Dynamics of self-gravitating particles in an expanding universe

Michael Joyce

LPNHE
Université Pierre et Marie Curie - Paris VI

Acknowledgements: my collaborators

T. Baertschiger (La Sapienza, Rome)
A. Gabrielli (SMC - INFM/CNR , Rome)
B. Marcos (Université de Nice)
F. Sylos Labini (Centro E. Fermi, Rome)
F. Sicard (Université de Bourgogne)
D. Benhaiem (LPNHE, Paris)

Outline

•**Infinite system** limit for Newtonian gravity

•The (contemporary) problem of cosmological structure formation:
cold dark matter models

•Evolution of these systems:
what is known and understood, and what isn't..

•The problem in **one dimension**

•Conclusions, questions, perspectives

From finite to infinite…

Finite and infinite

$$\ddot{\mathbf{r}}_i = -Gm \sum_{j \neq i} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

There are **two distinct problems** :

Finite system: N particles *in a finite region* of (infinite) space
("**astrophysics**")

Infinite system: an infinite number of particles distributed *throughout space*
("**cosmology**")

Defining the infinite system limit:

$$\ddot{\mathbf{r}}_i = -Gm \sum_{j \neq i} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Infinite system:

Even for a **uniform non-zero mass density** the force is badly defined..

Finite & Infinite: An Old Perspective …

Isaac Newton

Correspondence with G. Bentley, 1692

Bentley’s question: what happens

… If all the matter in the universe **is evenly scattered throughout all the heavens** and every particle has an innate gravity toward all the rest…

Finite & Infinite: Newton's reply (1)…

…**If the whole space throughout which all this matter was scattered was but finite**, the matter on the outside of the space would, by its gravity tend toward all the matter on the inside and by consequence fall down into the **middle of the whole space** and there compose one great spherical mass… .

Finite & Infinite: Newton's reply (2)

…**But if the mass was evenly disposed throughout an infinite space**, it could never convene into one mass; but some of it would convene into one mass and some other into another, so as to make an **infinite number of great masses**, scattered at great distances from one another throughout the infinite space….

Defining the infinite system limit: **Regularisation 1**

"Universe with a centre" : sum in spheres *about a chosen (arbitrary) point*

$$\ddot{\mathbf{r}}_i = -Gm \lim_{R_s \to \infty} \sum_{j \neq i, j \in S(R_s, \mathbf{r}_0)} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Uniform limit → expanding/contracting universe solutions of GR

Convenient to work in "comoving coordinates" $\mathbf{r}_i = a(t)\mathbf{x}_i$:

$$\frac{d^2\mathbf{x}_i}{dt^2} + 2H\frac{d\mathbf{x}_i}{dt} = -\frac{Gm}{a^3}\left[\lim_{R_s \to \infty} \sum_{j \neq i, j \in S(R_s, \mathbf{r}_0)} \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|^3} - \frac{4\pi}{3} n_0 \mathbf{x}_i\right]$$

where a(t) is the "scale factor" of the expanding/contracting universe, $H = \frac{\dot{a}}{a}$
(and n_0 the mean particle density)

Defining the infinite system limit: **Regularisation 2**

Universe without a centre: sum symmetrically *about each point*

$$\ddot{\mathbf{r}}_i = -Gm \lim_{R_s \to \infty} \sum_{j \neq i, j \in S(R_s, \mathbf{r}_i)} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Uniform limit → *static universe*

Mean density has been subtracted : “Jeans’ swindle”
Can also be written (cf. Kiessling 1999):

$$\mathbf{F}(\mathbf{r}_i) = -Gm \lim_{\mu \to 0} \sum_{j \neq i} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} e^{-\mu |\mathbf{r}_i - \mathbf{r}_j|}$$

Infinite uniform particle systems: equations of motion

Using a redefined time variable, *both regularisations* lead to eom

$$\frac{d^2\mathbf{x}_i}{d\tau^2} + \Gamma(\tau)\frac{d\mathbf{x}_i}{d\tau} = -Gm \lim_{\mu\to 0+} \sum_{j\neq i} \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|^3} e^{-\mu|\mathbf{x}_i - \mathbf{x}_j|}$$

where

$\Gamma = \sqrt{2\pi G\rho_0/3}$ for regularisation 1 ("critical" expanding universe)

$\Gamma = 0$ for regularisation 2 ("static universe")

Thus regularisations **give same equations modulo a viscous damping term** !

[Remark: It is GR which tells us which is the "right" regularisation of NG!!]

Infinite uniform particle systems: Definiteness of force

Have not shown that the "regularised sum" is actually defined !

→ Need first to specify the infinite distribution summed over!

1) **Infinite periodic system** (cube of side L)

Force (and potential) well defined (cf. Coulomb "gellium" or OCP model):
[equivalent to N body problem with a periodic pair potential,
implemented numerically using "Ewald summation"]

$$L \to \infty$$

Definiteness of force in a "real" infinite universe

Is use of periodicity a "cheat"? What happens as $L \to \infty$?

2) **Uniform stochastic point process in infinite space**

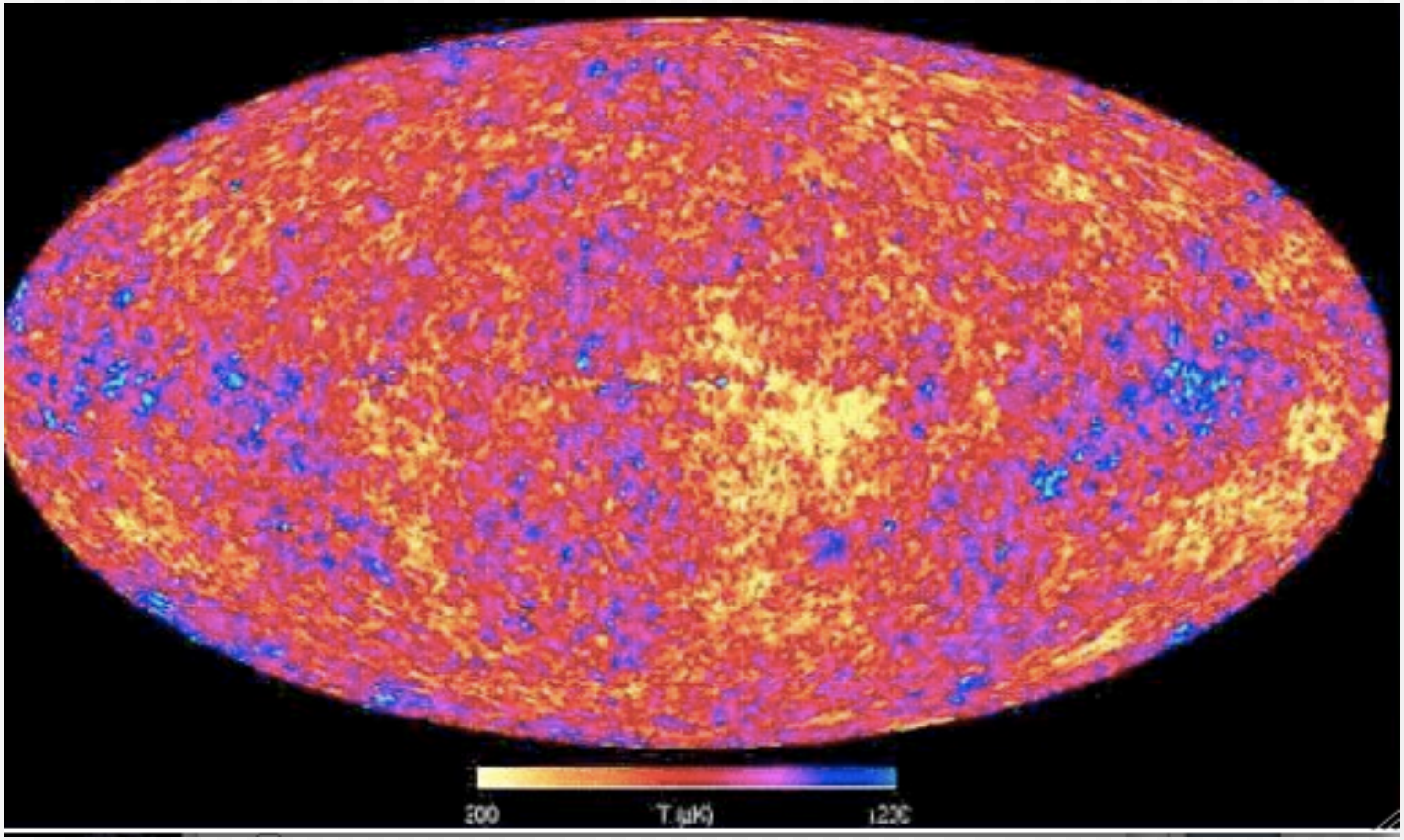
cf. A. Gabrielli, MJ, B. Marcos and F. Sicard, JSP(2011)

Show that **force PDF** are well defined under certain conditions..

Structure formation in an expanding universe with cold dark matter

Cosmology

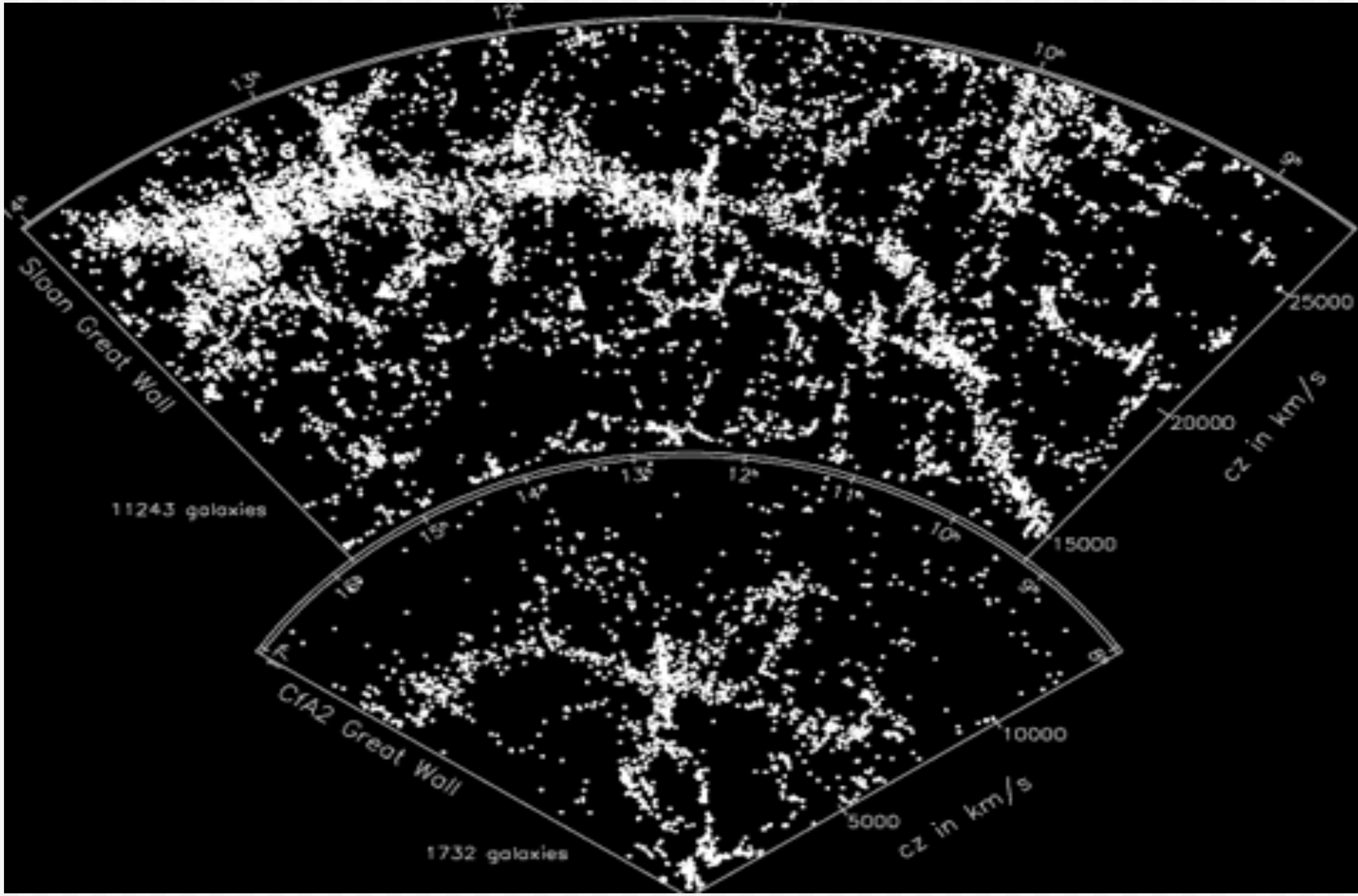
"WMAP" : the universe at ~10^5 years…



Density fluctuations ~**10^{-4} to 10^{-5}**

Cosmology

"SDSS" : the universe today (10^{10} years)



Fluctuations >> 1 at corresponding scales

Simulating the joint evolution of quasars, galaxies and their large-scale distribution

Volker Springel[1], Simon D. M. White[1], Adrian Jenkins[2], Carlos S. Fr
Naoki Yoshida[3], Liang Gao[1], Julio Navarro[4], Robert Thacker[5], Darre
John Helly[2], John A. Peacock[6], Shaun Cole[2], Peter Thomas[7], Hugh
August Evrard[8], Jörg Colberg[9] & Frazer Pearce[10]

[1]*Max-Planck-Institute for Astrophysics, Karl-Schwarzschild-Str. 1, 85740 Garching, Ger*
[2]*Inst. for Computational Cosmology, Dep. of Physics, Univ. of Durham, South Road, Du*
[3]*Department of Physics, Nagoya University, Chikusa-ku, Nagoya 464-8602, Japan*
[4]*Dep. of Physics & Astron., University of Victoria, Victoria, BC, V8P 5C2, Canada*
[5]*Dep. of Physics & Astron., McMaster Univ., 1280 Main St. West, Hamilton, Ontario, L8*
[6]*Institute of Astronomy, University of Edinburgh, Blackford Hill, Edinburgh EH9 3HJ, U*
[7]*Dep. of Physics & Astron., University of Sussex, Falmer, Brighton BN1 9QH, UK*
[8]*Dep. of Physics & Astron., Univ. of Michigan, Ann Arbor, MI 48109-1120, USA*
[9]*Dep. of Physics & Astron., Univ. of Pittsburgh, 3941 O'Hara Street, Pittsburgh PA 1526*
[10]*Physics and Astronomy Department, Univ. of Nottingham, Nottingham NG7 2RD, UK*

The cold dark matter model has become the leading theoretical para mation of structure in the Universe. Together with the theory of cos model makes a clear prediction for the initial conditions for structu predicts that structures grow hierarchically through gravitational in this model requires that the precise measurements delivered by gala compared to robust and equally precise theoretical calculations. Here framework for the quantitative physical interpretation of such survey the largest simulation of the growth of dark matter structure ever car techniques for following the formation and evolution of the visible com that baryon-induced features in the initial conditions of the Universe a torted form in the low-redshift galaxy distribution, an effect that can be the nature of dark energy with next generation surveys.

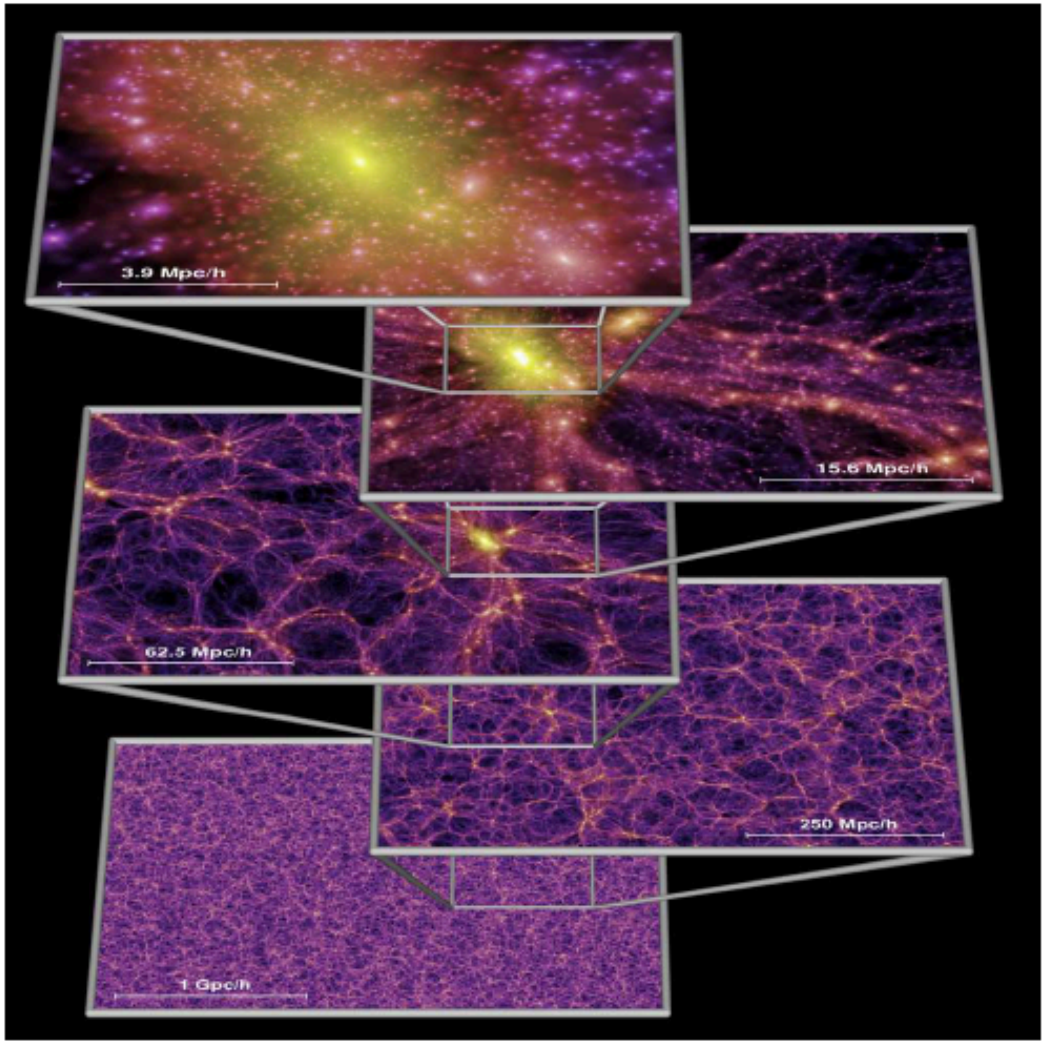


Figure 1: The dark matter density field on various scales. Each individual image shows the projected dark matter density field in a slab of thickness $15h^{-1}$Mpc (sliced from the periodic simulation volume at an angle chosen to avoid replicating structures in the lower two images), colour-coded by density and local dark matter velocity dispersion. The zoom sequence displays consecutive enlargements by factors of four, centred on one of the many galaxy cluster halos present in the simulation.

Structure formation in the standard cosmological model

"Cosmological N body simulations" solve equations of "regularisation 1"
(+ force "smoothing" at small scales)

Newtonian (purely gravitating) limit a valid approximation over large range of time and length scales

["Cold" → non-relativistic; "Dark" → just gravity; weak fields]

Cosmological models specify:

- **initial conditions**
- **damping term**

Dark matter in cosmology: continuum limit

Note: **simulation particles are not physical dark matter particles** !
They are "macro-particles" (astrophysical mass)!

Cosmologists would like to simulate continuum limit given by Vlasov-Poisson, adapted appropriately (infinite domain, regulated Newtonian force)

$$\frac{\partial f}{\partial t} + \frac{\mathbf{v}}{a^2} \cdot \nabla f - \nabla \Phi \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \Phi(\mathbf{x}) = \frac{4\pi G}{a^3} \int (f - f_0)\, d\mathbf{v}$$

[Rigorous derivation needed..?]

However currently not numerically feasible…

Cosmology: (typical) initial conditions

Cold dark matter fluid

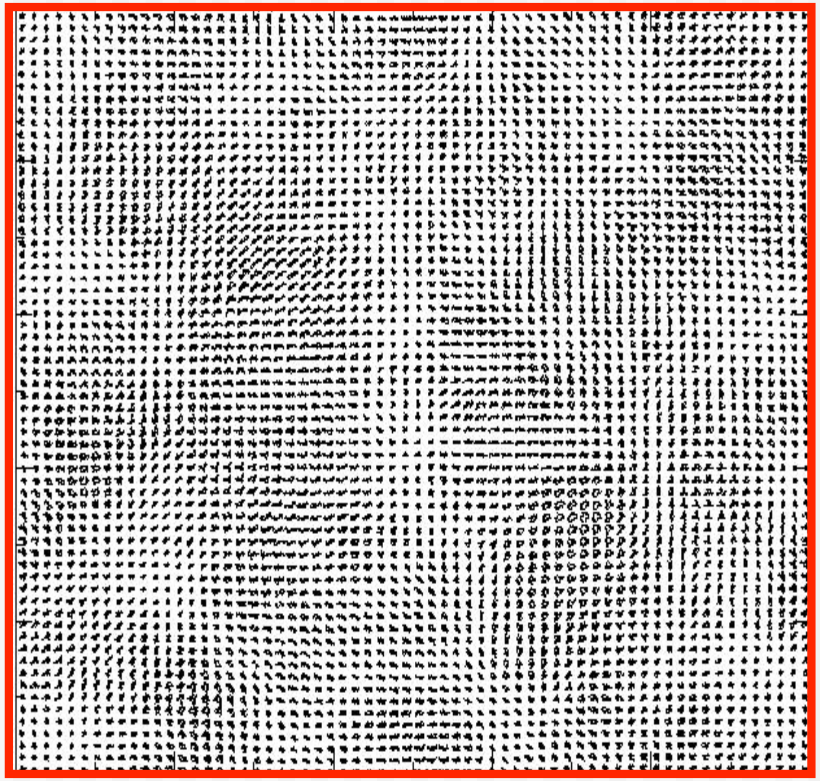
$$f(\mathbf{x}, \mathbf{v}) = \rho(\mathbf{x})\delta^{(3)}(\mathbf{v})$$

Density field : $\rho(\mathbf{x})$ realization of a *correlated gaussian process*

Fully characterized by power spectrum P(k)

e.g. “LambdaCDM” spectrum or variants

Initial conditions of N body simulations



IC are generated by **displacing particles off a lattice** (or “glass”)

Dynamics of infinite self-gravitating particle systems with cold initial conditions

Cosmological-like N body simulations: recap

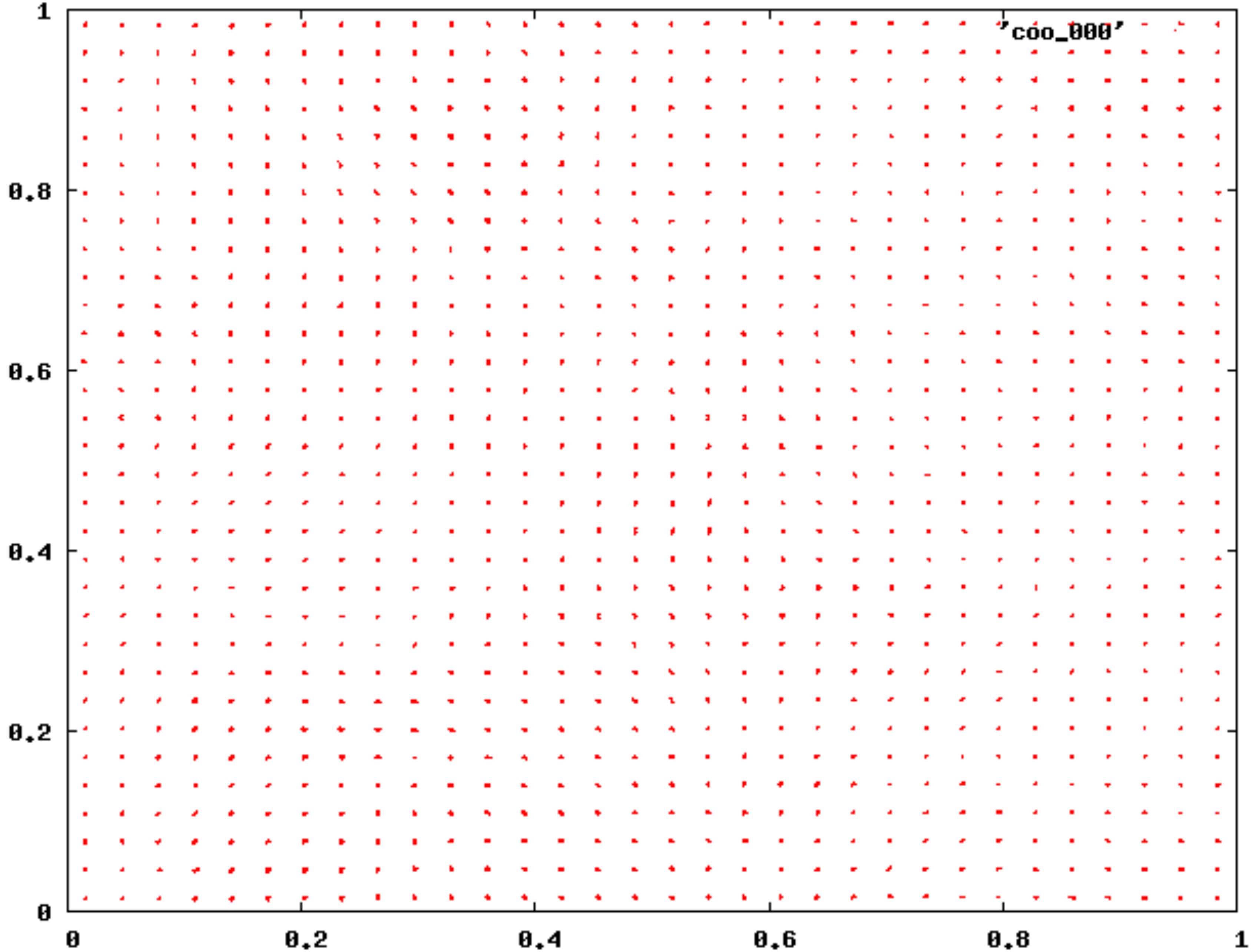
Equations of motion

$$\frac{d^2\mathbf{x}_i}{d\tau^2} + \Gamma(\tau)\frac{d\mathbf{x}_i}{d\tau} = -Gm \lim_{\mu\to 0+} \sum_{j\neq i} \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|^3} e^{-\mu|\mathbf{x}_i - \mathbf{x}_j|}$$

Γ constant or "slowly varying"

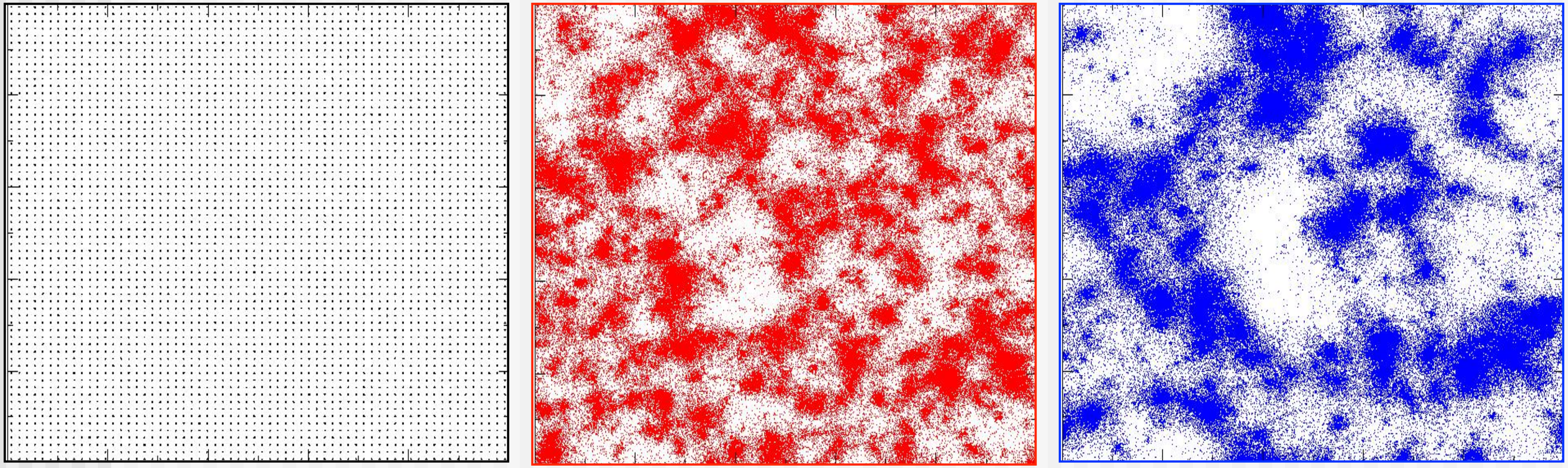
IC: **cold** particles, small displacements off lattice ("reasonable" P(k))

Evolution of an infinite (periodic) system



Evolution of an infinite (periodic) system

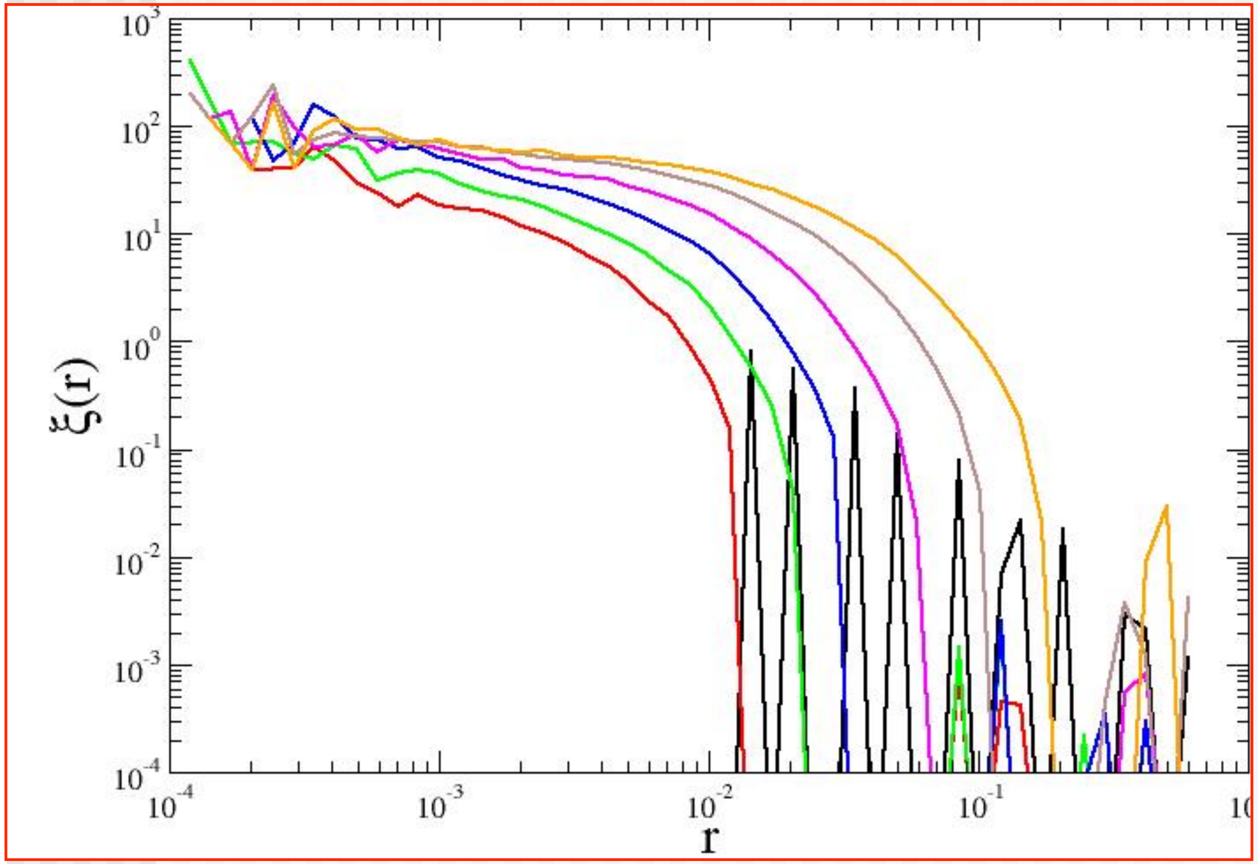
T. Baertchiger et al, PRE(2008)



Clustering develops first at small scales and "propagates" to larger scales

N.B. Simulation **represents the infinite system for a finite time**

Evolution of 2 point correlations



$\xi(r, t) > 1$ strong correlation

$\xi(r, t) < 1$ weak corrélation

Useful to define the scale

$$\xi(\lambda(t), t) = 1$$

$\lambda(t)$: "scale of non-linearity"

Hierarchical clustering in a nutshell

• Non-linearity scale propagates **from small to large scales,**
at a *rate predicted by linearised fluid theory* (Jeans instability)

• In non-linear regime "flow of power" **from large to small scales**
(via collapse dynamics exemplified by "spherical collapse model")

Non-linear fluctuations at a given scale are generated essentially by the evolution of *fluctuations initially at larger scales*

("linear theory amplification and then collapse")

Qualitative features c*ommon to all cold initial conditions and cosmologies*

The "challenge": the "non-linear regime"

- **How is non-linear clustering best characterized ?**

  (mathematical tools..)

- **How does it depend on initial conditions and cosmology?**

  (and can we understand and precisely characterize this..)

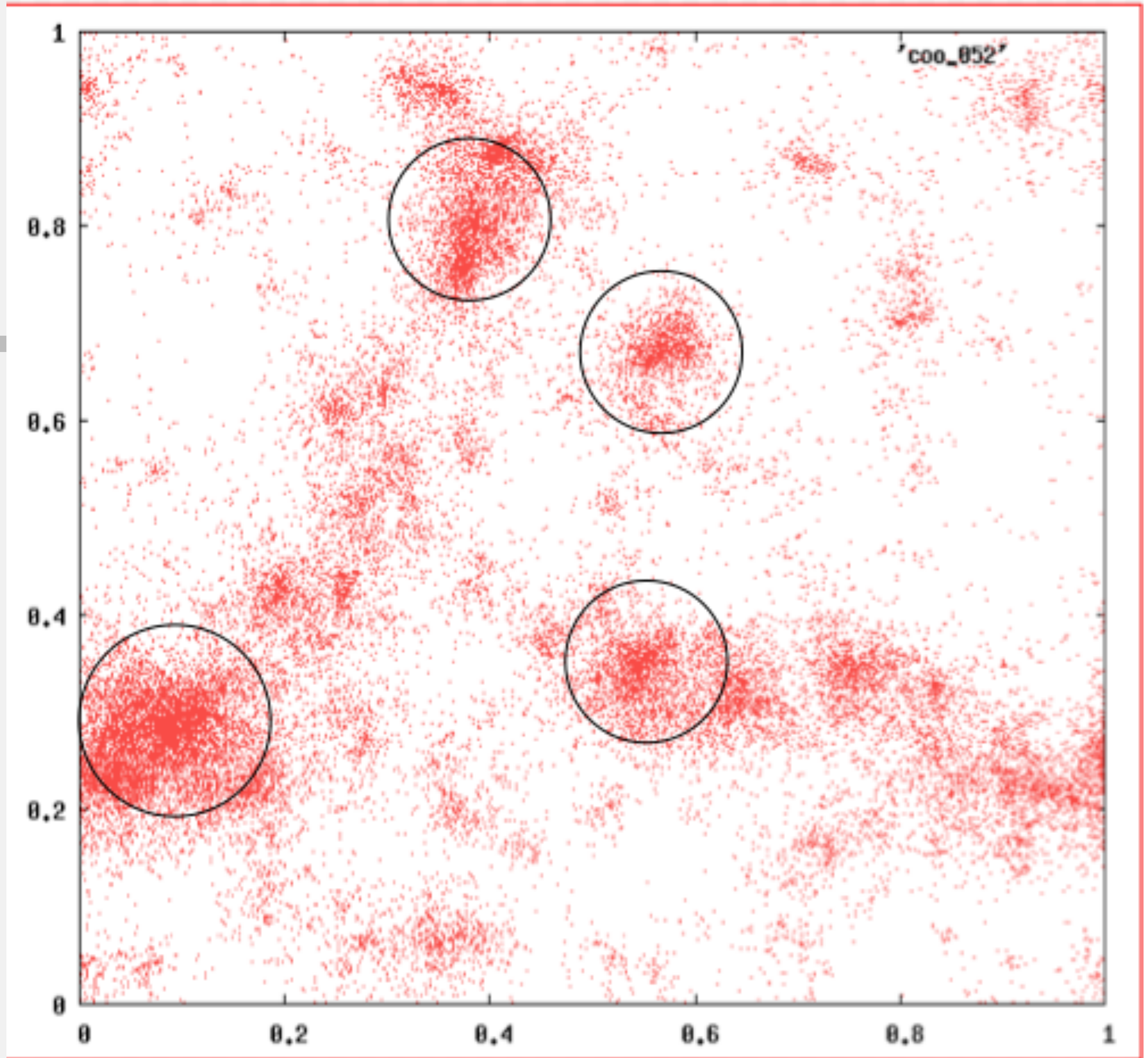
The non-linear regime
as now seen (understood?) by cosmologists

Huge studies focussed on “realistic” cosmological IC

→ phenomenological descriptions of the non-linear regime,

Currently “halo models” dominate

"Halo models" of non-linear clustering



Dark matter density field ≈

collection of (non-overlapping) spherical *smooth* virialized structures

Density profiles of these "halos" fitted by "universal" form, e.g.,

"NFW profile"

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}$$

However…

Problems…

- Halos are poorly defined objects
- The approximation of smoothness is problematic; increased resolution has revealed layer of “substructure”..
- Unclear what “universality” means, what is its origin

(Huge literature on each issue..)

The "challenge": the "non-linear regime"

- **How is non-linear clustering best characterized ?**

- **How does it depend on initial conditions and cosmology?**

Numerical simulations:

- enormous but still too small!
- debate and controversy about their claimed resolution/precision

Analytic approaches: Few and unsuccessful

These basic questions are still completely open..

1D models of cosmological structure formation

Infinite self-gravitating systems in 1D

In complete analogy to 3D, consider

$$\frac{d^2 x_i}{d\tau^2} + \Gamma \frac{dx_i}{d\tau} = -g \lim_{\mu \to 0^+} \sum_{j \neq i} \mathrm{sgn}(x_i - x_j) e^{-\mu |x_i - x_j|}$$

Advantages with respect to 3D:

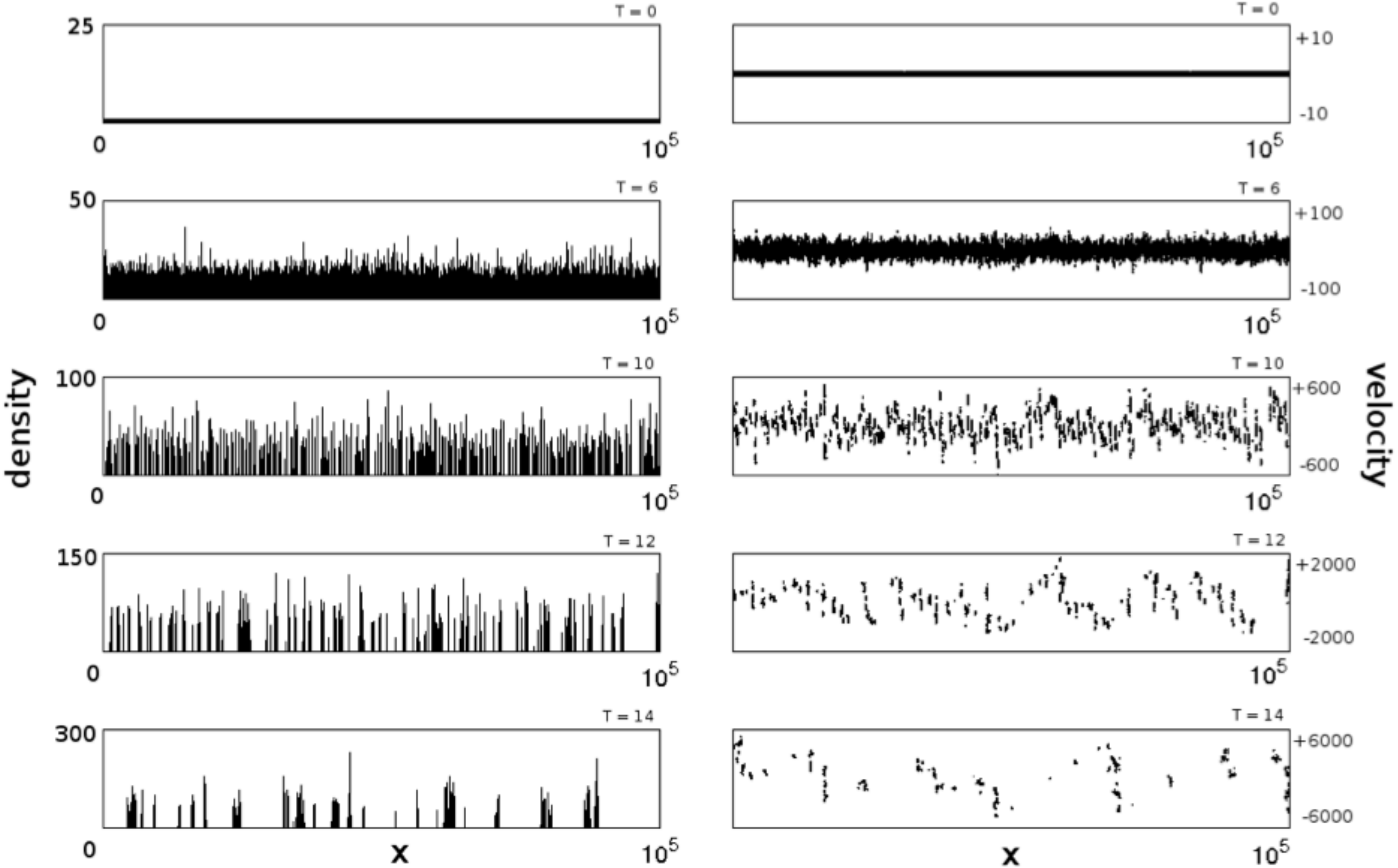
•**Force** can be **calculated exactly**

•The **equations of motion** can be integrated **"exactly"**

•Much **greater spatial resolution**
(no smoothing of force at small scale, scale resolved ~N, not ~ $N^{1/3}$)

IC as in 3D

Results: clustering in a 1-d universe



1D clustering from cold initial conditions:

Quantitative analyse reveals **behaviour completely analagous to 3D**

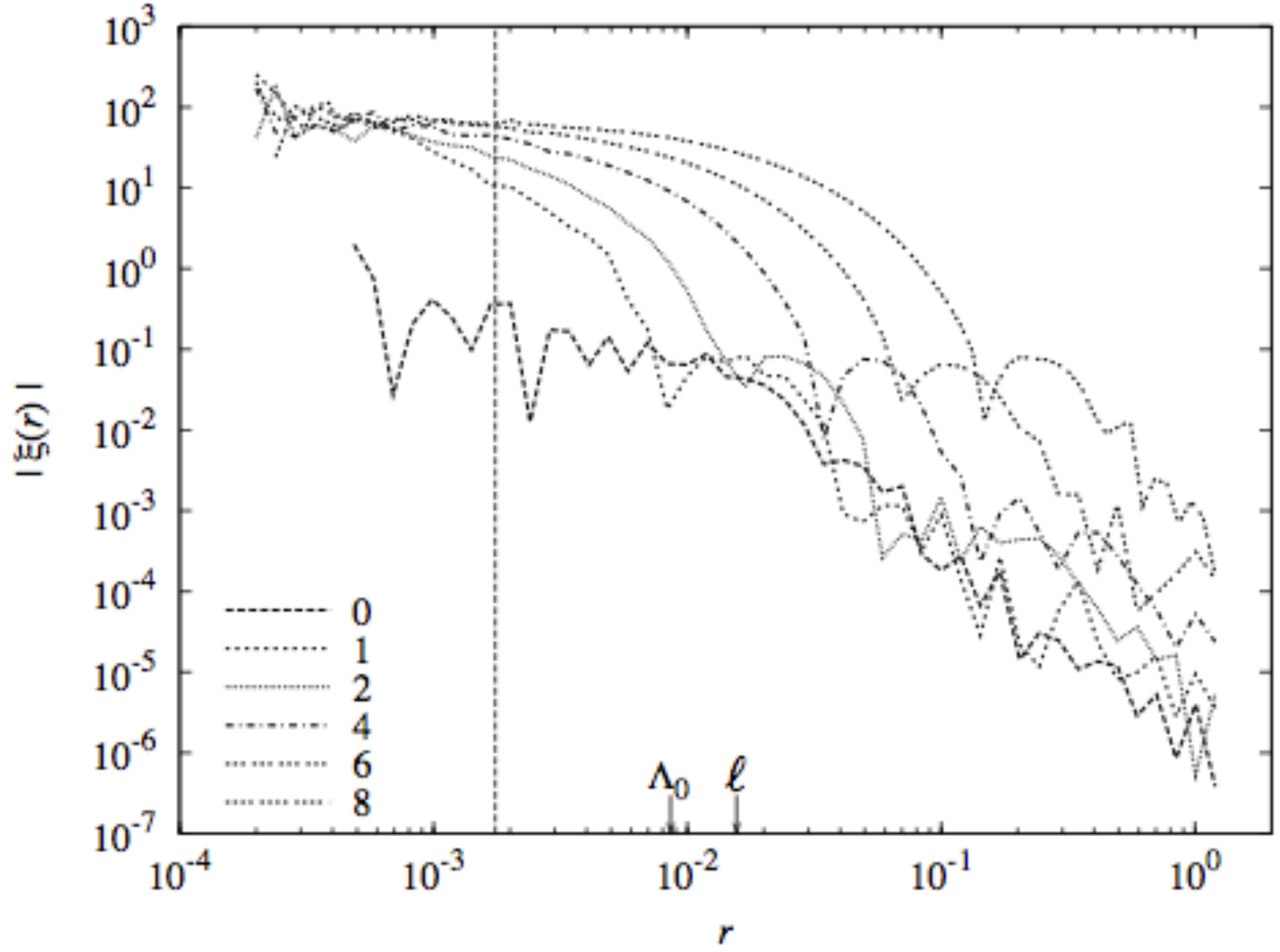
→ Hierarchical clustering (linear amplification + collapse)

As in 3D, for power law initial conditions, **“self-similarity” of correlations**:

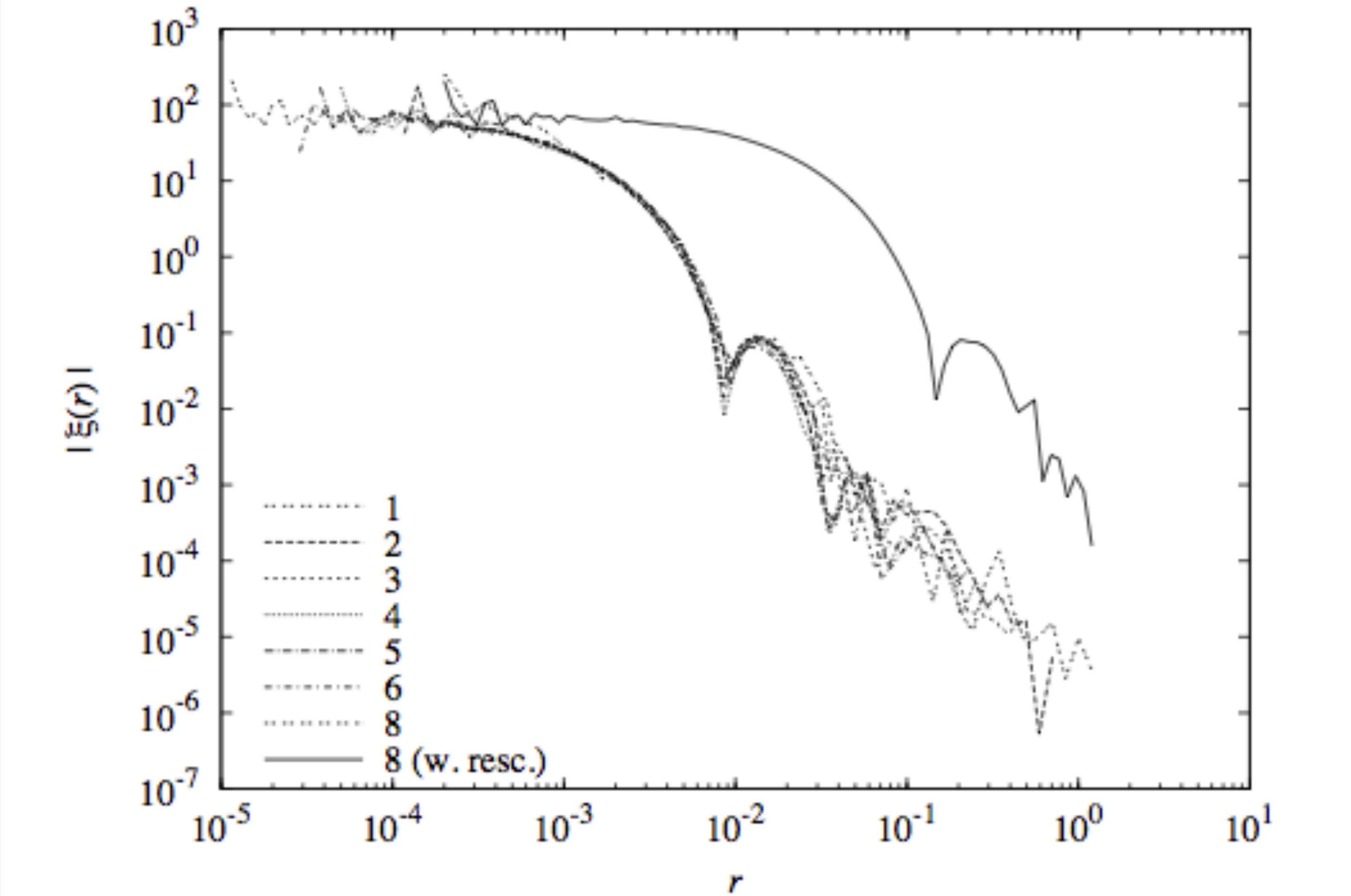
$$\xi(x,t) \approx \xi_0(\frac{x}{R_S(t)})$$

[$R_s(t)$ can be derived from linear theory, and fits simulations]

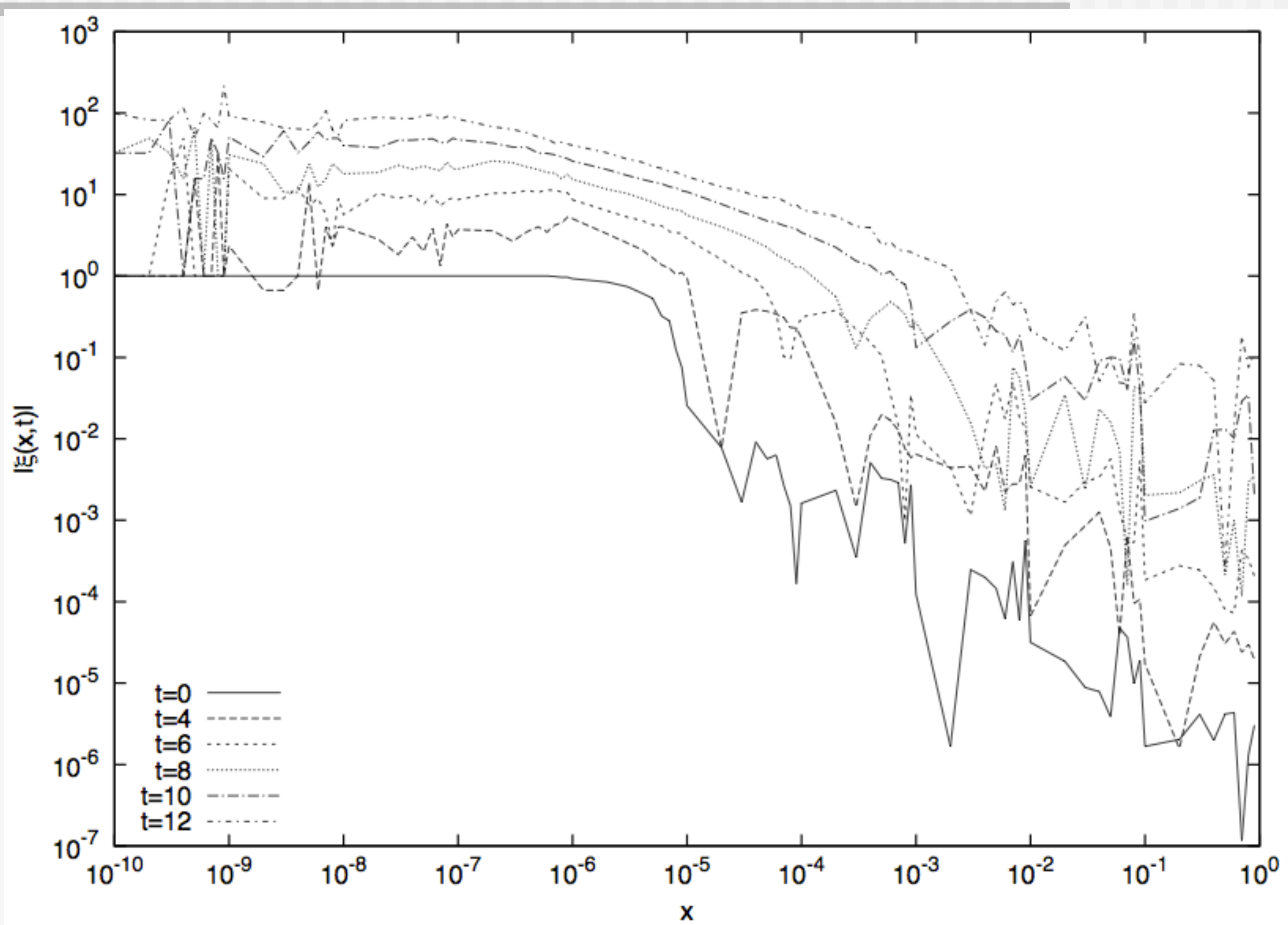
Self-similarity (3D)



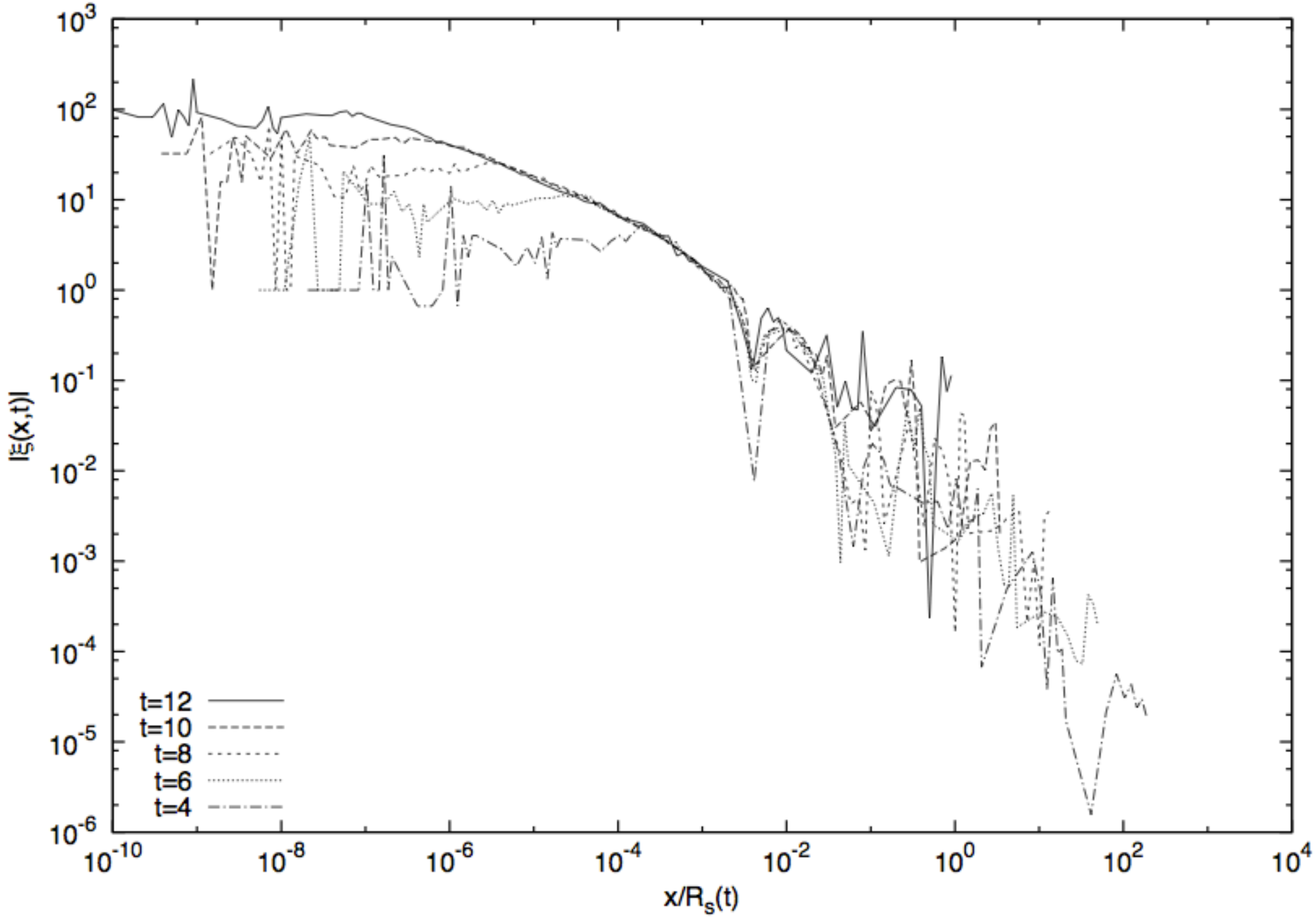
Self-similarity (3D)



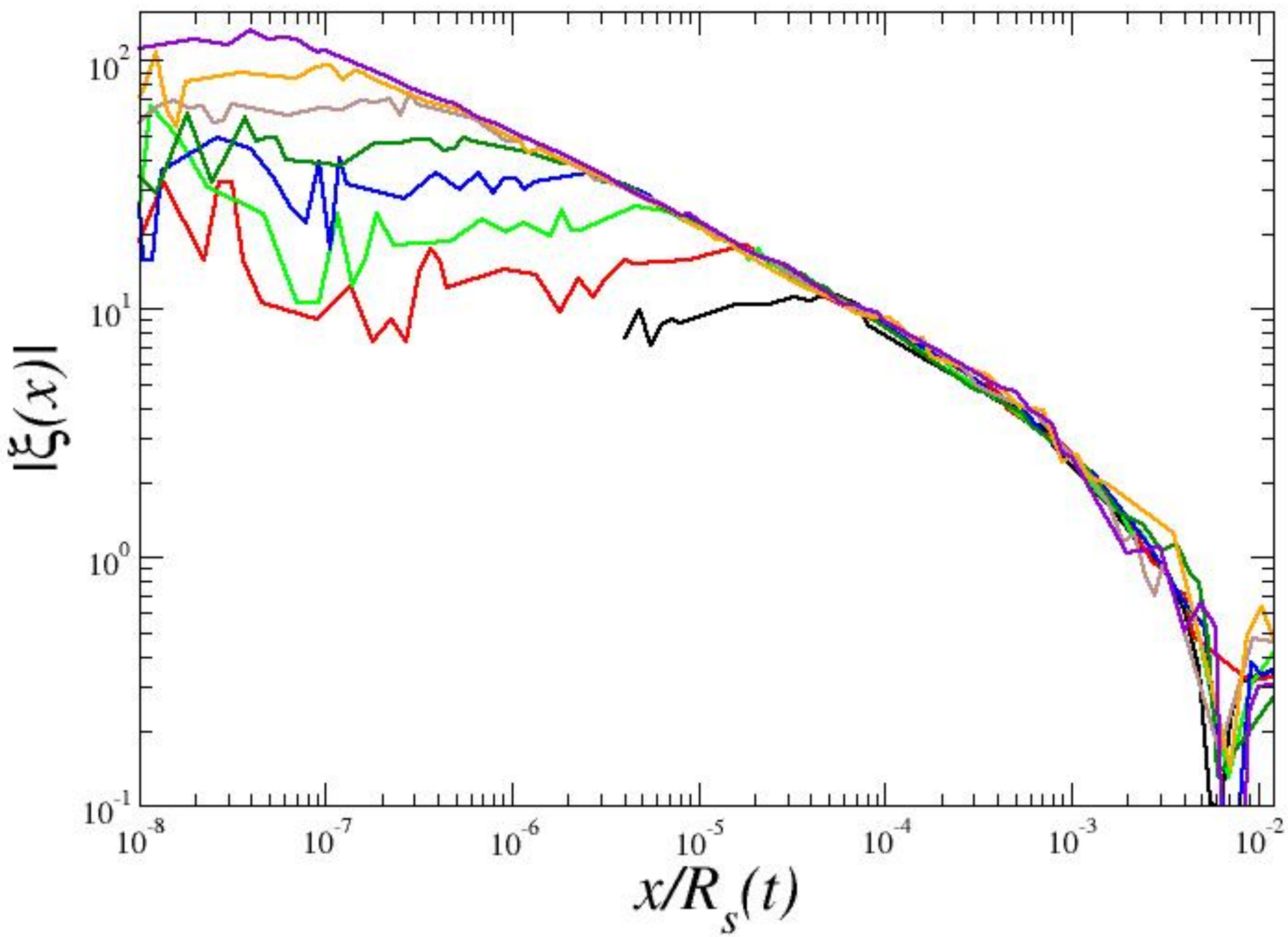
Self-similarity (1D)



Self-similarity (1D)



Self-similarity (1D)



Scale-invariance in 1D?

(MJ and F. Sicard MNRAS 2011)

Power law behaviour in spatial correlations, over 3-4 orders of magnitudes in expanding models

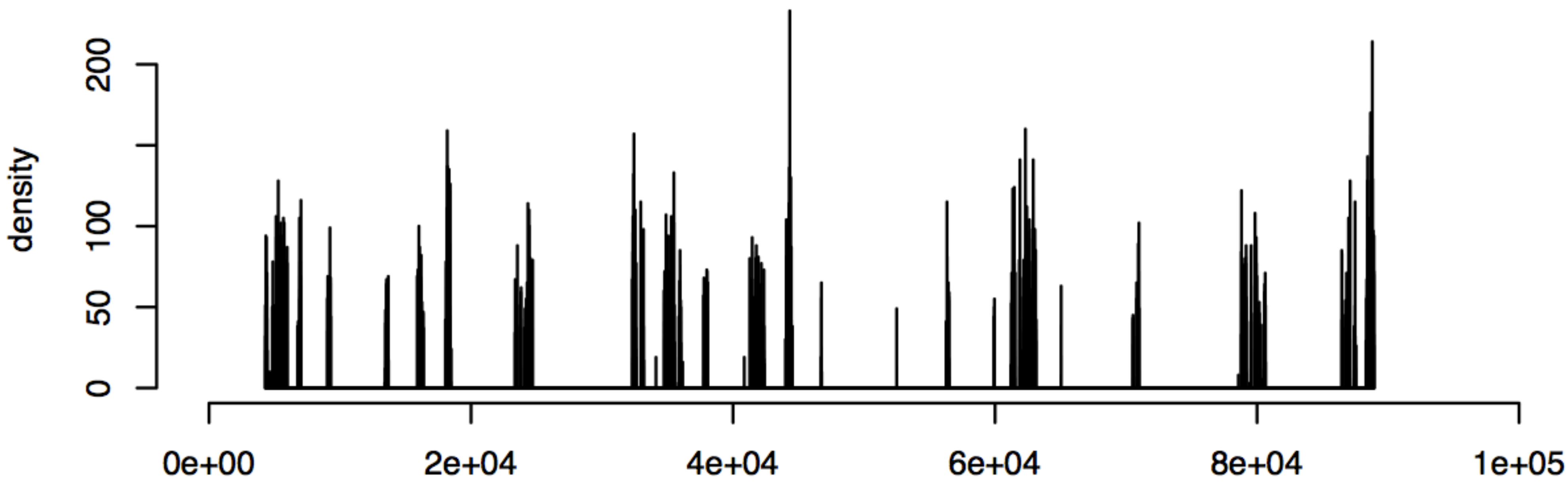
Appears to extend over an arbitrarily large range of scale, asymptotically apparently without limit..

Is it associated with an underlying scale invariance?

[cf. previous work of Miller, Rouet et al., Phy. Rev. E. (2007) and refs therein]

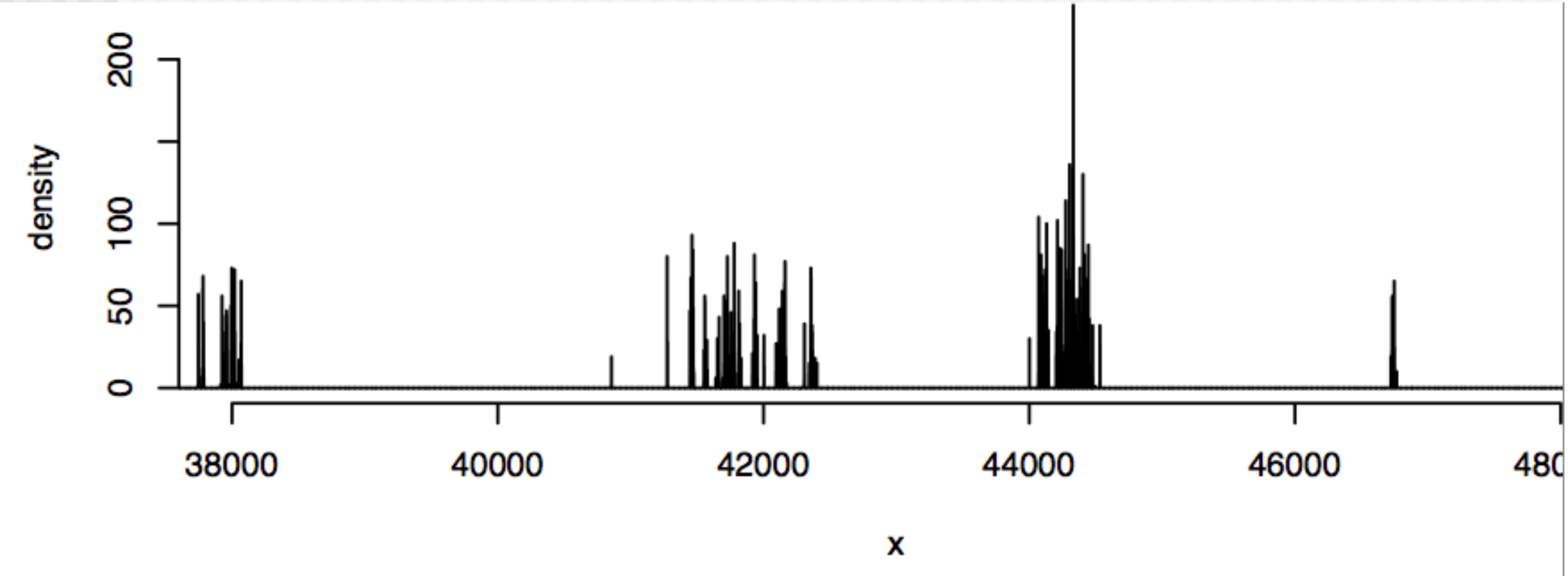
Non-linear clustering in 1D

Whole system (N=10^5 particles)



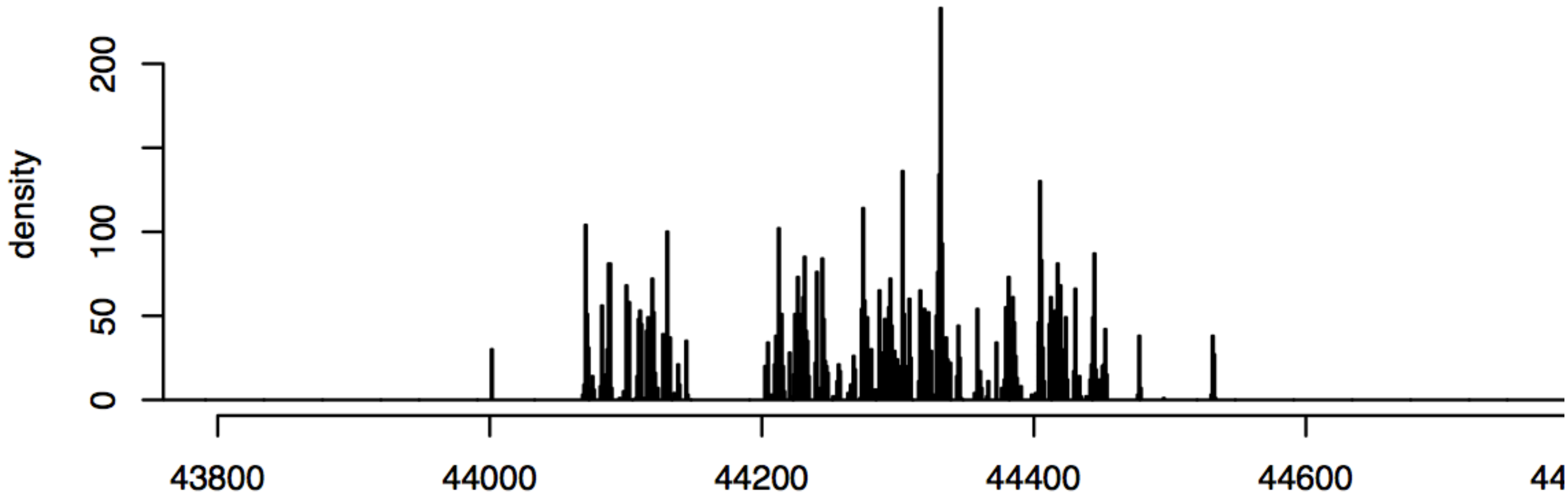
Non-linear clustering in 1D

1/10 th of system



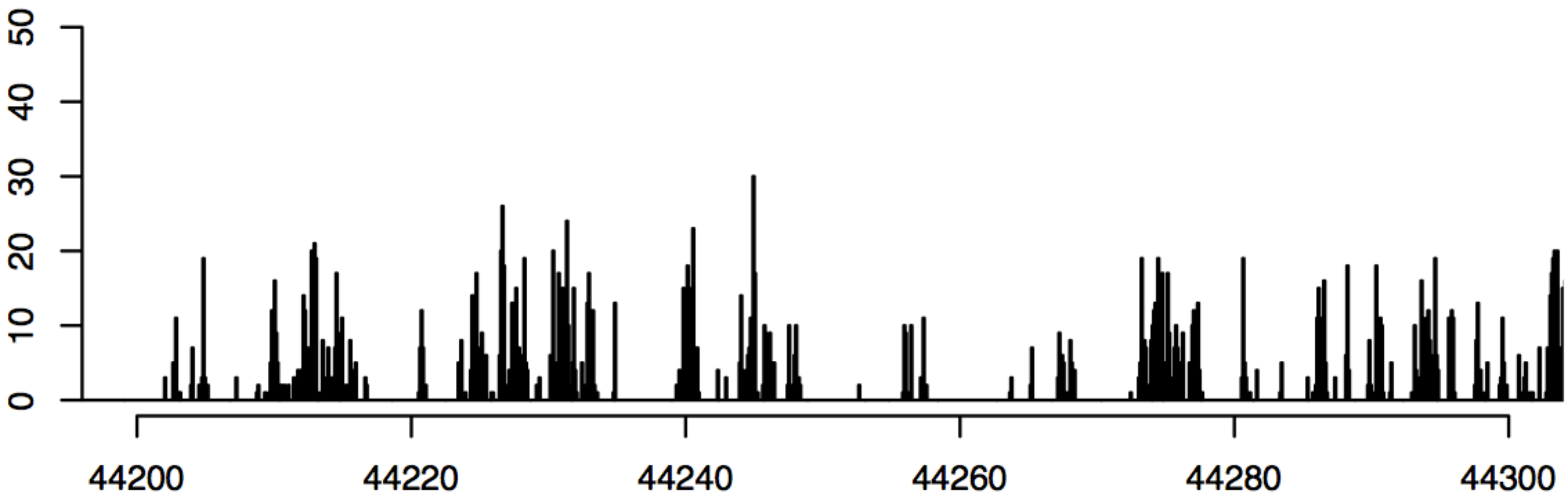
Non-linear clustering in 1D

1/10^2 of system



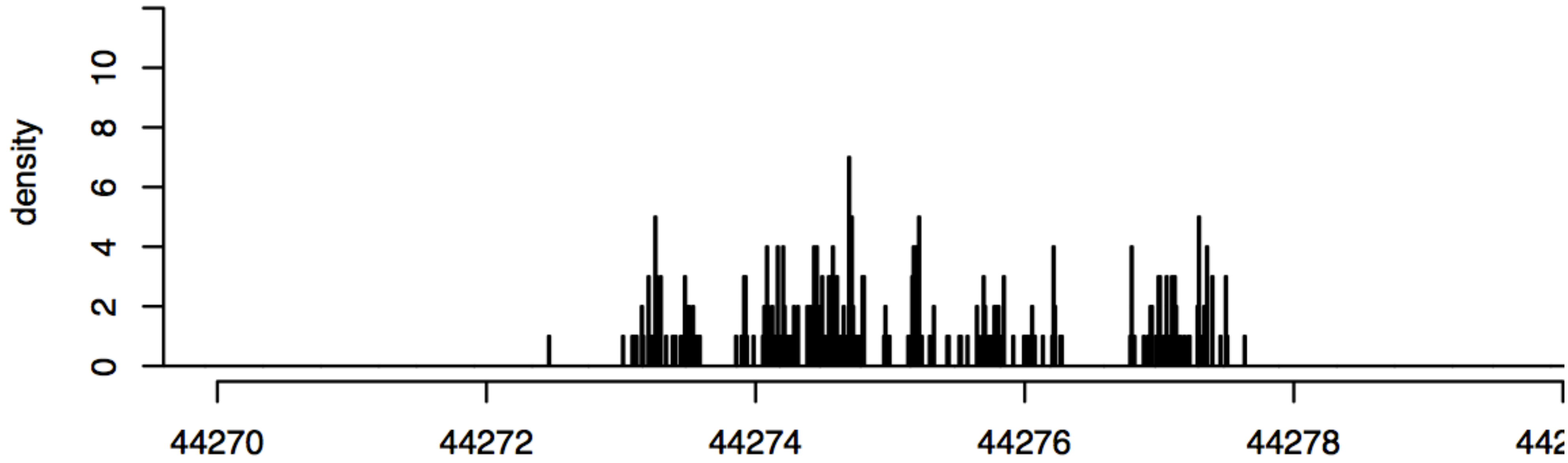
Non-linear clustering in 1D

1/10^3 of system



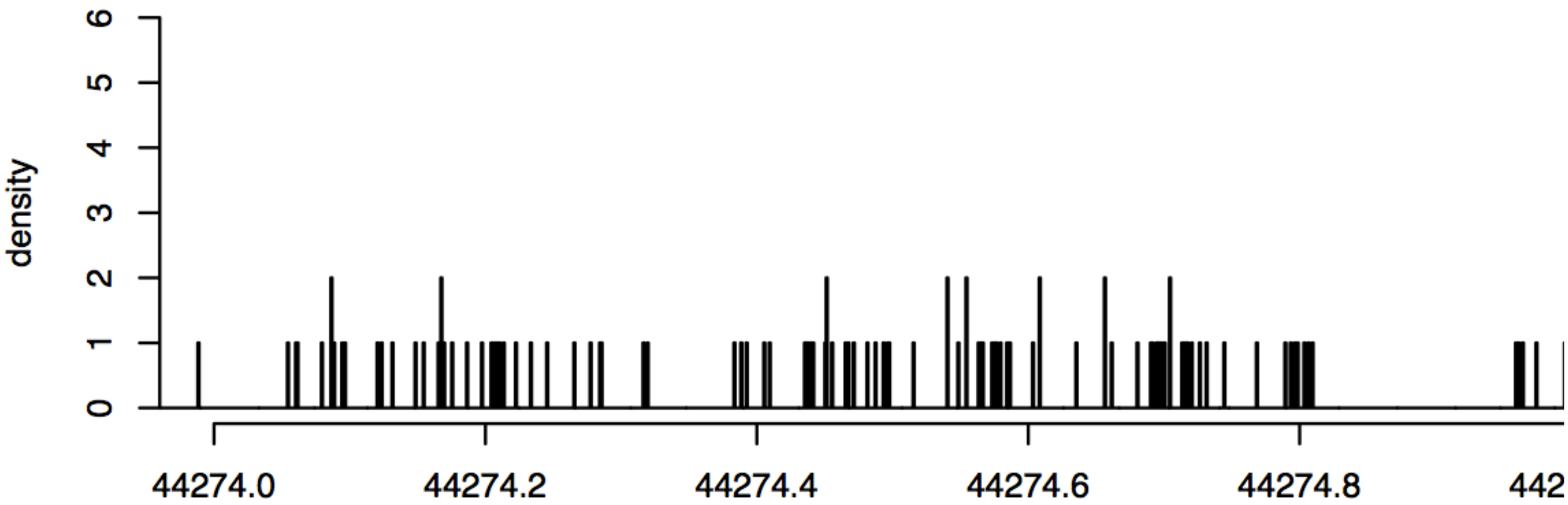
Non-linear clustering in 1D

1/10^4 of system



Non-linear clustering in 1D

1/10^5 of system



Results: scale-invariance in 1D?

(MJ, F. Sicard MNRAS 2011)

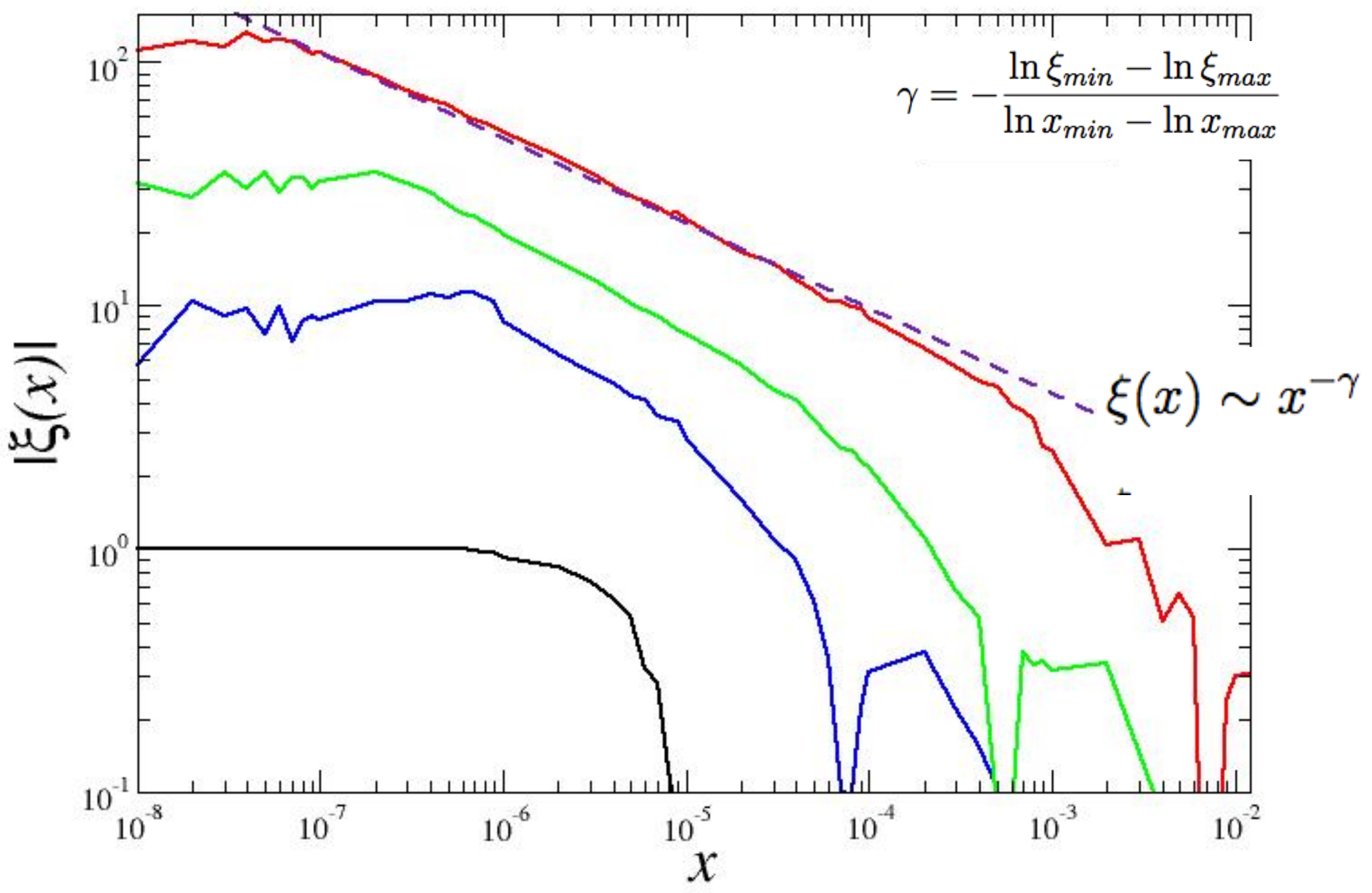
Study (multi-)fractal exponents using standard box-counting technique

Confirms findings of [Miller, Rouet et al., Phy. Rev. E. (2007) and refs therein]

strong evidence for fractal structure/scale-invariance

[over four to five orders of magnitude in scale in expanding models !]

Determination of correlation dimension (1D)



Correlation dimension in the "stable clustering" hypothesis

MJ, F. Sicard MNRAS 2011

"Stable clustering hypothesis" (Peebles 1974 for 3D EdS model) :

Assume strongly non-linear structures behave as isolated virialized objects

→ Clustering frozen in "physical coordinates"
→ Temporal evolution of lower cut-off

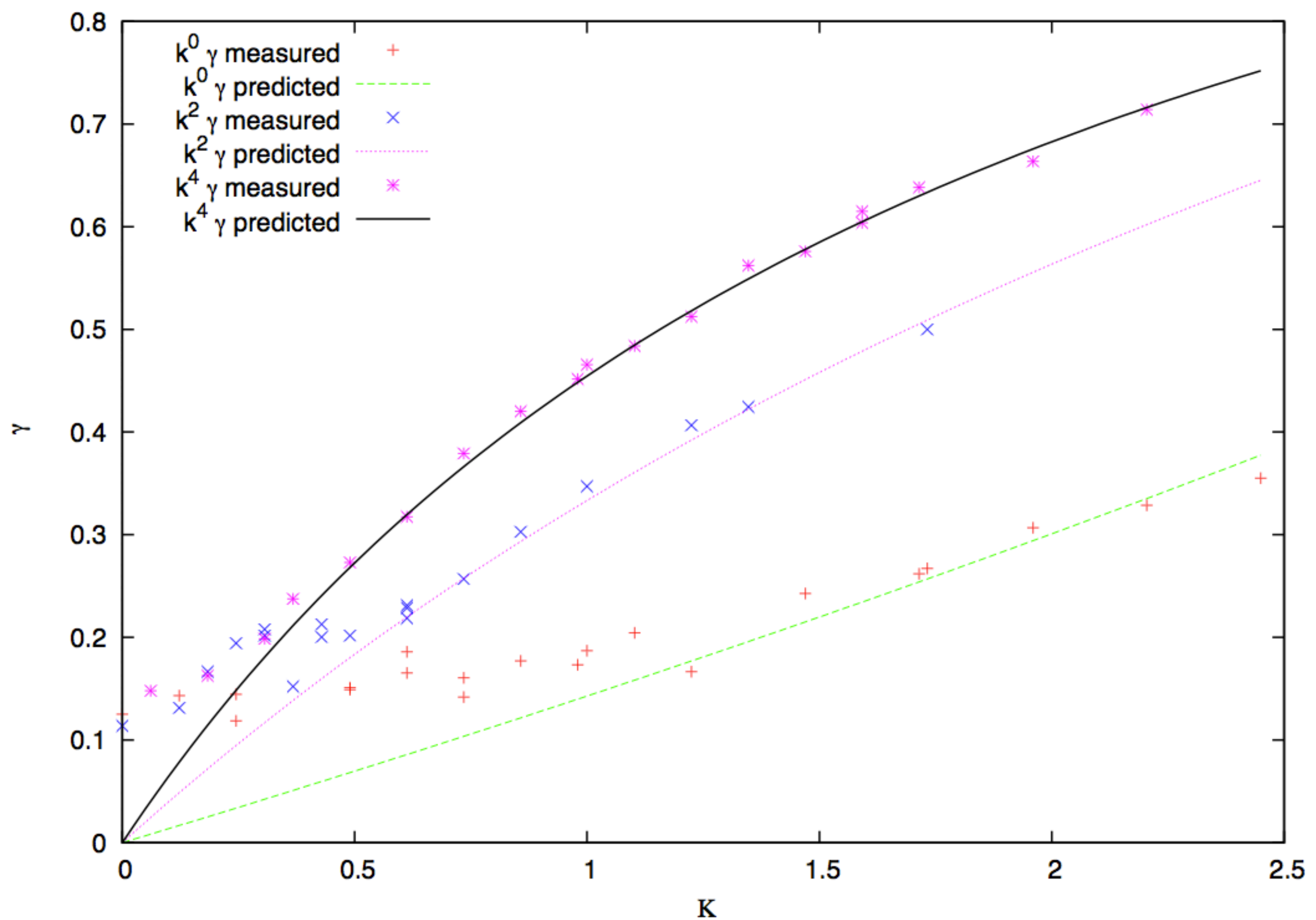
Using "self-similarity" to infer behaviour of upper cut-off, infer

$$\gamma_{\rm sc}\left(n,\kappa\right) = \frac{2\kappa\left(n+1\right)}{\kappa\left(2n-1\right)+3\sqrt{\kappa^2+24}}$$

where $\Gamma = \kappa\sqrt{2\pi G\rho_0/3}$

Exponents in 1D models: measurement from simulations

D. Benhaiem and MJ (2012, in preparation)



Exponents in 1D models: from stable clustering to universality

D. Benhaiem and MJ (2012, in preparation)

Excellent agreement with stable clustering when $\gamma_{sc}(n,\kappa) \gtrsim 0.2$

Otherwise exponent which is ~ independent of both expansion and IC
→ "universal" non-linear clustering

Why a critical value for validity of stable clustering?
Can show that

$$\left(\frac{L_2}{L_1}\right) = \left(\frac{L_2^0}{L_1^0}\right)^{-\frac{\gamma_{sc}}{1-\gamma_{sc}}} \left(\frac{L_2^0}{L_1^0}\right)$$

where $\left(\frac{L_2}{L_1}\right)$ is ratio of size of two structures when the larger one virializes,

while $\left(\frac{L_2^0}{L_1^0}\right)$ is the ratio of their initial sizes

Pour conclure…

Open questions about the "non-linear regime"

- **How is non-linear clustering properly characterized ?**
- **How does it depend on initial conditions and cosmology?**

1D suggests the space of cold IC and cosmologies breaks into two regions:

- fractal "virialized hierarchy", non-universal
- fractal "virialized hierarchy" (**or smooth, not so clear..**), universal

Second is possibly compatible with current model of "real cosmology"

Stable clustering in 3D revisited

(Work in progress with D. Benhaiem and B. Marcos)

(Theoretical and numerical) study of "Gamma cosmology" in 3D..

Generalisation of stable clustering prediction of Peebles:

$$\gamma_{\rm sc}(n,\kappa) = \frac{6(3+n)}{5+\sqrt{1+\frac{24}{\kappa^2}+2n}}$$

$$\Gamma = \kappa\sqrt{2\pi G\rho_0/3}$$

Preliminary results seem qualitatively in line with 1D
But *much attention needed to resolution issues*..

Power-law scaling in galaxy clustering

Observations:

Power law behaviours characterize galaxy correlations **in some range**

Is such power law clustering in galaxies indicative of scale-invariant phenomena?

If yes, is the purely gravitational dynamics giving rise to it?

Current standard model answer**: no, power-laws are an accident**

Standard model: power law correlations are an accident…..

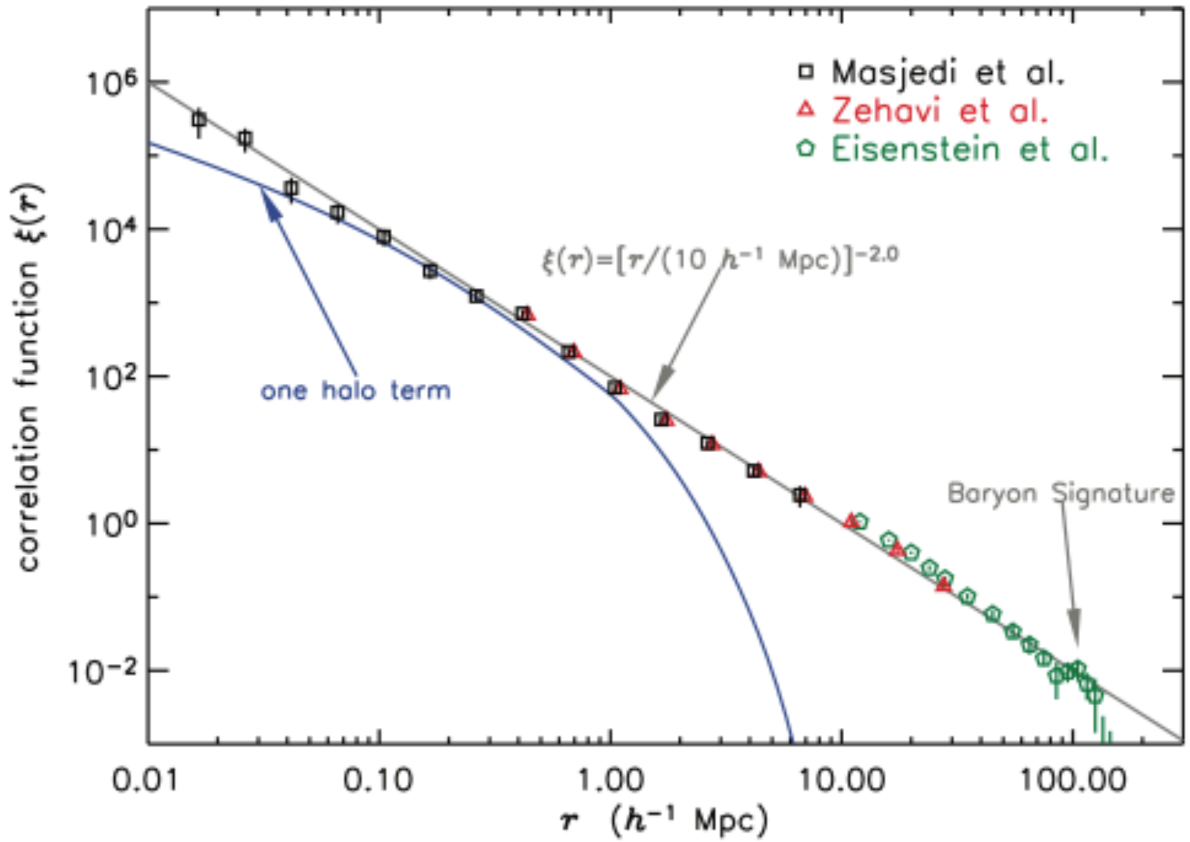


FIG. 4.—Real-space correlation function $\xi(r)$ for the LRG sample ($-23.2 < M_g < -21.2$ and $0.16 < z < 0.36$) calculated as described in the text on small scales, combined with real-space correlation function on intermediate scales from Zehavi et al. (2005a) and redshift-space correlation function $\xi(s)$ on large scales from Eisenstein et al. (2005; data points from Zehavi results are shifted by

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D. Benhaiem (LPNHE, Paris)

Leftovers…

And the "truly" infinite system limit?

A. Gabrielli, MJ, B. Marcos and F. Sicard, JSP(2011)

Describe configurations as **uniform** stochastic point process (SPP) (cf. Chandrasekhar)

Definiteness of force → existence of PDF of the force:

Depends **only** on the power spectrum of density fluctuations

Conclusion (for gravity, regulated)

PDF of force is defined for **sufficiently uniform SPP**
PDF of **force differences** is defined for **ALL uniform SPP**

But latter is the relevant physical criterion: only relative motion matters!
Note: this infinite system limit defined only for pair potentials with n geq d-2

From infinite back to finite

Finite subsystems behave as isolated finite systems **in physical coordinates**, in limit that

- their **density is large** (compared to mean density)

- they are **“far” from other matter** (compared to their own characteristic size)

Thus e.g. the solar system “shrinks” in comoving coordinates
(“non-linear systems do not feel the expansion of the universe”)

Theory: two key elements

Linear theory: linearisation of fluid equations gives

a *scale-independent amplification of density fluctuations*

$$\tilde{\delta\rho}(\vec{k},t) = A(t)\,\tilde{\delta\rho}(\vec{k},t=0)$$

Spherical collapse model: a spherical top-hat overdensity collapses to a singularity *in a finite time depending only on initial amplitude*

With simple assumptions gives predictions about the non-linear regime (e.g. number of virialized objects of given mass).