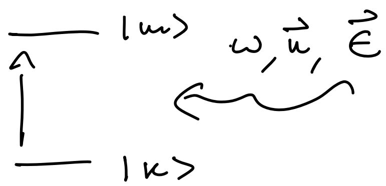
Interaction lumière – atomes

Absorption / émission stimulée



$$\mathcal{H}(t) = \mathcal{H}_{at} + \overbrace{\left[\left(\frac{eA_0}{2m}\right) \vec{\varepsilon}\cdot\vec{p}\, e^{i\vec{k}\cdot\vec{r}}\, e^{-i\omega t} + h.c.\right]}^{V}$$

$$\Gamma_{k\to m} = \frac{2\pi}{\hbar^2}\left(\frac{eA_0}{2m}\right)^2 \Big|\overbrace{\langle m|\, \vec{\varepsilon}\cdot\vec{p}\, e^{i\vec{k}\cdot\vec{r}}\, |k\rangle}^{M_{mk}}\Big|^2 \delta(\omega - \omega_{mk})$$

$$\omega_{mk} = \frac{E_m - E_k}{\hbar}$$

$M_{mk} \neq 0$?

$M_{mk} = 0$ transition "interdite"

$M_{mk} \neq 0 \Leftarrow$ règles de sélection

Approximation de dipôle (électrique)

$\vec{k}\cdot\vec{r} \sim 10^{-3} \div 10^{-2} \approx 0$

$r \sim a_0 \sim 10^{-10}\, m$

$k \sim \frac{2\pi}{\lambda}$ $\quad \lambda \sim 10^{-7}\, m$

$$M_{mk} = \langle m|\, \vec{\varepsilon}\cdot\vec{p}\, \overbrace{\cancel{e^{i\vec{k}\cdot\vec{r}}}}^{1}\, |k\rangle \simeq \vec{\varepsilon}\cdot\langle m|\vec{p}\,|k\rangle$$

$$\mathcal{H}_{at} = \frac{\vec{p}^2}{2m} + U_{pot} + \dots$$

états propres
de $\underline{\mathcal{H}_{at}}$

$$\frac{\vec{p}}{m} = \vec{v} = \dot{\vec{r}}$$

$$i\hbar\,\dot{\vec{r}} = [\vec{r}, \mathcal{H}_{at}]$$

$$\left[\underline{x}, \frac{\vec{p}^2}{2m}\right] = \frac{1}{2m}\left[x, p_x^2 + \cancel{p_y^2} + \cancel{p_z^2}\right]$$

$$= \frac{i\hbar}{m}\,\underline{\underline{p_x}}$$

$$[\underline{\underline{\vec{r}}}, \mathcal{H}_{at}] = \frac{i\hbar}{m}\,\underline{\vec{p}}$$

$$\underbrace{\vec{p} = \frac{m}{i\hbar}[\vec{r}, \mathcal{H}_{at}]}$$

$$\langle m|\,\vec{p}\,|n\rangle = \frac{m}{i\hbar}\langle m|\,[\vec{r}, \mathcal{H}_{at}]\,|n\rangle$$

$$(\vec{r}\,\mathcal{H}_{at} - \mathcal{H}_{at}\,\vec{r})$$

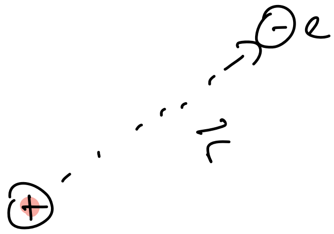
E_m

E_n

$$= -\frac{m}{i\hbar}(E_m - E_n)\,\langle m|\,\vec{r}\,|n\rangle$$

$$= i\,m\,\omega_{mn}\,\underline{\underline{\langle m|\,\vec{r}\,|n\rangle}} = -\frac{i}{e}\,m\,\omega_{mn}\,\vec{d}_{mn}$$

$$\vec{d}_{mn} = \langle m|\vec{d}|n\rangle$$



$$\vec{d} = -e\vec{r}$$

approx. de dipole

$$\Gamma_{n\to m} \simeq \frac{2\pi}{\hbar^2}\left(\cancel{\frac{eA_0}{2m}}\right)^2 \cancel{\frac{m^2}{e^2}}\,\omega^2\,|\vec{d}_{mn}\cdot\vec{\epsilon}|^2\,\delta(\omega-\omega_{mn})$$

$$\mathcal{E}_0 = A_0\omega$$

$$\vec{E}(\vec{r},t) = \frac{\mathcal{E}_0}{2}\left(e^{i(\cancel{\vec{k}\cdot\vec{r}}^{\,0}-\omega t)}\,\vec{\epsilon} + c.c.\right)$$

$$\Gamma_{n\to m} \simeq \frac{2\pi}{\hbar^2}\underbrace{\left(\frac{\mathcal{E}_0}{2}\right)^2|\vec{d}_{mn}\cdot\vec{\epsilon}|^2}\,\delta(\omega-\omega_{mn})$$

$$H(t) = H_{at} + \left(V e^{-i\omega t} + h.c.\right)$$

$$V = -\frac{\mathcal{E}_0}{2}\,\vec{\epsilon}\cdot\vec{d}$$

interaction champ électrique - dipole

$$= H_{at} - \vec{E}\cdot\vec{d}$$

$$\langle u | \vec{d} | u \rangle = 0$$

Règles de sélection en approximation de dipôle pour l'atome de H

$$\Rightarrow \langle m | \vec{d} \cdot \vec{\epsilon} | u \rangle \neq 0 \quad ?$$

H_{at} $\quad \vec{J}^2$

$$\underbrace{\langle n' l' s ; j' m_j' |}_{\langle m |} \vec{d} \cdot \vec{\epsilon} \underbrace{| n \, l \, s ; j \, m_j \rangle}_{| u \rangle} \quad J^z$$

$L^2 \quad S^2$

$$\begin{cases} \Delta n = n' - n = \text{quelconque} \; ? \\ \Delta j = j' - j = 0, \pm 1 \\ \Delta l = l' - l = \pm 1 \\ \Delta m_j = m_j' - m_j = q \end{cases}$$

$\vec{\epsilon} = \vec{u}_q$

pour quelles transitions ai-je $\langle m | \vec{d} \cdot \vec{e} | u \rangle \neq 0$?

Théorème de Wigner-Eckart

$$| n l s ; j m_j \rangle \rightarrow | \gamma ; j m_j \rangle$$

$\vec{J}^2, \; J^z$

opérateur vectoriel : $\vec{r}, \vec{p}, \vec{L}, \vec{J}, \ldots$

$\vec{V}$

opérateur qui se transforme comme un vecteur sous les rotations engendrées par $\vec{J}$

$$U = e^{-i\vartheta \vec{e}_n \vec{J}}$$

$\vec{e}_n$ vecteur unitaire

$V_q \qquad q = 0, \pm 1$

$$\vec{V} = (V_{-1}, V_0, V_1)$$

$$V_0 = V_z \qquad V_{\pm 1} = \frac{V_x \pm i V_y}{\sqrt{2}}$$

$$\langle \gamma' j' m_j' | V_q | \gamma j m_j \rangle = \langle \gamma' j' \| V \| \gamma j \rangle \cdot C^{j1}_{m_j q;\, j' m_j'}$$

(élément de matrice réduit)

$$\vec{v} \rightarrow \vec{r}$$

$$v_q \rightarrow r_q = \begin{cases} r_0 = z = r\cos\theta \\ r_1 = \frac{x+iy}{\sqrt{2}} = \frac{r}{\sqrt{2}}\sin\theta\, e^{i\phi} \\ r_{-1} = \frac{r}{\sqrt{2}}\sin\theta\, e^{-i\phi} \end{cases}$$

$$x = r\cos\phi\sin\theta$$

$$y = r\sin\phi\sin\theta$$

$$\underbrace{\langle j1; j' m_j' | \quad | j m_j \rangle \otimes | 1 q \rangle}$$

$$\rightarrow Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$\rightarrow Y_{1,\pm 1} = \sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}$$

$$r_q = \sqrt{\frac{4\pi}{3}}\, r\, Y_{1,q}(\theta, \phi)$$

$$\vec{\epsilon}\cdot\vec{d} = -e\sum_{q=0,\pm 1} \epsilon_q r_q$$

$$\langle n'l's; j' m_j' | r_q | nls; j m_j \rangle$$

$$= \langle n'l's j' \| r \| nls j \rangle \times C^{j1}_{m_j q; j' m_j'}$$

$l_1 \;,\; l_2 \qquad j$

$j \,,\, 1 \;\rightarrow\; j' = \underbrace{|j-1| \,,\; j \,,\; j+1}$

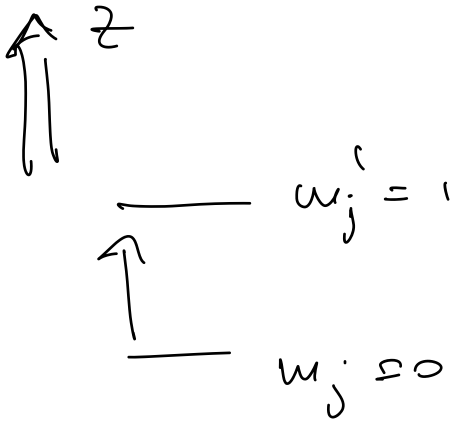
$j' - j = 0 \,, \pm 1$

$\Delta n =$ quelconque

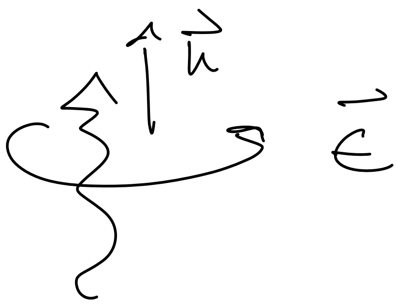
$m_j + q = m_j'$

$\Delta m_j = q$

$\vec{E} = \epsilon_q \vec{u}_q$



$q = 1$



$$|n l s; j m_j\rangle = \sum_{m m_s} C^{ls}_{m m_s; j m_j} |n l m\rangle \otimes |s m_s\rangle$$

$$\langle n' l' s; j' m_j' | r_q | n l s; j m_j \rangle$$

$$= \sum_{m m_s} \sum_{m' m_s'} C^{l's}_{m' m_s'; j' m_j'} C^{ls}_{m m_s; j m_j} \underbrace{\langle n' l' m' | r_q | n l m \rangle} \underbrace{\langle s m_s' | s m_s \rangle}_{\delta_{m_s, m_s'}}$$

$$\Rightarrow \langle \gamma' l' m' | r_q | \gamma l m \rangle \quad \overset{?}{=}$$

$j\, m_j$

$$= \langle \gamma' l' \| r \| \gamma l \rangle \qquad C^{l_1}_{mq;\, l'm'}$$

$$l' = |l-1|,\ l,\ l+1$$

$$\Delta l = l' - l = 0,\ \pm 1$$

$$r_q = \sqrt{\frac{4\pi}{3}}\, r\, Y_{1,q}(\theta, \phi)$$

$$\langle n' l' m' | r_q | n l m \rangle \neq 0$$

$$= \int dr\, r^2\, r\, R_{nl}(r)\, R_{n'l'}(r)$$

$$\underbrace{\int d\phi\, d\theta\, \sin\theta\ Y^*_{l'm'}(\theta, \phi)\, Y_{1q}(\theta, \phi)\, Y_{lm}(\theta, \phi)}$$

$$Y_{lm}(\vartheta, \phi) \xrightarrow{\text{inversion}} (-1)^{l}\, Y_{lm}(\vartheta, \phi)$$

$$\phi \to \pi + \phi$$

$$\vartheta \to \pi - \vartheta$$

parité sous inversion du produit

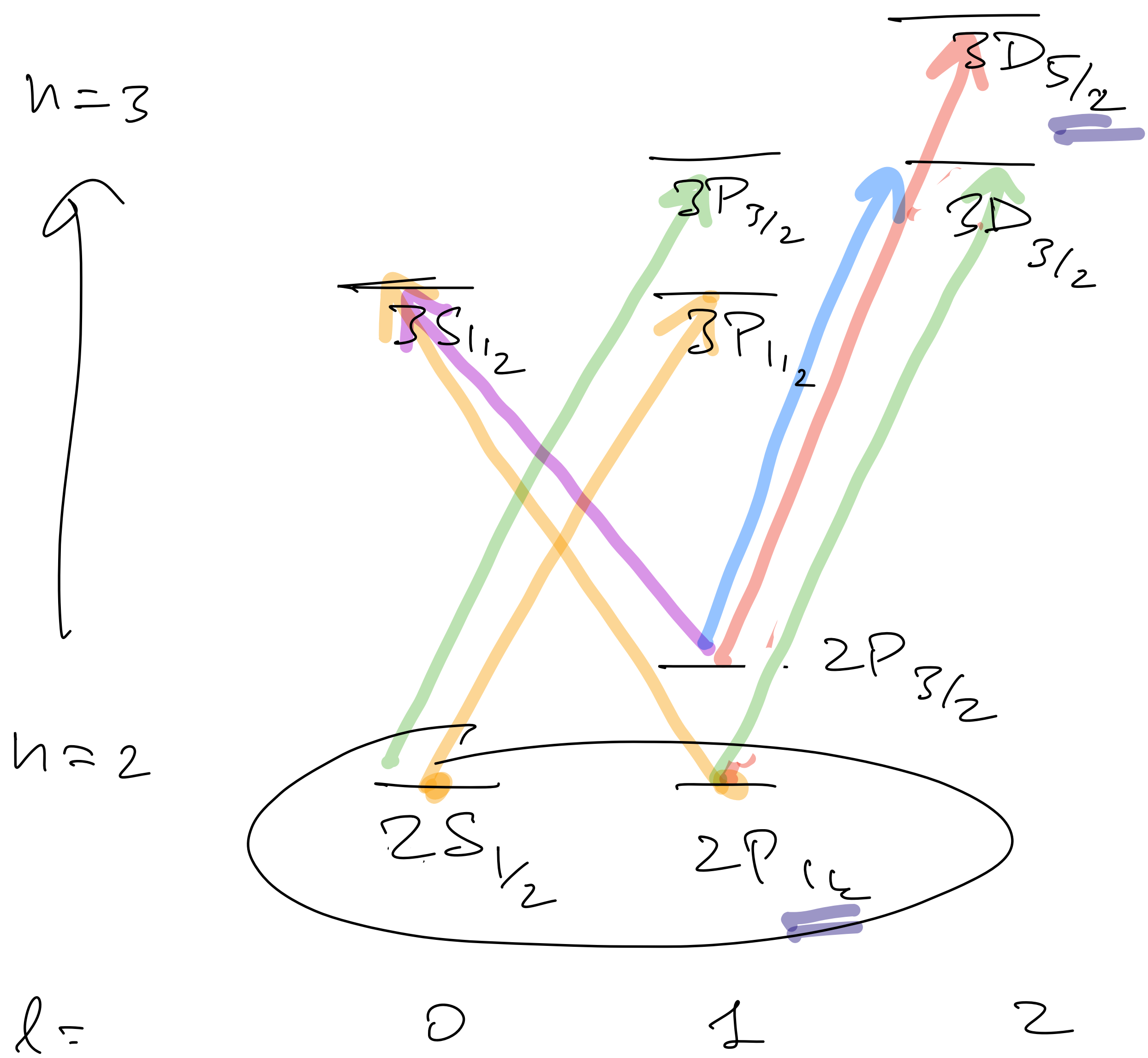
$$(-1)^{l' + l + 1} = 1$$

$$\underbrace{l' + l}_{\text{pair}} + 1 = \text{pair}$$

$$\Delta l = l' - l = \cancel{0} \pm 1$$

$$
\begin{cases}
\Delta n = n' - n = \text{quelconque} \; ? \\
\Delta j = j' - j = 0, \pm 1 \\
\Delta \ell = \ell' - \ell = \pm 1 \\
\Delta m_j = m_j' - m_j = q
\end{cases}
$$

$$\vec{\varepsilon} = \vec{u}_q$$



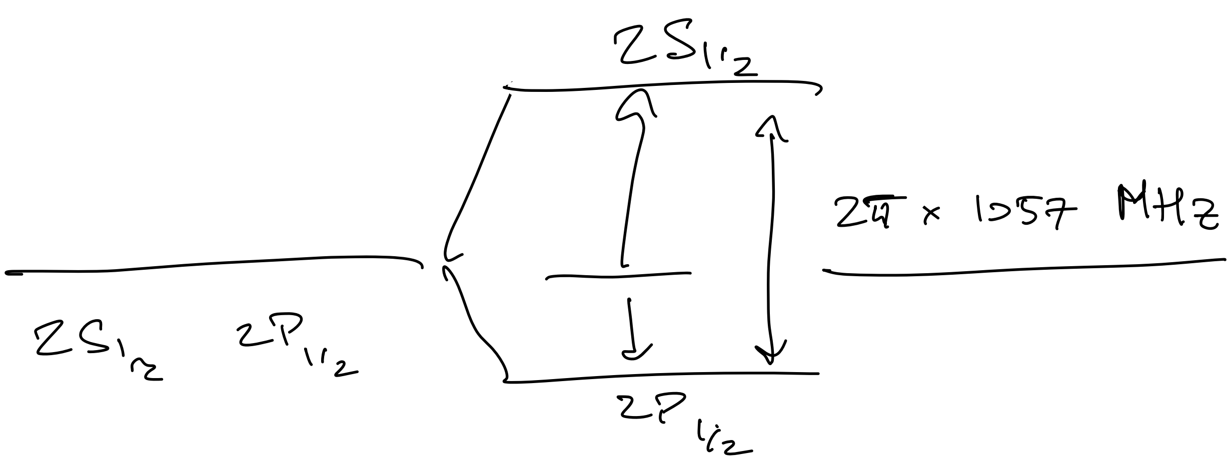
$n=1$

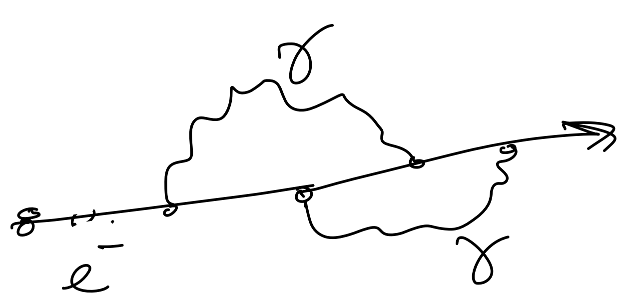
$1S_{1/2}$

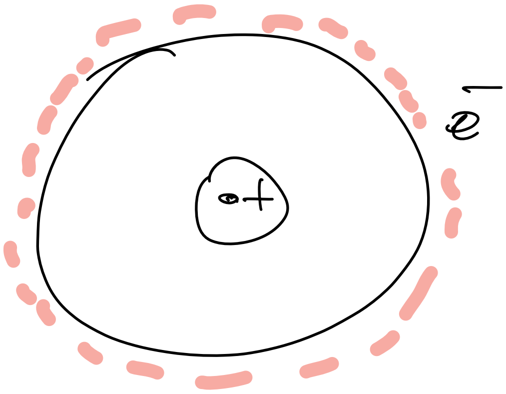
Décalage de Lamb

splitting entre nP_j, nS_j

nP_j, nD_j







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