SPIN 1925 Uhlenbeck, Goudsmit

- $\bar{e}$ charge $-e$
  masse m_e
  spin $\vec{S} = (S^x, S^y, S^z)$

  dimensions physique d'un moment angulaire

  $[S^\alpha] = \ell \cdot m \cdot v = E \cdot t$

$\vec{S} \to \vec{m}$ moment de dipôle magnétique

$\vec{m}$

e^- $\vec{m} = g \dfrac{\mu_B^{(e)}}{\hbar} \vec{S}$

$\hbar = 1.054 \times 10^{-34}$ J·s

$\mu_B^{(e)} =$ magnéton de Bohr électronique

$= \dfrac{e\hbar}{2m_e} = 9.27 \times 10^{-24}$ J/T

$g =$ facteur gyromagnétique

$\cong -2$ électron

p^+ : $\vec{m} = g \dfrac{\mu_B^{(p)}}{\hbar} \vec{S}$

$\mu_B^{(p)} = \dfrac{e\hbar}{2m_p} =$ m. d. B. nucléaire

$g \simeq 5.59$

n (neutron) $g \simeq -3.83$

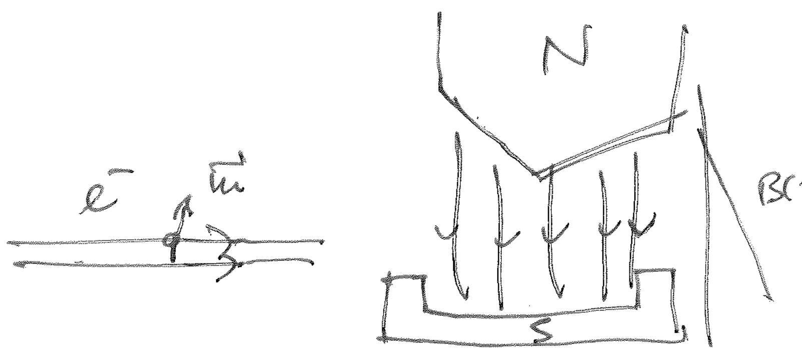
$|\vec{S}|^2 = \hbar^2 s(s+1)$ électron $s = \frac{1}{2}$ électron a un spin $\frac{1}{2}$

mesurer la composante z de $\vec{S}$

2 résultats possibles : $\pm \frac{\hbar}{2}$ pour n'importe quelle composante de $\vec{S}$

Expérience de Stern-Gerlach (1922)

$\uparrow z$



direction α : spin $\uparrow_\alpha$ si $S^\alpha = \frac{\hbar}{2}$

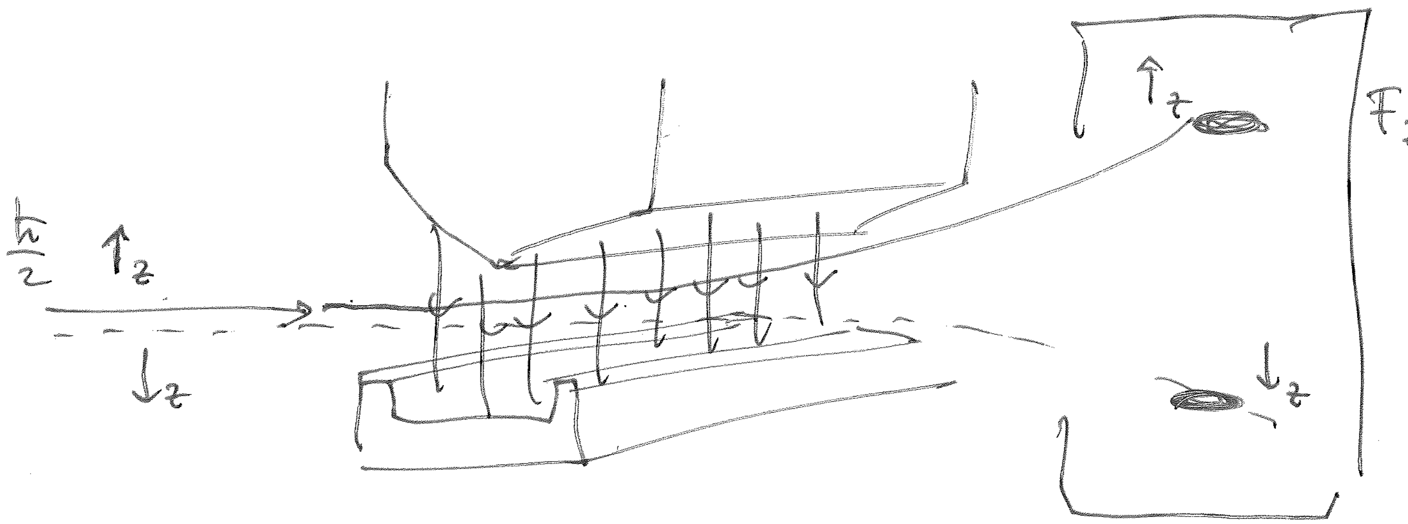
$\downarrow_\alpha$ si $S^\alpha = -\frac{\hbar}{2}$

$B(z)$ $\vec{B} = B(z)\,\vec{e}_z + \vec{B}_0$

$$E = -\vec{m}\cdot\vec{B}$$

$$\vec{F} = -\vec{\nabla} E = +\overbrace{\frac{\partial B}{\partial z}}^{<0} m_z \vec{e}_z$$

$$m_z \simeq -\cancel{2}\frac{e}{\cancel{2}m_e} S^z \simeq -\frac{e}{m_e} S^z$$

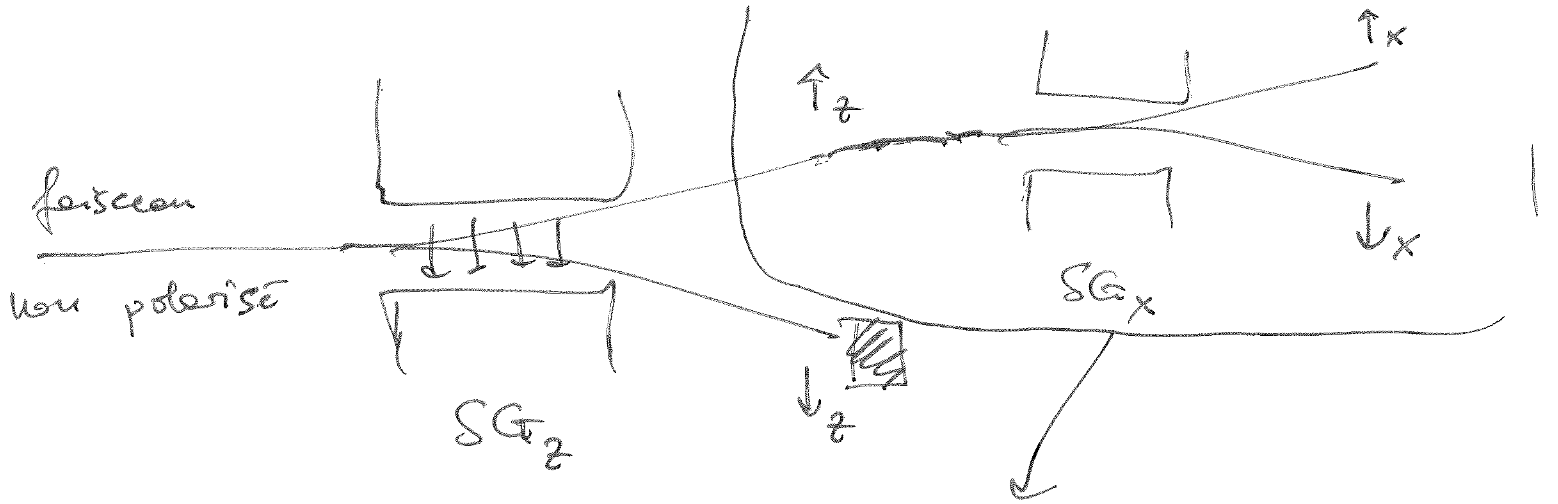


$$F_z = \left|\frac{\partial B}{\partial z}\right| \frac{e}{m_e} S^z$$

appareil de SG

no. 1

no. 2



$$P(\uparrow_z \to \uparrow_x) = \frac{1}{2}$$

$$= |\underbrace{\langle \uparrow_x | \uparrow_z \rangle}_{\text{amplitude de proba}}|^2$$

$$P(\uparrow_x \to \uparrow_z) = |\langle \uparrow_z | \uparrow_x \rangle|^2 = \frac{1}{2}$$

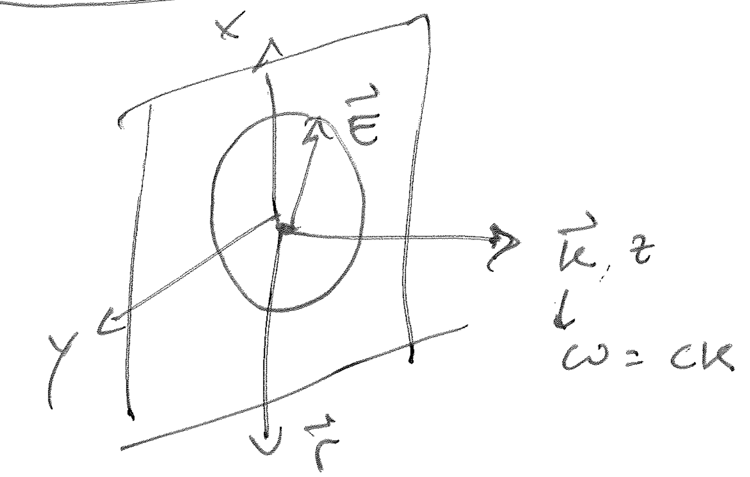
$$P(\uparrow_\uparrow \to \uparrow_x \to \uparrow_z) = |\langle \uparrow_x | \uparrow_z \rangle|^2 |\langle \uparrow_z | \uparrow_x \rangle|^2 = \frac{1}{4}$$

1) nature "ondulatoire" des états quantiques

2) observations incompatibles

3) mesure altère ~~de~~ l'état

Polarisation de la lumière



$$\vec{E}(\vec{r},t) = E_{0x} \cos(\vec{k}\cdot\vec{r} - \omega t + \delta_x)\,\vec{e}_x + E_{0y} \cos(\vec{k}\cdot\vec{r} - \omega t + \delta_y)\,\vec{e}_y$$

E_0 module de $\vec{E}$

$$\begin{cases} E_{0x} = \cos\vartheta \; E_0 \\ E_{0y} = \sin\vartheta \; E_0 \end{cases} \qquad E_{0x}^2 + E_{0y}^2 = E_0^2$$

$\vec{\mathcal{E}}(\vec{r},t)$: $\vec{E} = \mathrm{Re}[\vec{\mathcal{E}}]$

$\in \mathbb{C}^3$

$\vec{\mathcal{E}} \in \mathbb{C}^2$ deux composantes complexes $\boxed{\vec{r} = 0}$

$$\vec{\mathcal{E}}(\vec{r},t) = E_0 \left[\cos\vartheta \, e^{-i\omega t} e^{i\delta_x} \, \vec{e}_x + \sin\vartheta \, e^{-i\omega t} e^{i\delta_y} \, \vec{e}_y\right]$$

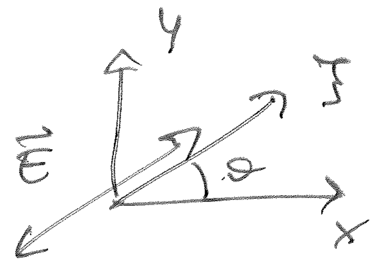
$$\delta_x = 0 \qquad \delta_y = \delta$$

$$= E_0 \left[\cos\vartheta \, e^{-i\omega t} \vec{e}_x + \sin\vartheta \, e^{-i\omega t} e^{i\delta} \vec{e}_y\right]$$

$|x\rangle$

$\boxed{\delta = 0}$ polarisation linéaire

$$\vec{\mathcal{E}}(\vec r, t) = \mathcal{E}_0 \left[\overbrace{\cos\vartheta \, \vec e_x + \sin\vartheta \, \vec e_y}^{\vec e_\xi} \right] e^{-i\omega t}$$



$$\vec E = \mathcal{E}_0 \cos(\omega t) \, \vec e_\xi$$

$\vartheta = 0$ $\quad \xi = x$ $\quad$ polarisation horizontale

$\vartheta = \frac{\pi}{2}$ $\quad \xi = y$ $\quad$ " verticale

$$\begin{cases} |H\rangle = \mathcal{E}_0 \, e^{-i\omega t} \, \vec e_x \\ |V\rangle = \mathcal{E}_0 \, e^{-i\omega t} \, \vec e_y \end{cases}$$ symboles

$|H\rangle$: vecteur en $\mathbb{C}^2 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$\alpha, \beta \in \mathbb{C}$

$\vec{\mathcal{E}}(t) = \cos\vartheta \, |H\rangle + \sin\vartheta \, |V\rangle$ $\quad$ polarisation linéaire

$\delta \neq 0$ $\quad$ polarisation elliptique

$\vec{\mathcal{E}}(t) = \cos\vartheta \, |H\rangle + e^{i\delta} \sin\vartheta \, |V\rangle$ $\qquad$ $\vec e_x^{\,T} \cdot \vec e_x = 1$

$\left(\mathcal{E}_0 \, e^{+i\omega t} \, \vec e_x\right)^T \cdot \left(\mathcal{E}_0 \, e^{-i\omega t} \, \vec e_x\right) = \mathcal{E}_0^2$ $\qquad$ $\vec e_x^{\,T} = (1, 0)$

$= \langle H | H \rangle$ $\qquad$ $\vec e_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\langle V | V \rangle = \mathcal{E}_0^2$ $\qquad$ $\langle H | V \rangle = \langle V | H \rangle = 0$

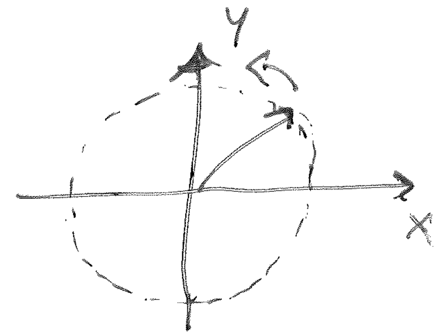
$\boxed{\delta = \pm\frac{\pi}{2}}$ polarisation circulaire $\vartheta = \pm\frac{\pi}{4}$

$$\vec{\mathcal{E}}(t) = \cos\vartheta\,|H\rangle \pm i\,\sin\vartheta\,|V\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|H\rangle \pm i\,|V\rangle\right)$$

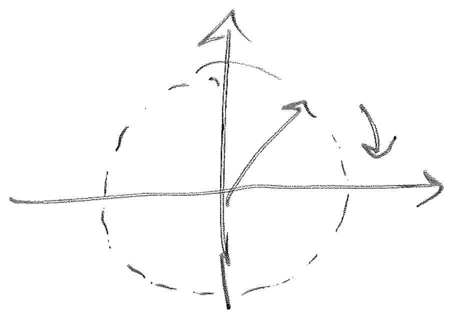
$$\mathrm{Re}[\vec{\mathcal{E}}] = \vec{E} = \frac{E_0}{\sqrt{2}}\left[\cos(\omega t)\,\vec{e}_x \pm \sin(\omega t)\,\vec{e}_y\right]$$

(+)



polarisation droite

(−)

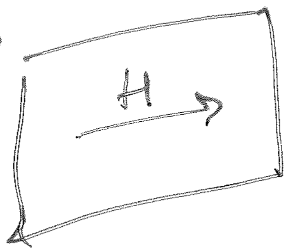


polarisation gauche

convention

$$\begin{cases} |D\rangle = -\frac{1}{\sqrt{2}}\left(|H\rangle + i|V\rangle\right) \\ |G\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - i|V\rangle\right) \end{cases}$$

$|\mathcal{E}\rangle = \cos\vartheta|H\rangle + \sin\vartheta|V\rangle \longrightarrow$



filtre polariseur

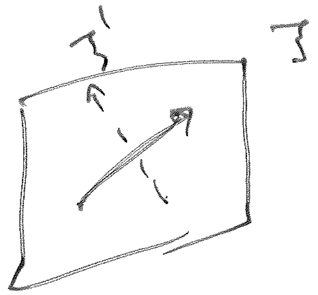
$$|\mathcal{E}'\rangle = \cos\vartheta\,|H\rangle = \frac{\langle H|\mathcal{E}\rangle}{E_0^2}\,|H\rangle$$

Intensité

$$I \sim |\mathcal{E}|^2 = \langle \mathcal{E}|\mathcal{E}\rangle$$

$$I' \sim \langle \mathcal{E}'|\mathcal{E}'\rangle$$

$|\mathcal{E}\rangle = |H\rangle$



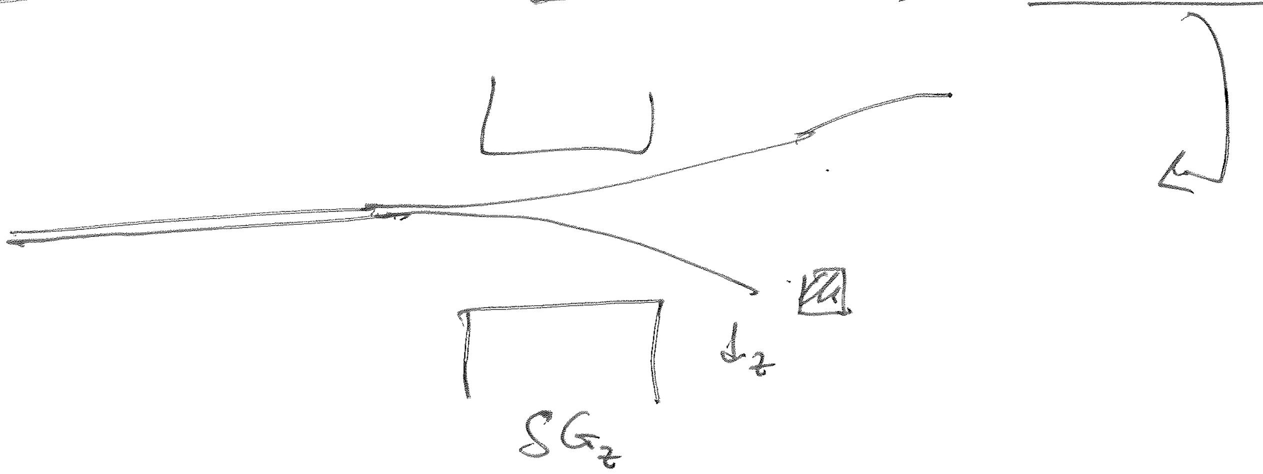
$$|\mathcal{E}'\rangle = \frac{\langle \xi|\mathcal{E}\rangle}{E_0^2}\,|\xi\rangle = \cos\vartheta\,|\xi\rangle \rightarrow \boxed{I' = I\cos^2\vartheta}$$

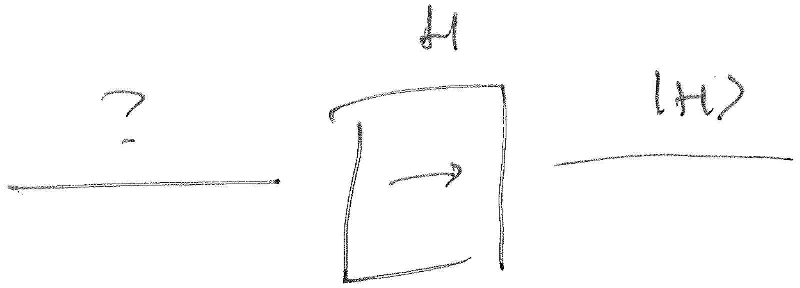
$$|H\rangle = \cos\vartheta\,|\xi\rangle - \sin\vartheta\,|\xi'\rangle$$

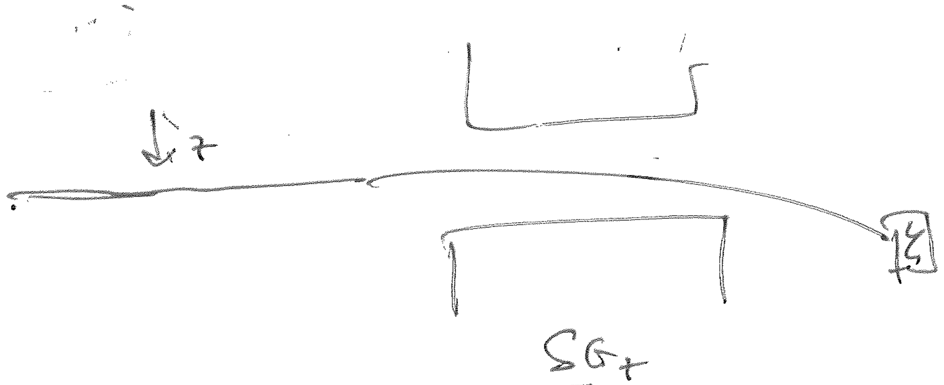
$$|V\rangle = \sin\vartheta\,|\xi\rangle + \cos\vartheta\,|\xi'\rangle$$

$$|\xi\rangle = \cos\vartheta\,|H\rangle + \sin\vartheta\,|V\rangle \qquad |\xi'\rangle = -\sin\vartheta\,|H\rangle + \cos\vartheta\,|V\rangle$$

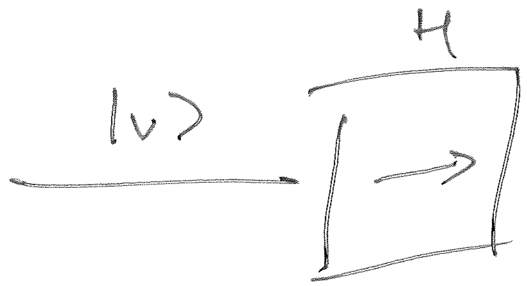
filtre polariseur $\Longleftrightarrow$ $\boxed{\text{polariseur de spin } \uparrow_z}$







rien

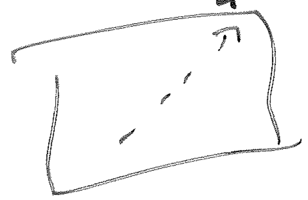


rien

$$\boxed{\begin{array}{l} |\uparrow_z\rangle \leftrightarrow |H\rangle \\ |\downarrow_z\rangle \leftrightarrow |V\rangle \end{array}}$$

$|\varepsilon\rangle = |H\rangle$

I



$|\zeta_{\pi/4}\rangle = \dfrac{|H\rangle + |V\rangle}{\sqrt{2}}$

$|\varepsilon'\rangle = \frac{1}{\sqrt{2}}|\zeta\rangle$

$I' = I/2$

$|\zeta_{\pi/4}\rangle$ $|\uparrow_x\rangle = \dfrac{|\uparrow_z\rangle + |\downarrow_z\rangle}{\sqrt{2}}$

$|\downarrow_x\rangle = \dfrac{-|\uparrow_z\rangle + |\downarrow_z\rangle}{\sqrt{2}}$

$|\zeta'_{3\pi/4}\rangle$

$$P(\uparrow_z \to \uparrow_x) = |\langle \uparrow_x | \uparrow_z \rangle|^2 = \frac{1}{2}$$

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$|\uparrow_y\rangle \to |D\rangle$

$|\downarrow_y\rangle \to |G\rangle$

$$\boxed{\begin{aligned} |\uparrow_y\rangle &= -\frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle + i|\downarrow_z\rangle\right) \\ |\downarrow_y\rangle &= \frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle - i|\downarrow_z\rangle\right) \end{aligned}}$$

$$\boxed{\begin{aligned} |\uparrow_x\rangle &= \frac{|\uparrow_z\rangle + |\downarrow_z\rangle}{\sqrt{2}} \\ |\downarrow_x\rangle &= \frac{-|\uparrow_z\rangle + |\downarrow_z\rangle}{\sqrt{2}} \end{aligned}}$$