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On the Nature of the P-Wave Velocity Gradient in the Inner Core beneath Central America

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ABSTRACT: We conduct an experiment to investigate whether linearity in the observed velocity gradient in the volume of the inner core sampled by the PKP ray paths beneath Central America is a robust approximation. Instead of solving an optimization problem, we approach it within the Bayesian inference. This is an ensemble approach, where model specification is relaxed so that instead of only one solution, groups of reasonable models are acceptable. Furthermore, in transdimensional Bayesian inference used here, the number of basis functions needed to model observations is by itself an unknown. Our modeling reveals that in the ensemble of models, the most likely are those containing only 2 nodes (linear trend). Thus our result justifies the assumption used for the determination of inner core rotation with respect to the rest of the mantle that the observed gradient is constant in its nature (linear). Recent observations in seismology suggest that it is likely that the spatial variability in elastic parameters is a widespread phenomenon in the inner core. Future array observations will further constrain spatial extent and magnitude of velocity changes and show whether there is a significant difference between these observations in the two quasi-hemispheres of the inner core.
KEY WORDS: solid earth physics, deep geophysics.

INTRODUCTION

There is growing evidence from seismological studies that the inner core is heterogeneous both in terms of seismic velocity and anisotropy. Many studies have focused on inner core velocity structure, however much less is known about regional variations of inner core elastic properties. Sparse ray-path coverage, particularly the lack of continuous spatial sampling on regional scales (several hundred km) within the inner core is the predominant reason for this ongoing problem. So far, it has been possible to collect datasets of differential travel times of core-sensitive seismic phases over a range of epicentral distances from a large number of events (e.g., Waszek and

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Deuss, 2011; Garcia et al., 2006; Tkalčić et al., 2002). These global data sets played an important role in inferring information about the anisotropic structure of the inner core. However due to the scarcity of dense regional arrays in seismology it has not yet been possible to collect travel time measurements from a single event and use them in a similar fashion in which the travel time data are used in exploration seismology.

The establishment of dense temporary arrays (WOMBAT in Australia, US Array in the US) and permanent arrays (Hi-net in Japan) with regional-scale apertures has opened a new era in seismology of the deep earth. Such recordings enable the construction of entire empirical travel time curves over the range of several tens of degrees and can serve as a powerful tool to map the deep earth structure. However, even the presence of dense regional scale arrays does not guarantee a good spatial coverage of the inner core by PKP waves, as the majority of large earthquakes are confined within the world's subduction zones and do not necessarily occur at the suitable epicentral distances from those arrays (i.e., from ~145º to 155º for PKP(bc-df) differential travel times).

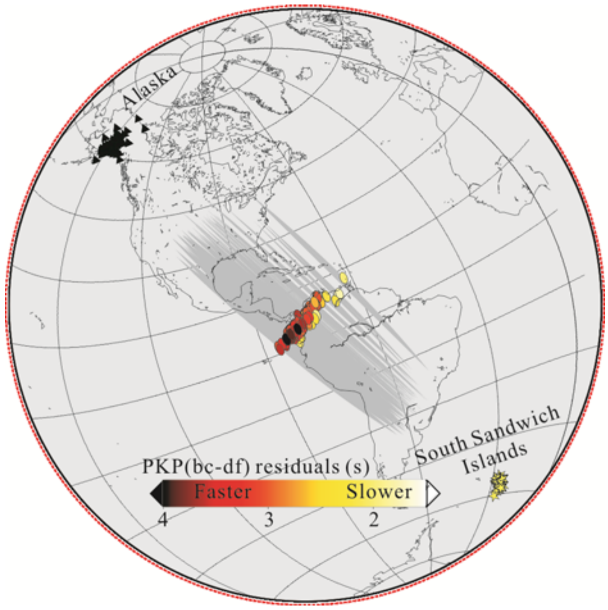

Figure 1. Locations of the South Sandwich Islands earthquakes (yellow stars), stations in Alaska (triangles) and bottoming points of PKP waves (ellipses) used in this study are projected to the surface of the globe. Bottoming points of PKP waves are shown in colours corresponding to the measured PKP(bc-df) differential travel time residuals. Great circle path projections of PKPdf waves in the inner core are shown with grey lines.

P-Wave Velocity Gradient in the Inner Core from South Sandwich Islands Earthquakes

The above-mentioned difficulty with regards to inner core sampling of PKP waves fortunately does not apply for a subduction zone in the South Atlantic Ocean beneath the South Sandwich Islands (SSI), characterized by a steady rate of seismicity. Given that most of the world's subduction zones are located between 45ºN and 45ºS latitude, SSI region represents a unique location. Opportunely, Alaskan stations are favourably located with respect to the SSI region to record PKP waves. The associated ray-paths sample the inner core in a unique way, creating quasi-polar angles with the Earth's spin axis (Fig. 1). The bottoming points of the ray-paths from SSI to Alaska are distributed in the uppermost inner core beneath Central America, approximately between 85°W and 60°W, and between 0°N and 15°N. Both the spatial distribution of the SSI earthquakes and the lateral extent of the Alaskan network act as a regional scale array.

Due to a significant number of earthquake sand the continuity of recording of the Alaskan network, it is not surprising that the inner core beneath Central America has been well mapped and documented in previous work. The existence of a linear spatial velocity gradient in the volume where ray paths from SSI travel through to the stations in Alaska was hypothesized in previous seismological work. The first seismological paper that addressed this subject was by Creager (1997). This simplicity of the result, however, does not seem to be widely accepted and a recent paper claimed that the velocity gradient is non linear (Lindner et al., 2010).

P-Wave Velocity Gradient as a Marker for Differential Rotation of the Inner Core

The assumption that underpins the inner core differential rotation with respect to the rest of the planet relies on the existence of a gradient in seismic velocities in the inner core with known properties. The axial rotation of the inner core that is recovered depends directly on this gradient. A slope in s/yr determined from temporal variations of seismic waveforms traversing the inner core can be converted to the inner core rotation rate (º/yr) using

$$\alpha = -\frac{\gamma \delta t}{\partial v / \partial L} \tag{1}$$

where γ is a dimensionless correction factor relating the change in source-receiver distance to the change of azimuth, δt is the slope (s/yr) normalized by the total time PKPdf waves spend in the inner core (%s/yr), and $\partial v/\partial L$ is the lateral velocity gradient in the inner core (%/º) (Creager, 1997). However, unless this large scale, linear gradient is convincingly demonstrated, it is likely that more complex velocity structure such as significant non-linear gradient would map into the estimated differential rotation rate. Since the seismic ray would travel through a more complex velocity structure as a function of time, the travel time change would also be complex. One could interpret these changes as accelerating or decelerating inner core, but it would be incorrect. Even if one could be certain that the inner core rotation is steady, it would be impossible to reconstruct its rate without knowing the nature of the velocity gradient. Since the work of Creager (1997), other workers demonstrated that a linearity of spatial velocity gradient along this particular path is a valid assumption (e.g., Zhang et al., 2005; Song, 2000). However, in their work, the linearity of the velocity gradient was assumed rather than statistically inferred. In this article we use Bayesian inference to investigate whether such a linear trend is supported by observations.

DATA AND METHOD

PKP(bc-df) Differential Travel Times

In order to test the hypothesis that the P-wave velocity gradient in the inner core is linear, we use PKPbc waves that traverse the lowermost part of the outer core, and PKPdf waves that traverse the inner core. Their relative (differential) travel times are used to infer inner core structure based on the assumption that the ray-paths are sufficiently similar in the mantle and that a well mixed outer core is extremely unlikely to contribute significantly toward relative travel times. The first assumption is based on the fact that the ray-paths of PKPdf and PKPbc waves are close to each other in the upper mantle and that the mid mantle does not seem to have significant heterogeneity. Zhao and Lei (2004) investigated the effects of ray-path deviations in the presence of sharp lateral velocity gradients in the mantle and found that they can be up to several tens of kilometers. However, the ray-path deviation due to the existence of a velocity gradient in the inner core is insignificant due to the fact that the gradient is smooth (0.5% per 27º according to Creager (1997)) in comparison with, say, abrupt changes in elastic properties as a function of the Earth's radius or the existence of strong lateral heterogeneity in the upper mantle. The stationarity of travel times is an excellent approximation for the PKPdf path perturbation. Heterogeneous structure in the lowermost mantle has been demonstrated to exist on many spatial scales, inter alia, scales smaller than the separation between PKPdf and PKPbc waves. Therefore lowermost mantle heterogeneity contributes toward explaining a large portion of PKP(ab-df) differential travel times (e.g., Tkalčić, 2010; Romanowicz et al., 2003; Tkalčić et al., 2002; Bréger et al., 2000) and remains to be a serious candidate for explaining at least a part of the signal in PKP(bc-df) differential travel times. This is at odds with some authors who arguethat inner core structure exclusively contributes to the PKP(bc-df) differential travel time residuals (e.g., Irving and Deuss, 2011).

For the purpose of studying the nature of the hypothesized velocity gradient in the inner core and under the assumption that the origin of the variability in PKP(bc-df) differential travel times comes predominantly from inner core structure, we analyze the differential travel time data from South Sandwich Island (SSI) earthquakes observed on stations in Alaska. The waveform data span the time interval of three decades. This dataset consists of 206 good to excellent quality measurements, subsets of which were previously presented by Leykam et al. (2010), Tkalčić (2010) and Tkalčić et al. (2002), which apart from new measurements also featured high quality data from Tanaka and Hamaguchi (1997) and McSweeney et al. (1997).

We define good and high quality measurements as those which consist of waveforms with cross-correlation coefficient of at least 0.6 for the window including both PKPdf and PKPbc waves, and only quality A and B out of all data classified from A to D were selected. The weights are assigned proportional to data quality. The residuals corresponding to COL station were corrected according to Creager (1997) Model 1 accounting for an aspherical structure under Alaska. Figure 2 shows normalized PKPbc-df travel time residuals (by the time they spend in the IC) with respect to ak135 model (Kennett et al., 1995) accounting for the Earth's ellipticity as a function of epicentral distance. Figure 3 shows the same as Fig. 2, but the data are plotted as a function of station longitude.

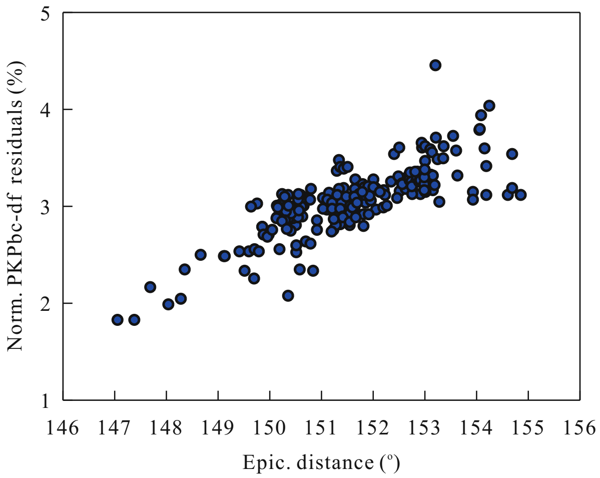

Figure 2. PKP(bc-df) differential travel time residuals used in this study, normalised by the time PKPdf waves spend in the inner core, plotted as a function of epicentral distance.

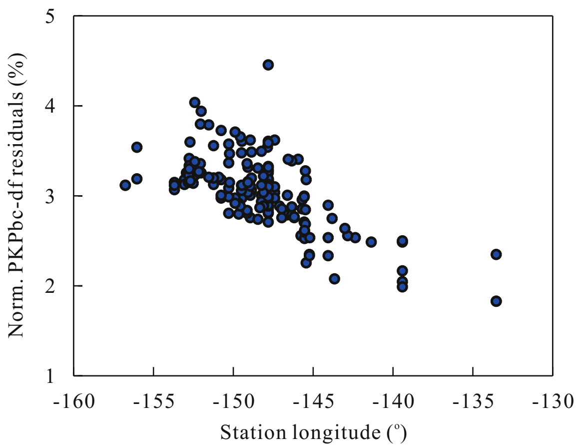

Figure 3. PKP(bc-df) differential travel time residuals used in this study, normalised by the time PKPdf waves spend in the inner core, and plotted as a function of the Alaskan stations' longitude.

Bayesian Transdimensional Inversion

One can argue qualitatively from Figs. 2 and 3 that a linear spatial gradient is a reasonable assumption in the modeling of the PKP differential travel time residuals. However, putting this on a quantitative basis requires further analysis. The crucial question we intend to answer is: "Is a linear trend with respect to azimuth (linear spatial velocity gradient in the inner core) justified for these data?" If this can be shown to be true, it would be a strong argument against the possibility that the velocity change in the inner core is random and the linearity of the gradient could be accepted with more confidence. Alternatively, the data might be better explained with a more complex trend that would be characterized by a number of change points. The ability to detect such change points and their statistical significance in data has previously been demonstrated in the medical statistics (e.g., Densison et al., 2002) and in a growing number of earth science contributions (e.g., Bodin et al., 2012; Gallagher et al., 2011).

We approach our regression problem within a Bayesian framework, where our solution will not be a single best-fit model as it would be within an optimisation framework, but will be an aggregate (ensemble) of a large number of realisations distributed according to the posterior probability density function, i.e., the probability of the model parameters given the observed measurements. Bayes' theorem (Bayes, 1763) is used to combine prior information about the model with the observed data to give the posterior probability density function

$$p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m}) \times p(\mathbf{m}) \quad (2)$$

where the formalism $a|b$ means: a given, or conditional on, b, i.e., the probability of having a when b is fixed. The $\mathbf{m}$ is the vector of the model parameters (i.e., number and position of nodes) and $\mathbf{d}$ is a vector defined by the set of observed data (i.e., differential travel times).

The term $p(\mathbf{d}|\mathbf{m})$ is the likelihood function, which is the probability of observing the measured data given a particular model. The likelihood is simply defined by a least square misfit function given by the distance between observed and estimated data

$$p(\mathbf{d}\,|\,\mathbf{m}) = \frac{1}{\prod_{i-1}^{N}\left(\sqrt{2\pi\sigma^2{}_i}\right)} \times \exp\left[\sum_{i=1}^{N} \frac{-(g(\mathbf{m})_i - \mathbf{d}_i)^2}{2(\sigma_i)^2}\right] \tag{3}$$

where $g(\mathbf{m})_i$ is the data i estimated from a given model $\mathbf{m}$, and σ_i is the standard deviation of an assumed random Gaussian noise for measurement i. The term $p(\mathbf{m})$ is the a priori probability density of $\mathbf{m}$, i.e., what we know about the model $\mathbf{m}$ before measuring the data $\mathbf{d}$. Here we assume unobtrusive prior knowledge by setting priors to uniform distribution with relatively wide bounds.

In our regression problem, we do not know the number of nodes, which means that the dimension of the model is itself variable, and hence the posterior becomes a transdimensional function (e.g., Sambridge et al., 2013; Green, 2003). In this scheme, the regression model is parameterized by a variable number of node points and the regression function is defined by linear interpolation between each consecutive pair of nodes. A model with two points is effectively a simple linear trend. Contrary to the classical regression problem, two different models can fit the same dataset perfectly well, and hence the solution is non-unique. This transdimensional posterior probability function can be sampled with a generalisation of the well-known Metropolis-Hasting algorithm (Hastings, 1970; Metropolis et al., 1953) termed the reversible jump Markov chain Monte Carlo (rj-McMC) sampler (e.g., Green, 1995), which allows inference on both model parameters and model dimensionality. The probabilistic regression algorithm produces a large ensemble of such models, each model having a variable number of nodes. From an ensemble of models distributed according to the posterior, we determine special properties such as the spatially average model.

RESULTS AND CONCLUSIONS

We analyze all differential travel time residuals as a function of azimuth from earthquake source (as in Creager, 1997). Figures 4 and 5 illustrate the results of our transdimensional inversion scheme. The red line in Fig. 4 is the expected model obtained by averaging all the models in the ensemble in each azimuth bin. The gray lines represent the 1-standard deviation limits of the ensemble at each azimuth. Note that this is a way to visualize the ensemble solution as a whole. The average model contains a slight "bend" at around 312° because it also accounts for higher order models, even though these models are less probable. This is illustrated in Fig. 5, which shows the probability for the number of nodes. In other words, in the ensemble of models, the most likely (60%) are those models containing only 2 nodes (linear trend). When only those models that have 2 nodes are kept, the posterior probability distribution of the gradient can be characterized by -0.023 7±0.002 1 s/º. This amounts to the travel time variation across the 27º longitude sampled here of 0.64 s, or a lateral gradient of about 0.019 %/º with the mean time of PKPdf waves through the inner core of 125 s.

During the regression scheme, data errors are considered as proportional to the inverse of the quality of data (expressed in weights), with the constant of proportionality being a parameter to be inferred by the data (hierarchical Bayesian inversion; see Bodin et al. (2012) for details). Therefore we account for the information about relative uncertainties between data points, but let the absolute level of data fit being a variable to be inverted for.

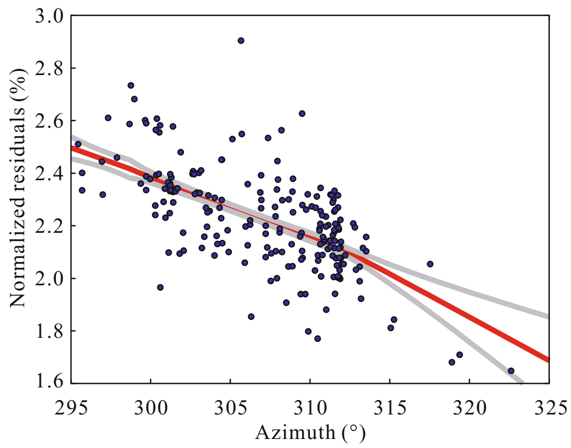

Figure 4. PKP(bc-df) differential travel time residuals used in this study (circles), normalised by the time PKPdf waves spend in the inner core, and plotted as a function of azimuth (measured from the source to the station). The central line is the expected regression model obtained by averaging all resulting models in the Bayesian inversion. The outward lines represent one standard deviation limits of the ensemble at each azimuth.

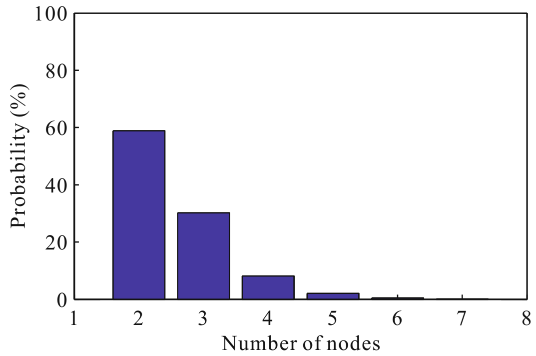

Figure 5. Posterior probability distribution of the number of nodes obtained in the transdimensional Bayesian inversion. A model with 2 points is the most likely model and it effectively represents a simple linear trend shown in Fig. 4.

The experiment conducted in this article shows that a linear velocity gradient in the volume of the inner core sampled by the ray paths between SSI earthquakes and stations in Alaska is a robust approximation. This has two immediate implications. Firstly, our result justifies the assumption and method used for the determination of inner core rotation with respect to the rest of the mantle that the lateral gradient is constant in its nature (linear). This gradient has been used as a marker to obtain differential rotation of the inner core with respect to the rest of the mantle (Tkalčić et al., 2013; Lindner et al., 2010; Zhang et al., 2005; Song, 2000; Creager, 1997).

Secondly, the sharpness of the velocity gradient (about 0.5% increase in velocity) across the lateral extent of about 27º (see Figs. 1 and 2 in Creager (1997)) is an important geodynamical constraint. In the inner core at the radius of about 1 000 km, the arc length of 27º translates to about 450 km, slightly above third of the inner core radius. Fresnel zone for a PKPdf phase with a dominant frequency of about 1 Hz for an epicentral distance of 150º is about 295 km (e.g., Calvet et al., 2006), which is well below the extent of the gradient zone discussed here. Concerning lateral resolution of features in the inner core, it is worth noting that scattering from scale-length even smaller than a Fresnel zone may effect travel times, although such simulations are beyond the scope of this article. Another significant velocity gradient in lateral and radial direction in the uppermost inner core has been inferred before beneath the Indian Ocean (Stroujkova and Cormier, 2004). Recent results from array analysis of PKP differential travel times and travel time curves suggest that these regional variations in the velocity structure at the UIC are more widespread than what we have previously been able to infer. If it can be demonstrated that the magnitude of velocity variation in the quasi-western hemisphere is stronger than in the quasi-eastern hemisphere, this, combined with robust observations from body waves and normal modes that the same hemisphere is also slower, might provide important clues on melting and freezing processes. As of now, melting and freezing processes could both be invoked to explain velocity differences in both hemispheres (Deguen et al., 2007).

It is highly likely that different regions of the inner core have different velocity gradients therefore if the temporal changes are not well constrained (such is the case if data from earthquake doublets are not present) one might obtain mutually inconsistent values for the differential rotation of the inner core (e.g., Mäkinen and Deuss, 2011).

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