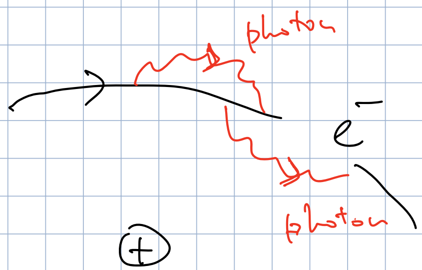
$$\underbrace{\underbrace{\langle j'm'|}\ \underbrace{r_q|jm\rangle}_{|1q\rangle\otimes}}_{(*)} = \underbrace{C^{1j}_{mq;j'm'}}_{\neq 0} \langle j'\| r \| j\rangle$$

condition nécessaire pour que $(*) \neq 0$

$$\Delta l = \pm 1$$

$$\langle n'l's;\, j'm' |\, r_q \,|\, nls;\, jm\rangle \neq 0 \iff \Delta l = \pm 1, \cancel{0}$$

Décalage de Lamb

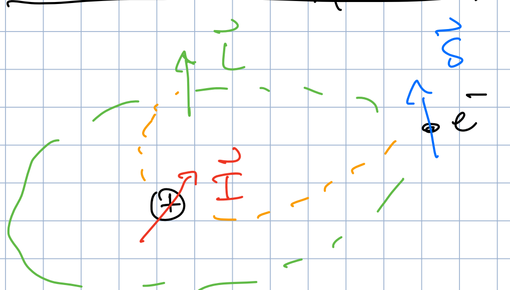


$$\Delta E_{FS} \sim \alpha^2\, Ry$$

structure fine

$$\Delta E_{Lamb} \sim \alpha^3 |\log \alpha|\, Ry$$

Structure hyperfine de l'atome de H



$\vec S$ él. $\to$ $\vec I$ noyau   moment cin. intrinsèque **nucléaire** noyau : proton ?

$I = \frac{1}{2}$

$$\vec{m}_N = g_p \, \mu_N \, \vec{I}/\hbar$$

$\vec{m}_N$: moment magnétique nucléaire

$g_p \downarrow 5.59$

μ_N : magnéton de Bohr nucléaire

$$\mu_N = \frac{e\hbar}{2m_p}$$

m_p = masse du proton $\sim 10^3 \, m_e$

$$\mu_N \sim 10^{-3} \mu_B$$

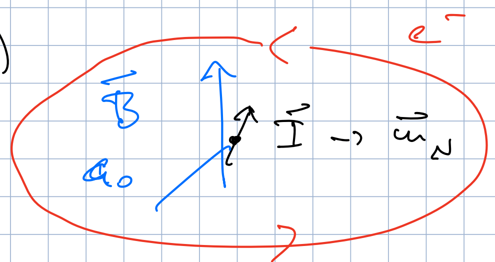
énergie associée au couplage spin nucléaire et → couplage hyperfin

→ 1) moment cinétique orbital } de e^-

2) " " intrinsèque

image semi-classique

1)



i courant

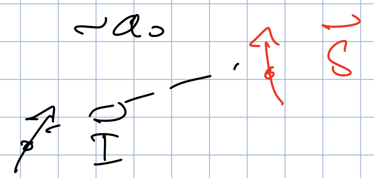
$$\Delta p = mv \simeq \frac{\hbar}{a_0} = \frac{\hbar}{\Delta x}$$

$$i \sim -e \frac{v}{a_0} = -e \underbrace{\frac{\hbar}{m a_0^2}}_{\sim \frac{Ry}{\hbar}}$$

$$B \simeq \frac{\mu_0 i}{2a_0} = - \frac{\mu_0}{2} \frac{e\hbar}{m a_0^3} = - \frac{\mu_0 \mu_B}{2 a_0^3}$$

$$E^{(1)}_{HF} \simeq - m_N B = + \frac{g_p \mu_N \mu_0 \mu_B}{2 a_0^3} \left(\frac{I}{\hbar} \right)$$

2)



$$E^{(2)}_{HF} = \frac{\mu_0}{4\pi}\left[\frac{\vec{m}_e\cdot\vec{m}_N}{|\vec{r}|^3} - \frac{3(\vec{r}\cdot\vec{m}_e)(\vec{r}\cdot\vec{m}_N)}{|\vec{r}|^5}\right]$$

$$\sim \frac{\mu_0}{4\pi}\frac{m_e m_N}{a_0^3} \sim \frac{\mu_0}{4\pi}\frac{g_e\mu_B\, g_p\mu_N}{a_0^3}$$

$$\Delta E_{HF} \sim \frac{\mu_0\mu_B\mu_N}{a_0^3} \sim \alpha^2 \overbrace{\left(\frac{m_e}{m_p}\right)}^{10^{-3}} Ry$$

sans faire de calculs

théorie perturbative du couplage hyperfin à partir d'états non perturbés

$$|n\, l\, s_i\, j\, m_j\rangle \otimes |I, m_I\rangle$$

$$\mathcal{H} \quad \vec{L}^2 \quad \vec{S}^2 \quad \vec{J}^2 \quad J^z \qquad \vec{I}^2 \quad I^z$$

couplage $\vec{I} \leftrightarrow \vec{L}$ (1)

$\vec{I} \leftrightarrow \vec{S}$ (2)

diagonaliser la perturbation

moment cinétique totale

$$|nls; jm_j\rangle \otimes |I m_I\rangle \to |nls\; jI; F m_F\rangle$$

$$\vec F = \vec J + \vec I \qquad \hbar^2 I(I+1) = \vec I^2$$

$$\vec J^2 = \hbar^2 J(J+1) \qquad \vec L_1^2 \quad \vec L_2^2$$

$$|l_1 m_1\rangle \otimes |l_2 m_2\rangle \to |l_1 l_2; l m\rangle$$

$$\vec J \cdot \vec I = \frac{1}{2}\left(\vec F^2 - \vec J^2 - \vec I^2\right)$$

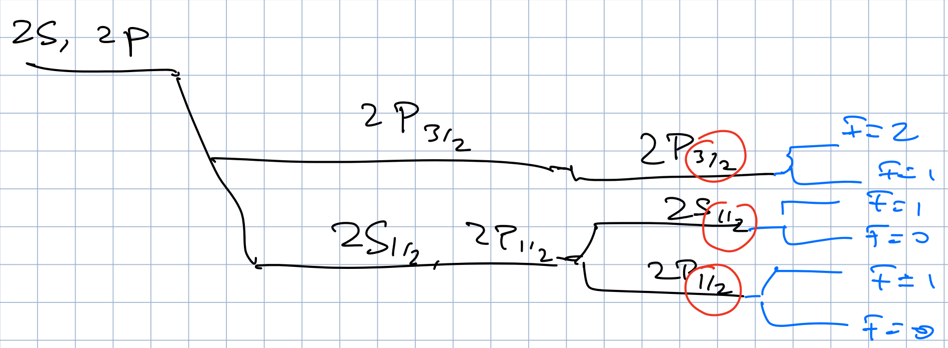
$$\Delta E^{(HF)}_{nljIF} = \frac{g_p \mu_0 \mu_N \mu_B}{4\pi a_0^3} Z^3 \frac{F(F+1) - j(j+1) - I(I+1)}{n^3\, j(j+1)\left(l+\frac{1}{2}\right)}$$

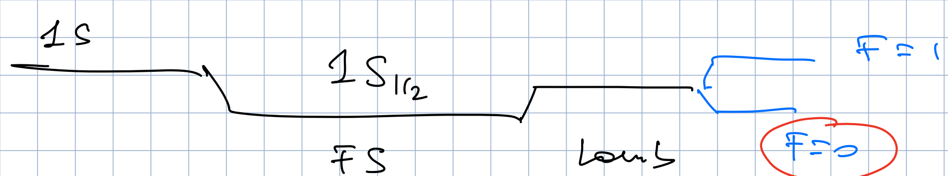
$$\left\langle \frac{1}{r^3} \right\rangle$$

$$F = |j - I|, \dots, j + I \qquad I = \frac{1}{2}$$

$$= j - \frac{1}{2},\; j + \frac{1}{2} \qquad j \geq \frac{1}{2}$$

$$\Delta E \sim F(F+1) - j(j+1) - I(I+1) = \begin{cases} -(j+1) < 0 & F = j - \frac{1}{2} \\ j > 0 & F = j + \frac{1}{2} \end{cases}$$





$j = \frac{1}{2}$

$F = 0, 1$

$$\left(\begin{array}{lll} {}^{133}Cs & I = \frac{7}{2} & j = \frac{1}{2} \\ & F = |j - I| = 3 & F = j + I = 4 \end{array} \right)$$

Transitions entre niveaux hyperfins : règles de sélection

$\vec{E} = \vec{u}_q$ (en approx. de dipôle électrique)

$|n l s ; j I ; F m_F\rangle \to |n' l' s ; j' I ; F' m'_F\rangle$

$$r_q = \begin{cases} z & q = 0 \\ \frac{x \pm iy}{\sqrt{2}} & q = \pm 1 \end{cases}$$

Δn = quelconque

$\Delta j = 0, \pm 1$

$\Delta l = \pm 1$

$\to \Delta F = 0, \pm 1$

$\Delta m_F = q$
$\pm$

$\langle j' F' m'_F | r_q | j F m_F \rangle$

$\sim C^{1 F}_{m_F q ; F' m'_F}$

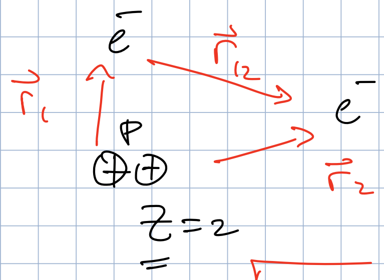
mais

$F = 0 \;\cancel{\to}\; F' = 0$

$F' = |F-1|, \cancel{F}, F+1$ $\quad F = 0$

Fin de H

Atome d'Hélium : atome à 2 électrons



$$H = \underbrace{-\frac{\hbar^2}{2m}\left(\vec\nabla_1^2+\vec\nabla_2^2\right) - \frac{Ze^2}{4\pi\varepsilon_0}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}_{H_0} + \underbrace{\frac{e^2}{4\pi\varepsilon_0\, r_{12}}}_{\text{perturbation}}$$

spectre imperturbé :

$$E_{n_1,n_2} = -Z^2 R_y\left(\frac{1}{n_1^2}+\frac{1}{n_2^2}\right)$$

États imperturbés : électrons discernables

$$\left(|n_1\,\ell_1\,m_1\rangle \otimes |s_1\,m_{s_1}\rangle\right) \otimes \left(|n_2\,\ell_2\,m_2\rangle \otimes |s_2\,m_{s_2}\rangle\right)$$

(2) ↓ (2) ↓

$$\underbrace{\psi_{n_1\ell_1m_1}(\vec r_1)\,\chi_{m_{s_1}}(\sigma_1)}_{\Psi_1(\vec r_1,\,\sigma_1)} \otimes \underbrace{\psi_{n_2\ell_2m_2}(\vec r_2)\,\chi_{m_{s_2}}(\sigma_2)}_{\Psi_2(\vec r_2,\,\sigma_2)}$$

$$\chi_{m_{s_1}}(\sigma_1) = \delta_{m_{s_1},\sigma_1}$$

$\sigma_1 = \uparrow,\downarrow$

$m_{s_1} = \uparrow,\downarrow$

Antisymétrisation de la fonction d'onde

(1)

$$\Psi(\vec{r}_1, \sigma_1 ; \vec{r}_2, \sigma_2) = \frac{1}{\sqrt{2}}\Big(\Psi_1(\vec{r}_1, \sigma_1)\,\Psi_2(\vec{r}_2, \sigma_2) - \Psi_2(\vec{r}_1, \sigma_1)\,\Psi_1(\vec{r}_2, \sigma_2)\Big)$$

(2)

$$\Psi(\vec{r}_1\, \sigma_1 ; \vec{r}_2\, \sigma_2) = \begin{cases} \frac{1}{2}\big(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2) - \psi_2(\vec{r}_1)\psi_1(\vec{r}_2)\big) \\ \underline{\big(\chi_1(\sigma_1)\chi_2(\sigma_2) + \chi_2(\sigma_1)\chi_1(\sigma_2)\big)} \\ + \quad \text{spatiale} \\ - \quad \underline{\text{spin}} \end{cases}$$

États symétriques de spin

$$\begin{cases} |\uparrow\uparrow\rangle = |S=1, m_s=1\rangle \\ \dfrac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} = |S=1, m_s=0\rangle \\ |\downarrow\downarrow\rangle = |S=1, m_s=-1\rangle \end{cases}$$

triplet

État antisymétrique

$$\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} = |S=0, m_s=0\rangle$$

singulet