{\partial \mu_q}$$



Phase transition in n+p+e matter

- Macroscopic charge neutrality: $\rho_q = 0$
- L and S phases do not have the same chemical potential
- They cannot coexist at equilibrium
- Density fluctuations can only appear at the microscopic level
- Continuous transition through an inhomogeneous phase?
- Need of a model beyond mean-field



Critical behavior

• • •

with a background of neutralizing charge

⇔ constraint on the order parameter $\rho_p = \rho_e$

C.Ducoin et al PRC 75 (2007) 065805.

P.Napolitani et al, Phys.Rev.Lett 98 (2007) 131102

Critical point fluctuations

- A diverging correlation length $\sigma_c(r) \propto r^{2-d-\eta}$,

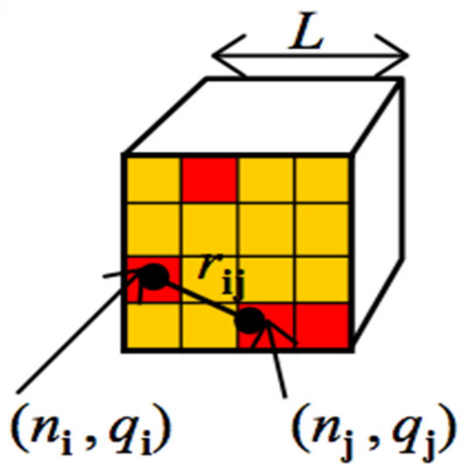
would imply a diverging energy density if charge density is an order parameter

$$\frac{\langle \hat{V}_c' \rangle}{\Omega} = \frac{\alpha}{2\Omega} \int \frac{\sigma_c(\vec{r},\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r} d\vec{r}' = 2\pi\alpha \int \sigma_c(r) r dr,$$

$$\sigma(\vec{r},\vec{r}') = \langle \delta\rho_c(\vec{r}) \delta\rho_c(\vec{r}') \rangle$$

=> No critical point in stellar matter

Long range Ising



- Frustrated Ising model with repulsive and attractive interactions

$$H^{(\ell)} = -\frac{\varepsilon}{2}\sum_{\langle ij\rangle} n_i n_j + \frac{\chi}{2}\sum_{ij} q_i q_j \frac{1}{r_{ij}}$$ **n = 0,1 occupation**

$$q_i = n_i - \frac{1}{N}\sum_{k=1}^{N} n_k$$ **electron background**

- Replicae: the lattice of finite size ℓ is repeated in all three directions N_R times $\vec{I} = \vec{i} + \vec{n}\ell \,;\, \vec{J} = \vec{j} + \vec{n}\ell$

$$H_c^{tot} = \frac{\chi}{2}\sum_{n=1}^{N_R}\sum_{ij} n_i n_j C_{IJ}$$

Replicae are equivalent to a renormalization of the long range coupling

$$= \frac{\chi}{2}\sum_{ij} n_i n_j C_{ij}^{N_R}$$

$$C_{ij}^{N_R} = (N_R - 1)\sum_{n=1}^{N_R}\left(\frac{1}{r_{\vec{i},\vec{j}+\vec{n}\ell}} - \frac{1}{2N}\sum_{k=1}^{N}\left(\frac{1}{r_{\vec{i},\vec{k}+\vec{n}\ell}} + \frac{1}{r_{\vec{k}+\vec{n}\ell,\vec{j}}}\right)\right)$$

Finite size scaling

- $L = \infty \quad \xi(t) \propto t^{-\nu}$ $\quad t = \frac{|T - T_c|}{T_c}$

  $Y(t) \propto t^{-x}$

- finite L $\quad Y = f\left(L/\xi(t)\right)|t|^{-x}$

  $\xi \sim L$ → f=cst

  $\xi \ll L$ → Y=cst

$$L^{-x/\nu} Y(T,L) \propto \begin{cases} cst & \xi \approx L \\ \left(L^{1/\nu} |t|\right)^{-x} & \xi \ll L \end{cases}$$

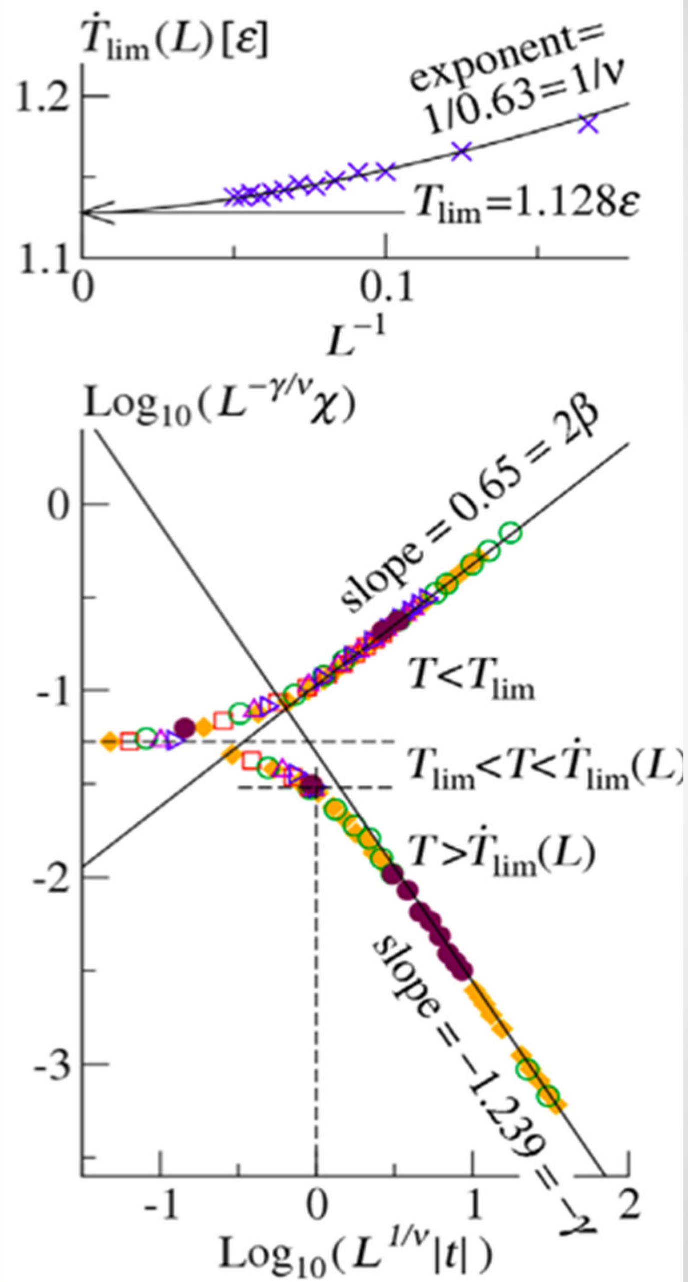
Ising

Finite size scaling

$$\left|T_c(L)-T_c\right| \propto L^{-1/\nu}$$

$$L^{\beta/\nu}\rho \propto \left(L^{1/\nu}t\right)^{\beta} \qquad \xi \gg L,\ T < T_c(L)$$

$$L^{-\gamma/\nu}\chi \propto \begin{cases} \left(L^{1/\nu}\,|t|\right)^{-\gamma} & \xi \gg L,\ T > T_c(L) \\ \left(L^{1/\nu}\,|t|\right)^{2\beta} & \xi \gg L,\ T < T_c(L) \end{cases}$$



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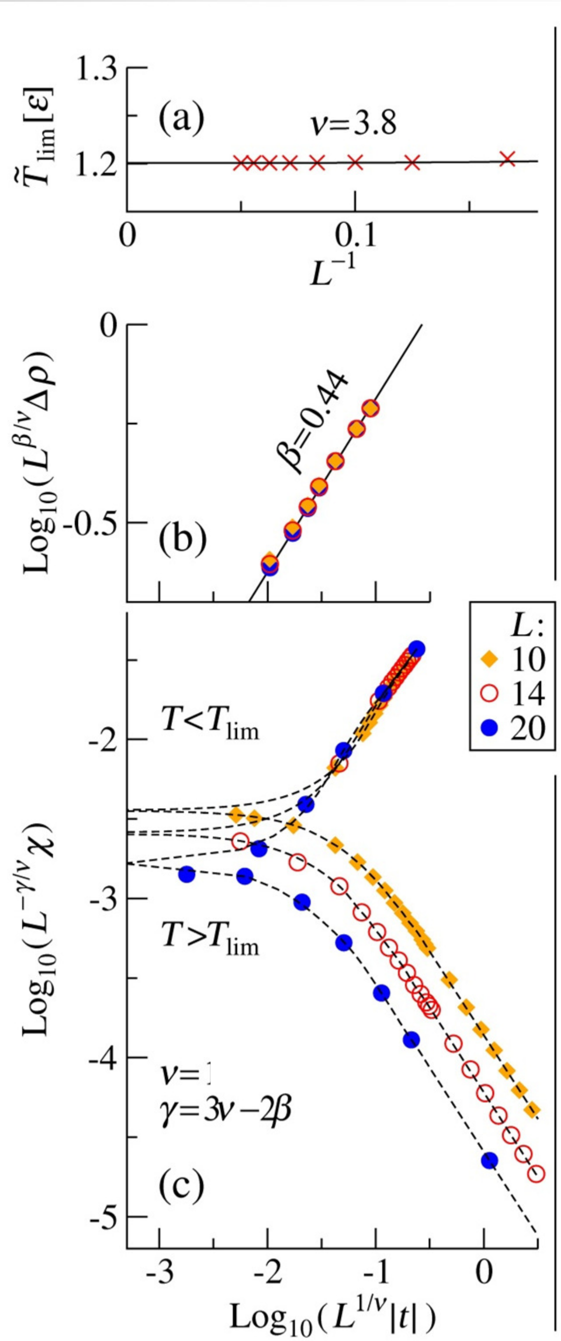
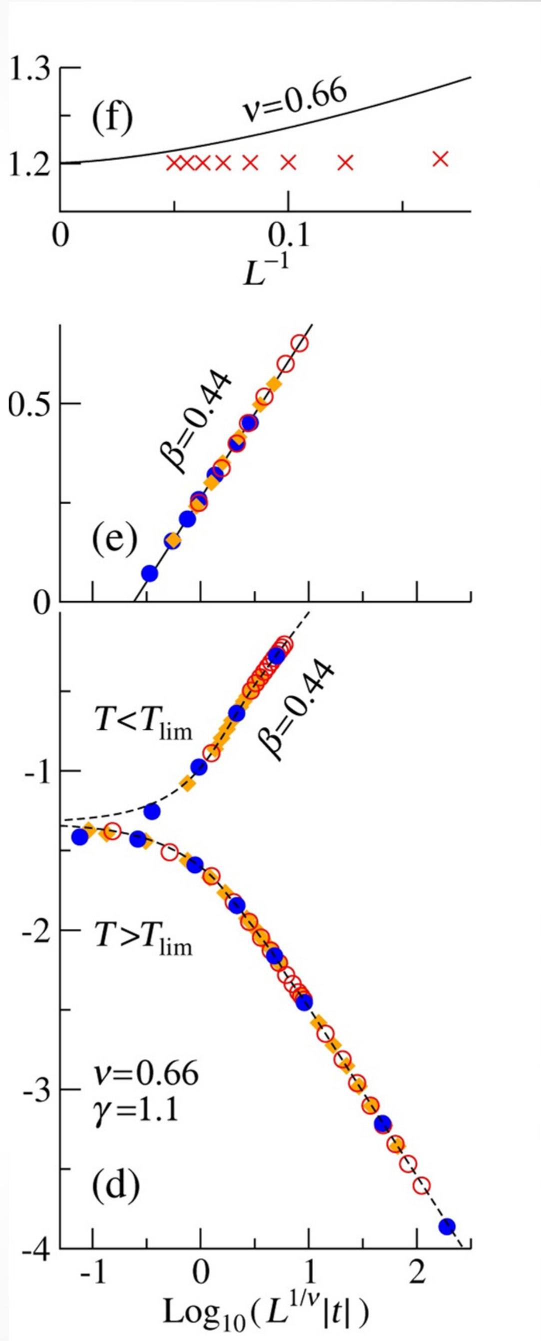


Ising*:
No scaling

The limiting temperature is a first order point



Instabilities of homogeneous matter

• • •

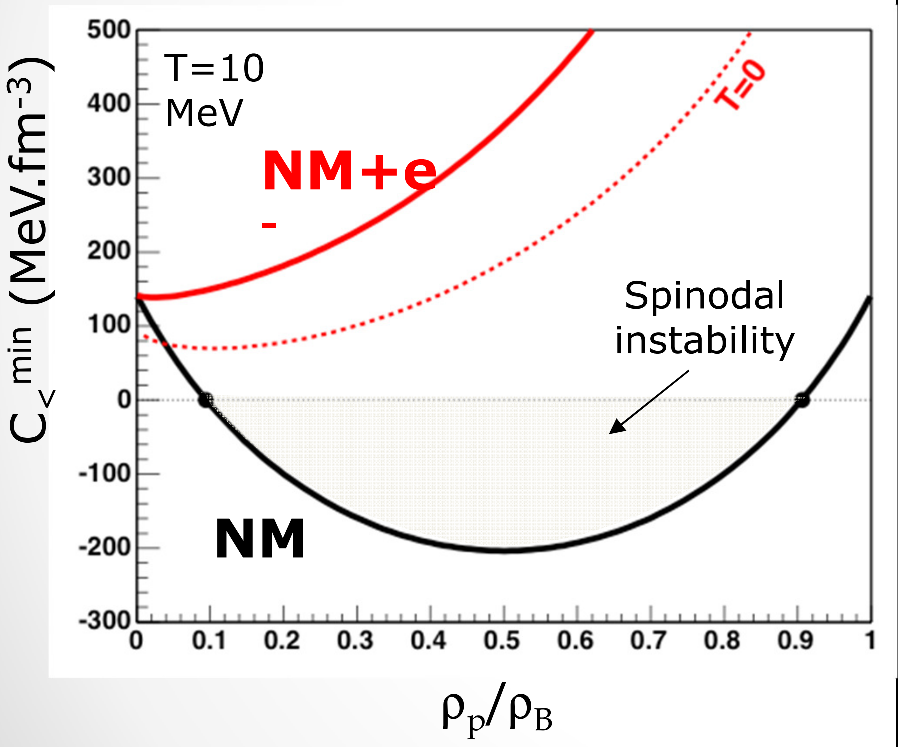
with a background of neutralizing charge

⇔ constraint on the order parameter $\rho_p = \rho_e$

C.Ducoin et al NPA 771 (2006) 68,
NPA 789 (2007) 403,
PRC 75 (2007) 065805.

Spinodal instability

Finite temperature HF with an effective energy functional (Skyrme 230a)



Free-energy curvature matrix: lower eigenvalue = $C_<$

- **Nuclear only ρ_n, ρ_p :**

$$C^h_{NM} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p \end{pmatrix} \quad \text{with} \quad \partial_{\rho_i}\mu_j = \frac{\partial^2 f}{\partial\rho_i \partial\rho_j}$$

- **+ electron gas $\rho_e = \rho_p$**

➔ *Curvature matrix remains 2D*

$$C^h_{NMe} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p + \partial_{\rho_e}\mu_e \end{pmatrix}$$

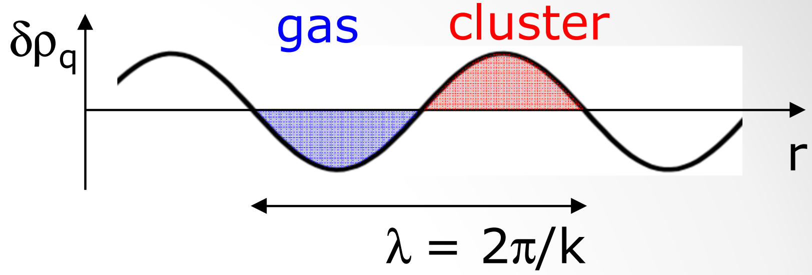
electron incompressibility

➔ High electron incompressibility suppresses the spinodal instability

Finite wavelength fluctuations

Plane-wave density fluctuations:

$\delta\rho_q\ (\mathbf{k.r}) = A_q e^{i\mathbf{k.r}} + A_q^{*} e^{-i\mathbf{k.r}}$ q=n,p,e



➔ Free-energy variation :

$$C^f_{NMe} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p & 0 \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p & 0 \\ 0 & 0 & \partial_{\rho_e}\mu_e \end{pmatrix}$$

Homogeneous Matter

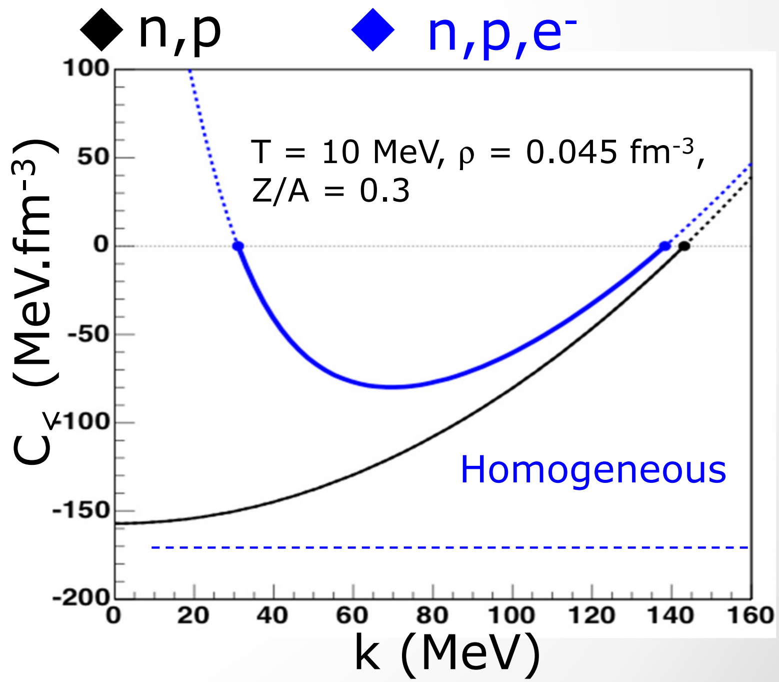
$$+ \begin{pmatrix} C^f_{nn} & C^f_{np} & 0 \\ C^f_{pn} & C^f_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} k^2$$

Density-gradient terms

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & -\alpha \\ 0 & -\alpha & \alpha \end{pmatrix} \frac{1}{k^2}$$

Coulomb between protons and electrons

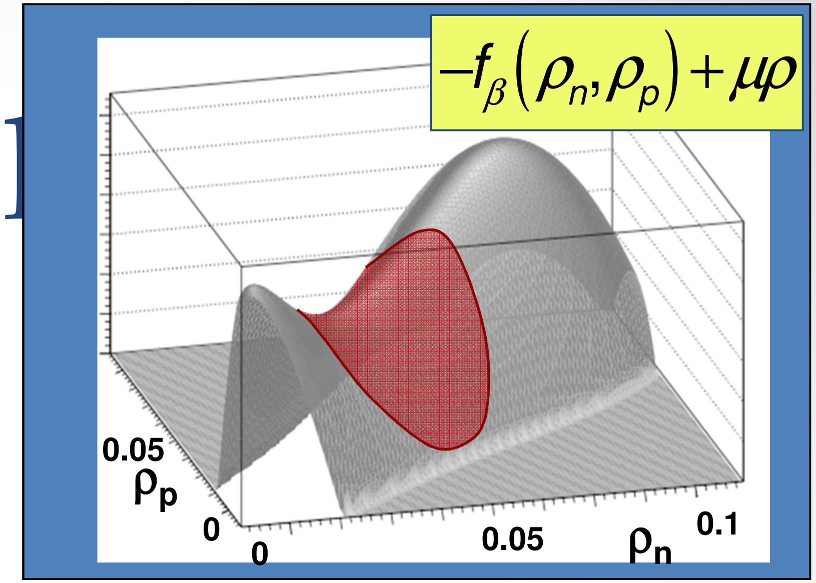
➔ k-dependent eigen-modes

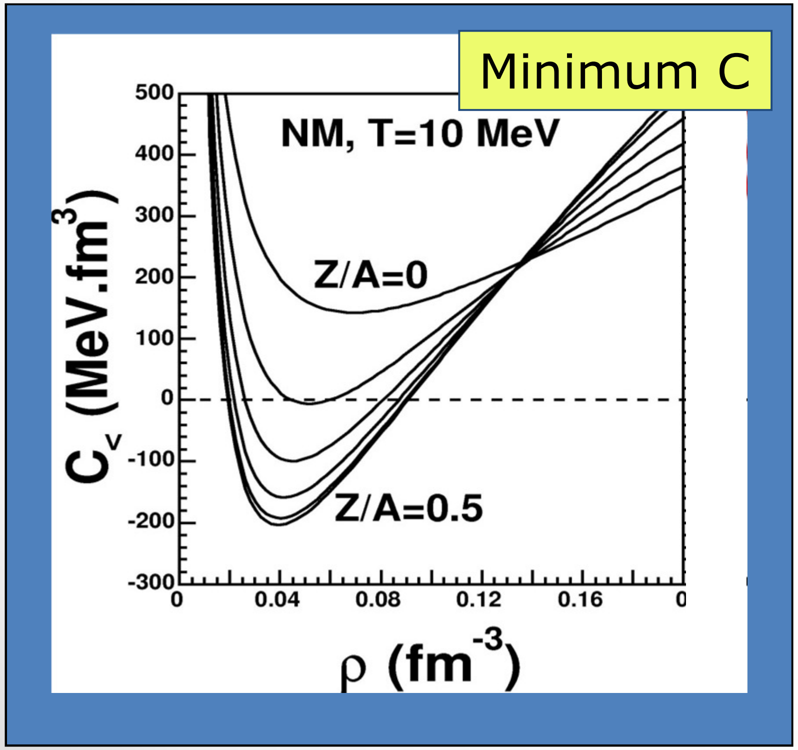


➔ Phase separation replaced by cluster formation

I - Spinodal

- A direction of negative curvature in the free energy $f_\beta(\rho_n,\rho_p)$ $C_<<0$



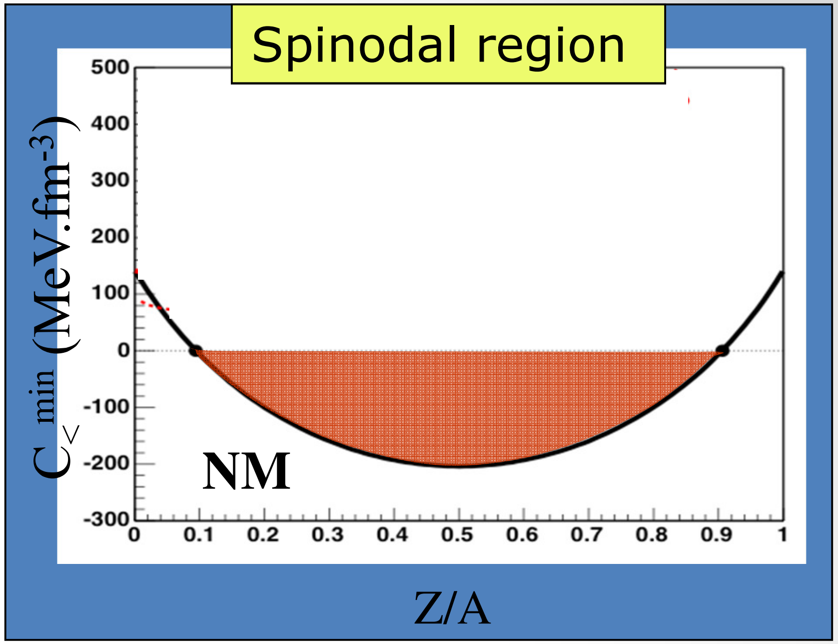


$$C_{ij} = \frac{\partial^2 f}{\partial\rho_i\partial\rho_j}$$

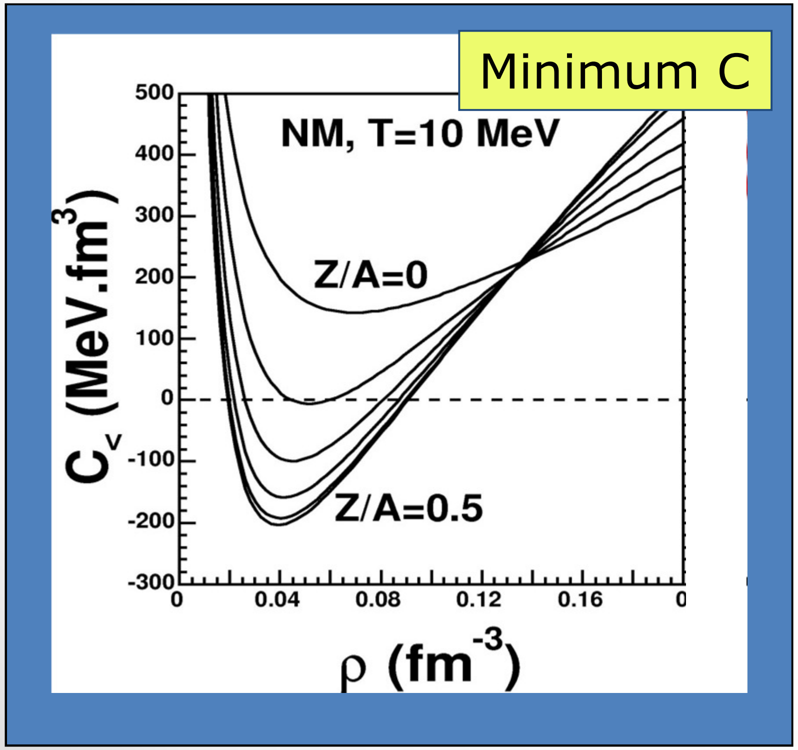
$$C = \begin{vmatrix} \frac{\partial\mu_n}{\partial\rho_n} & \frac{\partial\mu_n}{\partial\rho_p} \\ \frac{\partial\mu_p}{\partial\rho_n} & \frac{\partial\mu_p}{\partial\rho_p} \end{vmatrix}$$

I - Spinodal in NM

- A direction of negative curvature in the free energy $f_\beta(\rho_n,\rho_p)$ $C_<<0$



$$C_{ij} = \frac{\partial^2 f}{\partial \rho_i \partial \rho_j}$$



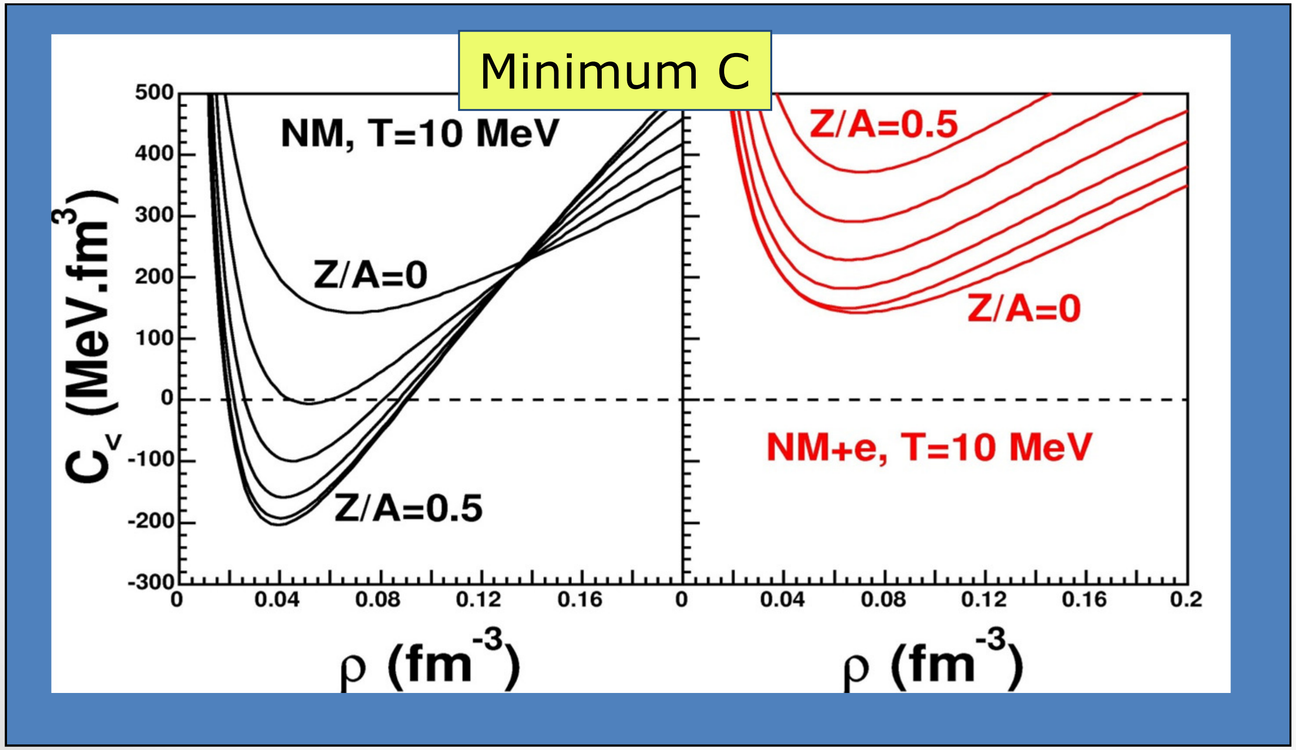
$$C = \begin{vmatrix} \frac{\partial \mu_n}{\partial \rho_n} & \frac{\partial \mu_n}{\partial \rho_p} \\ \frac{\partial \mu_p}{\partial \rho_n} & \frac{\partial \mu_p}{\partial \rho_p} \end{vmatrix}$$

I - Spinodal in NM+e

- Stellar matter (n,p,e)

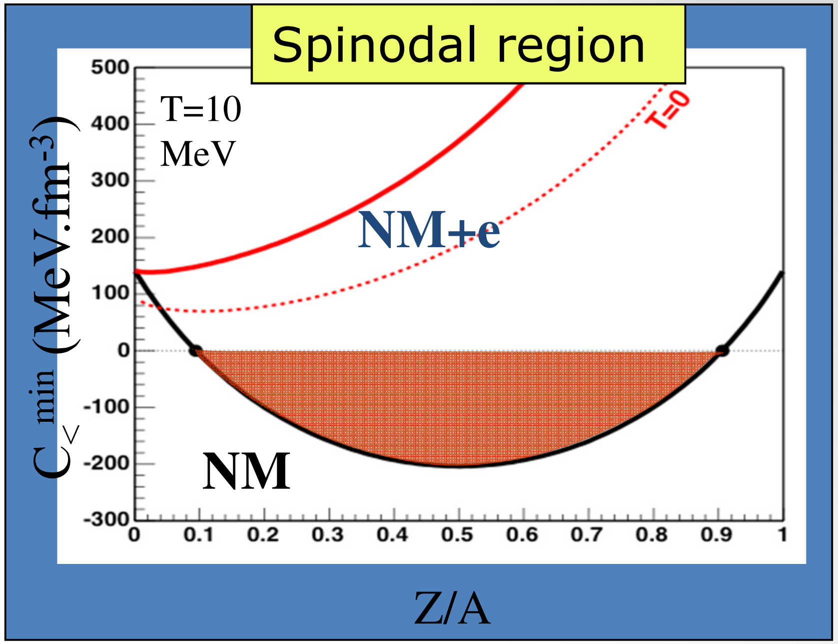
$$f_\beta(\rho_n,\rho) = f_\beta(\rho_n,\rho_p=\rho,\rho_e=\rho)$$

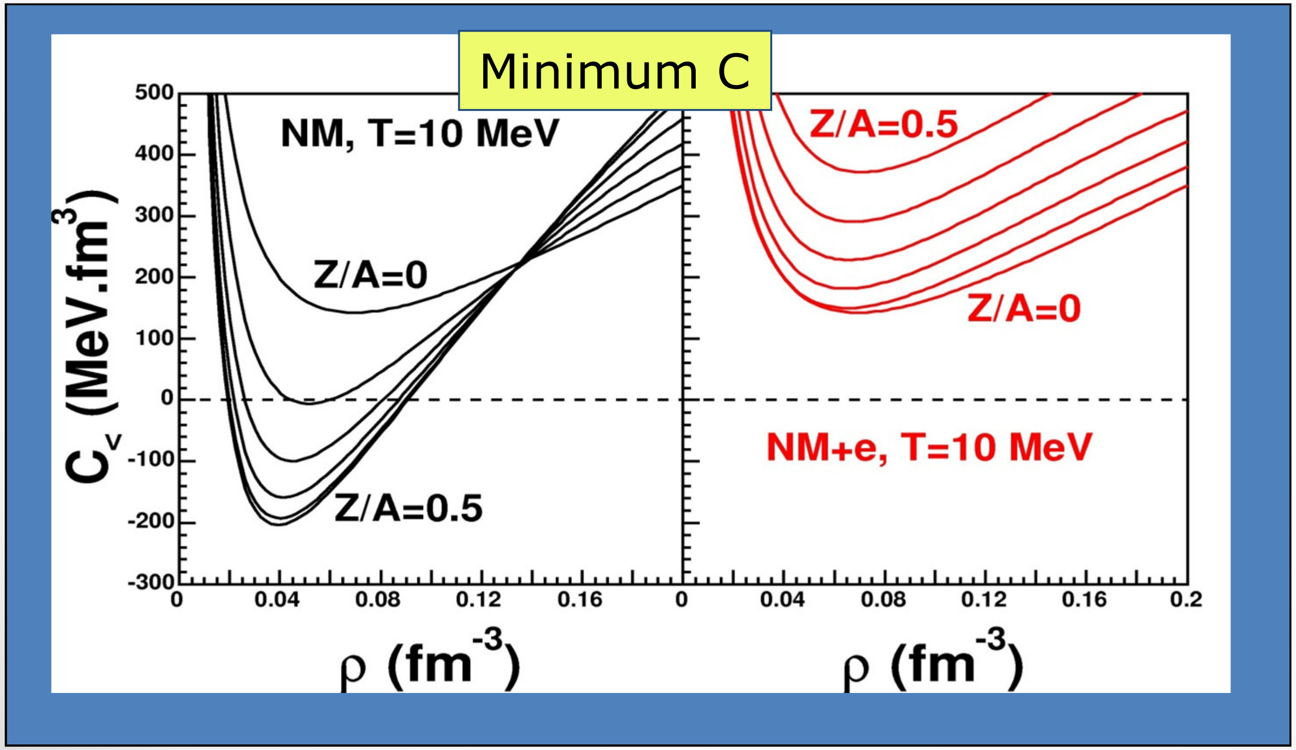
$$C = \begin{vmatrix} \dfrac{\partial\mu_n}{\partial\rho_n} & \dfrac{\partial\mu_n}{\partial\rho_p} \\ \dfrac{\partial\mu_p}{\partial\rho_n} & \dfrac{\partial\mu_p}{\partial\rho_p} + \dfrac{\partial\mu_e}{\partial\rho_e} \end{vmatrix}$$



I - Spinodal in NM+e

- Stellar matter : no macroscopic charge =>spinodal instabilities quenched





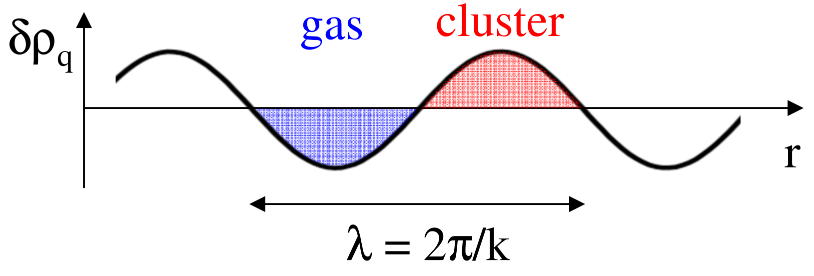
Finite λ fluctuations

- Independent density fluctuations of p,n,e
- Variation of the free energy

$$\delta f_{\beta} = \delta H - T\delta s$$

- Extra contribution of density gradient and Coulomb

$\delta\rho_q\,(\mathbf{k.r}) = A_q e^{i\mathbf{k.r}} + A_q^{*} e^{-i\mathbf{k.r}}$ with q=n,p,e



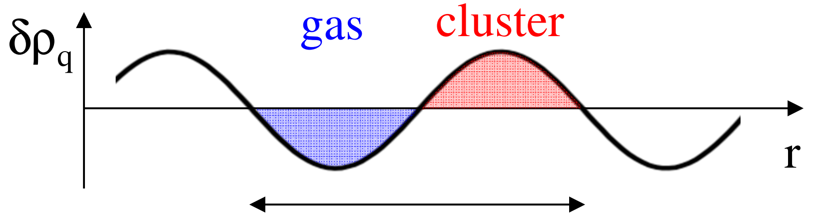
C.J.Pethick et al NPA 1995

$$C^{f}_{NMe} = \begin{pmatrix} \partial_{\rho_n}\mu_n & \partial_{\rho_n}\mu_p & 0 \\ \partial_{\rho_p}\mu_n & \partial_{\rho_p}\mu_p & 0 \\ 0 & 0 & \partial_{\rho_e}\mu_e \end{pmatrix} + \begin{pmatrix} C^{f}_{nn} & C^{f}_{np} & 0 \\ C^{f}_{pn} & C^{f}_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} k^2 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & -\alpha \\ 0 & -\alpha & \alpha \end{pmatrix} \frac{1}{k^2}$$

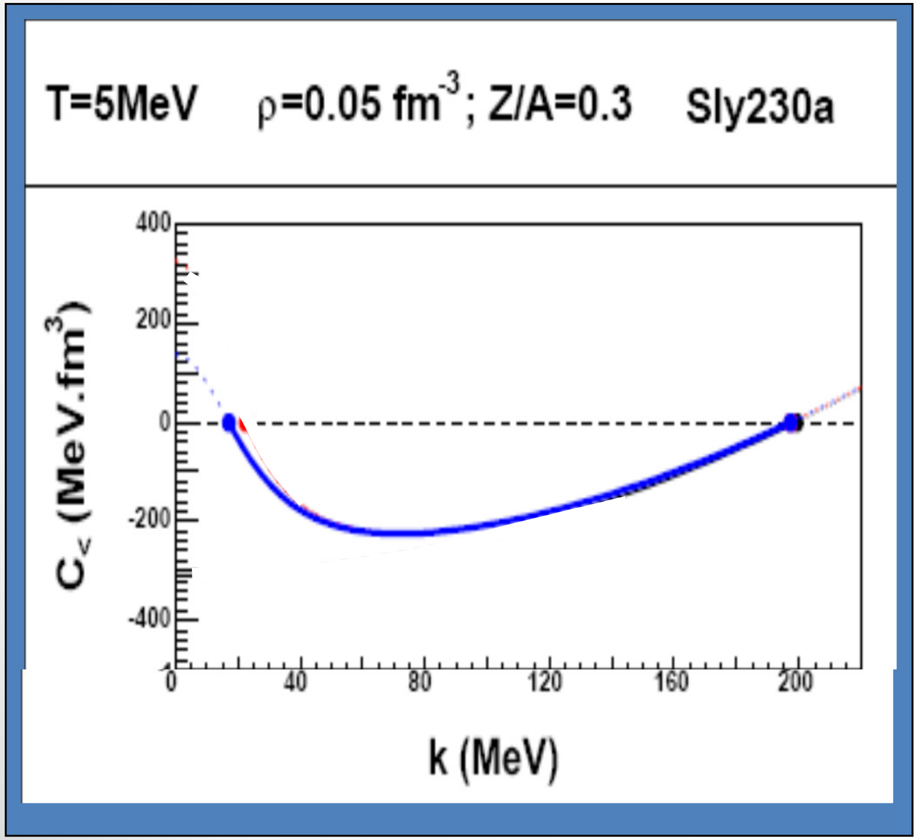
Finite λ fluctuations

$$\delta f = \delta H - T\delta s$$

$$\delta\rho_q\,(\mathbf{k.r}) = A_q e^{i\mathbf{k.r}} + A_q^{*} e^{-i\mathbf{k.r}} \text{ with } q=n,p,e$$



C.J.Pethick et al NPA 1995



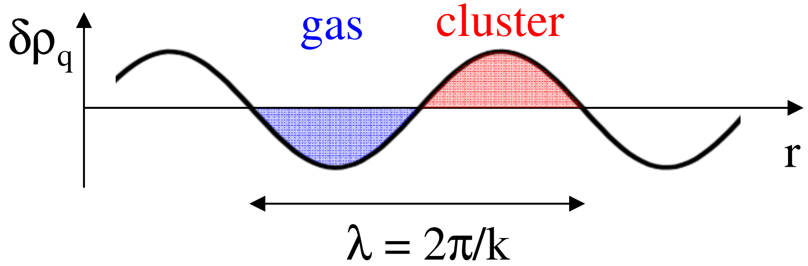
$$C' = C + \begin{vmatrix} C_{surf} & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{vmatrix} k^2 + \begin{vmatrix} 0 & 0 & 0 \\ 0 & C_{coul} & \\ 0 & & \end{vmatrix} \frac{1}{k^2}$$

- Coulomb hinders high λ fluctuations
- Surface hinders low λ fluctuations

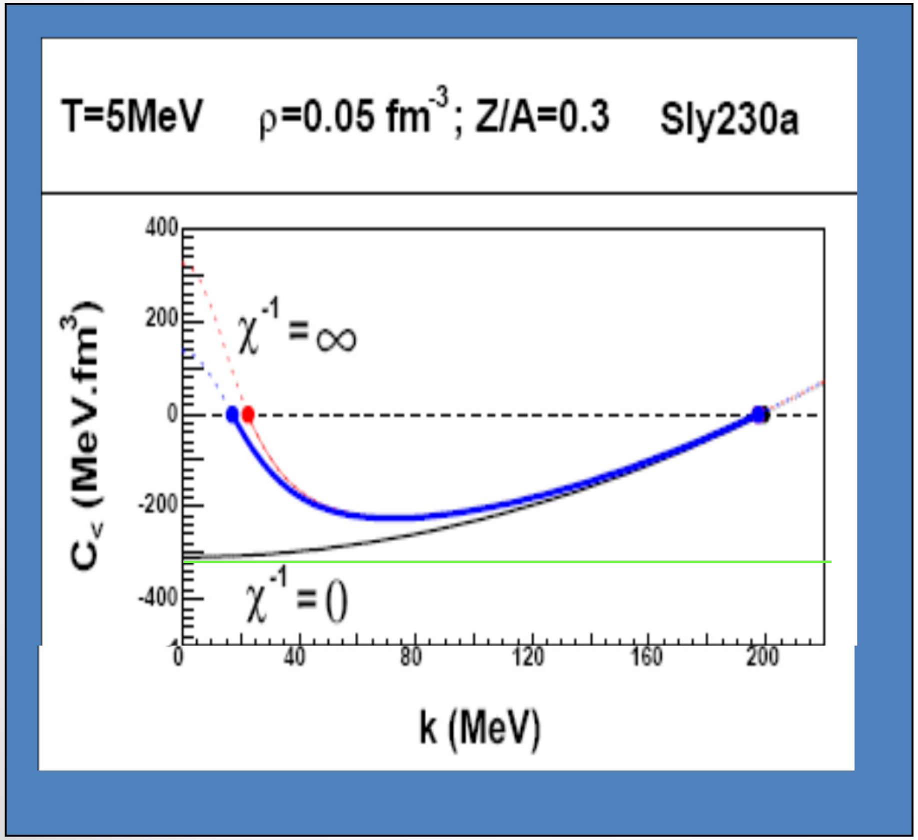
Finite λ fluctuations

$$\delta f = \delta H - T\delta s$$

$$\delta\rho_q(\mathbf{k.r}) = A_q e^{i\mathbf{k.r}} + A_q^{*} e^{-i\mathbf{k.r}} \text{ with } q=n,p,e$$



C.J.Pethick et al NPA 1995



$$C' = C + \begin{vmatrix} C_{surf} & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{vmatrix} k^2 + \begin{vmatrix} 0 & 0 & 0 \\ 0 & C_{coul} & \\ 0 & & \end{vmatrix} \frac{1}{k^2}$$

- $\chi^{-1} = \infty$ homogeneous electron gas =>no screening
- $\chi^{-1} = 0$ electrons in phase with protons => complete screening
- (green line) homogeneous NM