COMPOSITION DE 2 MOMENTS ANGULAIRES EN MQ

$\hat{\vec{L}}_1$, $\hat{\vec{L}}_2$

$\downarrow$ $\downarrow$

H_1 H_2

$\vec{L}$: ↗ moment ang. orbitalaire
↘ " " de spin

$$H = H_1 \otimes H_2$$

$\{|\psi_1^\alpha\rangle\}$ base de H_1

$\{|\psi_2^\beta\rangle\}$ base de H_2

$\dim(H_1) = D_1$

$\dim(H_2) = D_2$

$|\psi\rangle \in H$: engendré par $|\psi_1^\alpha\rangle \otimes |\psi_2^\beta\rangle$

$\downarrow$ $\downarrow$

$\mathbb{C}^{D_1}$ $\mathbb{C}^{D_2}$

$D_1 + D_2$ nombres complexes

$|\psi\rangle \in H$

$$|\psi\rangle = \sum_{\alpha=1}^{D_1} \sum_{\beta=1}^{D_2} c_{\alpha\beta} \underbrace{|\psi_1^\alpha\rangle}_{D_1} \otimes \underbrace{|\psi_2^\beta\rangle}_{D_2}$$

$c_{\alpha\beta}$ $\downarrow$ $D_1 \times D_2$

$\dim(H) = D_1 * D_2$

$$c_{\alpha\beta} \neq c_\alpha^{(1)} c_\beta^{(2)}$$

État intriqué ("entangled")

Meca classique

$d = 1$

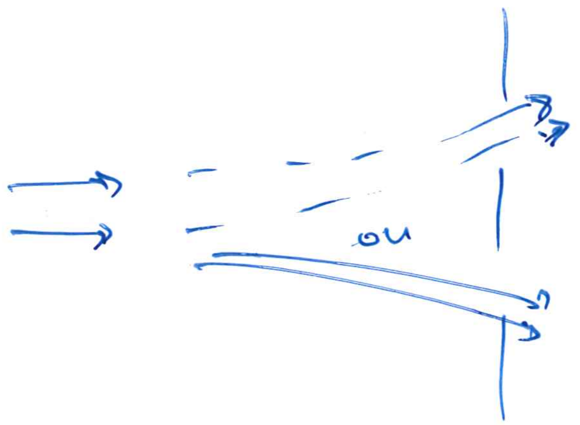
1 particule décrite par un vecteur $\mathbb{R}^2$

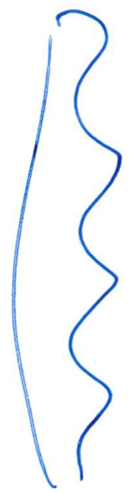
(x, p)

2 particules

(x_1, p_1, x_2, p_2)

$\mathbb{R}^4$





(2)

$\hat{\vec{L}} = \hat{\vec{L}}_1 + \hat{\vec{L}}_2$

$= \underbrace{\hat{\vec{L}}_1 \otimes \hat{\mathbb{1}}_2} + \underbrace{\hat{\mathbb{1}}_1 \otimes \hat{\vec{L}}_2}$

⊙ $\hat{\vec{L}}_1$ agit sur H_1

$\hat{\vec{L}}_1 \otimes \hat{\mathbb{1}}_2$ agit sur $H_1 \otimes H_2$

⊙ $\hat{\vec{L}}_2$ agit sur H_2

$\hat{\mathbb{1}}_1 \otimes \hat{\vec{L}}_2$ agit sur $H_1 \otimes H_2$

$\hat{\vec{L}}$: opérateur de moment angulaire :

$[\hat{L}^\alpha, \hat{L}^\beta] = i\hbar \sum_\gamma \epsilon_{\alpha\beta\gamma} \hat{L}^\gamma$

$[\hat{L}_1^\alpha, \hat{L}_2^\beta] = 0$

$[\hat{L}_1^\alpha + \hat{L}_2^\alpha, \hat{L}_1^\beta + \hat{L}_2^\beta] = [\hat{L}_1^\alpha, \hat{L}_1^\beta] + [\hat{L}_2^\alpha, \hat{L}_2^\beta]$

$= i\hbar \sum_\gamma \epsilon_{\alpha\beta\gamma} (\hat{L}_1^\gamma + \hat{L}_2^\gamma)$

$[\hat{\vec{L}}^2, \hat{L}^z] = 0$

$|\psi_1^\alpha\rangle \otimes |\psi_2^\beta\rangle$

$H_1 \otimes H_2$ engendré par $|l_1 m_1\rangle \otimes |l_2 m_2\rangle$

<u>état séparable</u>

$\hat{\vec{L}}_1^2 |l_1 m_1\rangle = \hbar^2 l_1(l_1+1) |l_1 m_1\rangle$

$\hat{L}_1^z |l_1 m_1\rangle = \hbar m_1 |l_1 m_1\rangle$

<u>base simultanée</u> de $\boxed{\hat{\vec{L}}_1^2, \hat{L}_1^z, \hat{\vec{L}}_2^2, \hat{L}_2^z}$

$[\hat{\vec{L}}^2_{1(2)}, \hat{\vec{L}}^2] = [\hat{\vec{L}}^2_{1(2)}, \cancel{\hat{\vec{L}}_1^2} + \cancel{2\hat{\vec{L}}_1 \cdot \hat{\vec{L}}_2} + \cancel{\hat{\vec{L}}_2^2}] = 0$

$[\hat{\vec{L}}^2_{1(2)}, \hat{L}^z] = [\hat{\vec{L}}^2_{1(2)}, \cancel{\hat{L}_1^z} + \cancel{\hat{L}_2^z}] = 0$

ensemble d'opérateurs compatibles

$\boxed{\hat{\vec{L}}_1^2, \hat{\vec{L}}_2^2, \hat{\vec{L}}^2, \hat{L}^z}$

3

$\hat{L}_1^2, \hat{L}_2^2, \hat{L}^2, \hat{L}^z \rightarrow |l_1 l_2; LM\rangle$ génériquement intriqué

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$l_1 \quad l_2 \quad L \quad M$

$\hat{L}^2 |l_1 l_2; LM\rangle = \hbar^2 L(L+1) |l_1 l_2; LM\rangle \qquad L = 0, \frac{1}{2}, 1, \ldots$

$\hat{L}^z |l_1 l_2; LM\rangle = \hbar M |l_1 l_2; LM\rangle \qquad -L \leq M \leq L$

si je fixe l_1, l_2 : $\underline{(2l_1+1) \times (2l_2+1)}$

$|l_1 m_1\rangle \otimes |l_2 m_2\rangle = |l_1 m_1; l_2 m_2\rangle \qquad |l_1 l_2; LM\rangle$

deux bases possibles pour $\underline{H_1 \otimes H_2}$.

$$|l_1 m_1; l_2 m_2\rangle = \underbrace{\sum_{L=0,\frac{1}{2},1,\ldots}^{\infty} \sum_{M=-L}^{L}}_{(2l_1+1)(2l_2+2)} \underbrace{\langle l_1 l_2; LM | l_1 m_1; l_2 m_2\rangle}_{\sim \delta_{M, m_1+m_2}} |l_1 l_2; LM\rangle$$

coefficients de Clebsch-Gordan

Exemple : somme de deux spins $s = 1/2$

$\hat{S}_1$: $|s_1 m_{s}\rangle = |\uparrow_1\rangle, |\downarrow_1\rangle \qquad \uparrow_z \quad \downarrow_z$

$\hat{S}_2$: $|\uparrow_2\rangle, |\downarrow_2\rangle$

$\hat{S} = \hat{S}_1 + \hat{S}_2 = \hat{S}_1 \otimes \hat{1}_2 + \hat{1}_1 \otimes \hat{S}_2$

$$\begin{cases} |\uparrow\uparrow\rangle = |\uparrow_1\rangle \otimes |\uparrow_2\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{cases}$$

$$\hat{S}^z = S_1^z + S_2^z = \begin{array}{c} |\uparrow\uparrow\rangle \\ \hbar|\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{array} \begin{pmatrix} 1 & & & 0 \\ & 0 & & \\ & & 0 & \\ 0 & & & -1 \end{pmatrix}$$

$M = -1, 0, 1$

$\uparrow$ 2 fois dégénéré

$$\hat{S}^x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad \hat{S}^y = \begin{pmatrix} 0 & -i & -i & 0 \\ i & 0 & 0 & -i \\ i & 0 & 0 & -i \\ 0 & i & i & 0 \end{pmatrix}$$

$$\hat{\vec{S}}^2 = \hbar^2 \begin{array}{c} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{array} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$\boxed{2 = L(L+1) = S(S+1) \Rightarrow S = 1}$ $\qquad L \to S'$

$\lambda = 2\hbar^2, \; 0\hbar^2 \to S = 0$

3 fois dég.

$(1-\lambda)^2 - 1 = 0$

$1 - \lambda = \pm 1 \qquad \lambda = 0, 2$

états propres

$S = 0 \qquad \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = |S=0, M=0\rangle$ $\quad$ état singulet

$S = 1 \qquad |\uparrow\uparrow\rangle = |S=1, M=1\rangle$

$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = |S=1, M=0\rangle$

$|\downarrow\downarrow\rangle = |S=1, M=-1\rangle$

états triplets

$\mathbb{1} + \sigma^x$

$\hat{H} = -(-H)\hat{S}^x + \mathbb{1}$

$\sigma^x \quad 1, -1$

$|\uparrow\rangle + |\downarrow\rangle$

$$\hat S_1^z \times \hat{\mathbb{1}}_2 = \frac{\hbar}{2}\begin{array}{c c}
 & \begin{array}{cccc} |\uparrow\uparrow\rangle & |\uparrow\downarrow\rangle & |\downarrow\uparrow\rangle & |\downarrow\downarrow\rangle \end{array} \\
\begin{array}{c} \langle\uparrow\uparrow| \\ \langle\uparrow\downarrow| \\ \langle\downarrow\uparrow| \\ \langle\downarrow\downarrow| \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \boxed{0} & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}
\qquad
\hat{\mathbb{1}}_1 \otimes \hat S_2^z = \frac{\hbar}{2}\begin{array}{c c}
 & \begin{array}{cccc} |\uparrow\uparrow\rangle & |\uparrow\downarrow\rangle & |\downarrow\uparrow\rangle & |\downarrow\downarrow\rangle \end{array} \\
\begin{array}{c} \langle\uparrow\uparrow| \\ \langle\uparrow\downarrow| \\ \langle\downarrow\uparrow| \\ \langle\downarrow\downarrow| \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}$$

$$\langle\downarrow\uparrow| \; S_1^z \otimes \hat{\mathbb{1}}_2 \; |\uparrow\downarrow\rangle = \langle\downarrow|\hat S_1^z|\uparrow\rangle \times \langle\uparrow|\hat{\mathbb{1}}_2|\downarrow\rangle$$

$$\left(\langle\downarrow| \otimes \langle\uparrow|\right) \hat S_1^z \otimes \hat{\mathbb{1}}_2 \left(|\uparrow\rangle \otimes |\downarrow\rangle\right)$$

$$\hat S_1^x \otimes \hat{\mathbb{1}}_2 = \frac{\hbar}{2}\begin{array}{c c}
 & \begin{array}{cccc} |\uparrow\uparrow\rangle & |\uparrow\downarrow\rangle & |\downarrow\uparrow\rangle & |\downarrow\downarrow\rangle \end{array} \\
\begin{array}{c} \langle\uparrow\uparrow| \\ \langle\uparrow\downarrow| \\ \langle\downarrow\uparrow| \\ \langle\downarrow\downarrow| \end{array} & \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}
\end{array}
\qquad
\hat S_1^x = \frac{\hat S_1^+ + \hat S_1^-}{2}$$

$$\hat S_1^y = \frac{\hbar}{2}\begin{array}{c c}
 & \begin{array}{cccc} & & |\downarrow\uparrow\rangle & |\downarrow\downarrow\rangle \end{array} \\
\begin{array}{c} \langle\uparrow\uparrow| \\ \langle\uparrow\downarrow| \\ \\ \\ \end{array} & \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{pmatrix}
\end{array}$$

$$\hat S_2^x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
\qquad
\hat S_2^y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

Somme de moments angulaires arbitraires

$$|\ell_1 m_1;\ \ell_2 m_2\rangle \quad \rightarrow \quad |\ell_1 \ell_2;\ LM\rangle$$

$$\hat{L}_1^2,\ \hat{L}_1^z \quad \hat{L}_2^2,\ \hat{L}_2^z \qquad \hat{L}_1^2,\ \hat{L}_2^2,\ \hat{L}^2,\ \hat{L}^z$$

$$\hat{L}^z = \hat{L}_1^z + \hat{L}_2^z \qquad \hat{L}^z\,|\ell_1 m_1;\ \ell_2 m_2\rangle = \hbar(m_1+m_2)\,\underbrace{|\ell_1 m_1;\ \ell_2 m_2\rangle}$$

$$\langle \underbrace{\ell_1 \ell_2;}\ LM\,|\,\ell_1 m_1;\ \ell_2 m_2\rangle \sim \delta_{m_1+m_2,\,M} \qquad \boxed{M = m_1 + m_2}$$

$$\hat{L}^z\,|\ell_1 \ell_2;\ LM\rangle = \hbar M\,\underbrace{|\ell_1 \ell_2;\ LM\rangle} \qquad |M_{max}| = \ell_1 + \ell_2$$

$$|\ell_1, m_1 = \ell_1;\ \ell_2;\ m_2 = \ell_2\rangle = |\ell_1 \ell_1;\ \ell_2 \ell_2\rangle$$

le seul état qui possède $M = \ell_1 + \ell_2$

$$|\ell_1 \ell_2;\ LM\rangle = |\ell_1 \ell_2;\ \ell_1+\ell_2,\ \ell_1+\ell_2\rangle$$

$$\rightarrow\ |\ell_1 \ell_1;\ \ell_2 \ell_2\rangle = |\ell_1 \ell_1;\ L = \ell_1+\ell_2,\ M = \ell_1+\ell_2\rangle$$

$$\hat{L}^+ = \hat{L}_1^+ + \hat{L}_2^+ \qquad \hat{L}^+\,|\ell_1 \ell_1;\ \ell_2\ell_2\rangle = (\hat{L}_1^+ + \hat{L}_2^+)\,|\ell_1\ell_1;\ \ell_2\ell_2\rangle = 0$$

$$\hat{L}^- = \hat{L}_1^- + \hat{L}_2^-$$

$$\hat{L}^+\,|\ell_1 \ell_2;\ \underset{\uparrow}{L},\ M = \ell_1+\ell_2\rangle = 0$$

$$L = \ell_1 + \ell_2$$

$$M_{max} = \ell_1 + \ell_2 \Rightarrow \boxed{L_{max} = \ell_1 + \ell_2}$$

$|\ell_1 \ell_1 ; \ell_2 \ell_2\rangle = |\ell_1 \ell_2 ; L_{max}, L_{max}\rangle$

$\underline{\ell_1 \ \ell_2 \quad \text{fixés}}$

$$\boxed{|\ell_1 \ell_2\rangle = |L_{max} = \ell_1 + \ell_2 , M = L_{max}\rangle}$$

$|\ell_1 m_1 ; \ell_2 m_2\rangle \rightarrow |m_1 m_2\rangle$

$|\ell_1 \ell_2 ; LM\rangle \rightarrow |LM\rangle$

$$\begin{cases} \hat{L}^{\pm} |LM\rangle = \hbar\sqrt{L(L+1) - M(M\pm 1)}\ |L, M\pm 1\rangle \\ \hat{L}_1^{\pm} |m_1 m_2\rangle = \hbar\sqrt{\ell_1(\ell_1+1) - m_1(m_1 \pm 1)}\ |m_1 \pm 1, m_2\rangle \\ \hat{L}_2^{\pm} \qquad\qquad\quad 2 \quad 2 \qquad\qquad 2 \quad 2 \qquad\quad |m_1, m_2 \pm 1\rangle \end{cases}$$

$$\hat{L}^{-} |L_{max}, L_{max}\rangle = \underbrace{\sqrt{L_{max}(L_{max}+1) - L_{max}(L_{max}-1)}}_{\sqrt{2L_{max}}}\ |L_{max}, L_{max}-1\rangle$$

$$= \sqrt{2L_{max}}\ |L_{max}, L_{max}-1\rangle$$

$$= (\hat{L}_1^- + \hat{L}_2^-)|\ell_1, \ell_2\rangle = \sqrt{2\ell_1}\ |\ell_1 - 1, \ell_2\rangle + \sqrt{2\ell_2}\ |\ell_1, \ell_2 - 1\rangle$$

$\underline{L_{max} = \ell_1 + \ell_2}$

$\underline{M = L_{max} - 1}$

$$|L_{max}, \overset{M}{L_{max}-1}\rangle$$

état orthogonal à celui ci

et qui a encore $M = L_{max} - 1 = \ell_1 + \ell_2 - 1$

$$|\overset{?}{L}, M = L_{max}-1\rangle \sim -\sqrt{2\ell_2}\ |\ell_1 - 1, \ell_2\rangle + \sqrt{2\ell_1}\ |\ell_1, \ell_2 - 1\rangle$$

$L = L_{max} - 1$

$\longrightarrow L$

$\downarrow M$

$$|L_{max}, L_{max}\rangle = |l_1, l_2\rangle$$

$$\Big\downarrow \hat{L}^-$$

$$|L_{max}, L_{max}-1\rangle = \frac{1}{\sqrt{2L_{max}}}\left(\sqrt{2l_1}\,|l_1-1, l_2\rangle + \sqrt{2l_2}\,|l_1, l_2-1\rangle\right) \xrightarrow{\text{orthogonaliser}} |L_{max}-1, L_{max}-1\rangle$$

$$\Big\downarrow \hat{L}^- \qquad\qquad \Big\downarrow \hat{L}^-$$

$$|L_{max}, L_{max}-2\rangle \qquad |L_{max}-1, L_{max}-2\rangle \xrightarrow{\text{ortho.}} |L_{max}-2, L_{max}-2\rangle$$

$$M = L_{max}-2$$

$$\Big\downarrow \hat{L}^- \qquad\qquad \Big\downarrow \hat{L}^-$$

$$|L_{max}, L_{max}-3\rangle \qquad |L_{max}-1, L_{max}-3\rangle$$

$\vdots$

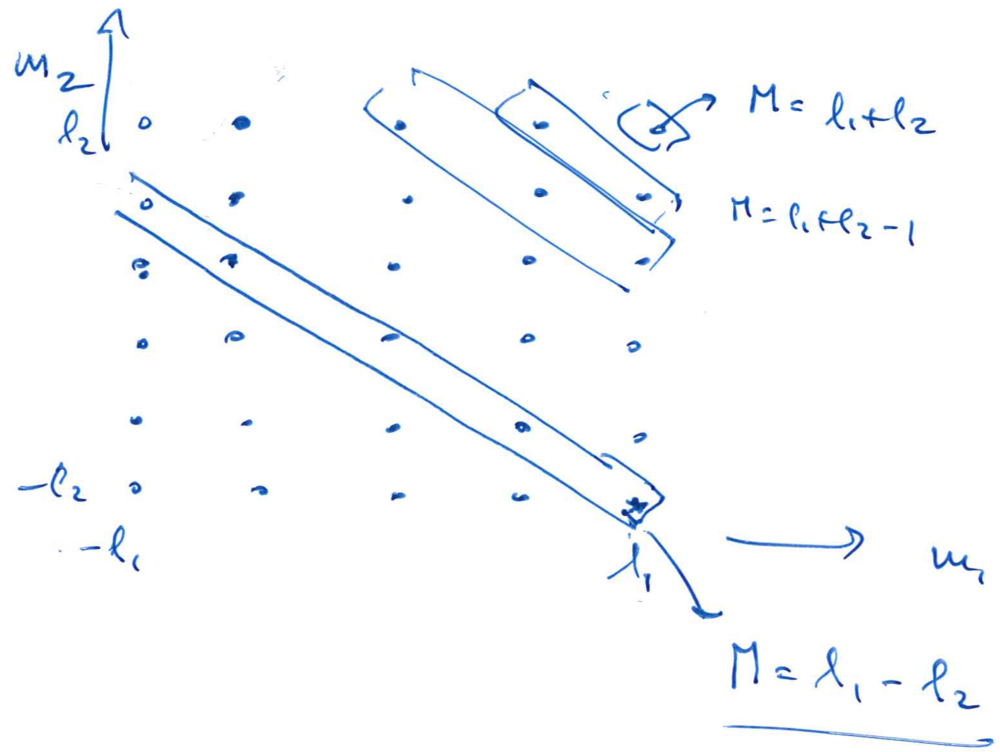
$$|L_{max}, -L_{max}\rangle$$

$(m_1\, m_2)$: 3 combinaisons possibles m_1, m_2 tel que $M = m_1 + m_2 = L_{max} - 2$

M : $|L_{max}, M\rangle, |L_{max} - 1, M\rangle$ —, $|L_{max} - n = M, M\rangle$

$n = L_{max} - M$ $\qquad |M|_{min} = |m_1 + m_2| = |l_1 - l_2|$

$\boxed{L_{max} - M + 1}$: # d'états qui ont M



valeur maximale : $L = L_{max}$

" minimale

$L = L_{max} - n_{max}$

$n_{max} = L_{max} - |l_1 - l_2|$

$L_{min} = L_{max} - L_{max} + |l_1 - l_2| = |l_1 - l_2|$

$$\boxed{L = |l_1 - l_2|,\ |l_1 - l_2| + 1,\ \ldots,\ l_1 + l_2}$$