Research Activities (2010-14)

Thierry Dauxois

Long-Range Interacting Systems

For systems with long-range interactions, the two-body potential decays at large distances as $V(r) \sim 1/r^\alpha$, with $\alpha \leq d$, where d is the space dimension. Examples are : gravitational systems, two-dimensional hydrodynamics, two-dimensional elasticity, charged and dipolar systems. Although such systems can be made extensive, they are intrinsically non additive : the sum of the energies of macroscopic subsystems is not equal to the energy of the whole system. Moreover, the space of accessible macroscopic thermodynamic parameters might be non convex. The violation of these two basic properties of the thermodynamics of short-range systems is at the origin of ensemble inequivalence.

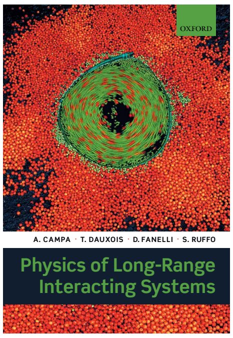

Figure 1: We have just finished writing a comprehensive book [B.10], on the recent advances on the statistical mechanics and out-of-equilibrium dynamics of solvable systems with long-range interactions. The abstract of the book is the following :

"Physics of Long-range Interacting Systems" deals with an important class of many-body systems : those where the interaction potential decays slowly for large inter-particle distance. In particular, systems where the decay is slower than the inverse inter-particle distance raised to the dimension of the embedding space. Gravitational and Coulomb interactions are the most prominent examples. However, it has become clear that long-range interactions are more common than previously thought. This has stimulated a growing interest in the study of long-range interacting systems, which has led to a much better understanding of the many peculiarities in their behaviour. The seed of all particular features of these systems, both at equilibrium and out-of-equilibrium, is the lack of additivity. It is now well understood that this does not prevent a statistical mechanics treatment. However, it does require a more in-depth study of the thermodynamic limit and of all related theoretical concepts. A satisfactory understanding of properties generally considered as oddities only a couple of decades ago has now been reached : ensemble inequivalence, negative specific heat, negative susceptibility, ergodicity breaking, out-of-equilibrium quasi-stationary-states, anomalous diffusion, etc. The book, intended for Master and PhD students, tries to gradually acquaint the reader with the subject. The first two parts describe the theoretical and computational instruments needed for addressing the study of both equilibrium and dynamical properties of systems subject to long-range forces. The third part of the book is devoted to discuss the applications of such techniques to the most relevant examples of long-range systems. The only prerequisite is a basic course in statistical mechanics.

In parallel to the redaction of the book, I have of course pursued the research on these systems with long-range interactions. The main directions were there folds : i) analyzing systems beyond the mean-field approximation, ii) considering physical systems (not only toy models) and iii) studying the effect of noise.

Generalizations to models [A.69] with both short and long-range interactions, and to models with weakly decaying interactions [A.70,A.72], show the robustness of the effects obtained for mean-field models.

We have also studied [A.81] needle-shaped three-dimensional classical spin systems with purely dipolar interactions in the microcanonical ensemble. We have observed and analytically explained spontaneous magnetization for different finite cubic lattices and first order transition from paramagnetic to ferromagnetic phases. These effects hint at performing experiments on isolated dipolar needles in order to verify some of the exotic properties of systems with long-range interactions in the microcanonical ensemble.

We have also studied [A.86] a system of hard spheres with gravitational interactions in a stationary state described by the microcanonical ensemble. Introducing a set of similar auxiliary systems with increasing sizes and numbers of particles, the masses and radii of the hard spheres of the auxiliary systems were rescaled in such a way that usual extensive properties are maintained despite the long-range nature of gravitational interactions, while the mass density and packing fractions are kept fixed. Within that scaling limit, we show that a local thermalization spontaneously emerges as a consequence of both extensive properties and relative smallness of fluctuations. The resulting mass density profile for the infinite system can be determined within a hydrostatic approach, where the gradient of the local hard-sphere pressure is balanced by the average gravitational field. The derivation sheds light onto the mechanisms which ensure that the local equilibrium in the in finite system is entirely controlled by hard-core interactions, while gravitational interactions can be treated at the mean-field level. This allows us to determine the conditions under which the hydrostatic approach is also valid for the genuine finite system of interest. We provide simple tests of such conditions for a few astrophysical examples.

Long-range interacting systems display an extremely slow relaxation towards thermodynamic equilibrium and, what is more striking, the convergence towards quasi-stationary states. The study of the effect of noise on this kind of systems is very important but is only at its infancy. We have studied long-range interacting systems driven by external stochastic forces [A.74,A.78] that act collectively on all the particles constituting the system, showing that the system reaches a stationary state where external forces balance dissipation on average. These states have an invariant probability that does not respect detailed balance, and are characterized by non-vanishing currents of conserved quantities. We introduced [A.82] also a model of long-range interacting particles evolving under a stochastic Monte Carlo dynamics, in which possible increase or decrease in the values of the dynamical variables is accepted with preassigned probabilities. For symmetric increments, the system at long times settles to the Gibbs equilibrium state, while for asymmetric updates, the steady state is out-of-equilibrium.

Internal Gravity Waves

The study of internal waves (IW) is of great interest owing to the evolving appreciation of their role in many geophysical systems. In addition to their particularly intriguing properties from a fundamental point of view, these waves play an important role in dissipating barotropic tidal energy in the ocean, are an important means of momentum transport in the atmosphere, while IW activity also impacts modern-day technology. However, many unanswered questions remain, particularly regarding the fate of internal waves and how much mixing do they generate in the ocean and via what processes ?

To tackle some of these questions, we have developed a *new wave generation* in stratified fluids. This innovative mechanism [A.67], which involves a tunable source composed of oscillating plates, generates well defined propagating plane wave beams, as shown in the left panel of Fig. 2. This generator has been used (and also copied in several laboratories throughout the world) for several studies described below.

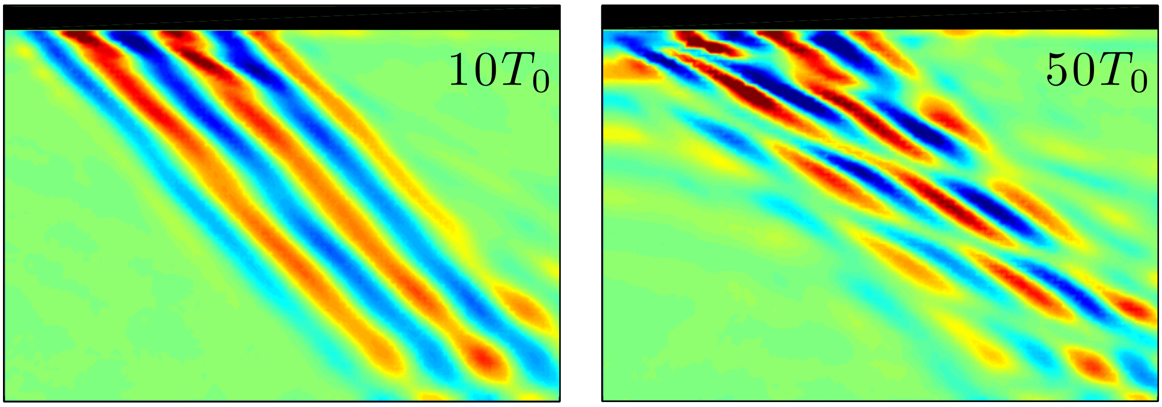

Figure 2. Snapshots of the experimental vertical density gradient field for $10T_0$ (a) and $50T_0$ (b) where T_0 is the primary wave period.

Using this device, we have also shown [A.77] the production of a robust horizontal mean flow induced by internal gravity waves, when a wave beam is forced at the lateral boundary of a tank.

However, the study which has attracted most of the attention was related to another important way that large-scale oceanic internal waves transfer their energy to small-scale mixing : through parametric subharmonic instability (PSI). Using this novel generator, we were indeed able to provide [A.75,A.79,A.87] the first experimental verifications of this nonlinear resonant interaction through which a primary wave (plane waves or vertical modes) excites pairs of waves whose frequencies and wave numbers add up to the frequency and wavenumber, respectively, of the primary wave. Right panel of Fig. 1 presents how the initial beam is destabilized and emit secondary waves out of the beam, since the unusual dispersion relation of IW implies a different angle of propagation when the frequency is modified.

Interestingly, we discovered that the disconnect between theory, which assumes the waves are periodic in space and time, and reality in which waves are transient and more importantly spatially localized, modifies drastically the result. We have thus shown theoretically, numerically and experimentally that the width of the internal wave beam is a key element, a feature totally overlooked previously, despite numerous numerical simulations. In particular, we have reported dramatic consequences on the triad selection mechanism. The subharmonic plane waves that are theoretically unstable can only extract energy from the primary wave if they do not leave the primary beam too quickly. This finite-width mechanism has two opposite consequences on the wave energy dissipation : it can hinder the PSI onset (reducing transfer and therefore dissipation), but when PSI is present it enhances the transfer towards small wavelengths, more affected by dissipation.

We have moreover shown [A.80] that PSI unexpectedly destroys the coherence of internal wave attractor in a confined fluid domain. The triadic resonance appears to be, moreover, a very efficient energy pathway from long to short length scales. This work provides an explanation of why attractors may be difficult or impossible to observe in natural systems subject to large amplitude forcing. Finally, let us stress also that we have also provided [A.73] an experimental study of PSI in the very similar context of inertial waves in a rotating homogeneous fluid.

We have also studied internal solitary waves first by revisiting [A.71] the deadwater phenomenon when a boat evolving on a two-layer fluid feels an extra drag due to waves being generated at the interface between the two layers whereas the free surface remains still. Second, by generating quasi-two-dimensional internal wave beam impinging on a pycnocline, which ends up by the generation of internal solitary waves at this interface. These experiments [A.76] were inspired by observations of internal solitary waves in the deep ocean from synthetic aperture radar (SAR) imagery.

In addition to above fundamental studies, we have started to study more realistic situations in which the different mechanisms studied separately occur simultaneously. The most challenging issue was related to the complex double-ridge system in the Luzon Strait in the South China Sea, which is one of the strongest sources of internal tides in the oceans. We have developed a large-scale laboratory experiment performed at the Coriolis platform in Grenoble. It was carefully designed so that the relevant dimensionless parameters closely matched the ocean scenario. The results [A.83] advocate that a broad and coherent weakly nonlinear, three-dimensional, M2 internal tide that is shaped by the overall geometry of the double-ridge system is radiated into the South China Sea and subsequently steepens ; it explains one of the strongest sources of internal tides in the oceans, associated with which are some of the largest amplitude internal solitary waves on record.

Impact

• The study [A.79] on the instability of internal gravity waves in stratified fluids has been chosen by the Journal of Fluid Mechanics as Focus on Fluid in 2013.

• The recent work published in Geophysical Research Letters [A.83] with an international team of researchers has been chosen by a CNRS press release and an article has been published in Le Monde in December 2013.

• The article published in Physical Review Letters [A.84] on the analysis of active matter models with dynamical systems tools has been selected as a Focus (2014).

• The book *Physics of Long-Range Interactions*, written by A. Campa, T. Dauxois, D. Fanelli, S. Ruffo is in press Oxford University Press (2014).

Publications' list : `http ://perso.ens-lyon.fr/thierry.dauxois/listepubli.html`

Main Research Interests (See Publications' list for references)

◇ SOLITARY WAVES : CREATION, STABILITY AND APPLICATIONS [B3,B4,B5,B6]

- Energy Localisation in Nonlinear Lattices, Breather modes [A1,A2,A6,A9,A18,A20,A36]
- Breathers, Equipartition of energy and chaos [A23,A25,A26,A44,A47,A48,A59,A60,A62]
- Application to Biophysical Problems [A3,A4,A5,A6,A7,A8,A11,A85]
- Diffusion of Atoms on Surfaces, Modelization with kink's concept, Mobility of defects [A15,A16,A17,A19,A21,A22,A24,A28]

◇ CONNECTIONS BETWEEN NL DYNAMICS AND STAT. MECHANICS

- Phase Transitions in simple one-dimensional systems [A12,A14,A29,A33]
- Lyapunov Exponents and Phase Transitions [A32,E5].
- Numerical Calculation of Lyapunov Exponents [E4,D7]

◇ PATH INTEGRALS AND STATISTICAL MECHANICS [A57].

◇ STABILITY OF PERIODIC ARRAY OF VORTICES [A10,A13]

◇ STOCHASTIC OSCILLATIONS in AUTOCATALYTIC REACTIONS [A64,A68]

◇ DYNAMICS AND THERMODYNAMICS OF SYSTEMS WITH LONG-RANGE INTERACTIONS

- Ensembles Inequivalence [A37,A43,A46,A51,A54,B2,C2,A61,A65,A69]. *Work in progress.*
- Birth and stabilisation of out-of-equilibrium states [A30,A31,A34,A35,C3].
- Application to Free Electron Laser (FEL) [A41].
- Relaxation toward equilibrium [A39,A42,A49,D8,A54,A56,A58,A65,A70,A72]. *Work in progress.*
- Effect of noise [A74,A78,A82]. *Work in progress.*
- Gravitational interaction [A46, A86]. *Work in progress.*
- Dipolar interaction [A81]. *Work in progress.*

◇ INTERNAL WAVES IN STRATIFIED FLUIDS

- Reflection of internal waves (Theory and Experiments) [A27,E3,A38,A40,A50,A63]. *Work in progress.*
- Internal tides and soliton generations [A55,A66,A71,A76,A83]. *Work in progress.*
- A novel internal waves generator [A53,A67]. *Work in progress.*
- Parametric Subharmonic Instability [A73,A75,A79]. *Work in progress.*
- Mean flow generation [A77]. *Work in progress.*
- Internal waves attractor [A80]. *Work in progress.*

◇ ACTIVE MATTER [A.84]