

#### MASS AGGREGATION IN A SELF-GRAVITATING ONE-DIMENSIONAL GAS

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Motto:

"Tis much better to do a little with certainty & leave the rest for others that come after, than to explain all things by conjecture without making sure of any thing."

#### Isaak Newton



"... The evolution of cold sticky matter ... is at least qualitatively like the evolution of self-gravitating matter in an expanding universe. The density distribution of matter becomes less homogeneous in the course of time. ..."

"... in a gravitating medium even without collisions there is effective sticking after the formation of multistream flows."

S.F. Shandarin, Ya. B. Zeldovich, *Large scale structure of the universe: Turbulence, intermittency, structures in a self-gravitating medium*, Rev. Mod. Phys. **61**, 185 (1989).

## SELF-GRAVITATING STICKY GAS IN ONE DIMENSION

Potential energy of a pair of particles

$$\gamma m_i m_j |x_i - x_j|$$

At a binary collision the particles merge instantaneously forming a single point mass

$$(m_i + m_j)$$

with momentum

 $(p_i + p_j)$ 

#### **INITIAL STATE**



N identical masses m starting from points (a, 2a, 3a, ..., Na) with uncorrelated velocities distributed according to some probability density  $\phi(v)$ .

Limit of a continuous mass distribution:

$$N \to \infty, \quad m \to 0, \quad a \to 0$$
  
 $M_{tot} = Nm = const, \quad \rho = \frac{m}{a} = const$ 

At the initial moment

$$\rho(x;0) = \theta(x)\theta\left(\frac{M_{tot}}{\rho} - x\right)\rho$$

## CHARACTERISTIC TIME OF GRAVITATIONAL INTERACTION

Static initial distribution:  $\phi(v) = \delta(v)$ . In the course of time distances between neighbouring particles shrink to  $(a - m\gamma t^2)$ . Then, all *N* particles merge simultaneously at the moment

$$t = t^* = \sqrt{\frac{a}{\gamma m}} = \frac{1}{\sqrt{\gamma \rho}}$$

In the continuum limit, for times  $t < t^*$ 

$$\rho(x;t) = \theta(x - \gamma M_{tot} \frac{t^2}{2}) \theta\left(\frac{M_{tot}}{\rho} - x - \gamma M_{tot} \frac{t^2}{2}\right) \frac{\rho}{1 - \rho \gamma t^2}$$

$$\lim_{t \nearrow t^*} \rho(x;t) = M_{tot} \delta\left(x - \frac{M_{tot}}{2\rho}\right)$$
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## PROBABILITY OF COMPLETE AGGREGATION



Initial configuration

 $x_1 < x_2 < \dots < x_N$ 

 $X^{r}(t)$  = center of mass trajectory of the cluster containing particles 1, 2, ..., r $X^{N-r}(t)$  = center of mass trajectory of the adjacent cluster containing particles r + 1, r + 2, ..., N

$$X^{r}(0) < X^{N-r}(0)$$

Probability of complete aggregation before time t

$$P_N(t) = \left\langle \prod_{r=1}^{N-1} \theta[X^r(t) - X^{N-r}(t)] \right\rangle$$

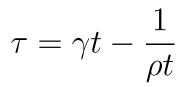
## GAUSSIAN INITIAL VELOCITY DISTRIBUTION

$$\phi_{\lambda}(v) = \frac{1}{\sqrt{2\pi\lambda}} \exp\left(-\frac{v^2}{2\lambda^2}\right)$$

$$P_N(\tau) = \sqrt{2\pi N} \int du_1 \dots \int du_{N-1} \phi(u_1) \phi(u_2 - u_1)$$

...
$$\phi(u_{N-1} - u_{N-2})\phi(-u_{N-1})\prod_{r=1}^{N-1} \theta\left\{u_r + \frac{m\tau}{2\lambda}r(N-r)\right\}$$

where



## COMPLETE AGGREGATION IN THE CONTINUUM LIMIT

At  $\tau = 0$  (or at  $t = t^*$ ), one finds a remarkably simple result

$$P_N(0) = \frac{1}{N}$$

Continuum limit

$$P(\tau) = \theta(\tau) \exp[-A(\tau)]$$

$$A(\tau) = 2 \sum_{n=1} \int_{(M_{tot}/2\lambda)\tau} dy \,\phi(\sqrt{n}\,y)$$

$$MASS DENSITY AT t = t^*$$

$$\rho(X; t^*) = M_{tot} \,\delta\left(X - \frac{M_{tot}}{2\rho}\right)$$

Configurations after  $t^*$  consist of a single macroscopic mass surrounded by a dust of non-extensive fragments composed of a finite number of initial particles. Probability  $\mu_k(t)$  to have exactly (k-1) fragments at  $t > t^*$ is given by the Poisson law

$$\mu_k(t) = \frac{[A(\tau)]^{k-1}}{(k-1)!} \exp[-A(\tau)]$$

 $A(\tau)$  = mean number of fragments.

# BEFORE THE APPEARANCE OF A MACROSCOPIC MASS



 $0 < t < t^*$ 

Most of the kinetic energy is rapidly dissipated by inelastic collisions. The subsequent evolution is dominated by gravitational forces. Conjectures to be proved:

- 6 mass density converges to the uniform density as in the static model
- 5 typical configurations consist of  $\sim \sqrt{N}$  aggregates with masses of the order  $m\sqrt{N}$

#### **Bibliography**



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