Stochastically forced long-range interacting systems

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Statistical Mechanics of self-gravitating particles

Fondation des Treilles, 25 October 2012

joint work with F. Bouchet, S. Gupta, T. Dauxois and S. Ruffo

CN, S. Gupta, S. Ruffo, T. Dauxois and F. Bouchet, Letter to JSTAT, L01002 (2012)

CN, S. Gupta, S. Ruffo, T. Dauxois and F. Bouchet, arXiv:1210.0492, submitted to JSTAT

d: spatial dimensions of the systems

$$r \gg r_{typ} \qquad v(r) \sim \frac{1}{r^{\alpha}} \qquad \alpha \leq d$$

- Gravitational system, one-component plasma

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2N} \sum_{i,j=1}^{N} v(q_i - q_j)$$

- 2D, Quasi-2D turbulence and geophysical flows
  - ex.: 2-d Euler:

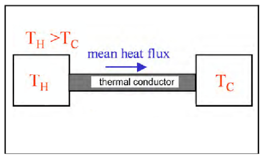
$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = 0 \qquad \text{with} \qquad \omega = \nabla \wedge \mathbf{v} \;, \; \omega = \triangle \psi$$

$$\mathcal{E} = -\int d\mathbf{r} d\mathbf{r}' \, \omega(\mathbf{r}) \, G(r, r') \omega(\mathbf{r}') \,, \qquad G \sim \log |\mathbf{r} - \mathbf{r}'|$$

  - Quasi-2D models: Shallow-water equation, Quasi-geostrophic equation, ...
- ...

Non-equilibrium statistical mechanics

Short-range systems



Non-equilibrium stationary states sustained by flux of conserved quantities,
broken detailed balance, ...
no analogous to Boltzmann-Gibbs theory

Long-range systems

forcing can act coherently on all the degrees of freedom:

- imposed electric fields on a plasma ?
- gravitational fields created by other galaxies ?
- ... ?

Fluid models

- 2D and Quasi-2D turbulence models
- Large scale structures in constrast with 3D turubulence

Geophysical systems

- Energy injection
  - Wind on oceans
  - Different layers in atmosphere
  - ...
- Dissipation on large scales
  - effect of boundaries

(Weak?) FLUX OF ENERGY from SMALL to LARGE SCALES

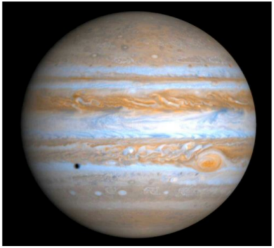
OUT OF EQUILIBRIUM PHENOMENA!

Stochastically forced quasi-Geostrophic equations

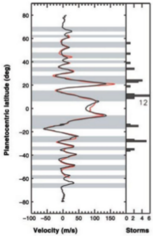
$$\partial_t q + \mathbf{v} \cdot \nabla q = -\alpha q + F(\mathbf{r}, t)$$

$$\langle F(\mathbf{r}, t) F(\mathbf{r}', t') \rangle = C(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

q: quasi-geostrophic potential vorticity



Jupiter atmosphere



Jupiter Zonal wind (Voyager and Cassini, from Porco et al 2003)

IN THIS TALK: stochastically forced Long-range PARTICLE systems

Work in progress (with F. Bouchet & T. Tangarife)

similar theoretical tecniques for stochastically forced 2d fluids

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = -\alpha \omega + F(\mathbf{r}, t)$$

Similarity between 2d Euler and Vlasov equation

- non-linear transport equations
- infinite number of conserved quantities (Casimirs)
- Hamiltonian structure

Outline

1. Isolated long-range particle systems

2. Stochastically forced particle systems: the model
3. Kinetic theory
4. Comparison between kinetic theory and numerical simulations
5. Bistability

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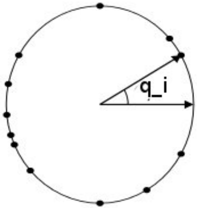
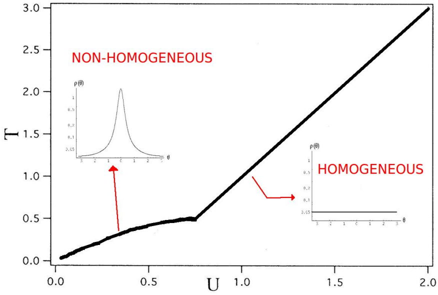
HMF model and its equilibrium behavior

Hamiltonian Mean Field model

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2} - \frac{1}{2N} \sum_{i,j} cos(q_i - q_j) =$$

$$= \sum_{i=1}^{N} \frac{p_i^2}{2} - \frac{N}{2} |m|^2$$





magnetization

$$m(t) = \sqrt{m_x^2 + m_y^2}$$

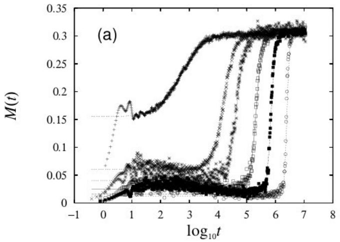
$$m_x(t) = \frac{1}{N} \sum_{i=1}^{N} \cos q_i$$

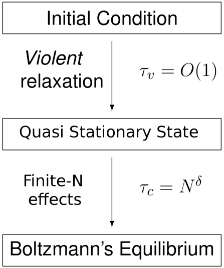
$$m_y(t) = \frac{1}{N} \sum_{i=1}^{N} \sin q_i$$

Computational cost of Molecular Dynamics $\sim N$

Antoni and Ruffo, PRE 52, 2361-2374 (1995)

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2N} \sum_{i,j=1}^{N} v(q_i - q_j) \qquad v(r) \sim \frac{1}{r^\alpha} \qquad \alpha \leq d$$





Relaxation to equilibrium

- Quasi-stationary states
  - Non-ergodicity
  - lifetime $\sim N$ (diverging with the system size!)

Y. Yamaguchi, J. Barré, F. Bouchet, T. Dauxois, and S. Ruffo, Physica A (2004) small parameter: $1/N$ $f(q,p,t)$: density in (q,p) at time t

Vlasov equation ($t << \tau_c \sim N^\delta$, $\delta > 0$)

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial q} - \frac{\partial f}{\partial p}\frac{\partial \Phi}{\partial q} = 0 \qquad \Phi(q) = \int dq'\, v(q-q')f(q')$$

- Mean field approximation
  - exact for $N \to \infty$
- Quasi-Stationary States: stable equilibria

Infinite number of QSS

Lenard-Balescu equation ... ($t \sim \tau_c$)

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial q} - \frac{\partial f}{\partial p}\frac{\partial \Phi}{\partial q} = \frac{1}{N}C[f]$$

Lowest order description of finite-N effects

- weak correlations cause slow evolution
  - analogous to Boltzmann equation

slow relaxation through Quasi-Stationary-States

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Isolated long-range systems: kinetic theory

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STOCHASTIC EQUATIONS OF MOTION

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i} - \alpha p_i + \sqrt{\alpha} F(q_i, t)$$

- F Gaussian stochastic process with $\langle F(q,t) \rangle = 0$

$$\langle dF(q,t) dF(q',t') \rangle = B(|q - q'|) \delta(t - t') dt$$

α: forcing and dissipation parameter

Coherent stochastic forces

- Coherent stochastic term: NOT $F_i(q_i, t)$
  - external stochastic field

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"Fourier expansion" of $F(q,t)$

$$g_k = \frac{1}{L}\int \mathrm{d}q\; B(q)e^{-ikq} > 0, \qquad F(q,t) = \sum_k g_k\, e^{ikq} W_k(t)$$

W_k: independent Weiner processes

g_k: forcing at the spatial scale $1/k$

Kinetic energy in a steady state: KINETIC TEMPERATURE

$$T = 2\langle K_{ss}\rangle = \frac{1}{2}\sum_k g_k^2$$

Detailed balance $\leftrightarrow$ $g_k = g \quad \forall k$

Can be far from detailed balance also for $\alpha \ll 1$

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- $N \gg 1$: number of degrees of freedoms
  - Plasma, self-gravitating systems
- $\alpha \ll 1$: weak forcing limit
  - Technical reason: small parameter

Timescales

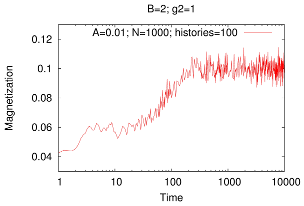
- Collective timescale: $\tau_c \sim N$
- Stochasticity: $\tau_s = 1/\alpha$

Fluctuations of intensive observables

- Finite-size effects: $\sim 1/\sqrt{N}$
- Stochasticity: $\sqrt{\alpha}$

Limits

- Continuous limit: $N\alpha \gg 1$
  - Negligible finite size effects: similar to 2D fluid models!
- $N\alpha \sim 1$ or $\ll 1$: simple generalization



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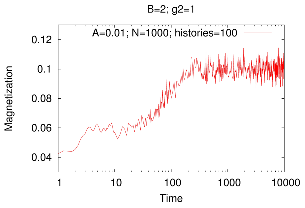

Perturbation theory in the SMALL PARAMETER: $\alpha \gg 1/N$

FOKKER-PLANCK for the N-particles distribution function f_N

$$\frac{\partial f_N}{\partial t} + \text{Liouville terms} = -\sum_{i=1}^{N} \frac{\partial(\alpha p_i f_N)}{\partial p_i} - \frac{\alpha}{2}\sum_{i,j=1}^{N} C(q_i - q_j)\frac{\partial^2 f_N}{\partial p_i \partial p_j}$$

$$\langle F(q,t)F(q',t')\rangle = C(|q-q'|)\delta(t-t')$$

Exact: too much information for a macroscopic description

Distribution functions

- $f_s = \int f_N \, d[s+1]...d[N]$
- $f_1 = f$: density in μ−space
- f_2: 2-particles correlations
- ....

$\Longrightarrow$

BBGKY hierarchy

$$\partial_t f_s = L[f_s, f_{s+1}]$$

How to close the BBGKY hierarchy?

Kinetic theory: a qualitative idea

First equation of the BBGKY

$$\frac{\partial f}{\partial t} + \text{Vlasov} - \frac{\partial}{\partial p_1}[\alpha p_1 f] - \alpha T \frac{\partial^2 f}{\partial p_1^2} = C[f_2]$$

- blue: order 1
- red: order α

non neglegible two-particle correlations

Analogy with finite size effects in Hamiltonian systems

CORRELATIONS	finite-N	stochastic forces

METHOD

- Minimal project: discard 3-particles correlations
- Solve the II eq. of BBGKY
- Plug the result in the I eq. of BBGKY

KIN. EQ.	Lenard-Balescu	"Our" kin. eq.

Hypothesis

f stationary stable solution of Vlasov equation at every time

- $-->$ Time-scale separation $\sim$ f evolves slowly w.r.t. g

homogeneous system: $f(p,q,t) = f(p,t)$

- $-->$ explicit form of the kinetic equation

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Kinetic theory. First step: closure of the BBGKY

connected components of correlations

$$f_2(1,2) = f(1)f(2) + \alpha g(1,2)$$
$$f_3(1,2,3) = f(1)f(2)f(3) + f(1)g(2,3) + \ldots + \ldots + h(1,2,3)$$
$$\ldots$$

it is SELF-CONSISTENT to suppose:

- $f \sim O(1)$
- $\alpha g \sim O(\alpha)$
- $h \sim \ll \alpha g$
- ...

Lowest order possible scheme if we want to describe the effect of the forcing

Discard three-particle and higher order correlations while taking into account two-particle correlations

- Analogous to derivation of Boltzmann eq. or Lenard-Balescu eq.

Kinetic theory. Second step: Time-scale separation

I equation BBGKY

$$\frac{1}{\alpha}\frac{\partial f}{\partial t} + \frac{1}{\alpha}\mathrm{Vlasov} - \frac{\partial}{\partial p_1}[p_1 f] - T\frac{\partial^2 f}{\partial p_1^2} = \frac{\partial}{\partial p_1}\int d[2]\, v'(q_1 - q_2) g(1,2,t)$$

II equation BBGKY

$$\frac{\partial g}{\partial t} + L_f^{(1)} g + L_f^{(2)} g = C(|q_1 - q_2|)\frac{\partial}{\partial p_1}\frac{\partial}{\partial p_2} f(1,t) f(2,t)$$

$L_f g$: linearized Vlasov operator around f acting on h

Time-scale separation

If f is a STATIONARY AND STABLE solution of the Vlasov equation

- f evolves on a timescale of order $1/\alpha$
- g evolves on a timescale of order 1

Bogoliugov “hypothesis”

We solve II supposing f fixed in time and we insert the STATIONARY solution in the r.h.s. of I

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Kinetic theory. Third step: solution of the II eq. BBGKY

I equation BBGKY

$$\frac{1}{\alpha}\frac{\partial f}{\partial t} - \frac{\partial}{\partial p_1}[p_1 f] - T\frac{\partial^2 f}{\partial p_1^2} = \lim_{t\to\infty}\frac{\partial}{\partial p_1}\int d[2]\, v'(q_1-q_2)g(1,2,t)$$

We have reduced the problem to find the stationary solution of II equation BBGKY

$$\frac{\partial g}{\partial t} + L_f^{(1)} g + L_f^{(2)} g = C(|q_1-q_2|)\frac{\partial f(1)}{\partial p_1}\frac{f(2)}{\partial p_2}$$

$L_f g$: linearized Vlasov operator around f acting on g

Very similar problem to solve the linear Vlasov equation

$$\frac{\partial h}{\partial t} + L_f h = 0$$

Easily doable when $f(q,p,t) = f(p,t)$

Remark: why we think that this is generalizable to fluids

$$\partial\omega + \mathbf{v}\cdot\nabla\omega = -\alpha\omega + \sqrt{\alpha}F$$

L_f $--->$ linear Euler operator

Kinetic theory: resume

Analogy with finite size effects in Hamiltonian systems

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KINETIC EQUATION: Non linear Fokker-Planck equation

$$\frac{1}{\alpha}\frac{\partial f(p_1,t)}{\partial t} - \frac{\partial}{\partial p_1}[p_1 f(p_1,t)] - \frac{\partial}{\partial p_1}\left[D[f](p_1)\frac{\partial f(p_1)}{\partial p_1}\right] = 0$$

α: time-rescaling

Diffusion coefficient $D[f](p_1)$

$$D[f](p) = T + 2\pi \sum_{k=1} v_k g_k \int^* \mathrm{d}p_1 \left[\frac{1}{|\epsilon(k,kp)|^2} + \frac{1}{|\epsilon(k,kp_1)|^2}\right] \frac{1}{p_1 - p} \left.\frac{\partial f}{\partial p}\right|_{p_1}.$$

$$T = \frac{1}{2}\sum_k g_k^2$$

$$\epsilon(k,\omega) = 1 - 2\pi i N k \varphi(k) \int_{-\infty}^{\infty} dp \, \frac{f'(p)}{-i\omega + ikp}$$

$\int^* dp$: Chauchy integral

v_k: Fourier components of the potential $v(q)$

Comparing the results: kinetic energy and $\langle p^4 \rangle$

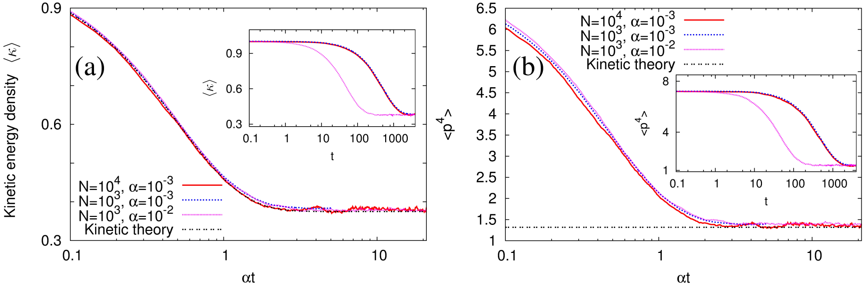
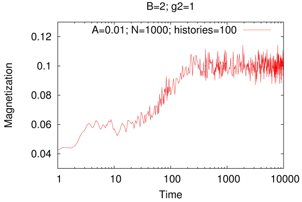
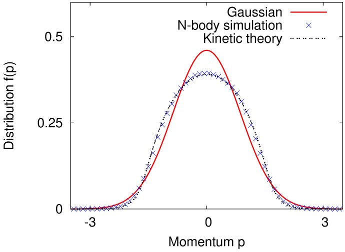


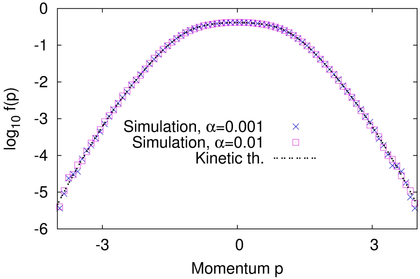
Figure: (a) Kinetic energy density $\langle \kappa \rangle$ and (b) $\langle p^4 \rangle$ as a function of αt, for the values $B_0 = 1.5$ and $g_1 = 0.75$. The data for different N and α values are obtained from numerical simulations of the stochastically forced HMF model, and involve averaging over 50 histories for $N = 10^4$ and 10^3 histories for $N = 10^3$. The data collapse implies that α is the timescale of relaxation to the stationary state. The inset shows the data without time rescaling by α.



NON-EQUILIBRIUM STATIONARY VELOCITY DISTRIBUTION

α-independent shape



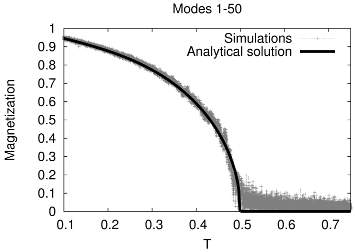


- $N = 10^4$
- kinetic temperature $T = 0.75$
- forced modes: $k = 1, 2$

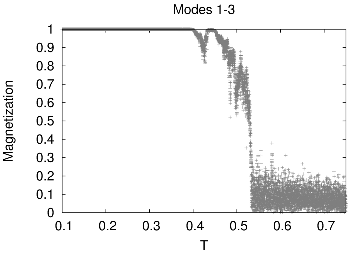
What happens close to the phase transition?

Adiabatic change of T

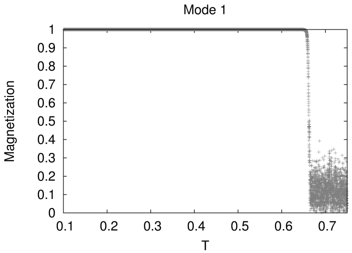
Close to detailed balance



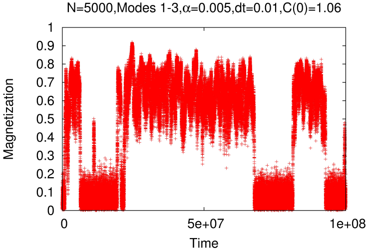
Far from detailed balance

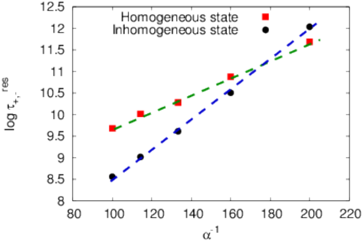


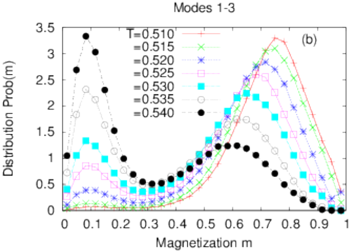
Even further from detailed balance



Bistability



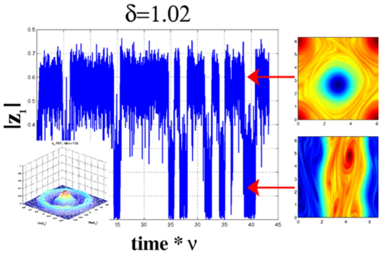




- lifetime $\sim e^{\lambda/\alpha}$
- Bistable behavior described by a Poisson process

2D Stochastic Euler

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = -\alpha \omega + \sqrt{2\alpha} F(\mathbf{r}, t)$$



Bouchet and Simonnet, PRL, 094504 (2009)

Analogous behavior to

- Reversal of earth magnetic field
- Path of ocean currents (ex: Kuroshio current)
- Experiments on rotating fluids

Stochastically perturbed particles interacting with a long-range potential

- Kinetic theory in the weak forcing limit

  Prediction of NON-EQUILIBRIUM homogeneous states

- Numerical observation of bistability

Ongoing works in kinetic theory

- 2d-turbulence: stochastic Euler equation

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = -\alpha \omega + \sqrt{2\alpha} F(q,t)$$

  - with F. Bouchet and T. Tangarife

(Long term) perspectives

- kinetic theory for geophysical models?
  - 2D stochastic Navier-Stokes, Shallow-water, Quasi-geostrophic equations ...
- Theoretical understanding of the bistability

Thank you!