MASS AGGREGATION IN A SELF-GRAVITATING ONE-DIMENSIONAL GAS

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Fondation des Treilles

”Statistical mechanics of self-gravitating particles”

22-27 October 2012

Motto:

”Tis much better to do a little with certainty

& leave the rest for others that come after,

than to explain all things by conjecture

without making sure of any thing.”

Isaak Newton

STICKY DUST

" ... The evolution of cold sticky matter ... is at least qualitatively like the evolution of self-gravitating matter in an expanding universe. The density distribution of matter becomes less homogeneous in the course of time. ..."

" ... in a gravitating medium even without collisions there is effective sticking after the formation of multistream flows."

S.F. Shandarin, Ya. B. Zeldovich, *Large scale structure of the universe: Turbulence, intermittency, structures in a self-gravitating medium*, Rev. Mod. Phys. **61**, 185 (1989).

SELF-GRAVITATING STICKY GAS IN ONE DIMENSION

Potential energy of a pair of particles

$$\gamma m_i m_j |x_i - x_j|$$

At a binary collision the particles merge instantaneously forming a single point mass

$$(m_i + m_j)$$

with momentum

$$(p_i + p_j)$$

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INITIAL STATE

N identical masses m starting from points $(a, 2a, 3a, ..., Na)$ with uncorrelated velocities distributed according to some probability density $\phi(v)$.
Limit of a continuous mass distribution:

$$N \to \infty, \qquad m \to 0, \qquad a \to 0$$

$$M_{tot} = Nm = const, \qquad \rho = \frac{m}{a} = const$$

At the initial moment

$$\rho(x;0) = \theta(x)\theta\left(\frac{M_{tot}}{\rho} - x\right)\rho$$

CHARACTERISTIC TIME OF GRAVITATIONAL INTERACTION

Static initial distribution: $\phi(v) = \delta(v)$.
In the course of time distances between neighbouring particles shrink to $(a - m\gamma t^2)$. Then, all N particles merge simultaneously at the moment

$$t = t^* = \sqrt{\frac{a}{\gamma m}} = \frac{1}{\sqrt{\gamma \rho}}$$

In the continuum limit, for times $t < t^*$

$$\rho(x;t) = \theta(x - \gamma M_{tot}\frac{t^2}{2})\theta\left(\frac{M_{tot}}{\rho} - x - \gamma M_{tot}\frac{t^2}{2}\right)\frac{\rho}{1 - \rho\gamma t^2}$$

$$\lim_{t \nearrow t^*} \rho(x;t) = M_{tot}\delta\left(x - \frac{M_{tot}}{2\rho}\right)$$

PROBABILITY OF COMPLETE AGGREGATION

Initial configuration

$$x_1 < x_2 < .... < x_N$$

$X^r(t)$ = center of mass trajectory of the cluster containing particles $1, 2, ..., r$
$X^{N-r}(t)$ = center of mass trajectory of the adjacent cluster containing particles $r+1, r+2, ..., N$

$$X^r(0) < X^{N-r}(0)$$

Probability of complete aggregation before time t

$$P_N(t) = \left\langle \prod_{r=1}^{N-1} \theta[X^r(t) - X^{N-r}(t)] \right\rangle$$

GAUSSIAN INITIAL VELOCITY DISTRIBUTION

$$\phi_\lambda(v) = \frac{1}{\sqrt{2\pi}\lambda} \exp\left(-\frac{v^2}{2\lambda^2}\right)$$

Interpretation in terms of the Brownian motion of the formula

$$P_N(\tau) = \sqrt{2\pi N} \int du_1 ... \int du_{N-1} \phi(u_1)\phi(u_2 - u_1)$$

$$...\phi(u_{N-1} - u_{N-2})\phi(-u_{N-1}) \prod_{r=1}^{N-1} \theta \left\{u_r + \frac{m\tau}{2\lambda} r(N-r)\right\}$$

where

$$\tau = \gamma t - \frac{1}{\rho t}$$

COMPLETE AGGREGATION IN THE CONTINUUM LIMIT

At $\tau = 0$ (or at $t = t^*$), one finds a remarkably simple result

$$P_N(0) = \frac{1}{N}$$

Continuum limit

$$P(\tau) = \theta(\tau)\exp[-A(\tau)]$$

$$A(\tau) = 2\sum_{n=1}^{\infty}\int_{(M_{tot}/2\lambda)\tau}^{\infty} dy\, \phi(\sqrt{n}\, y)$$

MASS DENSITY AT $t = t^*$

$$\rho(X; t^*) = M_{tot}\,\delta\left(X - \frac{M_{tot}}{2\rho}\right)$$

Configurations after t^* consist of a single macroscopic mass surrounded by a dust of non-extensive fragments composed of a finite number of initial particles.
Probability $\mu_k(t)$ to have exactly $(k-1)$ fragments at $t > t^*$ is given by the Poisson law

$$\mu_k(t) = \frac{[A(\tau)]^{k-1}}{(k-1)!}\exp[-A(\tau)]$$

$A(\tau) =$ mean number of fragments.

BEFORE THE APPEARANCE OF A MACROSCOPIC MASS

$$0 < t < t^*$$

Most of the kinetic energy is rapidly dissipated by inelastic collisions. The subsequent evolution is dominated by gravitational forces.
Conjectures to be proved:

- mass density converges to the uniform density as in the static model
- typical configurations consist of $\sim \sqrt{N}$ aggregates with masses of the order $m\sqrt{N}$

Bibliography

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