Interaction lumière – atome

$\omega, \vec{q}, \vec{\epsilon}$

lumière monochromatique

$|m\rangle$

$|k\rangle$

$$\mathcal{H}_0 |k\rangle = E_k |k\rangle$$

$$\omega_{mk} = \frac{E_m - E_k}{\hbar}$$

$$\mathcal{H}(t) = \mathcal{H}_0 + \underbrace{\frac{A_0 e}{2m}\left(e^{i\vec{q}\cdot\vec{r}}\, \hat{\vec{\epsilon}}\cdot\hat{\vec{p}}\, e^{-i\omega t} + h.c.\right)}_{\lambda \mathcal{H}'(t)}$$

$$|\psi(t=0)\rangle = |k\rangle$$

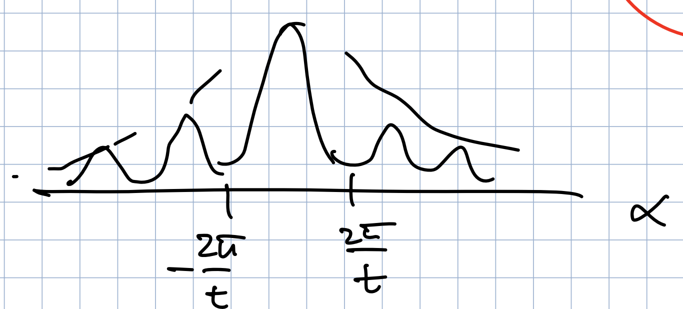
$$|\psi(t)\rangle = \sum_n e^{-iE_n t/\hbar}\, c_n(t)\, |n\rangle \qquad |n\rangle = |k\rangle, |m\rangle, \dots$$

$$c_n(t) = \delta_{kn} + \underbrace{\lambda\, c_n^{(1)}(t)} + \cancel{\lambda^2 c_n^{(2)}(t)} + \dots$$

$$c_m^{(1)}(t) = -\frac{A_0 e}{2m\hbar}\left(M_{mk}\, \frac{e^{i(\omega_{mk}-\omega)t}-1}{\omega_{mk}-\omega} + M^*_{km}\, \frac{e^{i(\omega_{mk}+\omega)t}-1}{\omega_{mk}+\omega}\right)$$

$$M_{mk} = \langle m|\, e^{i\vec{q}\cdot\vec{r}}\, \hat{\vec{e}}\cdot\vec{p}\, |k\rangle$$

$$\left|\frac{e^{i\alpha t}-1}{\alpha}\right|^2 = \underbrace{t^2 \operatorname{sinc}^2\left(\frac{\alpha t}{2}\right)} = \frac{\sin^2\left(\frac{\alpha}{2}t\right)}{(\alpha/2)^2}$$



$$\int_{-\infty}^{+\infty} dx \; \mathrm{sinc}^2(x) = \pi$$

$$\int_{-\infty}^{+\infty} d\alpha \; \frac{\sin^2\left(\frac{\alpha t}{2}\right)}{\left(\frac{\alpha}{2}\right)^2 t^2}\, t^2 = \frac{2}{t}\, t^2 \underbrace{\int_{-\infty}^{+\infty} d\left(\alpha \frac{t}{2}\right) \; \mathrm{sinc}^2\left(\frac{\alpha t}{2}\right)}_{\pi} = 2\pi t$$

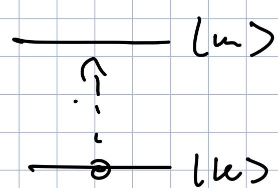
$$\frac{\sin^2\left(\frac{\alpha t}{2}\right)}{\left(\frac{\alpha}{2}\right)^2} \xrightarrow[t \to \infty]{} 2\pi t \;\; \delta(\alpha)$$

$$=$$

$$\left| \frac{e^{i(\omega_{mn} \pm \omega)t} - 1}{(\omega_{mn} \pm \omega)} \right|^2 \underset{t \to \infty}{\simeq} 2\pi t \;\; \delta(\omega \pm \omega_{mn})$$

2 cas : $\omega = cq > 0$

1) $\omega \simeq \omega_{mk} > 0$



Absorption

taux de transition

$$\Gamma_{k \to m} = \frac{|c_m^{(1)}(t)|^2}{t} = \left(\frac{A_0 e}{2m\hbar}\right)^2 2\pi \, |M_{mk}|^2 \, \delta(\omega - \omega_{mk})$$

taux d'absorption

$$\Rightarrow \quad = \frac{2\pi}{\hbar^2}\left|\langle m|\overbrace{\left(\frac{A_0 e}{2m}\, e^{i\vec{q}\cdot\vec{r}}\,\vec{\varepsilon}\cdot\vec{p}\right)}^{\hat{W}}|n\rangle\right|^2 \delta(\omega-\omega_{mn})$$

$$H'(t) = \underbrace{\left(\frac{A_0 e}{2m}\, e^{i\vec{q}\cdot\vec{r}}\,\vec{\varepsilon}\cdot\vec{p}\right)}_{\hat{W}} e^{-i\omega t} + h.c.$$

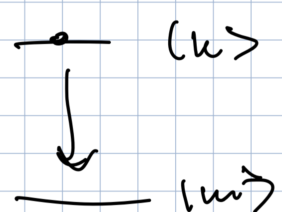
$$= \hat{W} e^{-i\omega t} + h.c.$$

règle d'or de Fermi

$$\Gamma_{n\to m} = \frac{2\pi}{\hbar^2}\left|\langle m|\hat{W}|n\rangle\right|^2 \delta(\omega-\omega_{mn})$$

$\frac{1}{(\varepsilon t)^2}$ $\quad\varepsilon^2\quad$ t

2) $\omega = -\omega_{mn}$



émission stimulée

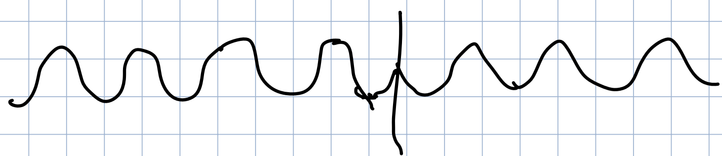
$$\Gamma_{n\to m} = \frac{2\pi}{\hbar^2}\left|\langle m|\hat{W}^{+}|n\rangle\right|^2 \delta(\omega+\omega_{mn})$$

Situation plus générale : lumière polychromatique

$$\vec{A}(\vec{r},t) = \sum_{\vec{q},\vec{\epsilon}} \frac{A_{\vec{q},\vec{\epsilon}}}{2}\left[e^{i\vec{q}\cdot\vec{r}}\, e^{-i\omega_{\vec{q}}t}\, e^{i\phi_{\vec{q},\vec{\epsilon}}}\,\vec{\epsilon} + c.c.\right]$$

$$M^{*}_{km} = \langle k|\, e^{i\vec{q}\cdot\vec{r}}\,\vec{\epsilon}\cdot\vec{p}\,|m\rangle^{*}$$

$$= \langle m|\, e^{-i\vec{q}\cdot\vec{r}}\,\vec{\epsilon}^{*}\cdot\vec{p}\,|k\rangle$$



$$c_m^{(1)}(t) = \sum_{\vec{q},\vec{\epsilon}} \frac{A_{\vec{q},\vec{\epsilon}}\, e}{2m\hbar}\Bigg(\underbrace{M_{mk}^{(\vec{q},\vec{\epsilon})}\, \overbrace{\frac{e^{i(\omega_{mk}-\omega_{\vec{q}})t}-1}{\omega_{mk}-\omega_{\vec{q}}}}^{1}\, e^{i\phi_{\vec{q},\vec{\epsilon}}}} + \underbrace{\left(M_{km}^{(\vec{q},\vec{\epsilon})}\right)^{*} \frac{e^{i(\omega_{mk}+\omega_{\vec{q}})t}-1}{\omega_{mk}+\omega_{\vec{q}}}\, e^{-i\phi_{\vec{q},\vec{\epsilon}}}}_{2} \Bigg)$$

$$M_{mk}^{(\vec{q},\vec{\epsilon})} = \langle m|\, e^{i\vec{q}\cdot\vec{r}}\,\vec{\epsilon}\cdot\vec{p}\,|k\rangle$$

$$|c_m^{(1)}(t)|^2 \underset{t\to\infty}{\simeq} 2\pi t \sum_{\vec{q},\vec{\epsilon}} \Bigg[\left(\frac{A_{\vec{q},\vec{\epsilon}}\, e}{2m\hbar}\right)^2 \left|M_{mk}^{(\vec{q},\vec{\epsilon})}\right|^2 \delta(\omega_{\vec{q}}-\omega_{mk})$$

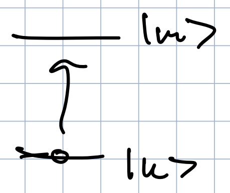
$$+ \left(\frac{A_{\vec{q},\vec{\epsilon}}\, e}{2m\hbar}\right)^2 \left|M_{km}^{(\vec{q},\vec{\epsilon})}\right|^2 \delta(\omega_{\vec{q}}+\omega_{mk})$$

$$\cancel{+ \sum_{\vec{q},\vec{\epsilon}} \sum_{\vec{q}',\vec{\epsilon}' \neq \vec{q},\vec{\epsilon}} \boxed{e^{i\left(\phi^{(t)}_{\vec{q},\vec{\epsilon}} - \phi^{(t)}_{\vec{q}',\vec{\epsilon}'}\right)}}\, (\ldots\ldots)}$$

$\omega_{\vec{q}} = cq$

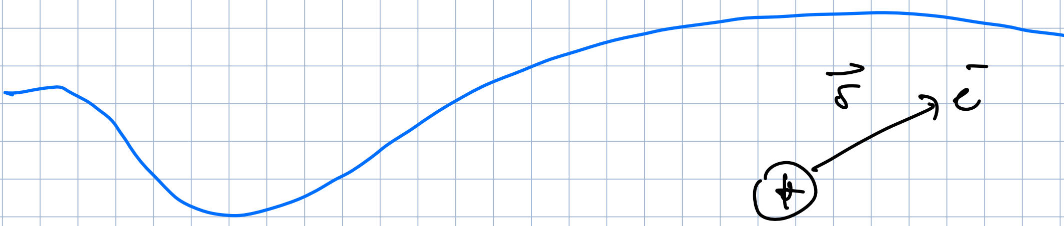
Absorption

$$\Gamma_{n\to m} = \frac{2\pi}{\hbar^2} \underbrace{\sum_{\vec q,\vec\epsilon} \left(\frac{A_{\vec q,\vec\epsilon}\, e}{2m}\right)^2 \left|M_{mn}^{(\vec q,\vec\epsilon)}\right|^2 \delta(\omega_{\vec q} - \omega_{mn})}$$



1) $A_{\vec q,\vec\epsilon} = \underline{A(\omega_{\vec q})}$

2) $M_{mn}^{(\vec q,\vec\epsilon)} = \langle m|\, \cancel{e^{i\vec q\cdot\vec r}}\, \vec\epsilon\cdot\vec p\, |n\rangle$ $\quad \cancel{e^{i\vec q\cdot\vec r}} \to 1$ "approximation du dipôle"

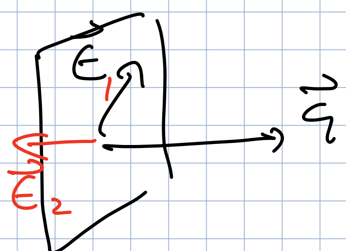


$r \sim 1\,\text{Å} = 10^{-10}\,m$

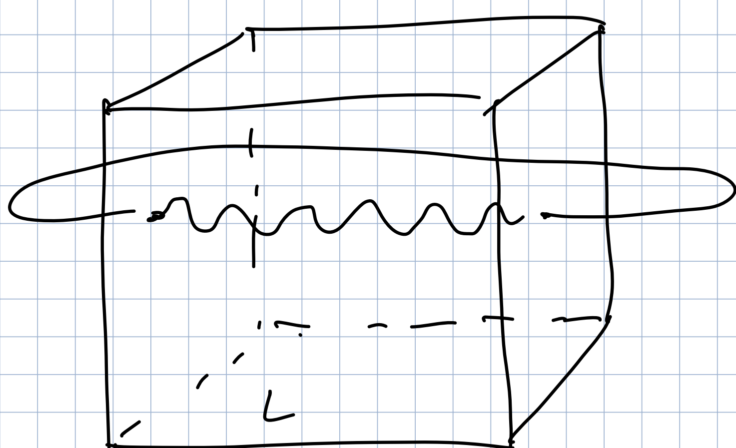
$q = \frac{2\pi}{\lambda}$ $\qquad qr = \cancel{2\pi}\,\frac{10^{-10}}{\cancel{5}\times 10^{-7}} \approx 10^{-3}$

$\lambda \simeq 500\ nm$

$= \underline{M_{nm}^{(\vec\epsilon)}}$



$$\Gamma_{n\to m} = \frac{2\pi}{\hbar^2}\frac{V}{(2\pi)^3}\sum_{\vec q}\left(\frac{2\pi}{L}\right)^3\sum_{\vec\epsilon}\frac{A^2(\omega_q)e^2}{(2m)^2}\left|M_{nm}^{(\vec\epsilon)}\right|^2\delta(\omega_{mn}-\omega_{\vec q})$$

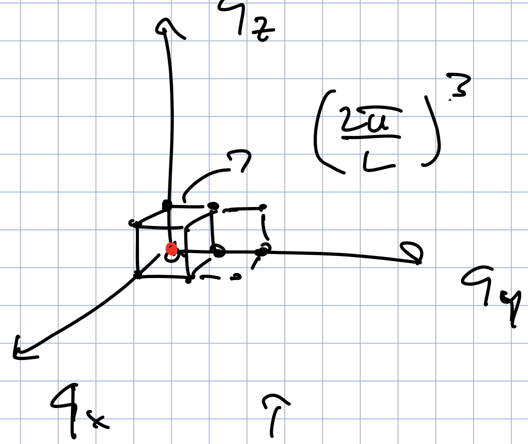


$V = L^3$

$$\boxed{\vec q = \frac{2\pi}{L}(n_x, n_y, n_z)}$$

$n_\mu \in \mathbb{Z}$

$\mu = x, y, z$



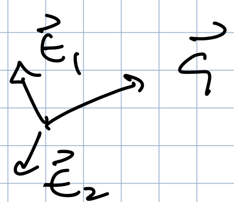
$$\sum_{\vec q} \left(\frac{2\pi}{L}\right)^3 (\dots) \underset{L\to\infty}{\simeq} \int d^3q \,(\dots)$$

$$\Gamma_{n\to m} \simeq \frac{2\pi}{\hbar^2} V \sum_{\vec\epsilon} \int \frac{d^3q}{(2\pi)^3} \left(\frac{A(\omega_q)\, e}{2m}\right)^2 \left|M^{(\vec\epsilon)}_{nm}\right|^2 \delta(\omega_q - \omega_{mn})$$

$$\omega_q = c|\vec q|$$

$$= \frac{2\pi}{\hbar^2} V \overset{2}{\sum_{\vec\epsilon}} \int \frac{dq\, q^2}{(2\pi)^3} \left(\frac{A(\omega_q)\, e}{2m}\right)^2 \delta(\omega_{mn} - \omega_q)$$

$$4\pi \, \frac{1}{4\pi}\int d\Omega_{\vec q} \left|M^{(\vec\epsilon)}_{nm}\right|^2$$



$$4\pi \, \overline{\left|M^{(\vec\epsilon)}_{nm}\right|^2}$$

$$= \frac{2\pi}{\hbar^2}\, 2\, V \, \overline{\left|M^{(\vec\epsilon)}_{nm}\right|^2}\, 4\pi \int \frac{dq}{(2\pi)^3}\, q^2 \left(\frac{A(\omega_q)\, e}{2m}\right)^2 \delta(\omega_{mn} - \omega_q)$$

$$\underbrace{2\, \frac{V}{(2\pi)^3}\, 4\pi q^2\, dq} = \underbrace{V g(\omega)\, d\omega}$$

$$q = \omega/c$$

$g(\omega) =$ densité de modes par unité de volume $= \dfrac{\omega^2}{\pi^2 c^3}$ $\leftarrow$

$$\underline{\underline{g(\omega)}}\, \underline{\underline{d\omega}} = \frac{dN[\omega, \omega + d\omega]}{V} = \#\text{ de fréquences par unité de volume}$$

$$A^2(\omega) = \frac{2\rho(\omega)}{\epsilon_0 \omega^2}$$

$\rho(\omega) =$ densité d'énergie $= \hbar\omega \dfrac{n(\omega)}{V}$

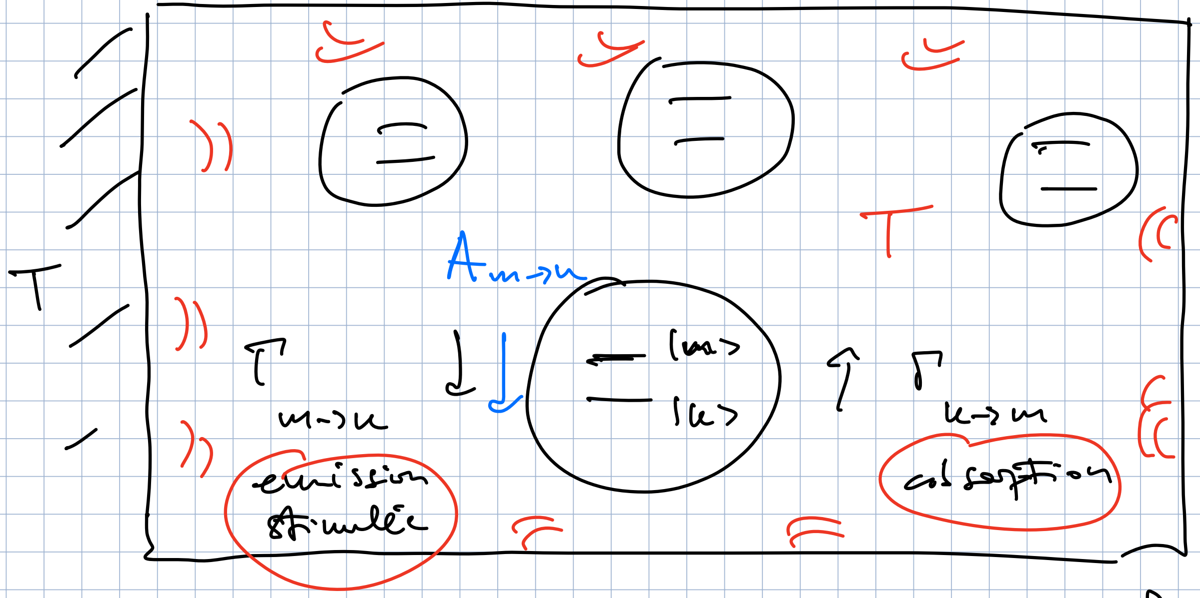
$$\Gamma_{k\to m} \simeq \frac{\cancel{2}\pi}{\hbar^2} V g(\omega_{mk}) \frac{e^2}{(\cancel{2}m)^2} \frac{\cancel{2}\rho(\omega_{mk})}{\epsilon_0 \omega_{mk}^2} \overline{|M^{(\vec{\epsilon})}_{mk}|^2}$$

$$= \underbrace{V g(\omega_{mk})\, \rho(\omega_{mk})}_{\text{propriété de la lumière}} \underbrace{\frac{\pi e^2 \overline{|M^{(\vec{\epsilon})}_{mk}|^2}}{\hbar^2 \epsilon_0 m^2 \omega_{mk}^2}}_{B_{mk}\ \to\ \text{propriété de l'atome}}$$

taux d'absorption sur l'effet de plusieurs composantes incohérentes du champ e.m.

$$\Gamma_{m\to k} = \Gamma_{k\to m}$$

taux d'émission stimulée



four

rayonnement de corps noir

population des atomes dans l'état m

$$\frac{dN_m}{dt} = \underbrace{\Gamma_{n\to m}}_{=} \underbrace{N_n}_{=} - \left(\Gamma_{m\to n} + A_{m\to n}\right) N_m = 0$$

régime stationnaire : $N_n = N_m$?

$$\frac{N_m}{N_n} = e^{-\frac{E_m - E_n}{k_B T}} \neq \quad N_m \sim P_m \sim e^{-\frac{E_m}{k_B T}}$$

$$= e^{-\frac{\hbar\omega_{mn}}{k_B T}} < 1 \qquad \omega_{mn} > 0$$

rayonnement de corps noir

Einstein, 1917 émission stimulée

$$\Gamma_{m\to n} \to \Gamma_{m\to n} + A_{m\to n}$$

émission spontanée

$$\frac{dN_m}{dt} = \Gamma N_n - (\Gamma + A) N_m = 0$$

$$\Gamma e^{\hbar\omega_{mn}/k_B T} - (\Gamma + A) = 0$$

$$A = \Gamma \left(e^{\hbar\omega_{mn}/k_B T} - 1\right)$$

$$= V g(\omega_{mn}) \rho(\omega_{mn}) \left(e^{\hbar\omega_{mn}/k_B T} - 1\right) B$$

$$\left[\frac{\omega_{mn}^2}{\pi^2 c^3}\right]$$

$$\Gamma$$

$$\hbar\omega_{mn}\, n(\omega_{mn})$$

de photons à la fréquence ω dans le rayonnement du corps noir

$$n(\omega) = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

Loi de Planck

(1900)

$$A_{m\to n} = g(\omega_{mn})\, \hbar\omega_{mn}\, B_{m\to n}$$

stimulé spontánní

$$\Gamma'_{m\to n} = \Gamma_{m\to n} + A_{m\to n}$$

$$= \cancel{V} g(\omega_{mn}) \frac{\hbar\omega_{mn} n(\omega_{mn})}{\cancel{V}} B + g(\omega_{mn})\hbar\omega_{mn} B$$

$$= g(\omega_{mn})\hbar\omega_{mn}\left(n(\omega_{mn}) + 1\right) B_{m\to n}$$

nemáme quant spontánní

$n=0$