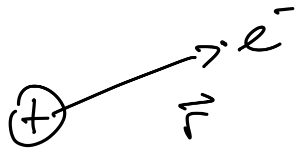
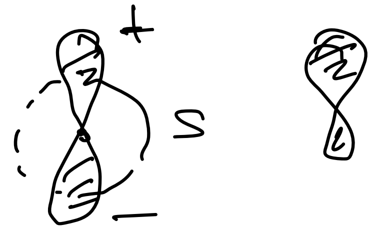


$\langle \vec{r} \rangle \neq 0$



$\quad n\,l\,m$

$\rightarrow \quad |2,0,0\rangle + |2,1,0\rangle$

$\qquad\quad S \qquad\qquad P$

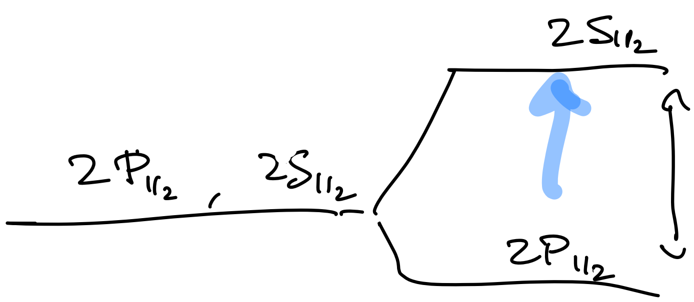
Effets ultérieurs : Spectre de H

→ décalage de Lamb

→ structure hyperfine de H

Décalage de Lamb

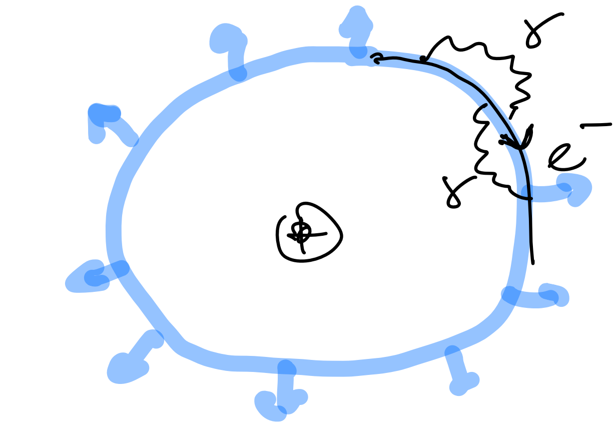
1947 : Lamb & Retherford $\Leftarrow$

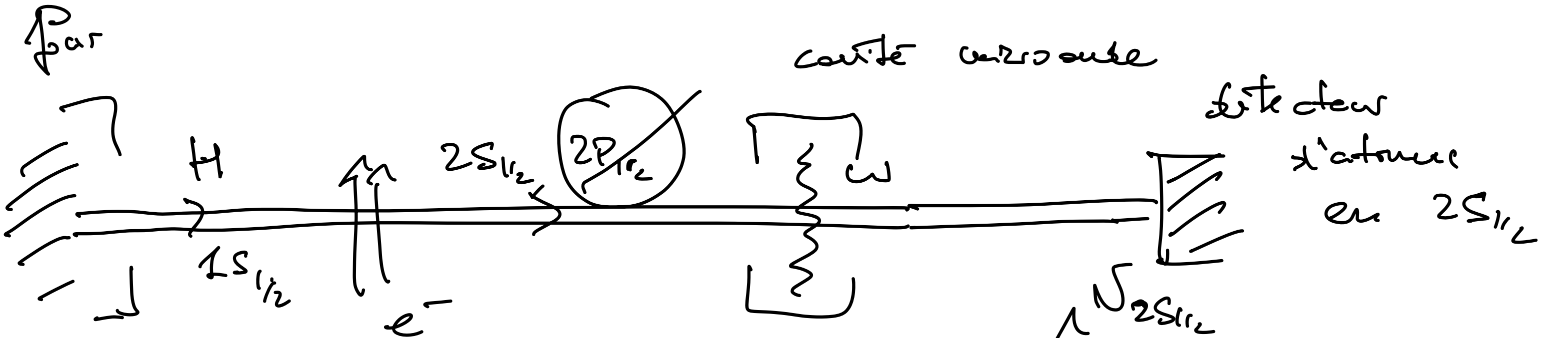


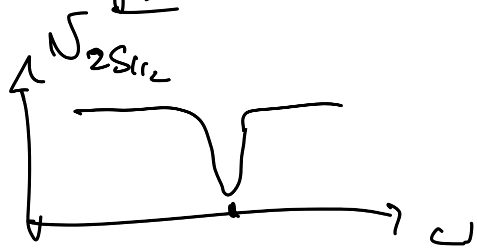
$2\pi \times 1057$ MHz

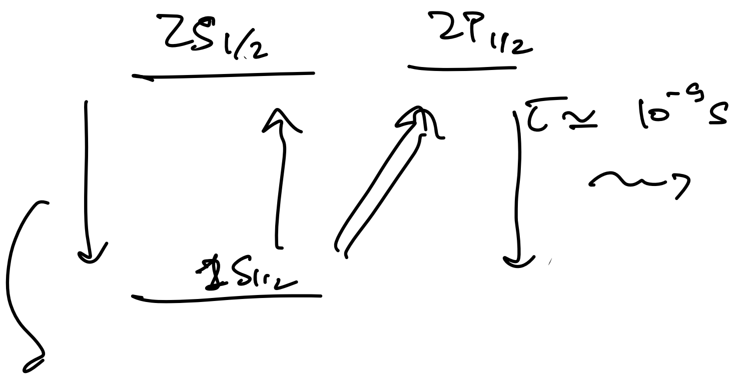
$\simeq \alpha^3 |\log \alpha|\ Ry$

$\alpha \simeq \frac{1}{137}$



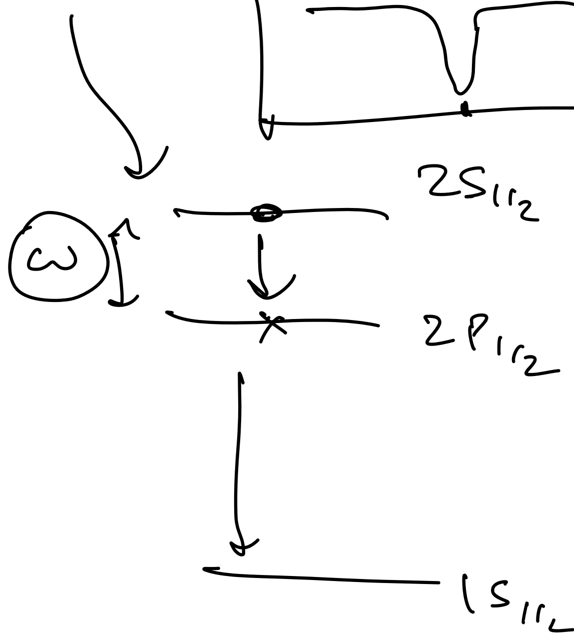




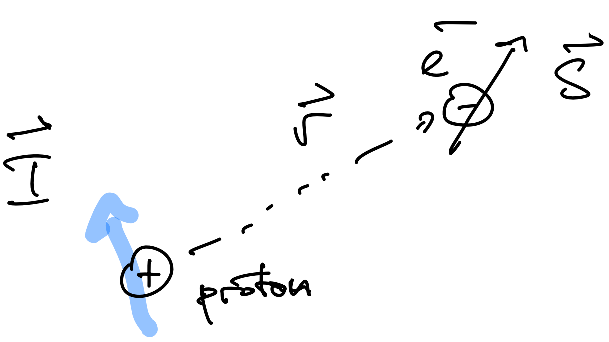


transition interdite

$\tau = \frac{1}{7}$ s



Structure hyperfine de H



interaction entre spin du proton et électron = interaction hyperfine (RMN)

$$\vec{m}_N = g_p \frac{\mu_N}{\hbar} \vec{I}$$

$$\mu_N = \frac{e\hbar}{2m_p}$$

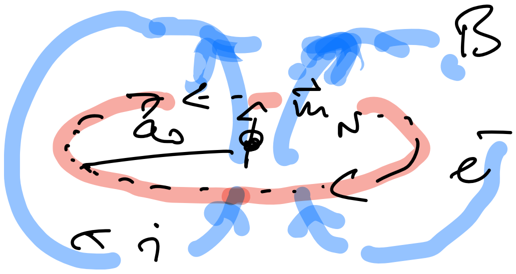
$$= \left(\frac{m_e}{m_p}\right) \mu_B$$

$$\simeq 10^{-3}$$

$g_p \simeq 5.59$
(proton)

Interaction hyperfine

1) interaction entre $\vec{I}$ et $\vec{L}$



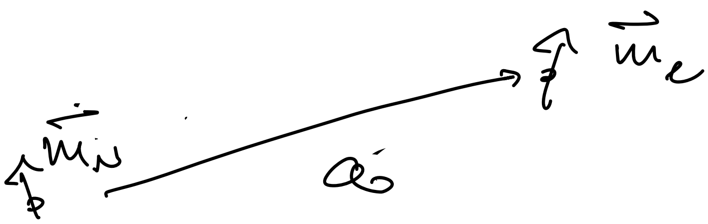
$$i = -e\omega \sim -\frac{e\hbar}{m a_0^2}$$

$$R_y = \frac{\hbar^2}{m a_0^2} \to \omega \simeq \frac{R_y}{\hbar}$$

$$B = \frac{\mu_0 i}{2 a_0} \sim \underbrace{\frac{\mu_0}{2} \frac{e\hbar}{m}}_{\mu_B} \frac{1}{a_0^3} = \frac{\mu_0 \mu_B}{a_0^3}$$

$$E = - \overset{\mu_N}{\vec{m}_N} \cdot \vec{B} \sim \frac{\mu_0 \mu_N \mu_B}{a_0^3}$$

2) Interaction entre $\vec{I}$ et $\vec{S}$



$$E_{dip-dip} \sim \frac{\mu_0}{4\pi} \frac{\mu_N \mu_B}{a_0^3}$$

str. hyperfine

$$E_{HF} \sim \frac{\mu_0 \mu_N \mu_B}{a_0^3}$$

$$\epsilon_0 \mu_0 = \frac{1}{c^2}$$

$$\frac{\mu_0 \mu_N \mu_B}{a_0^3 \, Ry} = \dots = \pi \alpha^2 \frac{m_e}{m_p}$$

10^{-3}

(Corrs : $\alpha^3 |\log\alpha|$)

Traiter l'interaction hyperfine comme perturbation de la structure fine + décalage de Lamb

$$\left[\, |n\, l\, S\, ;\, j\, m_j\rangle \otimes |I\, m_I\rangle \,\right]$$

$\mathcal{H}_0$ $\quad \vec{L}^2 \quad \vec{S}^2 \quad \vec{J}^2 \quad J_z \quad \vec{I}^2 \quad I_z$

proton $I = \frac{1}{2}$

$$E^{(FS)}_{nj} \longrightarrow E^{(FS+\text{Lamb})}_{njl}$$

=

États dégénérés en m_j et m_I

$\mathcal{H}_{HF}$

Base de la somme de $\vec{J}$ et $\vec{I}$

$$\vec{F} = \vec{J} + \vec{I}$$

↳ moment cinétique total de l'atome (spin hyperfin)

$$|n \, l \, s_i \, j m_j\rangle \otimes |I m_I\rangle$$

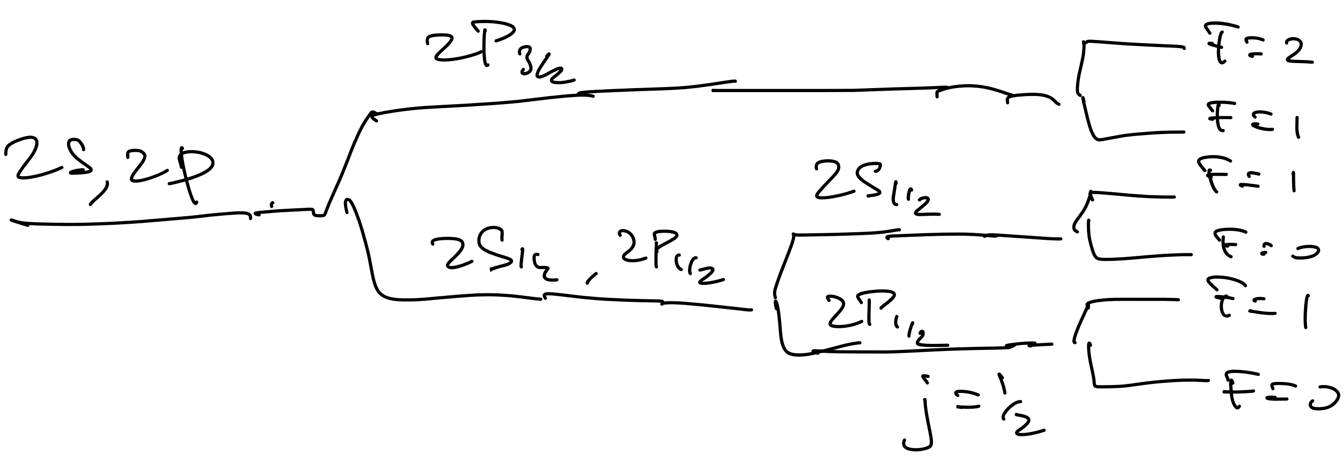
$$\to |n \, l \, s_i \, j \, I_i \, F m_F\rangle \to F_z$$

$$H_0 \quad \vec{L}^2 \quad \vec{S}^2 \quad \vec{J}^2 \quad \vec{I}^2 \quad \vec{F}^2$$

$$\Delta E^{(HF)}_{nljIF} = g_p \frac{\mu_0 \mu_N \mu_B}{4\pi a_0^3} Z^3 \times$$

$$\to \frac{F(F+1) - j(j+1) - I(I+1)}{n^3 \, j(j+1)\left(l + \frac{1}{2}\right)}$$

3/4



$\to j = \frac{1}{2} \qquad m_j = \pm\frac{1}{2}$

$\to I = \frac{1}{2} \qquad m_I = \pm\frac{1}{2}$

$F = 0, 1$

↓

$$F(F+1) - j(j+1) - \frac{3}{4} = \begin{cases} j & F = j + \frac{1}{2} \\ -(j+1) & F = j - \frac{1}{2} \end{cases}$$

$$F = \left|j - \frac{1}{2}\right|, \cancel{-}, j + \frac{1}{2}$$

↓

Règles de sélection pour transitions entre états hyperfins

↓ ↓

$$\langle \underbrace{n'l's; j'I}_{\gamma'}; F'm_F' | r_q | \underbrace{nls; jI}_{\gamma}; Fm_F \rangle$$

$\Delta n =$ quelconque

$\Delta l = \pm 1$

$(\Delta S = \Delta I = 0)$

$\Delta F = F' - F = 0, \pm 1$ $\qquad F' = (F-1, F, F+1)$

$\langle \gamma; F' m_F' | r_q | \gamma; F m_F \rangle$

$\sim$ $1F$

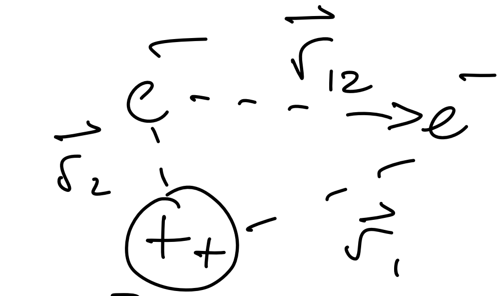
$q\, m_F; F' m_F'$

$\Delta m_F = m_F' - m_F = q$

$\boxed{F = 0 \;\cancel{\rightarrow}\; F' = 0}$ $\qquad F' = N$

Fin de H

Atome d'Hélium



$\rightarrow Z = 2$

$$\mathcal{H} = \underbrace{-\frac{\hbar^2}{2m}\left(\vec{\nabla}_1^{\,2} + \vec{\nabla}_2^{\,2}\right) - \frac{Ze^2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}_{\mathcal{H}_0}$$

$$\left[\underbrace{+\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_{12}}}_{\mathcal{H}_{ee}}\right]$$

États propres de $\underline{\mathcal{H}_0}$:

$$|n_1 \ell_1 m_1\rangle \otimes |n_2 \ell_2 m_2\rangle$$

$$\downarrow$$

$$E_{n_1 n_2} = -R_y Z^2\left(\frac{1}{n_1^2} + \frac{1}{n_2^2}\right)$$

$$\qquad\qquad\quad 1 \qquad\qquad\qquad 2$$

$$|\Psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|n_1 \ell_1 m_1\rangle \otimes |n_2 \ell_2 m_2\rangle \pm \right.$$

$$\qquad\qquad\quad 1 \qquad\qquad\qquad 2$$

$$|n_2 \ell_2 m_2\rangle \otimes |n_1 \ell_1 m_1\rangle \Big)$$

$$\otimes \, |\chi^{(\mp)}\rangle \rightarrow \underline{\underline{\text{Spin}}}$$

$$|\chi^{(-)}\rangle = \frac{|\uparrow_1 \downarrow_2\rangle - |\downarrow_1 \uparrow_2\rangle}{\sqrt{2}}$$

$$= |S_{tot} = 0, \, M_S = 0\rangle \qquad \underline{\text{singulet}}$$

$$|\chi^{(+)}\rangle = \begin{cases} |\uparrow_1 \uparrow_2\rangle = |S_{tot} = 1, \, M_S = 1\rangle \\ \frac{1}{\sqrt{2}}\Big(|\uparrow_1 \downarrow_2\rangle + |\downarrow_1 \uparrow_2\rangle\Big) = |S_{tot} = 1, \, M_S = 0\rangle \\ |\downarrow_1 \downarrow_2\rangle = |S_{tot} = 1, \, M_S = -1\rangle \end{cases}$$

<u>triplets</u>

Etats non perturbés de <u>H_0</u>.

H_{ee} comme perturbation

1) État fondamental

1s, 1s

$$|100\rangle \otimes |100\rangle \otimes |S=0, M_S=0\rangle \quad |\chi^{(-)}\rangle$$

n, l, m,

$$\Delta E_{ee} = \langle 100| \langle 100| \frac{e^2}{4\pi\epsilon_0 r_{12}} |100\rangle |100\rangle$$

$$= \dots = \int d^3r_1 \, d^3r_2 \, |\psi_{100}(\vec{r}_1)|^2 |\psi_{100}(\vec{r}_2)|^2 \frac{e^2}{4\pi\epsilon_0 r_{12}}$$

$$\simeq \underline{34 \text{ eV}} \sim 2 \text{ Ry}$$

$$E_{ee} = \frac{e^2}{4\pi\epsilon_0 a_0} \simeq 27 \text{ eV}$$

$$E_0 = -4 \text{ Ry} \times 2 = -8 \text{ Ry}$$