SOMME DES MOMENTS ANGULAIRES

$(2l_1+1)(2l_2+1)$

$$|l_1 m_1 ; l_2 m_2\rangle \rightarrow |l_1 l_2 ; L M\rangle$$

$$\hat{L}_1^2, \hat{L}_1^z ; \hat{L}_2^2, \hat{L}_2^z \qquad \hat{L}_1^2, \hat{L}_2^2, \hat{L}^2, \hat{L}^z$$

$$\begin{cases} \hat{\vec{L}} = \vec{L}_1 + \vec{L}_2 \\ \hat{L}^z = \hat{L}_1^z + \hat{L}_2^z \end{cases}$$

$$\hat{L}^- = \hat{L}^x - i\hat{L}^y$$

$$\hat{L}^- = \hat{L}_1^- + \hat{L}_2^-$$

$$L_{max} = l_1 + l_2$$

$$|l_1 \overset{(m_1)}{l_1} ; l_2 \overset{(m_2)}{l_2}\rangle = |l_1 l_2 ; L_{max} L_{max}\rangle$$

$$\perp \quad |l_1 l_2\rangle \qquad |L_{max} L_{max}\rangle$$

$$\longrightarrow L$$

M

$$|L_{max} L_{max}\rangle$$

$$\downarrow \hat{L}^-$$

$$|L_{max}, \overset{M}{L_{max}-1}\rangle \xrightarrow{\text{ortho}} |L_{max}-1, \overset{M}{L_{max}-1}\rangle$$

$$\downarrow \hat{L}^- \qquad \downarrow \hat{L}^-$$

$$|L_{max}, L_{max}-2\rangle \qquad |L_{max}-1, L_{max}-2\rangle \xrightarrow{\text{ortho}} |L_{max}-2, L_{max}-2\rangle \qquad |L_{min}, L_{min}\rangle$$

$$\downarrow \qquad \downarrow \hat{L}^- \qquad \downarrow \hat{L}^- \qquad \downarrow \hat{L}^-$$

$$\boxed{L_{min} = L_{max} - n_{max} + 1}$$

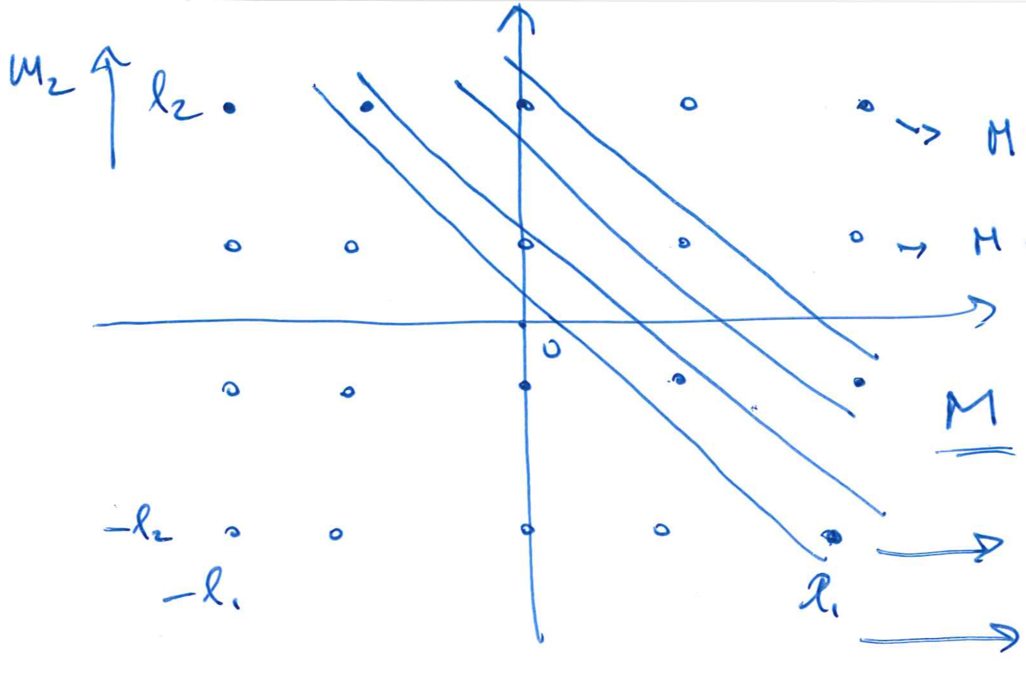
$$|L_{max}-2, -L_{max}+2\rangle$$

$$|L_{max}-1, -L_{max}+1\rangle$$

$$|L_{max}, -L_{max}\rangle$$

$$2(l_1+l_2)+1$$

L_{min}



$\to M = l_1 + l_2 = L_{max}$

$\to M = L_{max} - 1$

$\underline{M} = L_{max-2}$

$\to \boxed{M = l_1 - l_2}$

$$\underline{M = m_1 + m_2}$$

$$\underline{m_2 = -m_1 + M}$$

$$\langle l_1 m_1 ; l_2 m_2 | l_1 l_2 ; L M \rangle \sim \delta_{m_1+m_2, M}$$

$M = L_{max}$ 1

$M = L_{max} - 1$ 2

$$n_M = L_{max} - |M| + 1$$

$$n_{max} = L_{max} - |l_1 - l_2| + 1$$

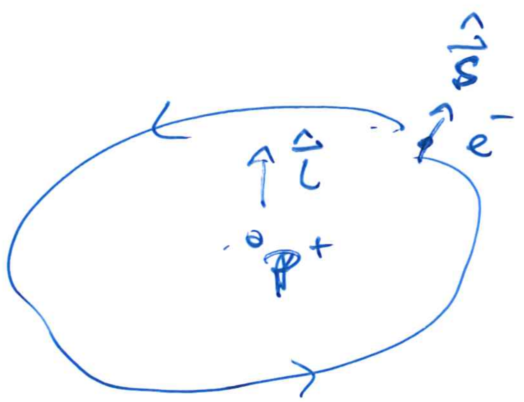
$$\boxed{|l_1 - l_2| \le L \le l_1 + l_2}$$

$$\boxed{L = |l_1 - l_2|, |l_1 - l_2| + 1, \ldots, l_1 + l_2}$$

$$L_{min} = L_{max} - L_{max} + |l_1 - l_2| - 1 + 1$$
$$= |l_1 - l_2|$$

<u>Couplage spin-orbite</u>

$$\hat{H} = \hat{H}_0 + \hat{H}_{SO} + \ldots$$



$$\hat{H}_{SO} = v(r)\, \hat{\vec{L}} \cdot \hat{\vec{S}}$$

$$\underbrace{\hat{L}^2, \hat{L}_z}_{|l m\rangle}, \quad \hat{S}^2, \hat{S}_z \quad |S, m_s\rangle$$

moment angulaire total électronique

$$\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$$

$$\hat{\vec{J}}^2 = (\hat{\vec{L}} + \hat{\vec{S}})^2 = \hat{\vec{L}}^2 + \hat{\vec{S}}^2 + 2\hat{\vec{L}}\cdot\hat{\vec{S}}$$

$$\hat{\vec{L}}\cdot\hat{\vec{S}} = \frac{1}{2}\left(\hat{\vec{J}}^2 - \hat{\vec{L}}^2 - \hat{\vec{S}}^2\right)$$

base $|\ell\, s;\, J M_J\rangle$ qui diagonalise

$\hat{L}^2, \hat{\vec{S}}^2, \hat{\vec{J}}^2, \hat{J}_z$

Remarque mathématique

$\hat{\vec{L}}_1, \hat{\vec{L}}_2 \rightarrow \hat{\vec{L}} = \hat{\vec{L}}_1 + \hat{\vec{L}}_2$

$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$

$|\ell_1 m_1\rangle \otimes |\ell_2 m_2\rangle$

$|\ell_1, \ell_2; LM\rangle$ sous espaces $|LM\rangle$: L fixé

$-L \leq M \leq L$

$$\mathcal{H} = \bigoplus_{L=|\ell_1-\ell_2|}^{\ell_1+\ell_2} \mathcal{H}_L = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$(2\ell_1+1)(2\ell_2+1)$$

$$\sum_{L=|\ell_1-\ell_2|}^{\ell_1+\ell_2} (2L+1) =$$

$$|\psi\rangle_{12} = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$= \alpha|\psi_1\rangle \otimes |\psi_2\rangle + \beta|\phi_1\rangle \otimes |\phi_2\rangle$$

$$\langle \phi_1 | \psi_1\rangle = 0$$

$$\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$\mathcal{H}_L$ engendré par $|LM\rangle$

PARTICULES IDENTIQUES EN MQ

Deux particules identiques : même charge q
masse m
spin S
⋮

$\hat{H} = \hat{H}_1 + \hat{H}_2$ 2 particules non interagissantes

$\hat{H}_{1,2} = \frac{\hat{\vec{p}}^{\,2}_{1(2)}}{2m} + U(\hat{\vec{r}}_{1(2)})$ $\hat{H}_1 |\psi_\alpha\rangle = E_\alpha |\psi_\alpha\rangle$

$|\Psi_{12}\rangle = \sum_{\alpha\beta} c_{\alpha\beta} |\psi_\alpha^{(1)}\rangle \otimes |\psi_\beta^{(2)}\rangle$

mesure l'énergie : $E = E_0 + E_1$

$|\Psi\rangle = \alpha\, |\psi_0\rangle \otimes |\psi_1\rangle + \beta\, |\psi_1\rangle \otimes |\psi_0\rangle$ $|\alpha|^2 + |\beta|^2 = 1$

Principe : $\boxed{\text{particules quantiques identiques sont INDISCERNABLES}}$

opérateur permutation $\hat{P}_{12} |\psi_1\rangle \otimes |\psi_2\rangle = |\psi_2\rangle \otimes |\psi_1\rangle$

$\hat{P}_{12} |\Psi\rangle = e^{i\phi} |\Psi\rangle$

$\boxed{d \geq 3}$: seulement deux valeurs possibles pour ϕ :

$\boxed{\phi = 0}$ S entier : BOSONS

$S=0$ Boson de Higgs

$S=1$ photon, gluon, ...

$S=2$ graviton (?)

$\hat{P}|\Psi\rangle = |\Psi\rangle$

symétrique

Théorème de spin-statistique

$\boxed{\phi = \pi}$ S demi-entier : FERMIONS

$S = 1/2$ electron, proton, neutron, ...

$\hat{P}|\Psi\rangle = -|\Psi\rangle$

antisymétrique

$\boxed{d = 2}$ ϕ quelconque : ANYONS

Exemple : 2 électrons

$|\psi\rangle = \underset{\text{spatiale}}{|\phi\rangle} \otimes \underset{\text{spin}}{|\chi\rangle} \rightarrow \phi(\vec{r}) \otimes |\chi\rangle$

$|\uparrow\rangle, |\downarrow\rangle$

$|\Psi_{12}\rangle = \Phi(\vec{r}_1, \vec{r}_2) \otimes |\chi_{12}\rangle$

$(\neq \phi_1(\vec{r}_1)\, \phi_2(\vec{r}_2))$

$\hat{P}|\Psi_{12}\rangle = -|\Psi_{12}\rangle$ $\rightarrow$ 1) $\Phi(\vec{r}_1, \vec{r}_2) = -\Phi(\vec{r}_2, \vec{r}_1)$ $|\chi_{12}\rangle$ symétrique

$\downarrow$

$\rightarrow$

$2)\quad \Phi(\vec r_1, \vec r_2) = \Phi(\vec r_1, \vec r_2)$

$|\uparrow\uparrow\rangle, \; |\downarrow\downarrow\rangle, \; \frac{1}{\sqrt 2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

triplets $\quad S = 1$

$\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt 2}$

singulet $\quad S=0$

Exemple : N particules sans interaction (identiques)

indice de particule

$$|\Psi_N\rangle = |\phi^{(1)}_{\alpha_1}\rangle \otimes |\phi^{(2)}_{\alpha_2}\rangle \otimes \dots \otimes |\phi^{(N)}_{\alpha_N}\rangle$$

$|\phi_\alpha\rangle$ base de H_1

$H_N = H_1^{\otimes N}$

Bosons $\quad$ symétrisation de l'état

$$|\Psi_N^{(B)}\rangle = C \sum_P |\phi^{(1)}_{\alpha_{P(1)}}\rangle \otimes |\phi^{(2)}_{\alpha_{P(2)}}\rangle \otimes \dots \otimes |\phi^{(N)}_{\alpha_{P(N)}}\rangle$$

$P(1) \to 2$
$P(2) \to 1$
$P(i) = j$

Fermions $\quad$ anti-symétrisation

$$|\Psi_N^{(F)}\rangle = \frac{1}{\sqrt{N!}} \sum_P (-1)^{p(P)} |\phi^{(1)}_{\alpha_{P(1)}}\rangle \otimes |\phi^{(2)}_{\alpha_{P(2)}}\rangle \otimes \dots \otimes |\phi^{(N)}_{\alpha_{P(N)}}\rangle$$

$$= \frac{1}{\sqrt{N!}} \begin{vmatrix} |\phi^{(1)}_{\alpha_1}\rangle & |\phi^{(1)}_{\alpha_2}\rangle & \cdots & |\phi^{(1)}_{\alpha_N}\rangle \\ \vdots & & & \vdots \\ |\phi^{(N)}_{\alpha_1}\rangle & |\phi^{(N)}_{\alpha_2}\rangle & \cdots & |\phi^{(N)}_{\alpha_N}\rangle \end{vmatrix}$$

<u>Determinant de Slater</u>

$\phi_{\alpha_i} = \phi_{\alpha_j} \rightarrow \underline{|\ \ | = 0}$

$\boxed{\text{Principe d'exclusion de Pauli}}$