Hierarchical Bayesian inversion of marine CSEM data over the Scarborough gas field - a lesson in correlated noise

Anandaroop Ray and Kerry Key, Scripps Institution of Oceanography, UC San Diego; Thomas Bodin, UC Berkeley*

SUMMARY

Uncertainty in the transmitter position, theory error and insufficient model parameterization amongst various other factors can lead to significant correlated error in observed controlled source electromagnetic data. These errors come to light by an examination of the residuals after performing inversion. Since correlated error violates the assumption of independent data noise it can manifest in spurious structure in inverted models. We demonstrate this using both synthetic data and real data from Scarborough gas field, North West Australia. In this work we propose a method which uses a hierarchical Bayesian framework and reversible jump Markov chain Monte Carlo to account for correlated error. We find that this removes suspect structure from the inverted models and within reasonable prior bounds, provides information on the resolution of resistivity at depth.

INTRODUCTION

Useful inferences from marine controlled source electromagnetic methods (CSEM) are obtained through inversion of the observed data, and not directly from the data itself (Weiss, 2007). Through stacking of observed data and the central limit theorem, the data errors are often justified as being Gaussian distributed and independent. Owing to various kinds of observational error (such as transmitter positioning), insufficient model physics or parametrization, correlated error can arise and violate the independent Gaussian assumption. Since an inversion that assumes such errors depends solely on the χ^2 misfit value to navigate the misfit space, incorrect assumptions about a diagonal data covariance matrix will lead to incorrectly inverted structure and spurious interpretation of subsurface resistivity.

THEORY

In this work we utilize a reversible jump Markov chain Monte Carlo (RJ-MCMC) method (Green, 1995) where the model parameterization is flexible. The number of layers, their resistivities and the positions of the layers are not fixed *a priori* and are treated as unknowns during the inversion. Owing to the inherent non-uniqueness and non-linearity of the CSEM inverse problem, a flexible parameterization using RJ-MCMC provides a more realistic estimate of resistivity uncertainty at depth (Ray and Key, 2012). The uncertainty estimate is provided by the posterior model distribution $p(\mathbf{m}|\mathbf{d})$ where

$$p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m}) \times p(\mathbf{m}) \tag{1}$$

$p(\mathbf{m})$ is the prior model distribution and $p(\mathbf{d}|\mathbf{m})$ is the misfit-dependent likelihood for the model $\mathbf{m}$ given by

$$p(\mathbf{d}|\mathbf{m}) = \frac{1}{\det(\pi \mathbf{C_d})} \exp\Big(-[\mathbf{d}-f(\mathbf{m})]^{\dagger} \mathbf{C_d}^{-1} [\mathbf{d}-f(\mathbf{m})]\Big) \tag{2}$$

for circularly symmetric complex errors. † indicates a Hermitian transpose and $\mathbf{C_d}$ is the data covariance matrix.

Hierarchical approaches to estimating data noise where $\mathbf{C_d}$ can itself be 'sampled' are gaining popularity in geophyics (e.g. Malinverno and Briggs, 2004; Bodin et al., 2012; Dettmer et al., 2012; Steininger et al., 2013). From equation 2 it can be seen that $\mathbf{C_d}$ in the determinant and in the argument of the exponential have opposite effects on the likelihood, thus ensuring that sampled data error values are not too large or too small. Thus, a hierarchical approach allows for inverted models not to be overly simple or overfitting the data, with posteriori uncorrelated residuals. This leads to a rigorous uncertainty estimate of resistivity at depth, a challenging task in CSEM inversion.

SYNTHETIC DATA EXAMPLE

To demonstrate the effects of correlated error, exponentially correlated noise was added to synthetic data generated from the red model (Trainor and Hoversten, 2009) shown in Figure 1 (left pane). In the middle and right panes, solid colored lines represent the true model responses, the grey dashed lines show the black background model response, and the noisy data realizations are shown with 1σ error bars on either side. Gaussian noise with standard deviations equal to 3%, 5% and 7% of the amplitude was added to the data at 0.125 Hz, 0.25 Hz and 1.25 Hz. The generated noise was correlated such that correlation falls off exponentially with range. The offset-to-offset correlation coefficients were 0.7, 0.4 and 0.2 at 0.125 Hz, 0.25 Hz and 1.25 Hz. This is not an unrealistic model, as it is natural that the lower frequencies will exhibit greater spatial correlation owing to their longer wavelengths / skin depths.

The results of an RJ-MCMC inversion using a diagonal covariance matrix, ignoring off-diagonal terms is shown in Figure 2(a). In the left panel, hotter colors correspond to a higher probability of resistivity at depth. The 5% and 95% quantile lines of resistivity at depth are shown with thick white lines. The true model is shown with a thick black line. The right panel shows the probability of the presence of interfaces (resistive contrasts) at depth. Using the hierarchical RJ-MCMC Bayesian method, with both the data variance and the correlation coefficient at each frequency as unknowns to be sampled during the inversion, we obtain the posterior model distributions shown in Figure 2(b). It is clear that ignoring the off-diagonal covariance terms leads to diminished sensitivity to the middle target layer and spurious half space structure. Clearly the hierarchical Bayesian scheme does a better job of

DOI http://dx.doi.org/10.1190/segam2013-1439.1

quantifying the subsurface resistivity uncertainty more accurately.

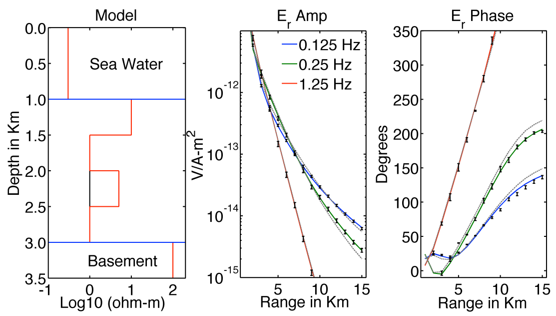


Figure 1: Synthetic model (red) from Trainor and Hoversten (2009) for study.

SCARBOROUGH CSEM EXAMPLE

The Scarborough gas field in the NW Australian shelf presents an approximately 1D yet challenging CSEM inversion target. The difficulty lies in the resistive Gearle siltstone formation overlying the gas reservoir section with both sections characterized by roughly equal resistivity-thickness product (Myer et al., 2012). Using the nominally calculated stacking errors in a diagonal covariance matrix for RJ-MCMC inversion of on-reservoir CSEM data, we obtain Figure 3(a). Though it seems that high probabilities of reservoir-like resistivity are found at depth (left pane), the interface probability curves (right pane) are highly jagged at the Gearly level (blue horizontal lines) and reservoir level (red horizontal lines). The black resistivity model overlain on the left panel is an Occam's inversion (Constable et al., 1987) model over the same location from Myer et al. (2012), where the location of the Gearle and the reservoir had to be fixed a priori with appropriate prejudices applied to obtain a sensible model. In Figure 3(b), a random selection of normalized residuals from 1000 RJ-MCMC inverted models have been plotted at each offset, for each frequency (color images). The top row shows the real residuals and the bottom row the imaginary residuals. Hotter colors correspond to a higher number of residuals at a given value, and the vertical white line represents the zero line. Alongside the color images are shown histogram plots of the mean residuals at each frequency, along with the plots for a theoretical standard normal Gaussian (black) and the PDF of all residuals (dashed red). From the color panels, we can clearly see a lot of correlation and the histogram plots alongside show that the data errors are not very Gaussian. Thus, the posterior model distributions shown in Figure 3(a) are not very robust.

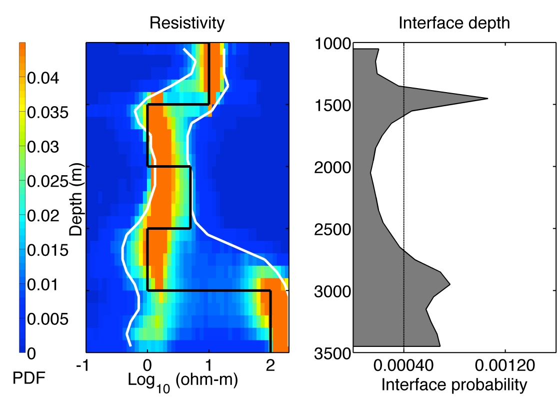
When using a hierarchical Bayesian RJ-MCMC scheme with a sampled correlation matrix, we obtain the results shown in Figure 4(a). A lot of shallow structure has been removed by the hierarchical inversion. This is expected, given that we included a correlation model that explicitly includes short offset correlation. There is also a booming probability of interfaces at the reservoir depth (red horizontal lines), and kinks in the probability of interfaces at the depth of the Gearle layer (blue horizontal lines). Further, from Figure 4(b), the data errors are now uncorrelated and far more Gaussian, making Figure 4(a) a far more robust and rigorous inference.

CONCLUSIONS

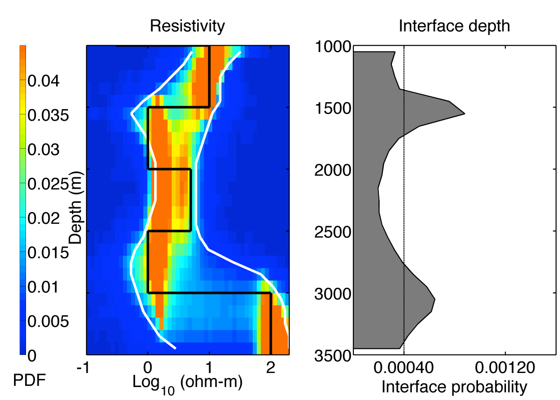
In this work we have shown the benefits of explicitly dealing with correlated noise and the inevitable pitfalls which crop up if they are disregarded. The procedure outlined here is not restricted to 1D problems alone and can be applied to 2D/3D problems as well, given that correlated errors crop up even if the model physics are made more complicated. Since we may have folded subtle 2D reservoir signals into correlated error, we are currently working on 2D methods of inverting Scarborough CSEM data within a hierarchical RJ-MCMC Bayesian framework to provide an even more rigorous and robust measure of sub-seafloor resistivity uncertainty.

ACKNOWLEDGEMENTS

This work was supported by the Scripps Seafloor Electromagnetic Methods Consortium at the Scripps Institution of Oceanography, UC San Diego.



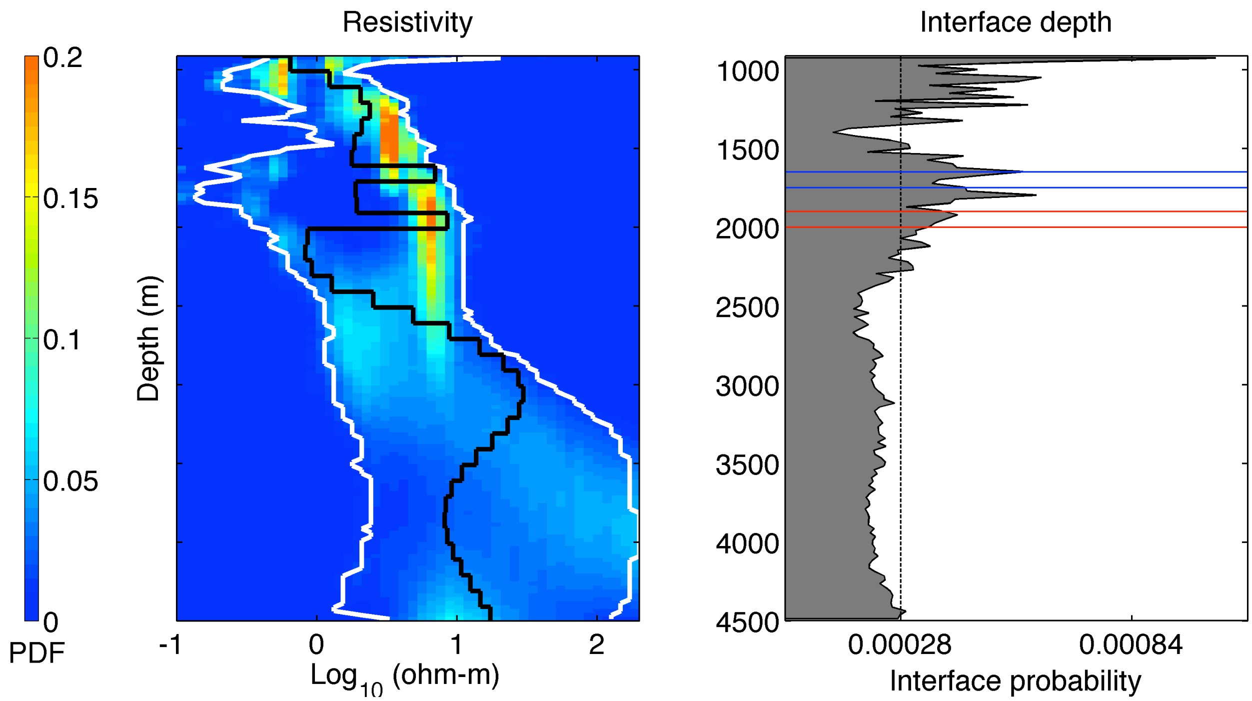
(a)



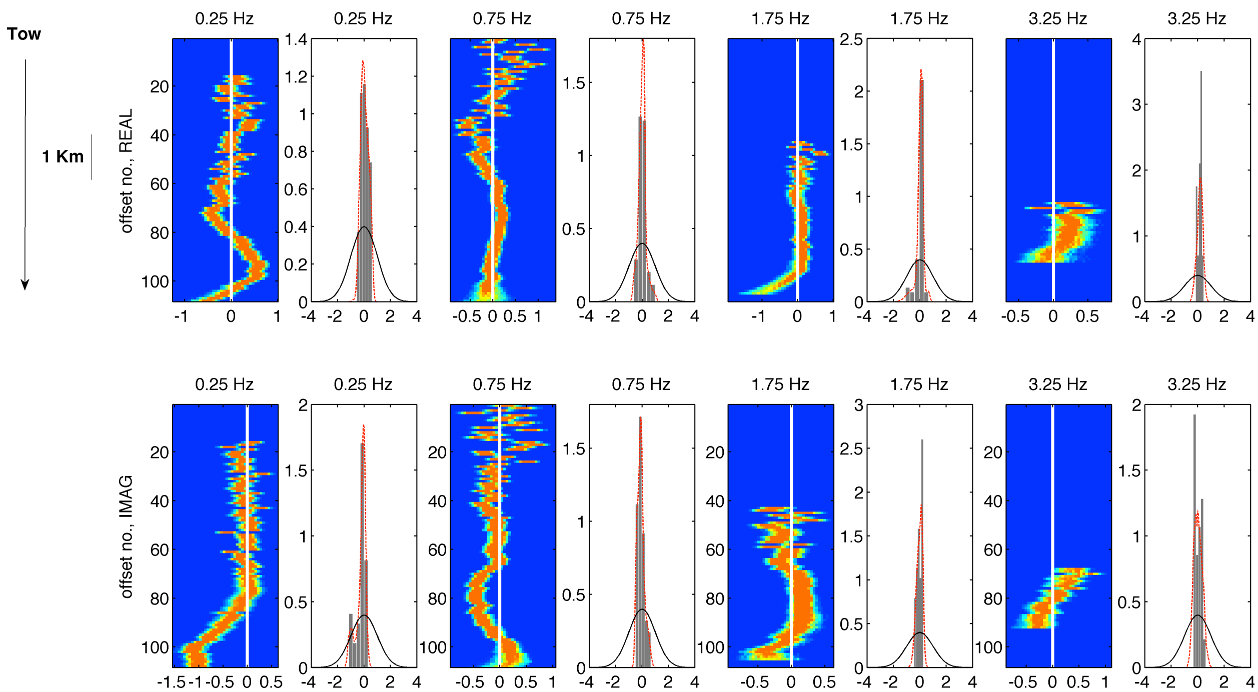
(b)

Figure 2: a) RJ-MCMC inversion of synthetic CSEM data ignoring correlated error. b) Hierarchical Bayes RJ-MCMC inversion accounting for unknown noise and correlated error.

DOI http://dx.doi.org/10.1190/segam2013-1439.1



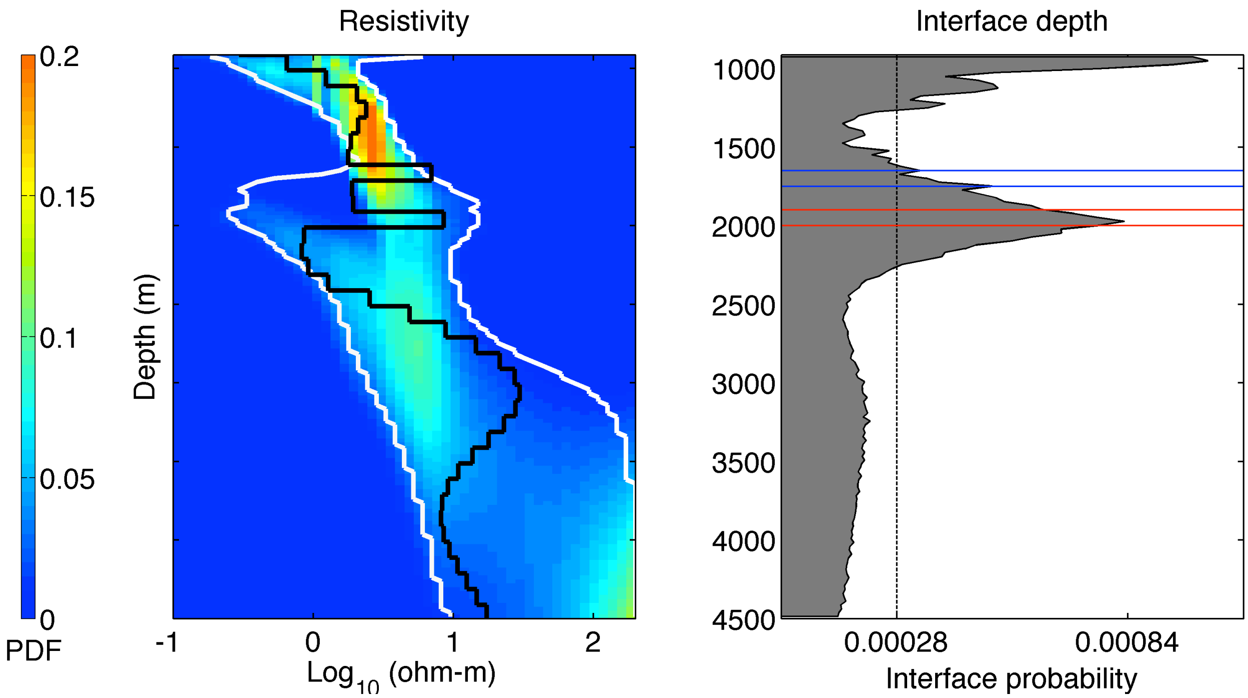
(a)



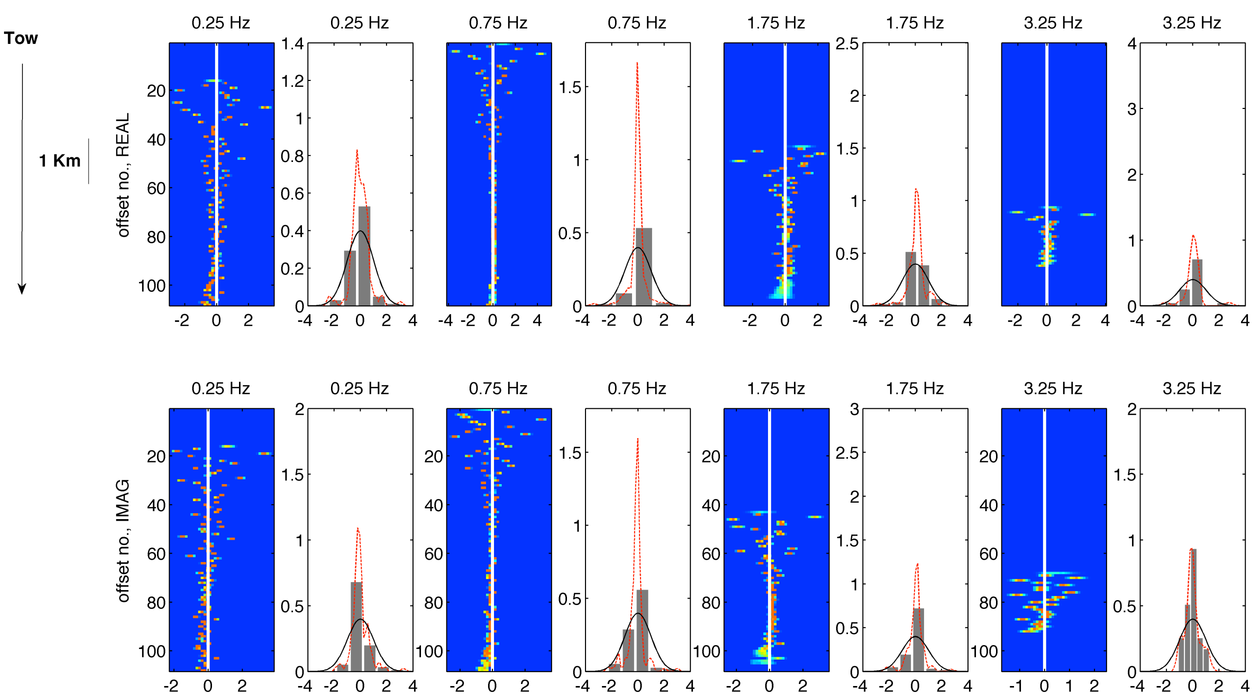
(b)

Figure 3: a) RJ-MCMC inversion of Scarborough gas field CSEM data ignoring correlated error. b) Color images are normalized residuals at each frequency at each offset from 1000 randomly chosen inverted models. Histogram plots are of the mean residuals at each frequency, the black curve is a standard normal Gaussian. Highly correlated un-Gaussian error is evident. Inferred structure in Figure 3(a) may be spurious.

DOI http://dx.doi.org/10.1190/segam2013-1439.1



(a)



(b)

Figure 4: a) Hierarchical Bayes RJ-MCMC inversion of Scarborough CSEM data. There is a booming probability of interfaces (right pane) at both the Gearle level (blue lines) and reservoir level (red lines). b) Correlated errors and un-Gaussianity much reduced compared to Figure 3(b). Figure 4(a) is now a much more robust inference about resistivity uncertainty.

http://dx.doi.org/10.1190/segam2013-1439.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Bodin, T., M. Sambridge, H. Tkalcic, P. Arroucau, K. Gallagher, and N. Rawlinson, 2012, Transdimensional inversion of receiver functions and surface wave dispersion: Journal of Geophysical Research, **117**, B2.

Constable, S. C., R. L. Parker, and C. G. Constable, 1987, Occams inversion: A practical algorithm for generating smooth models from electromagnetic sounding data: Geophysics, **52**, 289–300, http://dx.doi.org/10.1190/1.1442303.

Dettmer, J., S. Molnar, G. Steininger, S. E. Dosso, and J. F. Cassidy, 2012, Trans-dimensional inversion of microtremor array dispersion data with hierarchical autoregressive error models: Geophysical Journal International, **188**, no. 2, 719–734, http://dx.doi.org/10.1111/j.1365-246X.2011.05302.x.

Green, P. J., 1995, Reversible jump Markov chain Monte Carlo computation and Bayesian model determination: Biometrika, **82**, no. 4, 711–732, http://dx.doi.org/10.1093/biomet/82.4.711.

Malinverno, A., and V. A. Briggs, 2004, Expanded uncertainty quantification in inverse problems: Hierarchical Bayes and empirical Bayes: Geophysics, **69**, 1005–1016, http://dx.doi.org/10.1190/1.1778243.

Myer, D., S. Constable, K. Key, M. E. Glinsky, and G. Liu, 2012, Marine CSEM of the Scarborough gas field, Part 1: Experimental design and data uncertainty: Geophysics, **77**, no. 4, E281–E299, http://dx.doi.org/10.1190/geo2011-0380.1.

Ray, A., and K. Key, 2012, Bayesian inversion of marine CSEM data with a trans-dimensional self parametrizing algorithm: Geophysical Journal International, **191**, 1135–1151.

Steininger, G., J. Dettmer, S. E. Dosso, and C. W. Holland, 2013, Trans-dimensional joint inversion of seabed scattering and reflection data: The Journal of the Acoustical Society of America, **133**, no. 3, 1347–1357, http://dx.doi.org/10.1121/1.4789930.

Trainor, W., and G. M. Hoversten, 2009, Practical challenges of stochastic inversion implementation for geophysical problems: 79th Annual International Meeting, SEG, Expanded Abstracts, 734–738.

Weiss, C., 2007, The fallacy of the "shallow-water problem" in marine CSEM exploration: Geophysics, **72**, no. 6, A93–A97, http://dx.doi.org/10.1190/1.2786868.