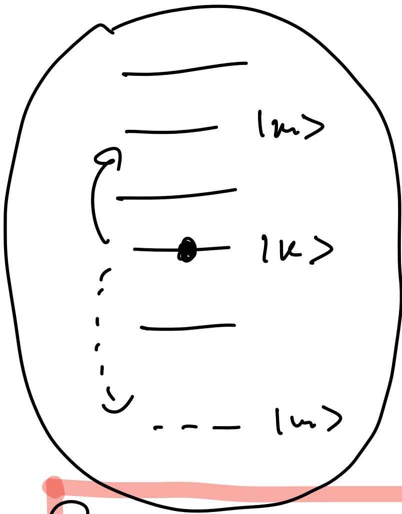
Interaction lumière - atome → cohérente



$\omega, \vec{k}, \vec{\varepsilon}$ Sources monochromatique

absorption $\quad \omega_{mk} = \dfrac{E_m - E_k}{\hbar} > 0 \qquad \vec{A}, \phi$

émission stimulée $\quad \omega_{nk} < 0$

$$\boxed{\vec{A}(\vec{r},t) = \frac{A_0}{2}\left[\vec{\varepsilon}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)+i\phi} + c.c.\right]}$$

Jauge de Coulomb

$$\vec{\nabla}\cdot\vec{A} = 0$$

$$\nabla^2 \phi = \frac{\rho(\vec{r},t)}{\varepsilon_0}$$

$$\phi(\vec{r},t) = \int d^3r' \frac{\rho(\vec{r}',t)}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|} \simeq 0$$

$$|\vec{r}-\vec{r}'| \to \infty$$

$$H_0 \psi_n = E_n \psi_n$$

$$\psi(\vec{r},t)\Big|_{t=0} = \psi_k(\vec{r})$$

$$\psi(\vec{r},t)\Big|_{t>0} = \sum_m c_m^{(1)}(t)\, e^{-\frac{i}{\hbar}E_m t}\, \psi_m(\vec{r})$$

$$c_m^{(1)}(t) = \frac{eA_0}{2m}\left[M_{mk} \frac{e^{i(\omega_{mk}-\omega)t}-1}{i(\omega_{mk}-\omega)} + M^*_{km}\left(e^{i(\omega_{mk}+\omega)t}-1\right)/i(\omega_{mk}+\omega)\right]$$

$$M_{mk} = \langle m | \vec{\varepsilon} \cdot \vec{p} \, e^{i \vec{k} \cdot \vec{r}} | k \rangle$$

proba d'être en ψ_m

$$\lim_{t \to \infty} \frac{|c_{mk}^{(1)}(t)|^2}{t} = \Gamma_{k \to m}$$

taux de transition de k à m $\omega_{mk} > 0$

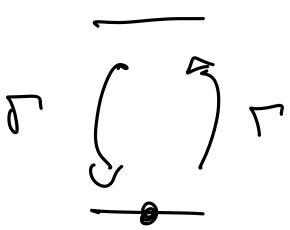
$$= \frac{2\pi}{\hbar^2} \left(\frac{e A_0}{2m} \right)^2 \left[|M_{mk}|^2 \delta(\omega - \omega_{mk}) + |M^*_{km}|^2 \delta(\omega + \omega_{mk}) \right]$$

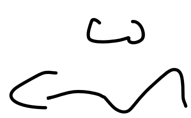
Règle d'or de Fermi 1

$$H(t) = H_0 + V e^{-i\omega t} + V^\dagger e^{i\omega t}$$

$$V = \frac{e A_0}{2m} \vec{\varepsilon} \cdot \vec{p} \, e^{i \vec{k} \cdot \vec{r}}$$

$$\Gamma_{k \to m} = \frac{2\pi}{\hbar^2} |\langle m | V | k \rangle|^2 \delta(\omega - \omega_{mk})$$





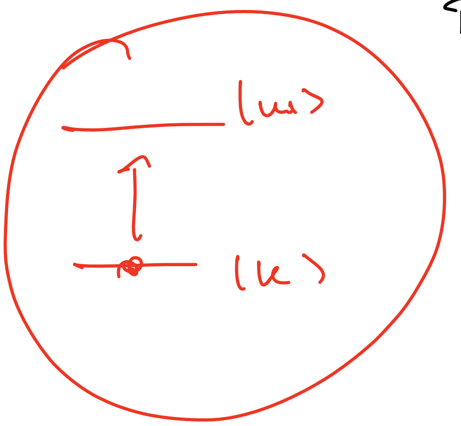
lumière incohérente & polychromatique :

$$\vec{A}(\vec{r},t) = \sum_{\vec{k},\vec{\epsilon}} A_{\vec{k},\vec{\epsilon}} \left[\vec{\epsilon}\, e^{i(\vec{k}\cdot\vec{r} - \omega_{\vec{k}} t)}\, e^{i\phi_{\vec{k},\vec{\epsilon}}} + c.c. \right]$$

$$\omega_{\vec{k}} = ck$$

$$\langle m | \vec{\epsilon}\cdot\vec{p}\, e^{i\vec{k}\cdot\vec{r}} | n \rangle$$

$$c_m^{(1)}(t) = \sum_{\vec{k},\vec{\epsilon}} \frac{eA_{\vec{k},\vec{\epsilon}}}{2m} \left[e^{i\phi_{\vec{k},\vec{\epsilon}}}\, M_{mn}^{(\vec{k},\vec{\epsilon})}\, \frac{e^{i(\omega_{mn}-\omega)t} - 1}{i(\omega_{mn}-\omega)} + e^{-i\phi_{\vec{k},\vec{\epsilon}}} \left(M_{nm}^{(\vec{k},\vec{\epsilon})}\right)^* \frac{e^{i(\omega_{mn}+\omega)t} - 1}{i(\omega_{mn}+\omega)} \right]$$



$$\lim_{t\to\infty} \frac{|c_m^{(1)}(t)|^2}{t} = \left(\frac{2\pi}{\hbar^2}\right) \sum_{\vec{k},\vec{\epsilon}} \left(\frac{eA_{\vec{k},\vec{\epsilon}}}{2m}\right)^2 \left|M_{mn}^{(\vec{k},\vec{\epsilon})}\right|^2 \delta(\omega_{\vec{k}} - \omega_{mn})$$

$$+ \sum_{\vec{k},\vec{\epsilon}} \left(\frac{eA_{\vec{k},\vec{\epsilon}}}{2m}\right)^2 \left|M_{mn}^{(\vec{k},\vec{\epsilon})}\right|^2 \delta(\omega_{\vec{k}} + \omega_{mn})$$

$$\cancel{+ \sum_{\vec{k},\vec{\epsilon}} \sum_{(\vec{k}',\vec{\epsilon}') \neq (\vec{k},\vec{\epsilon})} \lim_{t\to\infty} \frac{1}{t} \frac{eA_{\vec{k},\vec{\epsilon}}}{2m} \frac{eA_{\vec{k}',\vec{\epsilon}'}}{2m} M_{mn}^{(\vec{k},\vec{\epsilon})} \left(M_{mn}^{(\vec{k}',\vec{\epsilon}')}\right)^* \frac{e^{i(\omega_{mn} - \omega_{\vec{k}})t} - 1}{i(\omega_{mn} - \omega_{\vec{k}})} \times \left(\frac{e^{i(\omega_{mn} - \omega_{\vec{k}'})t} - 1}{i(\omega_{mn} - \omega_{\vec{k}'})t}\right)^* e^{i\left(\phi_{\vec{k},\vec{\epsilon}} - \phi_{\vec{k}',\vec{\epsilon}'}\right)}}$$

$$\hookrightarrow \; \cancel{(- \;\; -) + (+ \;\; +) + (+ \;\; -) + (- \;\; +)}$$

1

$$\lim_{t\to\infty} \frac{1}{t} \left| \frac{e^{i(\omega_{mn} \pm \omega_{\vec{k}})t} - 1}{i(\omega_{mn} \pm \omega_{\vec{k}})} \right|^2$$

$$= 2\pi\, \delta(\omega_{\vec{k}} \pm \omega_{mn})$$

Termes mutuellement exclusifs

$\omega_{mn} - \omega_{\vec{k}} \simeq 0$ si $\omega_{\vec{k}} \neq \omega_{\vec{k}'}$

(2) $e^{i(\Phi_{\vec{k},\vec{\epsilon}} - \Phi_{\vec{k}',\vec{\epsilon}'})}$

$\sim$ aléatoire

$\omega_{mn} > 0$ absorption

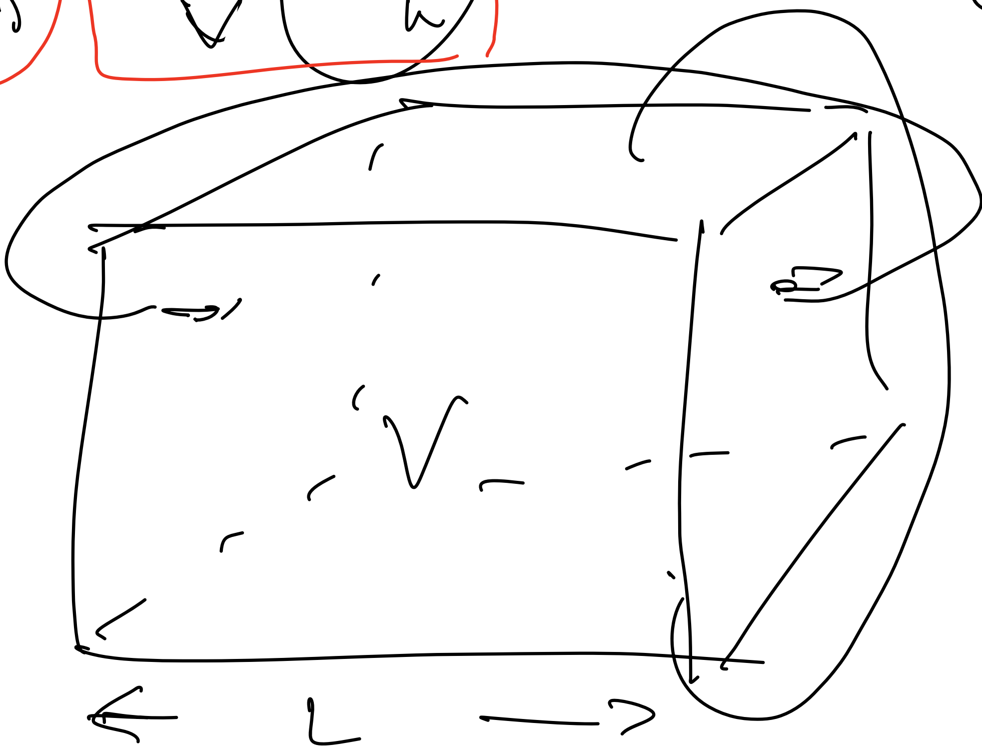
$$\underbrace{\Gamma_{n\to m} = \left(\frac{2\pi}{\hbar^2}\right) \sum_{\vec{k},\vec{\epsilon}} \left(\frac{e A_{\vec{k},\vec{\epsilon}}}{2m}\right)^2 \left|M_{mn}^{(\vec{k},\vec{\epsilon})}\right|^2 \delta(\omega_{\vec{k}} - \omega_{mn})}$$

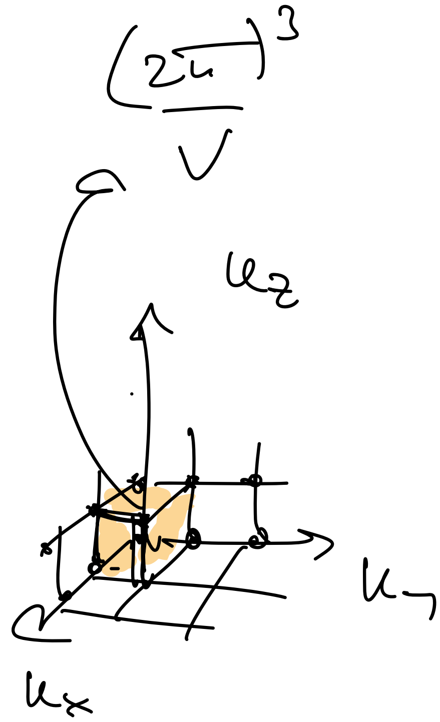
. hypothèse

$\cancel{A}_{\vec{k},\vec{\epsilon}} = A_{\underline{\omega}}$ lumière uni directionnelle ni polarisée

$\downarrow$

$$\frac{V}{(2\pi)^3}\ \boxed{\frac{(2\pi)^3}{V}\sum_{\vec{k}}}\ (\dots) \overset{V\to\infty}{\simeq} \frac{V}{(2\pi)^3}\int d^3k\,(\dots)$$





$$V = L^3$$

$$\boxed{\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z)}$$

$$n_x, n_y, n_z \in \mathbb{Z}$$

$$L \gg \lambda \leftarrow \vec{k} \quad : \quad \omega_{\vec{k}} \simeq \omega_{mn}$$

$$\Gamma_{n\to m} \frac{2\pi}{\hbar^2} \left|\frac{V}{(2\pi)^3}\sum_{\vec{\epsilon}}\int d^3k \left(\frac{eA_\omega}{2m}\right)^2 \left|M_{mn}^{(\vec{k},\vec{\epsilon})}\right|^2 \delta(\omega_{\vec{k}} - \omega_{mn})\right.$$

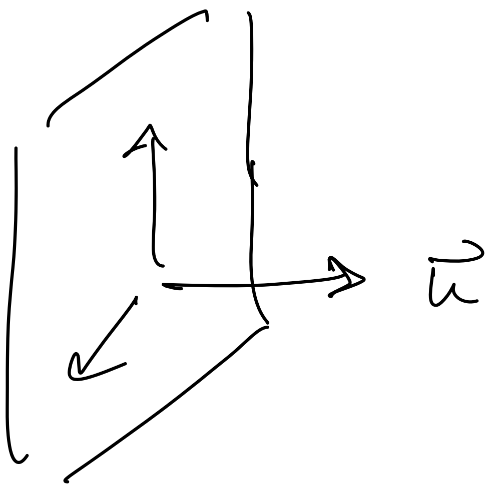
$$\omega_{\vec{k}} = c|\vec{k}|$$

$$= \frac{2\pi}{\hbar^2} \frac{V}{(2\pi)^3} \int_0^\infty dk\, k^2 \left(\frac{eA_\omega}{2m}\right)^2 \delta(\omega_k - \omega_{mn})$$

$$\sum_{\vec{\epsilon}} 4\pi \underbrace{\frac{1}{4\pi}\int d\Omega \left|M_{mn}^{(\vec{k},\vec{\epsilon})}\right|^2}$$

(2)

$$\overline{\left|M_{mn}^{(\vec{k},\vec{\epsilon})}\right|^2}$$ moyenne angulaire



$$\Gamma_{n\to m} = \frac{2\pi}{\hbar^2}(2)\frac{V}{(2\pi)^3}\int_0^\infty dk\,4\pi k^2 \frac{(eA_\omega)^2}{(2m)^2}\delta(\omega_k - \omega_{mn})\;\overline{\left|M_{mn}^{(\cancel{\vec{k}},\vec{\epsilon})}\right|^2}$$

$$1$$

$$M_{mn}^{(\vec{k},\vec{\epsilon})} = \langle m|\, \cancel{e^{i\vec{k}\cdot\vec{r}}}\, \vec{\epsilon}\cdot\vec{p}\, |n\rangle = M_{mn}^{(\vec{\epsilon})}$$

$\vec{k}\cdot\vec{r} \ll 1$ "approximation de dipole"

$k \simeq \frac{2\pi}{\lambda}$ $\lambda \simeq 10^{-7}\,m$

$|\vec{r}| \simeq a_0$

$$2\, dk\; 4\pi k^2 \frac{V}{(2\pi)^3} = V\, g(\omega)\, d\omega$$

$\uparrow$ $\uparrow$ $c\,dk$

densité d'états (par unité de volume)

$\omega = ck$

$$\boxed{g(\omega) = \frac{\omega^2}{\pi^2 c^3}}$$

$$\Gamma_{k\to m} = \frac{2\pi}{\hbar^2} V \int_0^\infty d\omega\; g(\omega)\, \overline{\left|M^{(\vec{\varepsilon})}_{mk}\right|^2}\, \delta(\omega-\omega_{mk})\left(\frac{eA(\omega)}{2m}\right)^2$$

$$= V g(\omega_{mk}) \left|M^{(\vec{\varepsilon})}_{mk}\right|^2 \frac{(e\,A(\omega_{mk}))^2}{(2m)^2}$$

$$A(\omega) = \sqrt{\frac{2\rho(\omega)}{\epsilon_0 \omega^2}}$$

$$\rho(\omega) = \frac{\hbar\omega\, n(\omega)}{V}$$

$$\Gamma_{k\to m} = V_g(\omega_{mk})\, \rho(\omega_{mk})\, \frac{\cancel{2}\pi}{\hbar^2}\, \frac{\cancel{2} e^2 \overline{\left|M_{mk}^{(\vec\epsilon)}\right|^2}}{\cancel{4}\epsilon_0 \omega_{mk}^2 m^2}$$

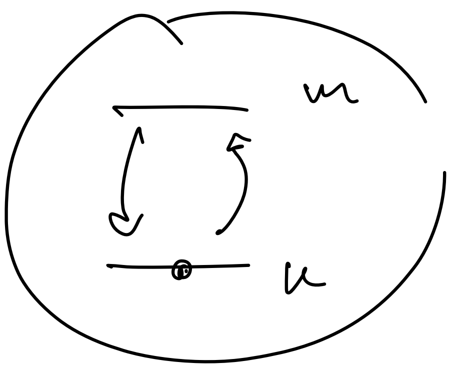
absorption

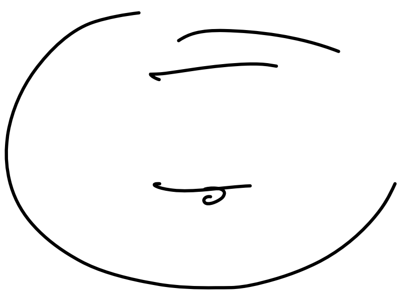
$B_{k\to m}$

$$B_{k\to m} = \frac{\pi e^2 \overline{\left|M_{mk}^{(\vec\epsilon)}\right|^2}}{\hbar^2 \epsilon_0 \omega_{mk}^2 m^2}$$

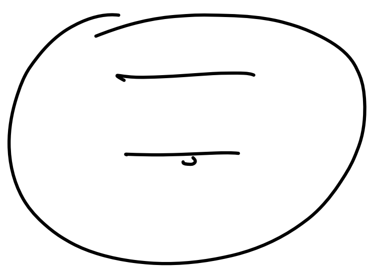
$= \Gamma_{m\to k}$

emission stimulée





ensemble d'atomes



$\underline{\underline{N_n}}$, $\underline{\underline{N_m}}$

$$\frac{dN_n}{dt} = \Gamma_{m\to n} N_m - \Gamma_{n\to m} N_n = 0$$

$$\Gamma(N_m - N_n) = 0$$

$$N_m = N_n \qquad \text{absurd}$$

<u>Radiation de corps noir</u>

T temp.

$$N_m = N\, p_m(T) \sim N e^{-\frac{E_m}{k_B T}}$$

$k_B = 1.38 \times 10^{-23}$ J/K

$$\frac{N_m}{N_k} = e^{-\frac{E_m - E_n}{k_B T}} = e^{-\frac{\hbar \omega_{mn}}{k_B T}} < 1$$

$\omega_{mn} > 0$

Einstein (1916-17)

$$\frac{dN_m}{dt} = \Gamma (N_k - N_m) - \underbrace{A_{m \to n} N_m}_{}$$

émission spontanée

$$= (\Gamma + A_{m \to n}) - \Gamma \frac{N_n}{N_m} = 0$$

$$(\Gamma + A_{m \to n}) \frac{N_m}{N_n} = \Gamma$$

$$\Gamma + A_{m \to n} = \Gamma \left(1 - e^{-\frac{\hbar \omega_{mn}}{k_B T}}\right)$$

$$= \Gamma + A = \Gamma\, e^{\frac{\hbar\omega_{mn}}{k_B T}}$$

$$\Gamma\left(1 + e^{\frac{\hbar\omega_{mn}}{k_B T}}\right) = A_{m\to n}$$

$$A_{m\to n} = \sqrt{g(\omega_{mn})}\, \rho_T(\omega_{mn})\, B_{m\to n}$$

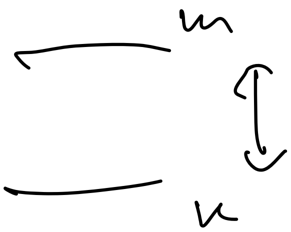
$$\to \left(1 + e^{\hbar\omega_{mn}/k_B T}\right)$$

indépendant de T

$$\boxed{\rho(\omega_{mn}) = \frac{\hbar\omega_{mn}}{e^{\hbar\omega_{mn}/k_B T} - 1}}$$

Distribution de Planck

$$A_{m\to n} = \frac{V\hbar\,\omega_{mn}^3}{\pi^2 c^3}\, B_{m\to n}$$



$$E_m - E_n = \hbar\omega_{mn}$$

stimulée

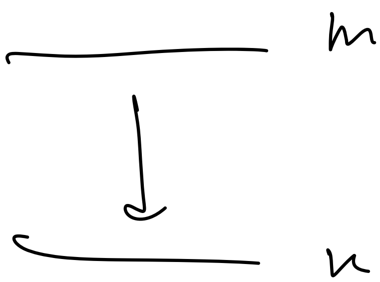
$$\Gamma'_{m\to n} = \Gamma_{m\to n} + A_{m\to n}$$

$$= \cancel{V} g(\omega) \frac{n(\omega)\hbar\omega}{\cancel{V}} B_{m\to n} + V g(\omega) \frac{1}{V} \hbar\omega B$$

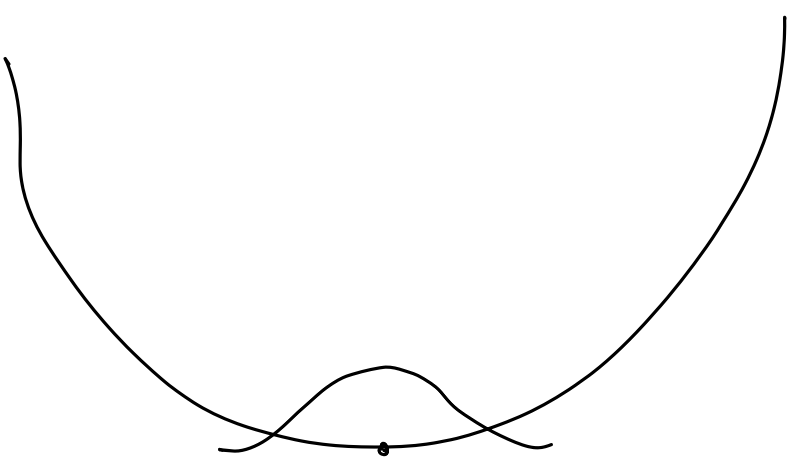
$$= g(\omega)\hbar\omega \left(n(\omega) + 1\right) B_{m\to n}$$

spontanée

Stimulée



électrodynamique quantique



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$