Quantum simulation with cold atoms

An experimental and theoretical challenge

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ENS DE LYON

IUF

GdR Atomes Froids

AGENCE NATIONALE DE LA RECHERCHE

PSMN PÔLE SCIENTIFIQUE DE MODÉLISATION NUMÉRIQUE

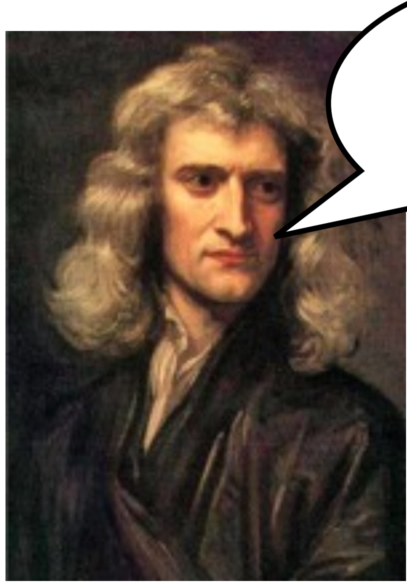
cnrs

First vs. second quantum revolution

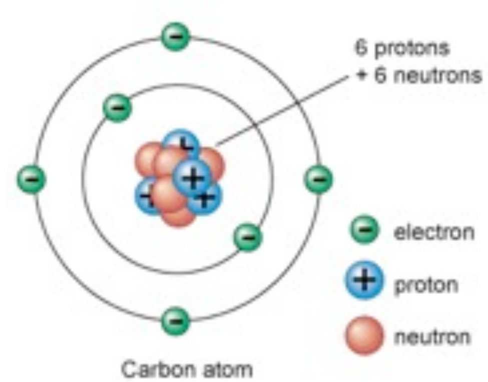
The first quantum revolution

Paradigm: macroscopic objects are classical
elementary constituents obey new laws (quantum mechanics)

F=ma



Isaac Newton

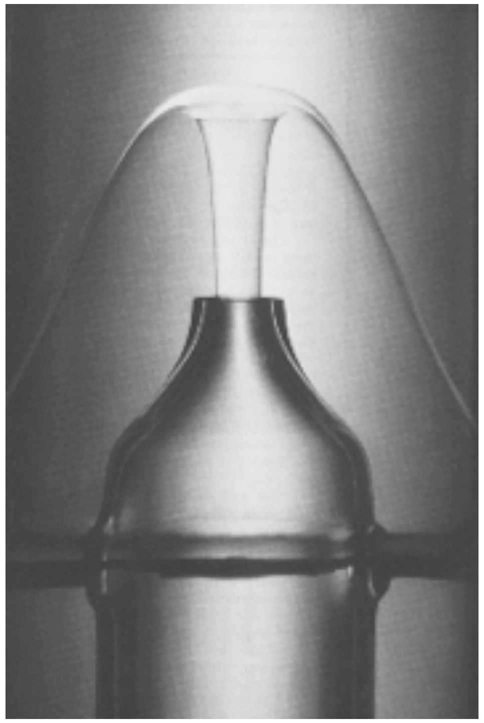


$$i\hbar \frac{d}{dt}|\psi\rangle = \mathcal{H}|\psi\rangle$$

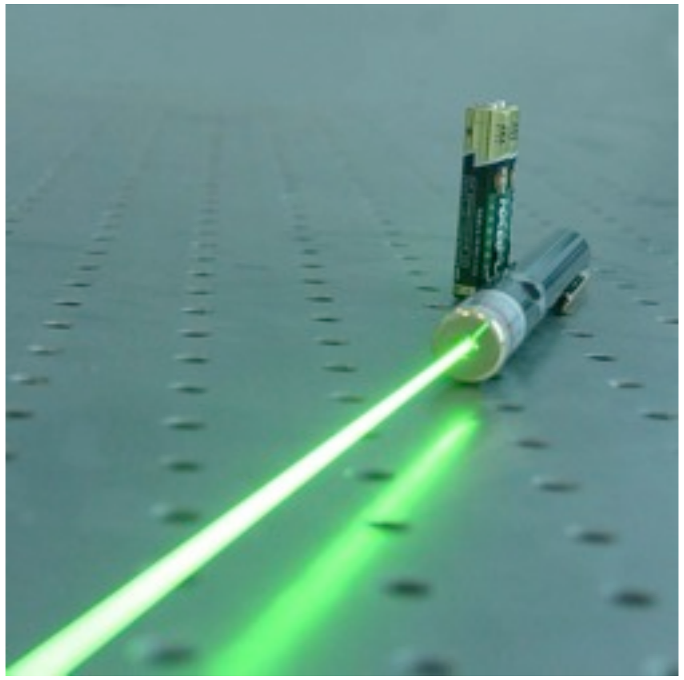
Erwin Schrödinger

Macroscopic quantum effects

Superconductivity *(1913)*



Superfluidity of He-4 *(1938)*

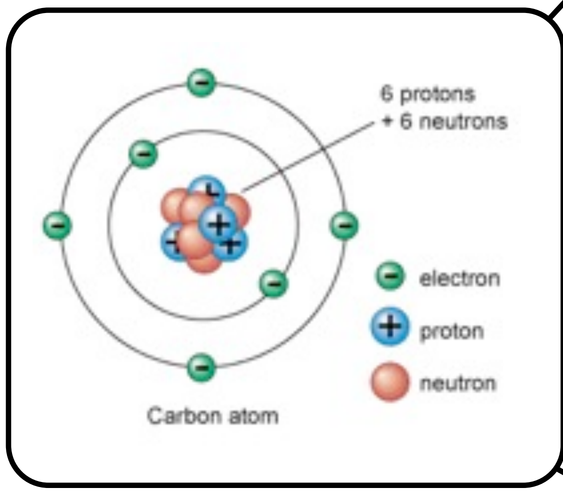


Lasers *(1960)*

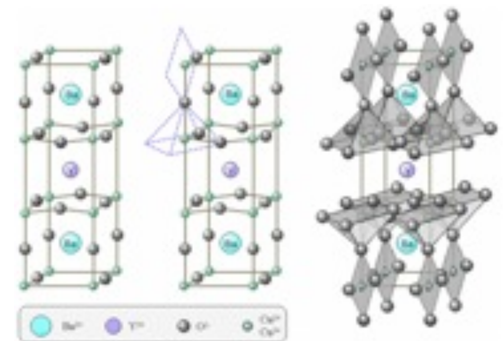
etc....

Quantum matter: *the 2nd quantum revolution*

Design of (meta-)materials
dominated by quantum effects



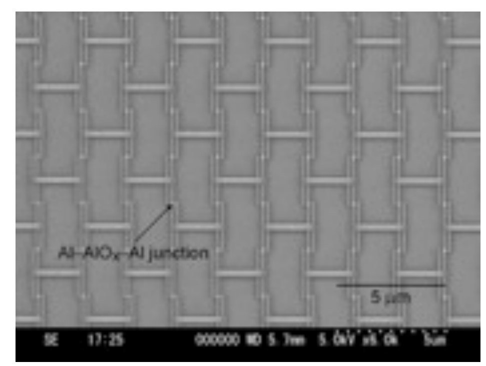
Complex oxides:
novel superconductors/
quantum magnets
(~1987)



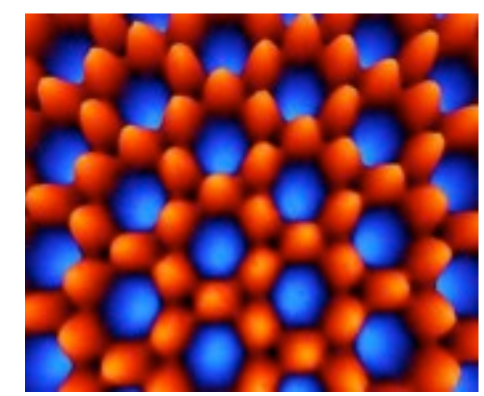
chemical synthesis

nanotechnologies

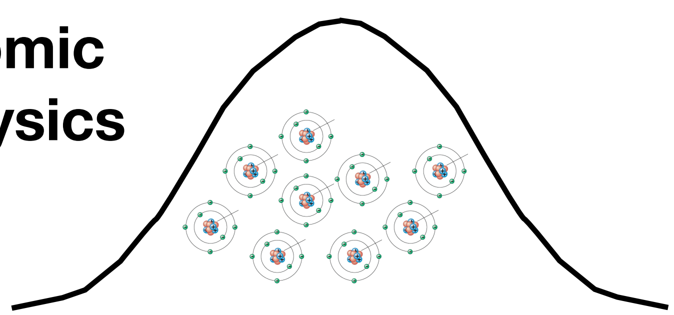
Nano-patterned
superconductors
(~1980)

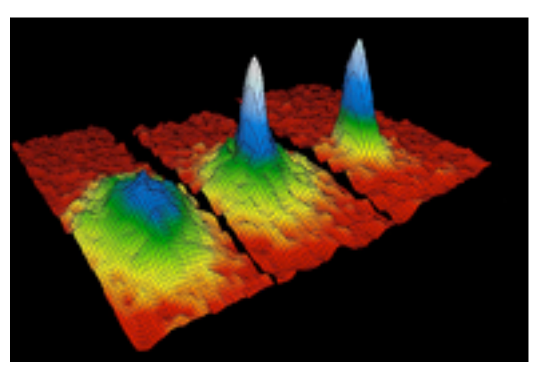


quantum-dot
arrays
on a surface
(~2000)

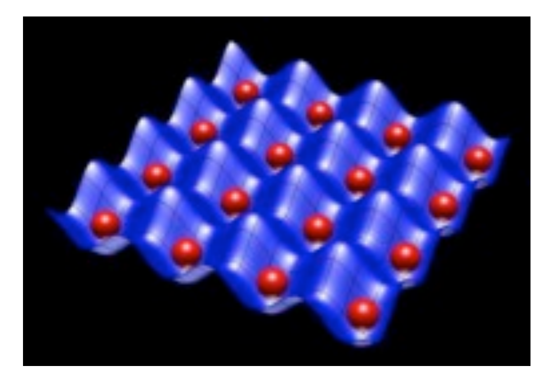


atomic physics





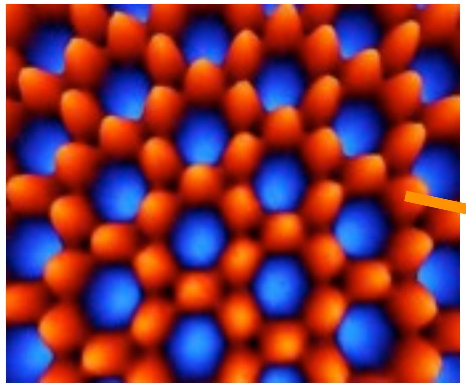
Optical lattices:
artificial solids of light
(~2000)



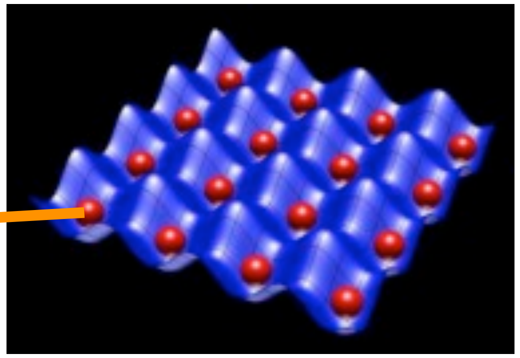
Ultra-cold atoms: Bose-Einstein condensates *(1995)*

The complexity of quantum matter

Quantum matter = quantum fields



$$\hat{\psi}_i$$



operator-valued field (field of matrices)

Fermions: 2x2 matrix

$$\hat{\psi} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

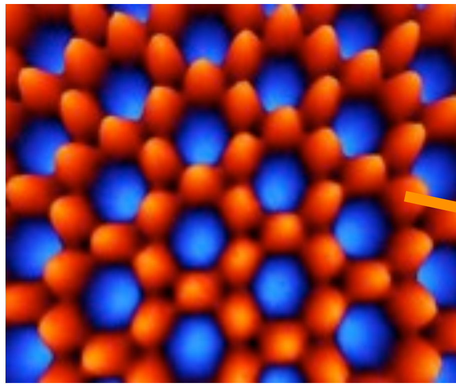
Bosons: $\infty \times \infty$ matrix

$$\hat{\psi} = \begin{pmatrix} 0 & 1 & 0 & 0 & \ldots \\ 0 & 0 & \sqrt{2} & 0 & \ldots \\ 0 & 0 & 0 & \sqrt{3} & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

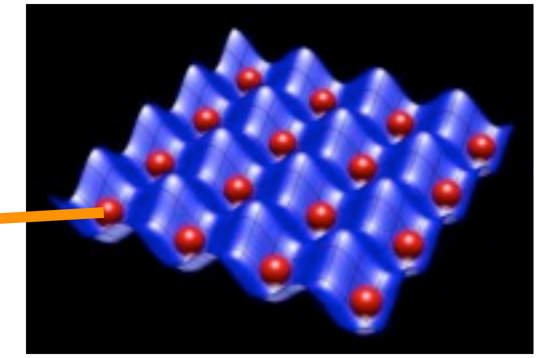
$\hat{\psi}_i^\dagger \hat{\psi}_i$ = particle number at site *i*

$$\hat{\psi}_i^\dagger \hat{\psi}_i \neq \hat{\psi}_i \hat{\psi}_i^\dagger$$

Quantum matter = quantum fields



$$\hat{\psi}_i$$

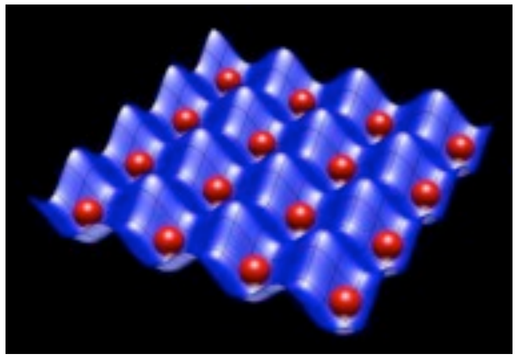


interacting quantum fields: energy is also an operator (*Hamiltonian*)

$$\hat{\mathcal{H}} = \sum_{ij} \left(J_{ij}\ \hat{\psi}_i^\dagger \hat{\psi}_j + \mathrm{h.c.} \right) + \boxed{\sum_{ij} V_{ij}\ \hat{\psi}_i^\dagger \hat{\psi}_i\ \hat{\psi}_j^\dagger \hat{\psi}_j}$$

Quantum many-body problem: condensed matter
quantum chemistry
quantum field-theory and particle physics
nuclear physics
etc..

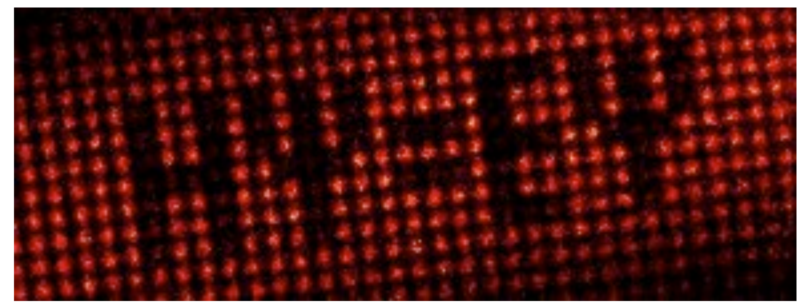
Numerics: matrix diagonalization



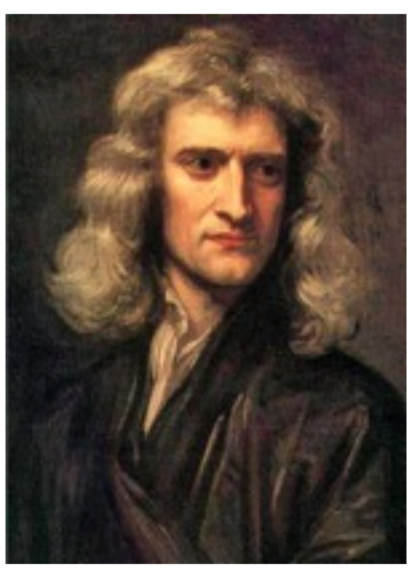
$$\hat{\mathcal{H}} = \sum_{ij} \left(J_{ij}\ \hat{\psi}_i^\dagger \hat{\psi}_j + \mathrm{h.c.} \right) + \sum_{ij} V_{ij}\ \hat{\psi}_i^\dagger \hat{\psi}_i\ \hat{\psi}_j^\dagger \hat{\psi}_j$$

Time-independent Schrödinger's equation

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle$$



(written with atoms in optical lattices)



That's easy!

You just have to solve a simple **linear** *eigenvalue problem!*

My **F**=m**a** *is a set of* **coupled nonlinear partial differential equations!** *(now, that's hard...)*

Numerics: matrix diagonalization

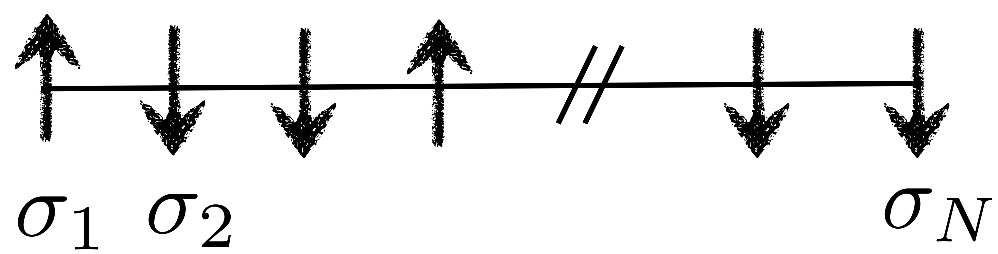
Then just take the biggest supercomputers and crunch numbers...

PSMN (Lyon)
(~ 1000 cores)

Tianhe-2 (China)
(> 3 million cores)

“The Hilbert space is a big place”

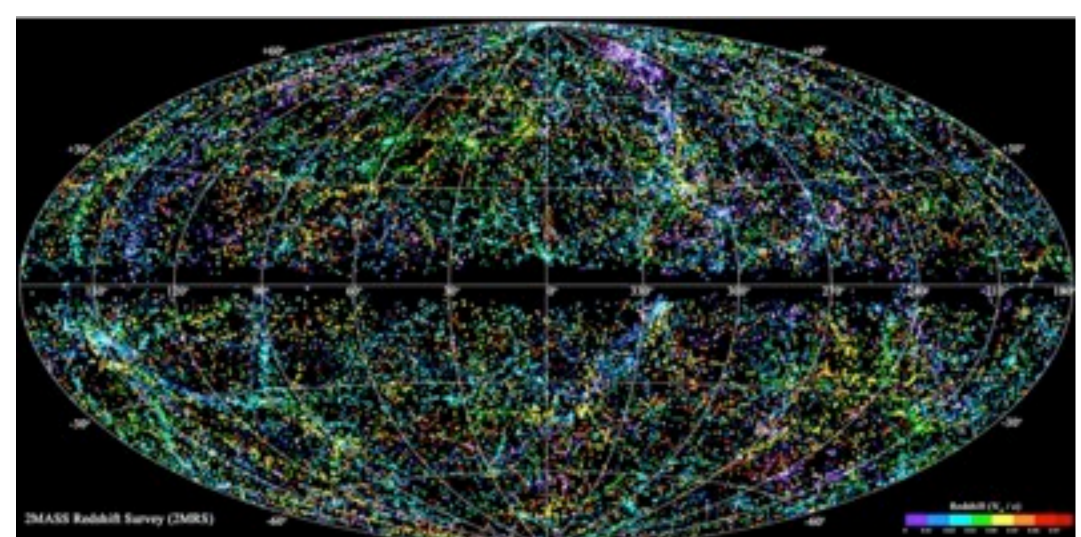
Take the simplest quantum field: the S=1/2 spin (two values on each site)



$$\sigma_i = \pm 1$$

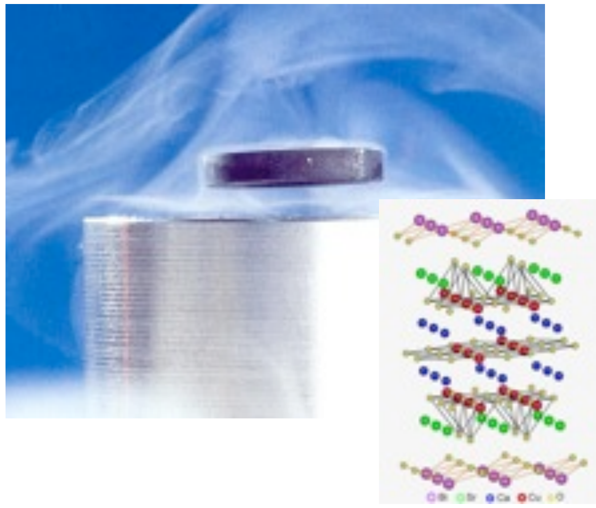
Hamiltonian describing a system of N interacting spins: $2^N \times 2^N$ matrix

$N = 250 :$ $2^{250} >$ **number of atoms in the universe**



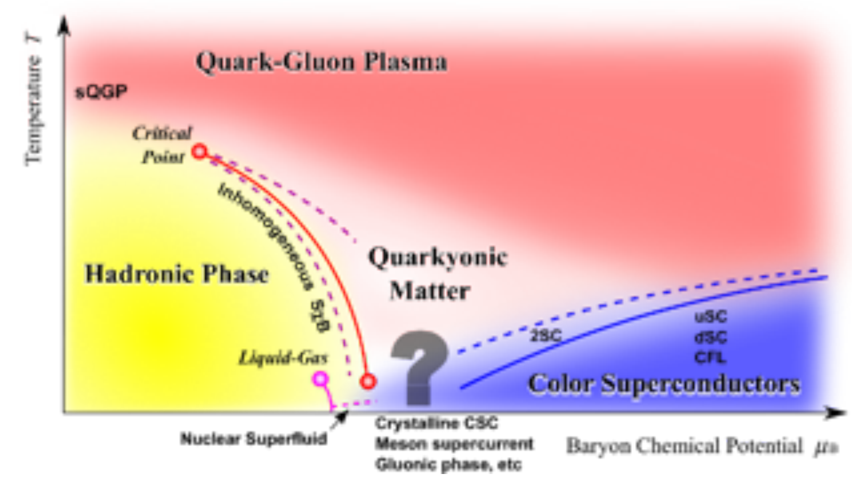
“Things we know we don’t know”

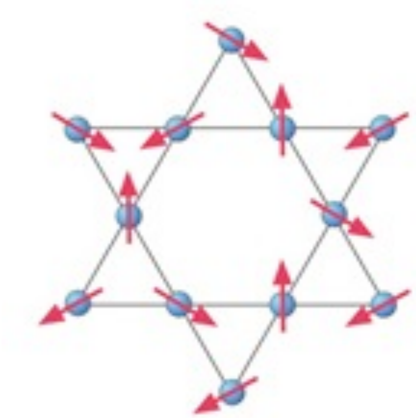
http://en.wikipedia.org/wiki/List_of_unsolved_problems_in_physics



We do not know the **mechanism of superconductivity of *most* superconducting materials**
(in particular, of materials with a high critical temperature)

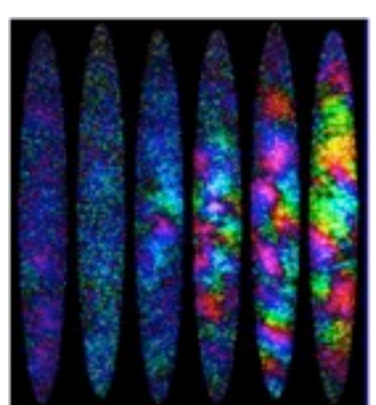
We do not know **the phase diagram of nuclear matter (Quantum Chromodynamics)**





We do not know the **ground state of “frustrated” quantum spin systems**

We do not know **how a closed quantum system relaxes towards its equilibrium state**



fermions + interaction + ...

bosons + interaction + a gauge field + ...

= ??

etc...

The big idea: use quantum machines...

Analog quantum simulators

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.

Classical computer

digital machine

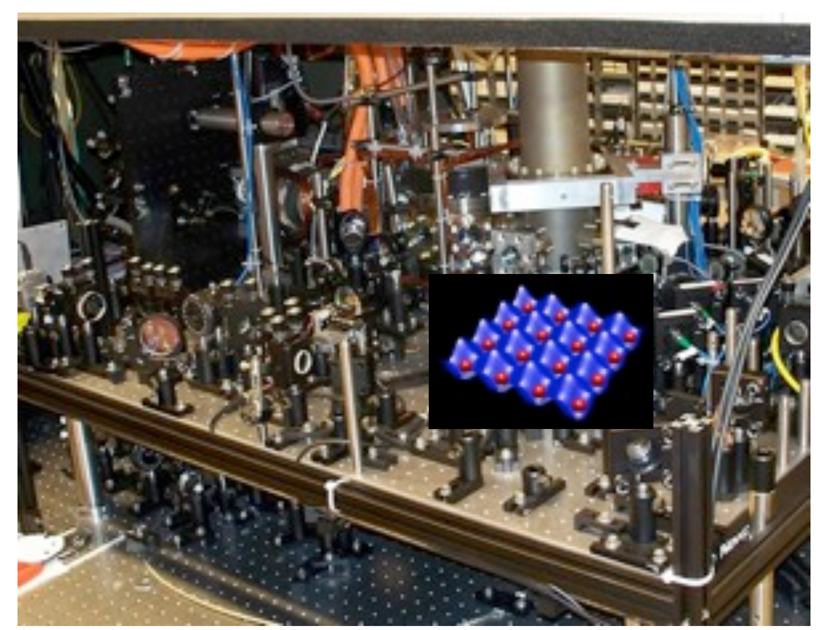
which *simulates any other system*

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle$$

Quantum simulator

analog machine

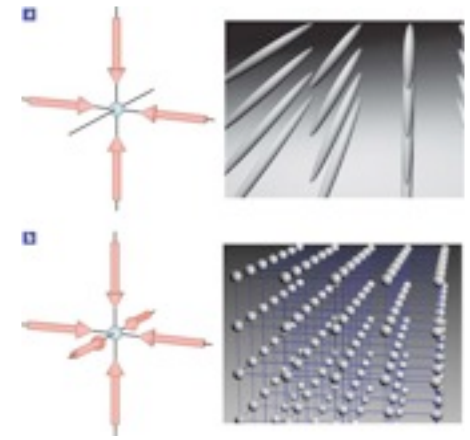
which *efficiently simulates* *itself*



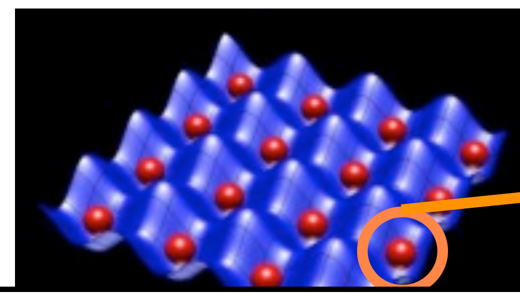
$$\hat{\mathcal{H}} = \sum_{ij}\left(J_{ij}\ \hat{\psi}_i^\dagger\hat{\psi}_j + \mathrm{h.c.}\right) + \sum_{ij} V_{ij}\ \hat{\psi}_i^\dagger\hat{\psi}_i\ \hat{\psi}_j^\dagger\hat{\psi}_j$$

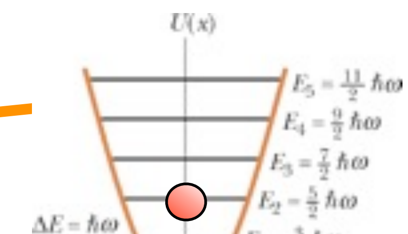
Exquisite experimental control on cold atoms

Optical trapping: fine control on **geometry**



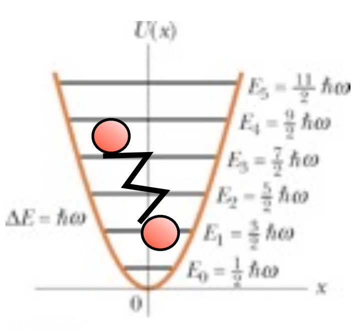
Simplicity of building blocks

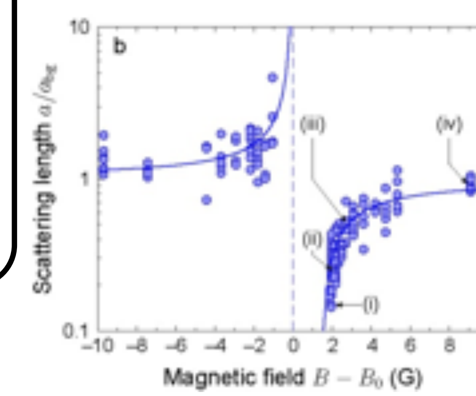




Hamiltonian engineering

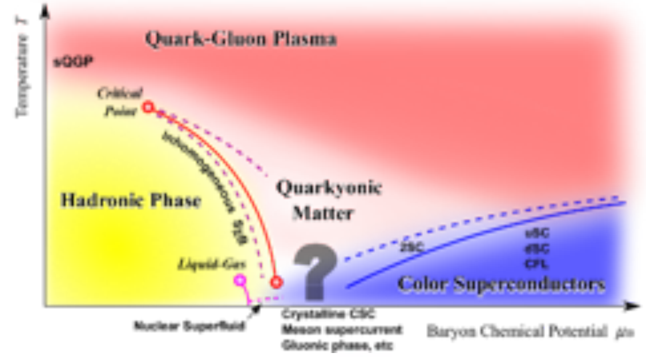
$$\hat{\mathcal{H}} = \sum_{ij} \left(J_{ij} \; \hat{\psi}_i^\dagger \hat{\psi}_j + \mathrm{h.c.} \right) + \sum_{ij} V_{ij} \; \hat{\psi}_i^\dagger \hat{\psi}_i \; \hat{\psi}_j^\dagger \hat{\psi}_j$$

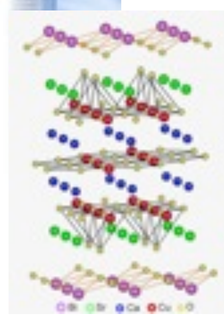


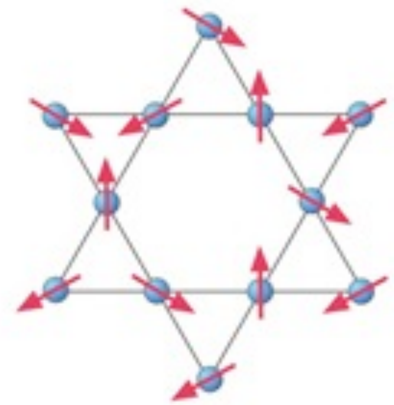


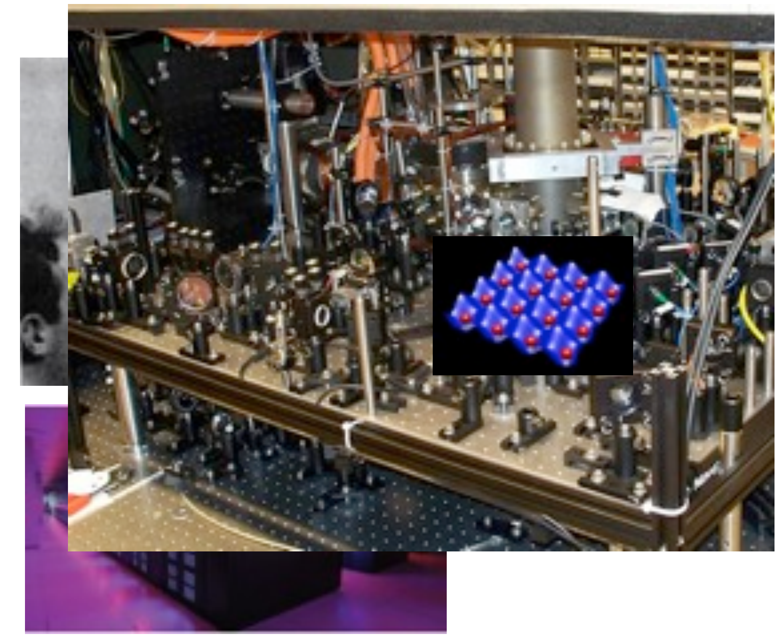
Control on the **nature of the quantum fields** (bosons/fermions, spinful/spinless)

Is a quantum machine going to solve it all?



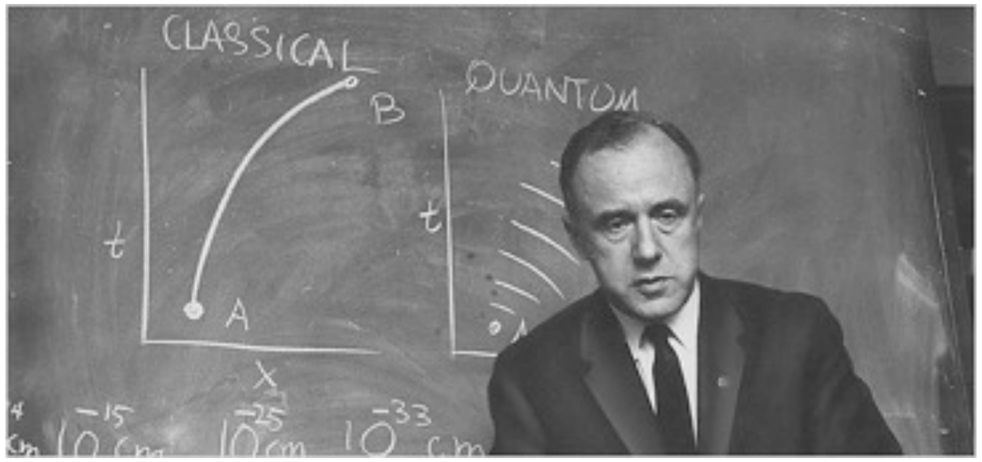






Is a quantum machine going to solve it all?

“Never make a calculation until you know the answer”



John Wheeler

Never make a quantum simulation until you know the answer

The D-Wave machine

512 superconducting qubits

Feb. 17, 2014

How "Quantum" is the D-Wave Machine?

Seung Woo Shin*, Graeme Smith†, John A. Smolin†, and Umesh Vazirani*

*Computer Science division, UC Berkeley, USA.
†IBM T.J. Watson Research Center, Yorktown Heights, NY 10598, USA.

arXiv:1401.7087v1 [quant-ph] 28 Jan 2014

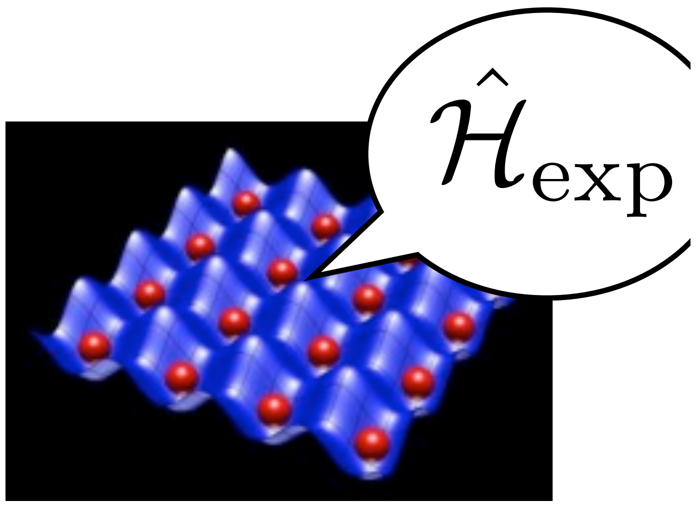
Distinguishing Classical and Quantum Models for the D-Wave Device

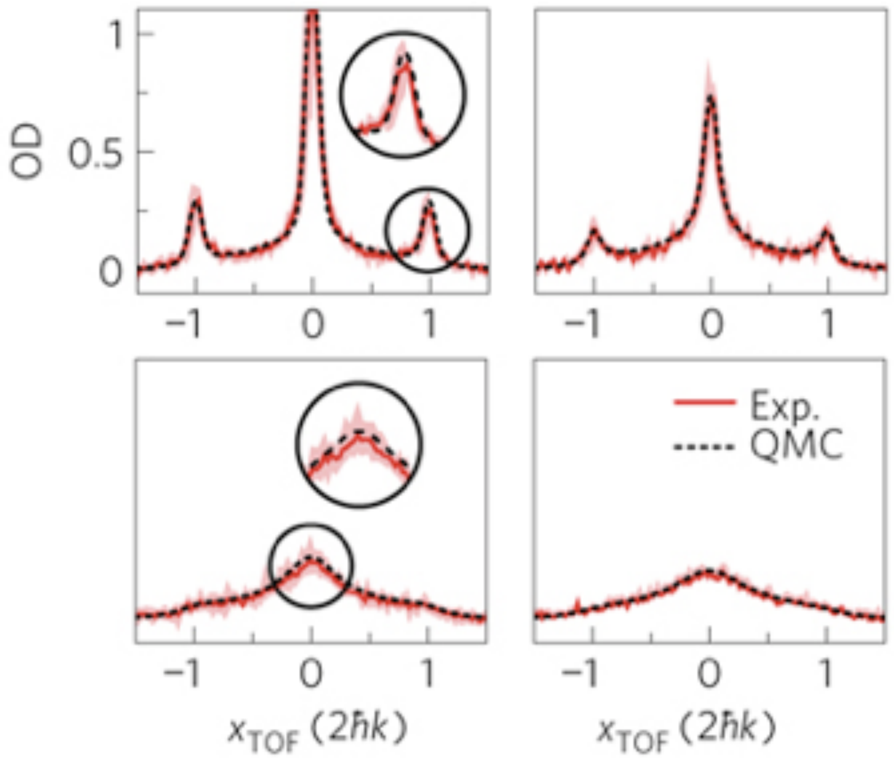
Walter Vinci,[1,2,*] Tameem Albash,[3,4,*] Anurag Mishra,[3,4] Paul A. Warburton,[1,5] and Daniel A. Lidar[3,4,6,7]

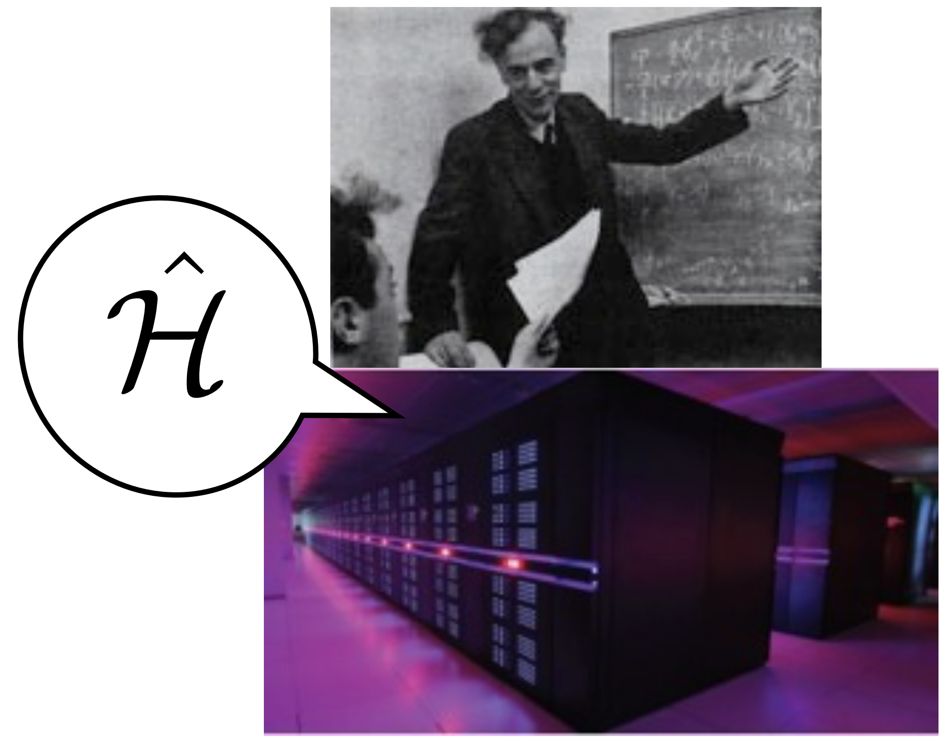
arXiv:1403.4228v1 [quant-ph] 17 Mar 2014

Quantum simulators need “classical simulators”
(and viceversa)

“Classical simulators” can test the experimental Hamiltonians





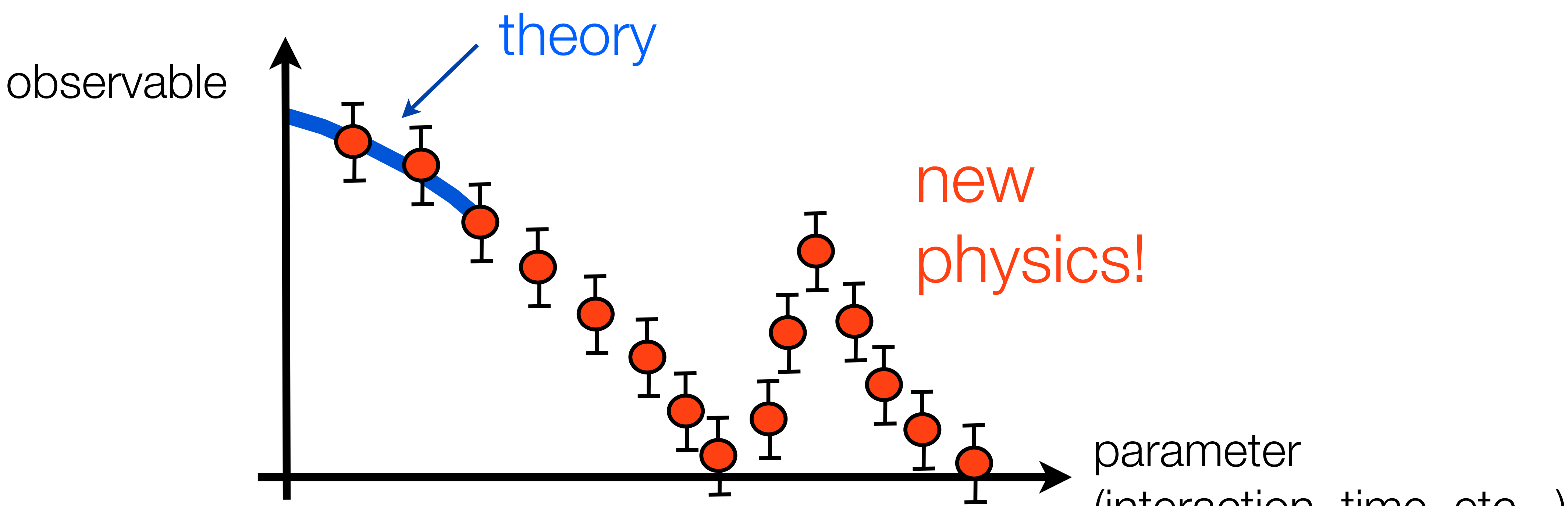


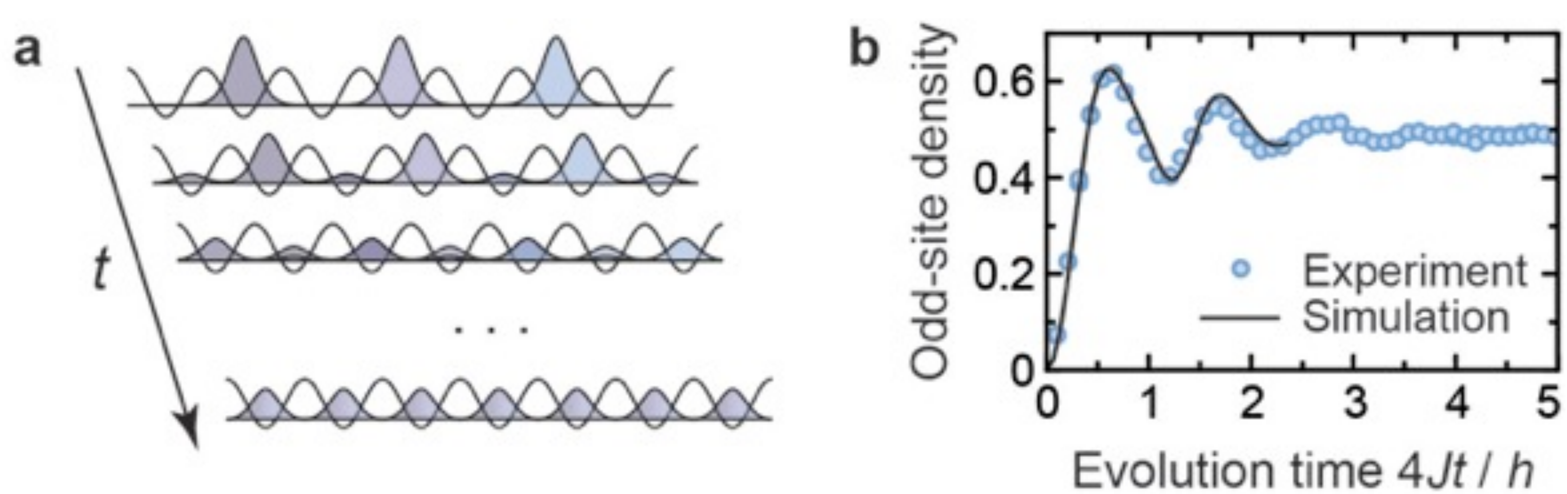
S. Trotzky et al., Nat. Phys. 2010

Quantum simulators can test approximate theories (and approx. classical simulation methods)

“Quantum *stimulators*” for the study of complex quantum phenomena *(A. Aspect)*

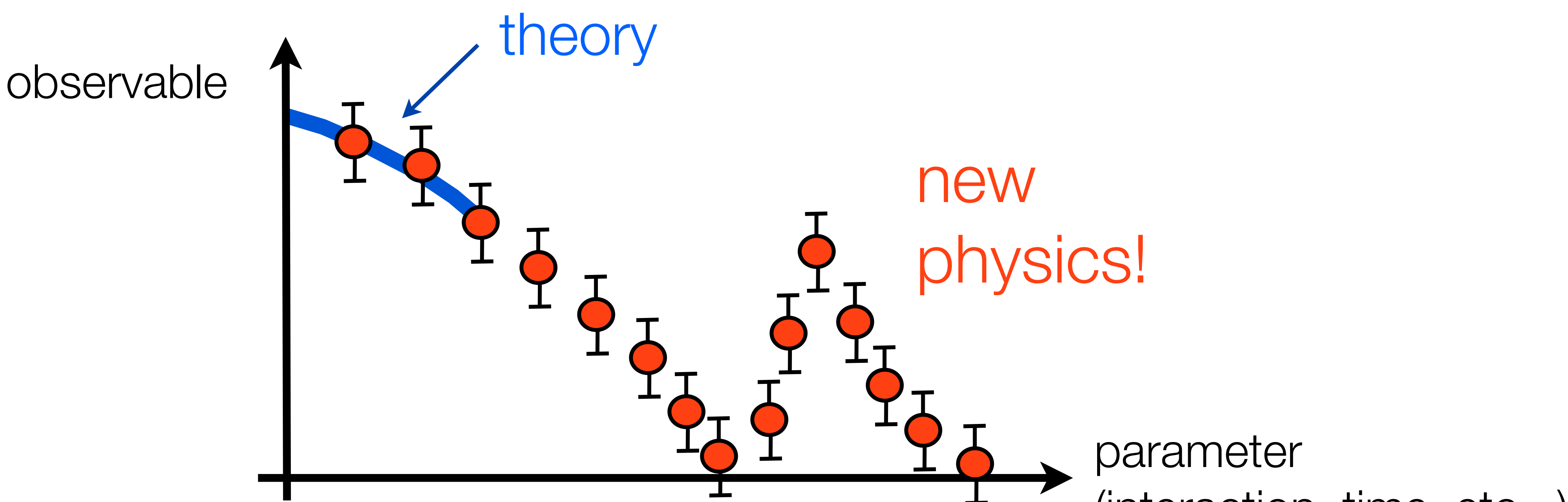
Quantum simulation in practice: *extension of theory*

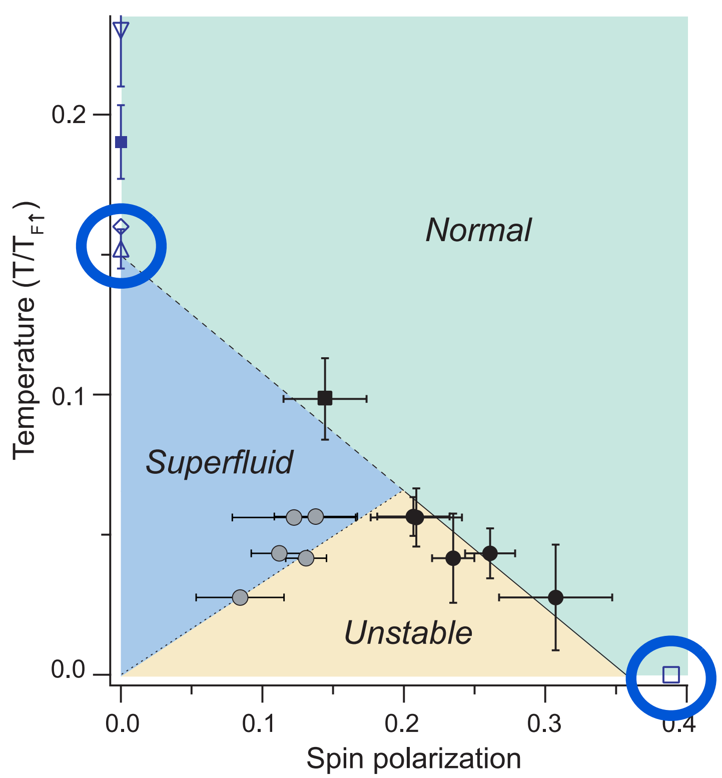




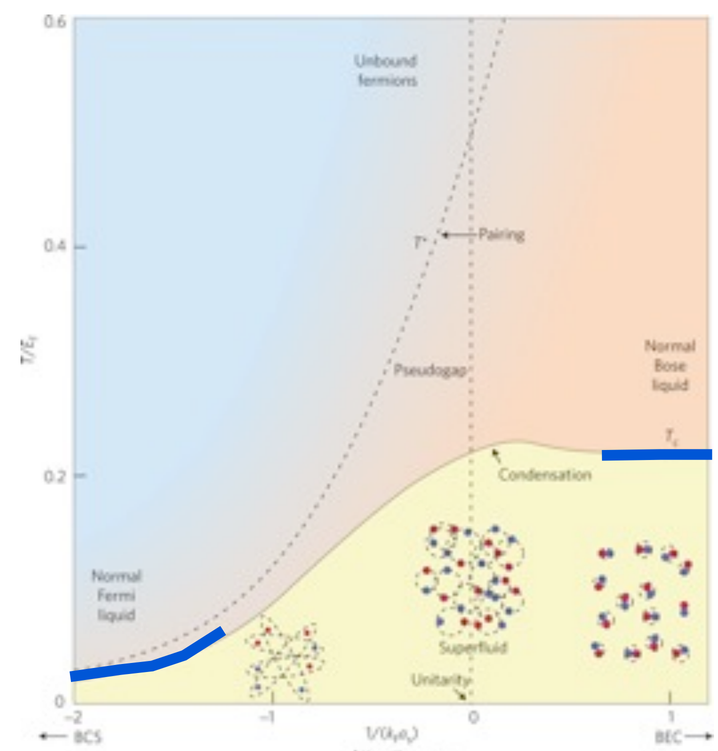
S. Trotzky et al.
Nat. Phys. 2012

Quantum simulation in practice: *extension of theory*





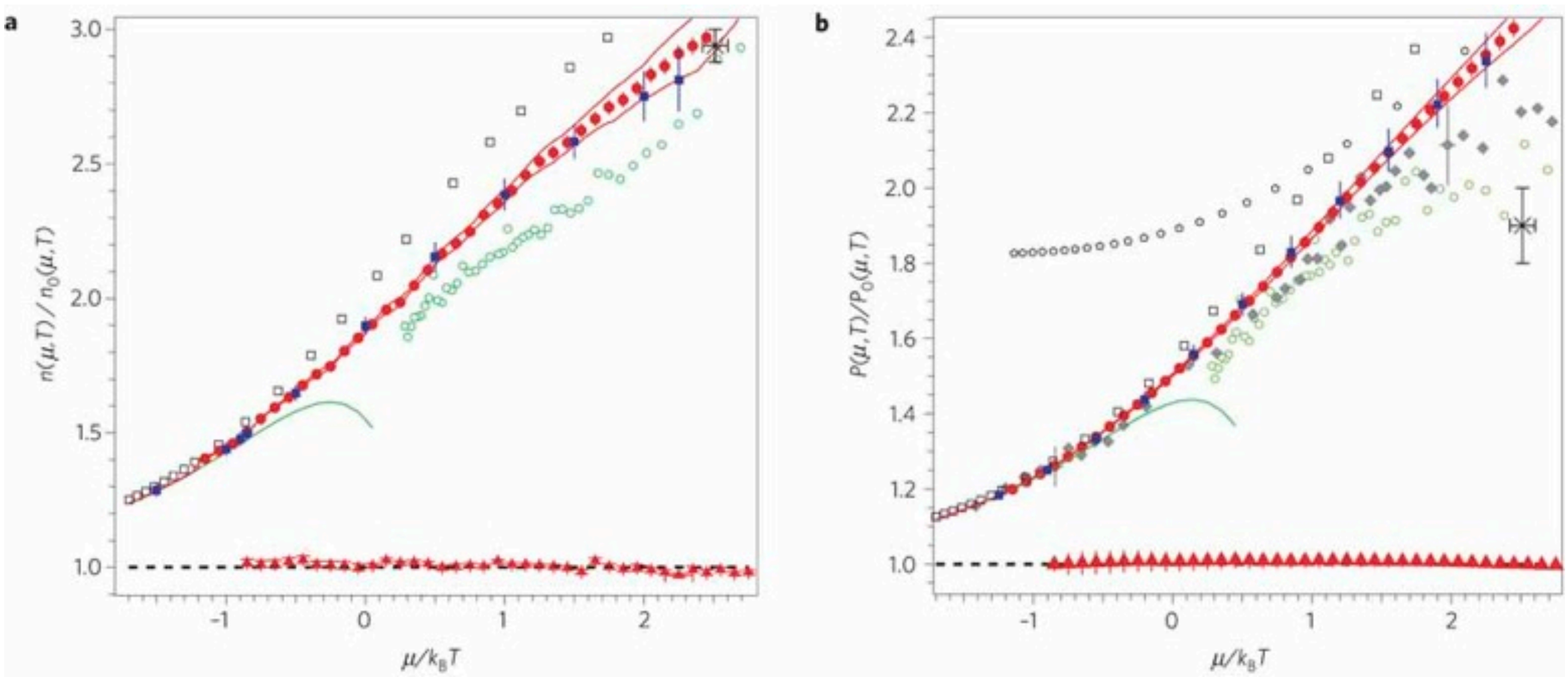
Y. Shin et al., Nature 2008



M. Randeria, Nat. Phys. 2010

Quantum simulation in practice: *test of theory*

theory: numerical resummation of a non-simply convergent infinite series



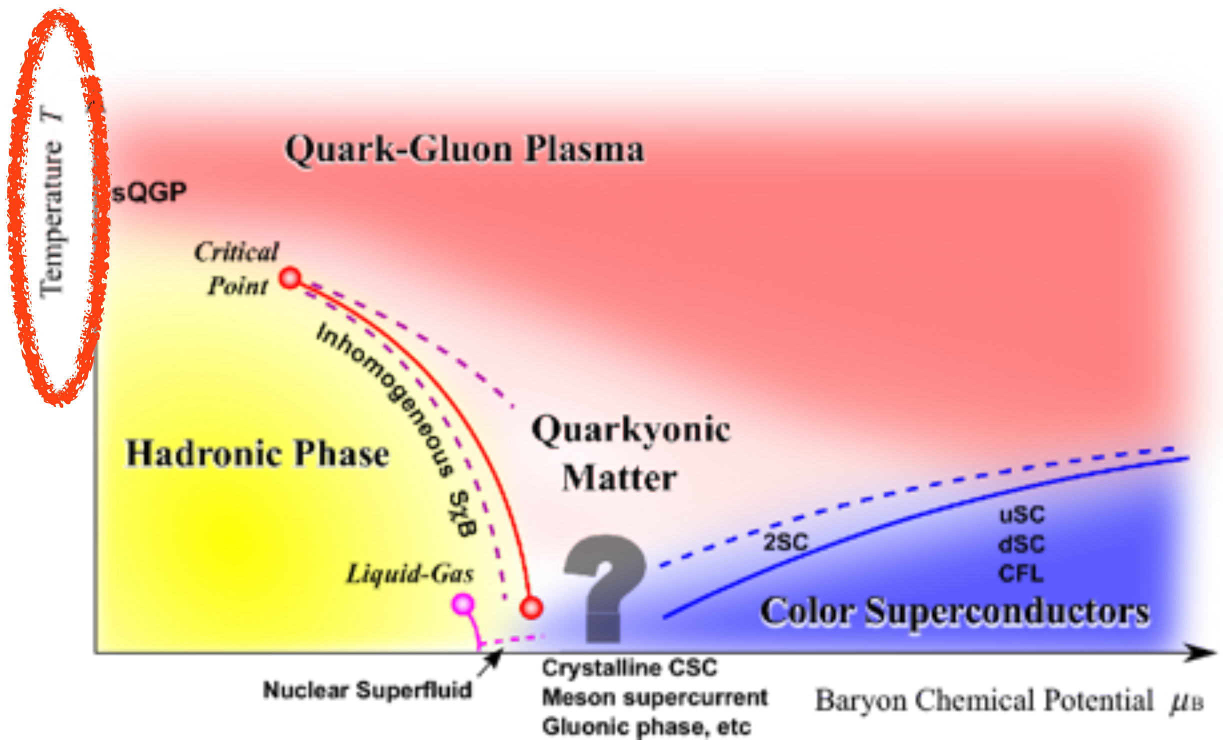
K. van Houcke et al., Nat. Phys. 2012

exp: ultracold gas of K-40 fermions

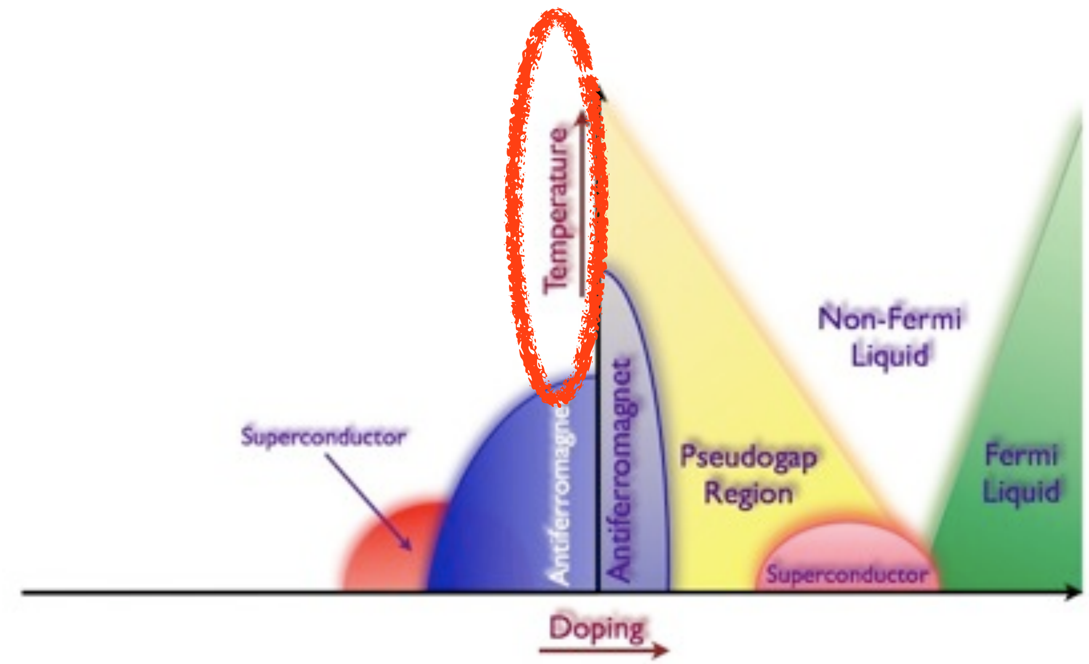
“Calibrating” quantum simulators

(and learning new physics while doing so)

Quantum simulation of complex phase diagrams



quantum chromodynamics



high-Tc superconductors

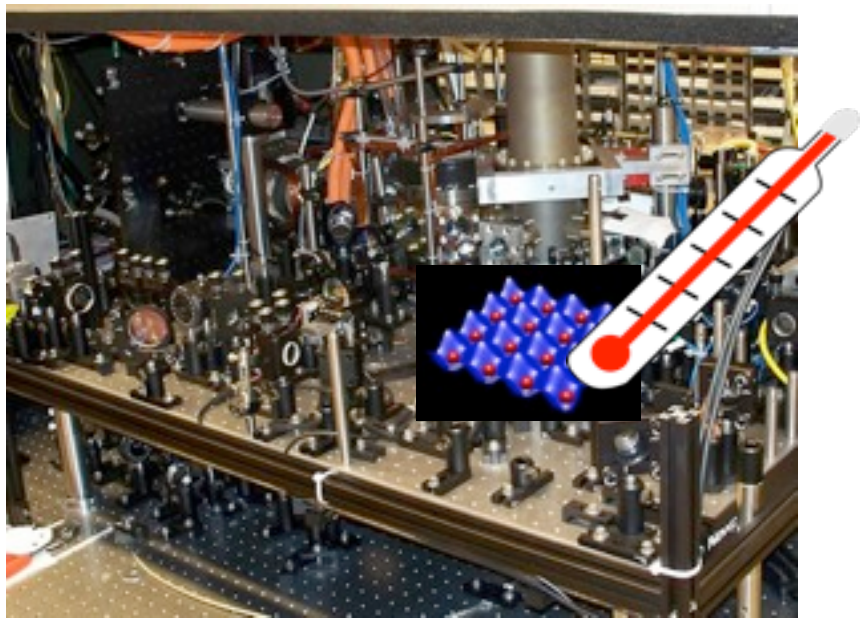
All condensed matter experiments are performed in a **heat bath**

Thermometry in a cold-atom quantum simulator

Where's the heat bath here?

The system is its own heat bath
(microcanonical setting)

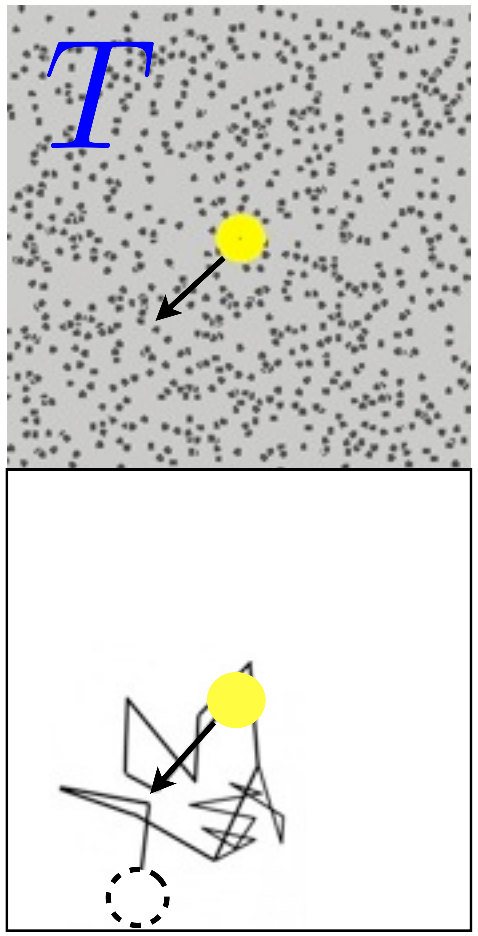
Temperature has to be *measured*



Thermometer = "gentle probe" whose thermodynamics is perfectly known

Atomic physic quantum simulators:
~100,000 particles at *T* ~ 10 nK !!

Idea: use the noise!



Brownian motion: Einstein's relation (1905)

$$D = \mu_\gamma \boxed{k_B T}$$

diffusion constant

mobility

A. Einstein

$$\mu_\gamma = 1/\gamma \qquad \mathbf{F}_{\text{frict}} = -\gamma \mathbf{v}$$

R. Kubo

Fluctuation-dissipation relation (~1950's) *(R. Kubo)*

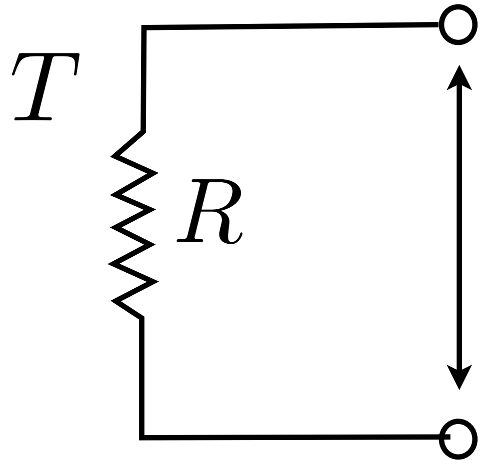
$$D = \boxed{\int_0^\infty dt \; \langle \delta \mathbf{v}(0) \cdot \delta \mathbf{v}(t) \rangle} = |\delta \tilde{\mathbf{v}}(\omega = 0)|^2 = \frac{\boxed{k_B T}}{\boxed{\gamma}}$$

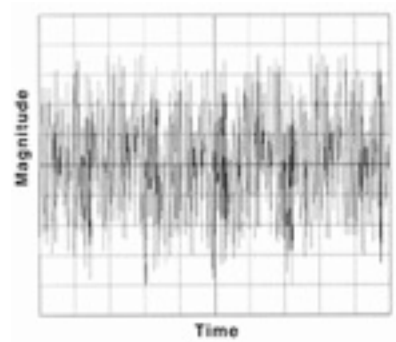
fluctuation of the particle's velocity

dissipation term

Noise thermometry

Thermal noise in a resistor: *Johnson-Nyquist noise* (1928)





J. B. Johnson

H. Nyquist

$$|\tilde{V}(\omega)|^2 = \int dt\; e^{-i\omega t} \langle V(0)\; V(t) \rangle$$

noise power spectrum

$$\boxed{|\tilde{V}(\omega)|^2} = 4\, \boxed{R}\, \boxed{k_B T}$$

fluctuation *dissipation*

(classical) Nyquist theorem

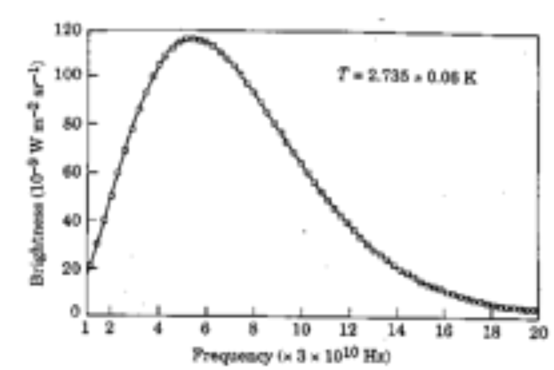
Johnson-noise thermometer

THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 17, NUMBER 7 JULY, 1946

The Measurement of Thermal Radiation at Microwave Frequencies

R. H. DICKE*
*Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts***

(Received April 15, 1946)



cosmic black-body radiation

Thermal vs. quantum noise

$$|\tilde{V}(\omega)|^2 = 4\ R\ k_B T$$

(classical) Nyquist theorem

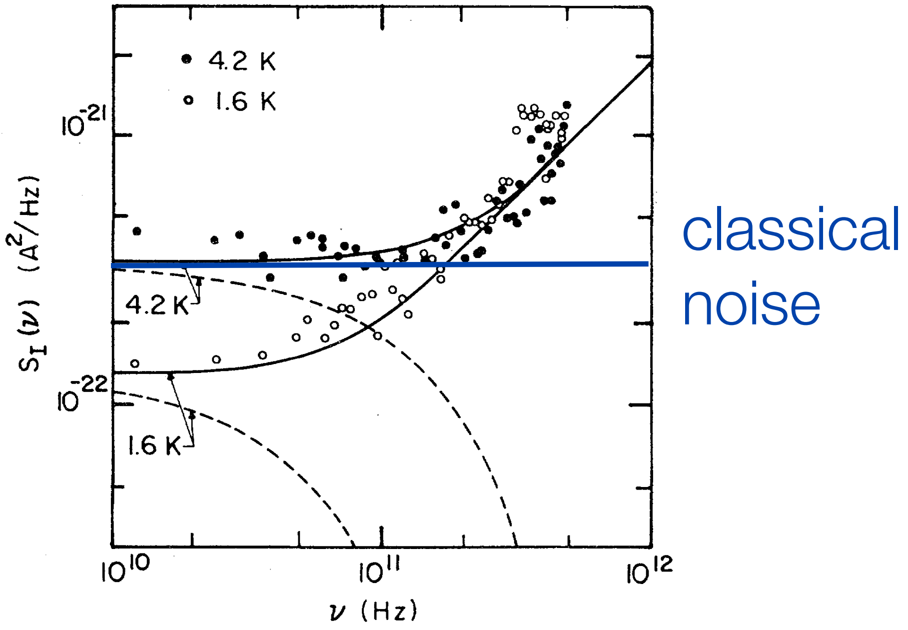
$$|\tilde{V}(\omega)|^2 = 4\ R\ \hbar\omega \overbrace{\left(n(\omega) + \frac{1}{2} \right)}$$

quantum Nyquist theorem

thermal noise

quantum noise (zero-point fluctuations)

$$n(\omega) = \frac{1}{e^{\hbar\omega/(k_B T)} - 1}$$

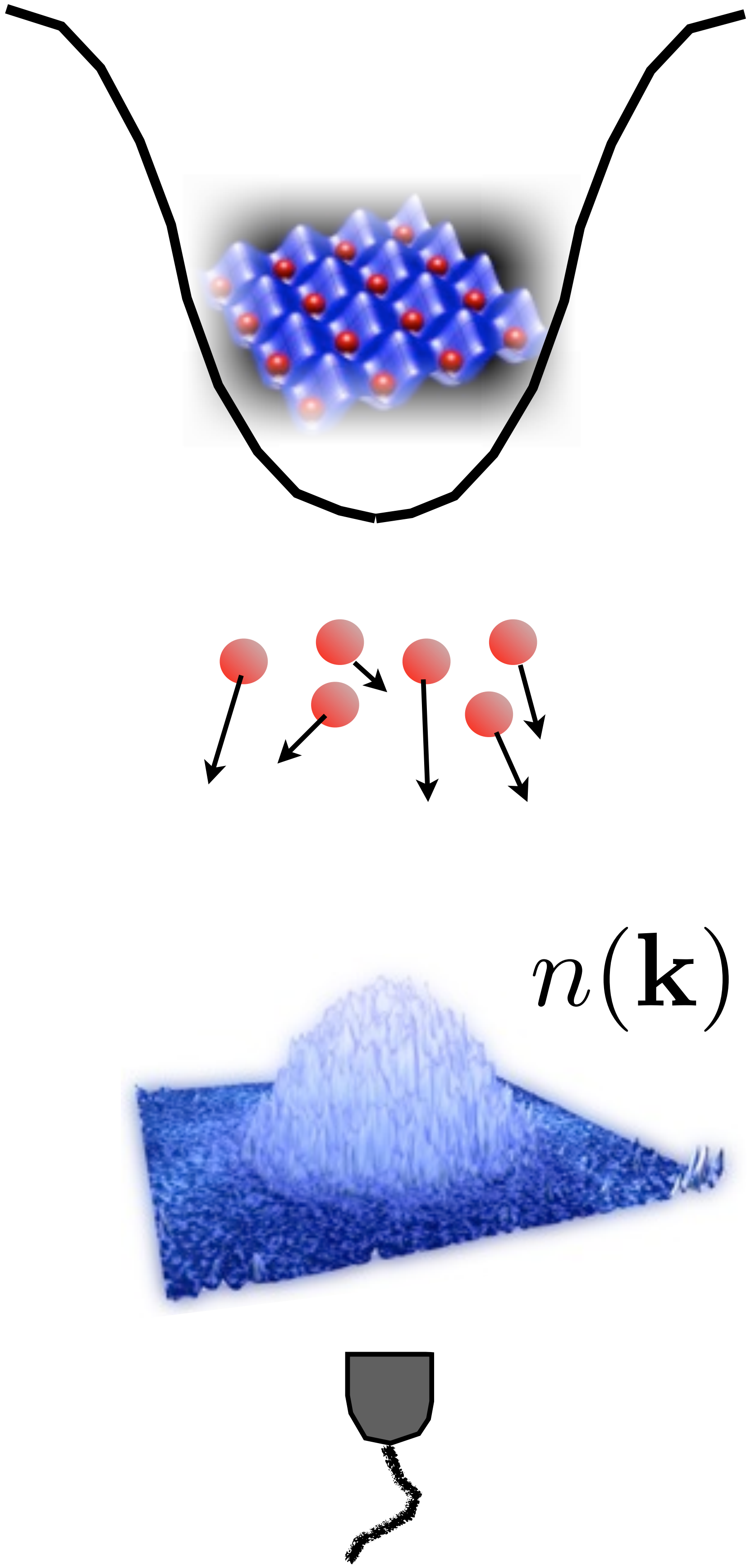


Koch et al, PRB 1982

$$\hbar\omega \ll k_B T$$

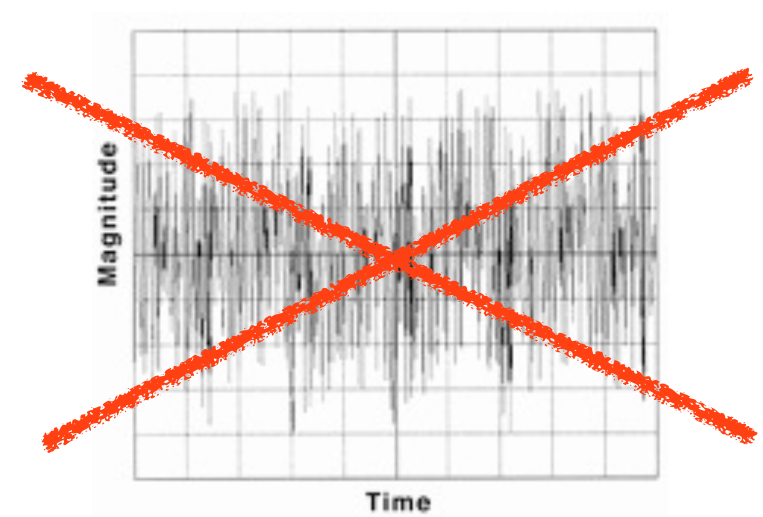
classical limit

What noise in a cold atom experiment?



measurements in cold atoms
are generally destructive

temporal fluctuations
cannot be monitored

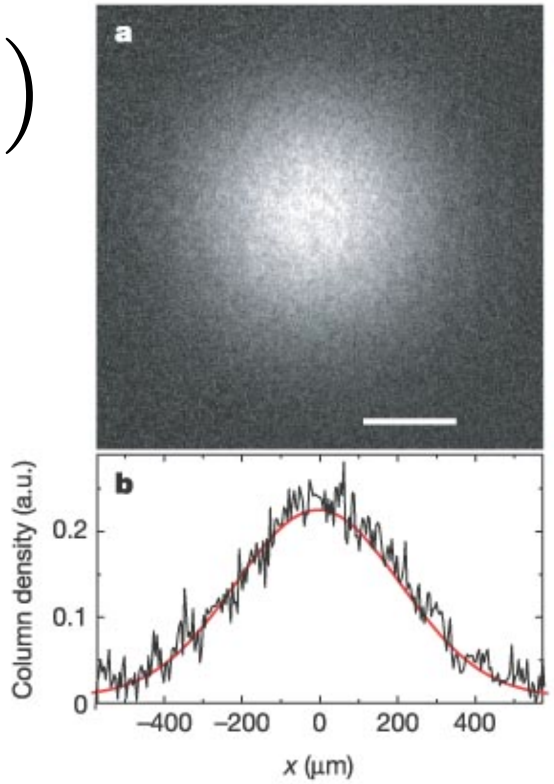


only single snapshots ->
frequency-integrated noise

What noise in a cold atom experiment?

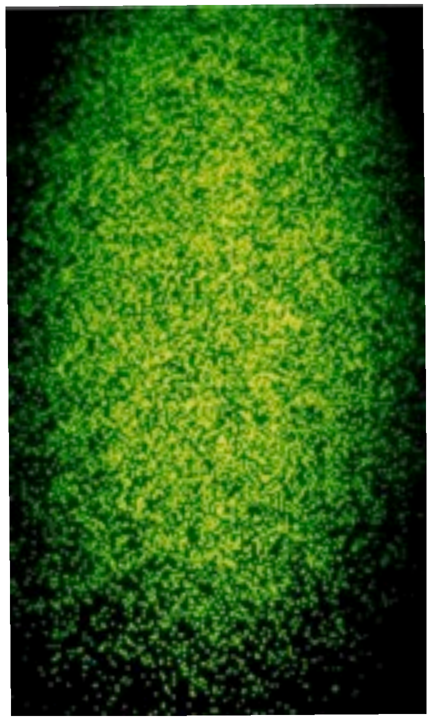
time-of-flight image

$n(\mathbf{k})$



S. Fölling et al., Nature 2005

in-situ microscopy image



$n(\mathbf{r})$

W. Bakr et al., Nature 2009

$$G(\mathbf{k},\mathbf{k}') = \langle \delta n(\mathbf{k}) \delta n(\mathbf{k}') \rangle$$

$$C(\mathbf{r},\mathbf{r}') = \langle \delta n(\mathbf{r}) \delta n(\mathbf{r}') \rangle$$

Correlation functions for fluctuations

Fluctuation-dissipation relations

$$\mathcal{H} = \mathcal{H}_0 - hA$$ static perturbation

$$\langle A \rangle = -\frac{\partial F}{\partial h}$$

response
(*"dissipation"*)

fluctuation

$$\boxed{\chi_{AA}} = \frac{\partial \langle A \rangle}{\partial h} = \frac{\boxed{\langle (\delta A)^2 \rangle}}{\boxed{k_B T}} \qquad [A, \mathcal{H}_0] = 0$$

$$\langle \delta^2 A \rangle = \chi_{AA} \; k_B T$$

thermal
noise

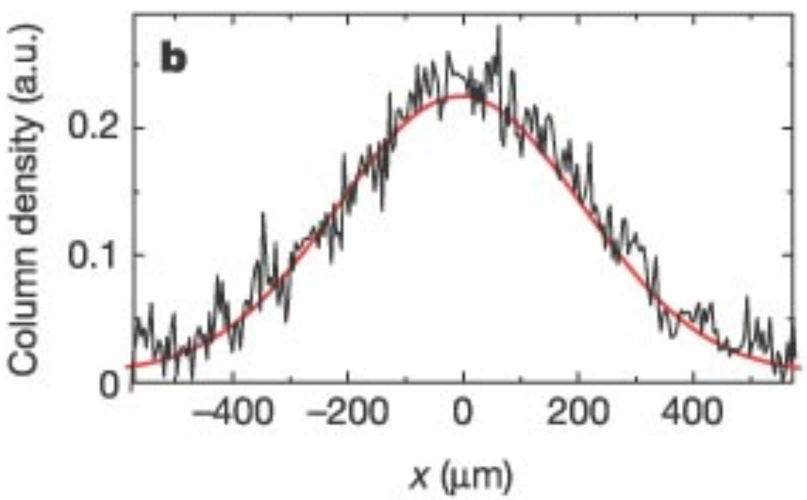
but if $\boxed{[A, \mathcal{H}_0] \neq 0}$

$$\langle (\delta A)^2 \rangle \; = \; \langle (\delta A)^2 \rangle_T + \langle (\delta A)^2 \rangle_Q \; \geq \; \chi_{AA} \; k_B T$$

thermal noise

quantum noise

Momentum-noise thermometry



$$\hat{P}_n = \sum_{\mathbf{k}} (\hbar k_n)\ \hat{n}(\mathbf{k})$$ *total momentum along n*

$$\langle \hat{P}_n \rangle = 0 \qquad \boxed{\langle \hat{P}_n^2 \rangle \neq 0}$$

$$[\hat{\mathcal{H}}, \hat{P}_n] = 0 \qquad \frac{\langle \hat{P}_n^2 \rangle}{2mN} = \frac{1}{2} k_B T$$

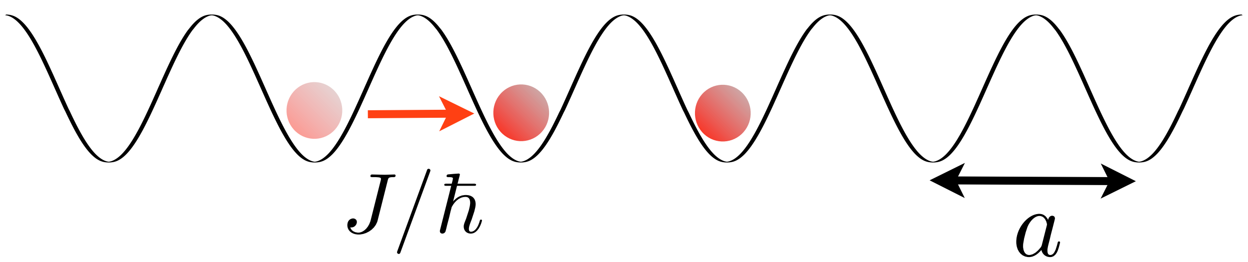
kinematic def. of T
beyond equipartition

$$\langle \delta^2 \hat{P}_n \rangle = N \langle \hat{p}_n^2 \rangle \qquad \frac{\langle p_n^2 \rangle}{2m} = \frac{1}{2} k_B T$$

ideal classical gas (equipartition)

$$[\hat{\mathcal{H}}, \hat{P}_n] \neq 0 \qquad \frac{\langle \hat{P}_n^2 \rangle}{2mN} = \frac{\langle \hat{P}_n^2 \rangle_T + \langle \hat{P}_n^2 \rangle_Q}{2mN} \geq \frac{1}{2} k_B T$$

In an optical lattice



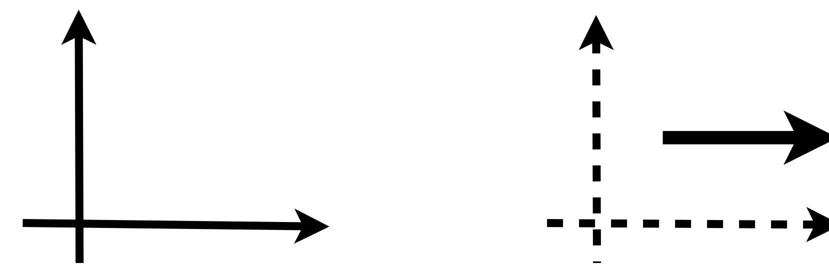
$$\hat{P}_n \quad \rightarrow \quad \hat{\mathcal{J}}_n$$

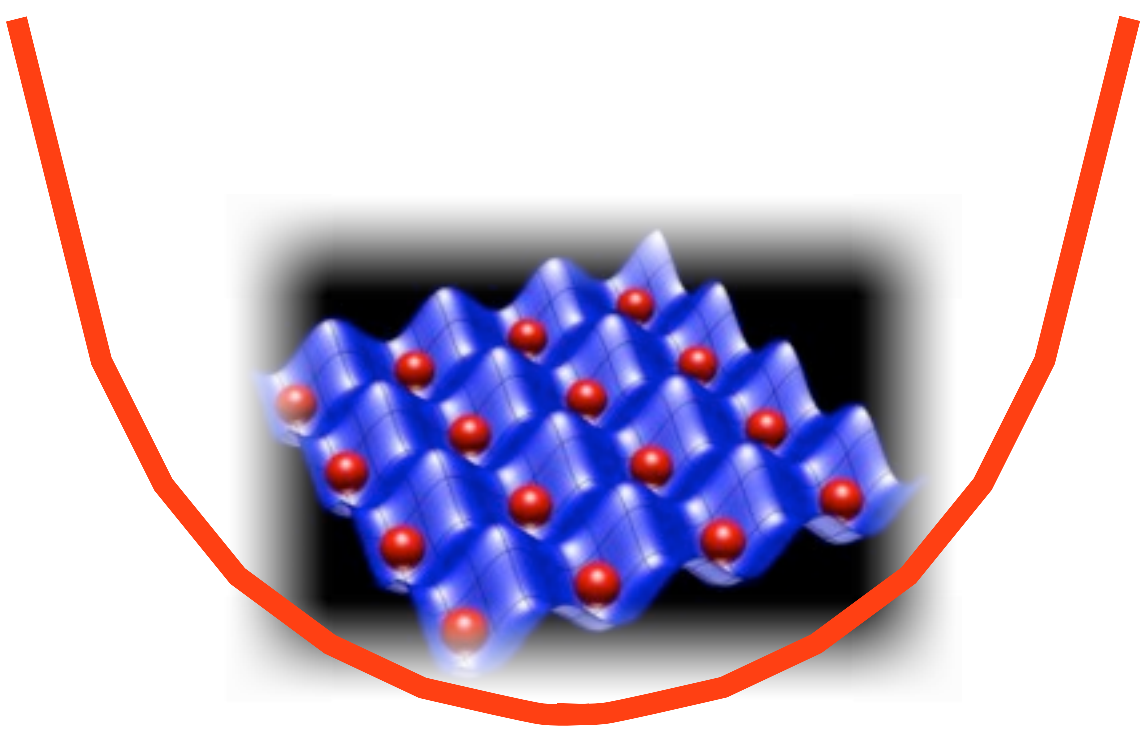
current along the *n*-direction

$$[\hat{\mathcal{H}}, \hat{\mathcal{J}}_n] = 0$$

$$\frac{\langle (\delta \mathcal{J}_n)^2 \rangle}{\partial_\phi \langle \mathcal{J}_n \rangle} = \frac{k_B T}{J}$$

$$\mathbf{v} = \frac{\hbar \phi}{m a} \hat{n}$$





non-interacting fermions in an optical lattice

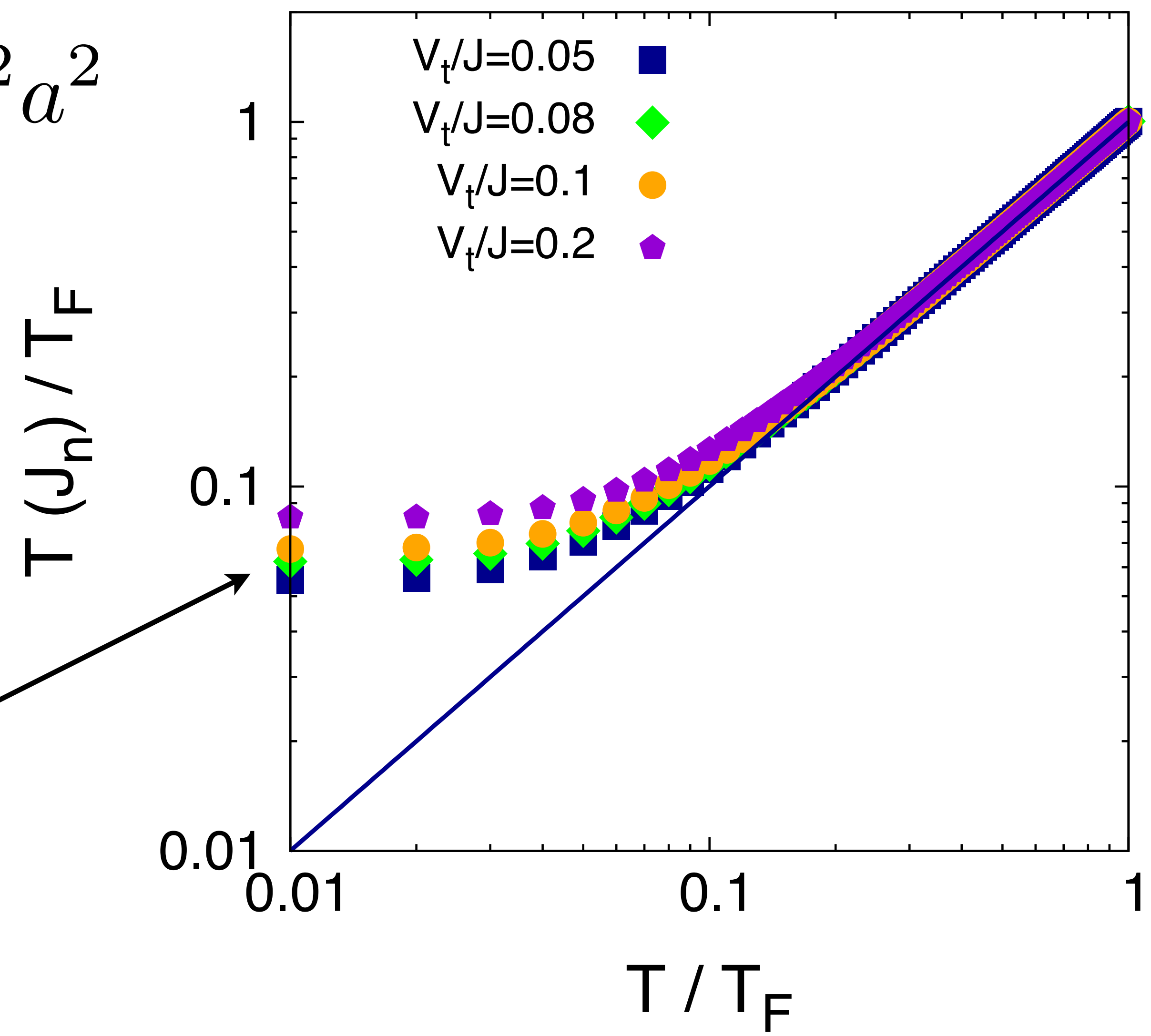
$$[\hat{\mathcal{H}}, \hat{\mathcal{J}}_n] \neq 0$$

quantum fluctuations of the current *due to the parabolic trap*

Thermal vs. quantum fluctuations

$$\frac{\langle (\delta \mathcal{J}_n)^2 \rangle}{\partial_\phi \langle \mathcal{J}_n \rangle} = \frac{k_B T_{\mathcal{J}_n}}{J} \geq \frac{k_B T}{J}$$

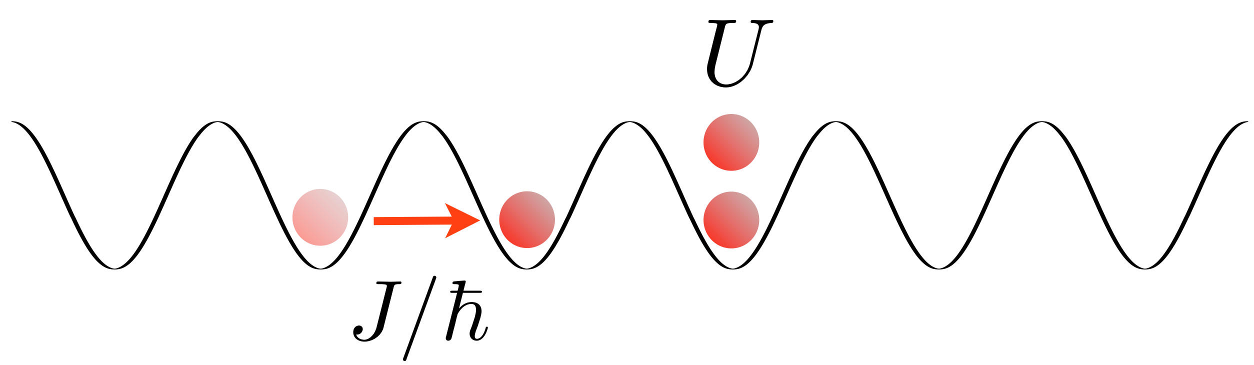
$$V_t = m\omega^2 a^2$$

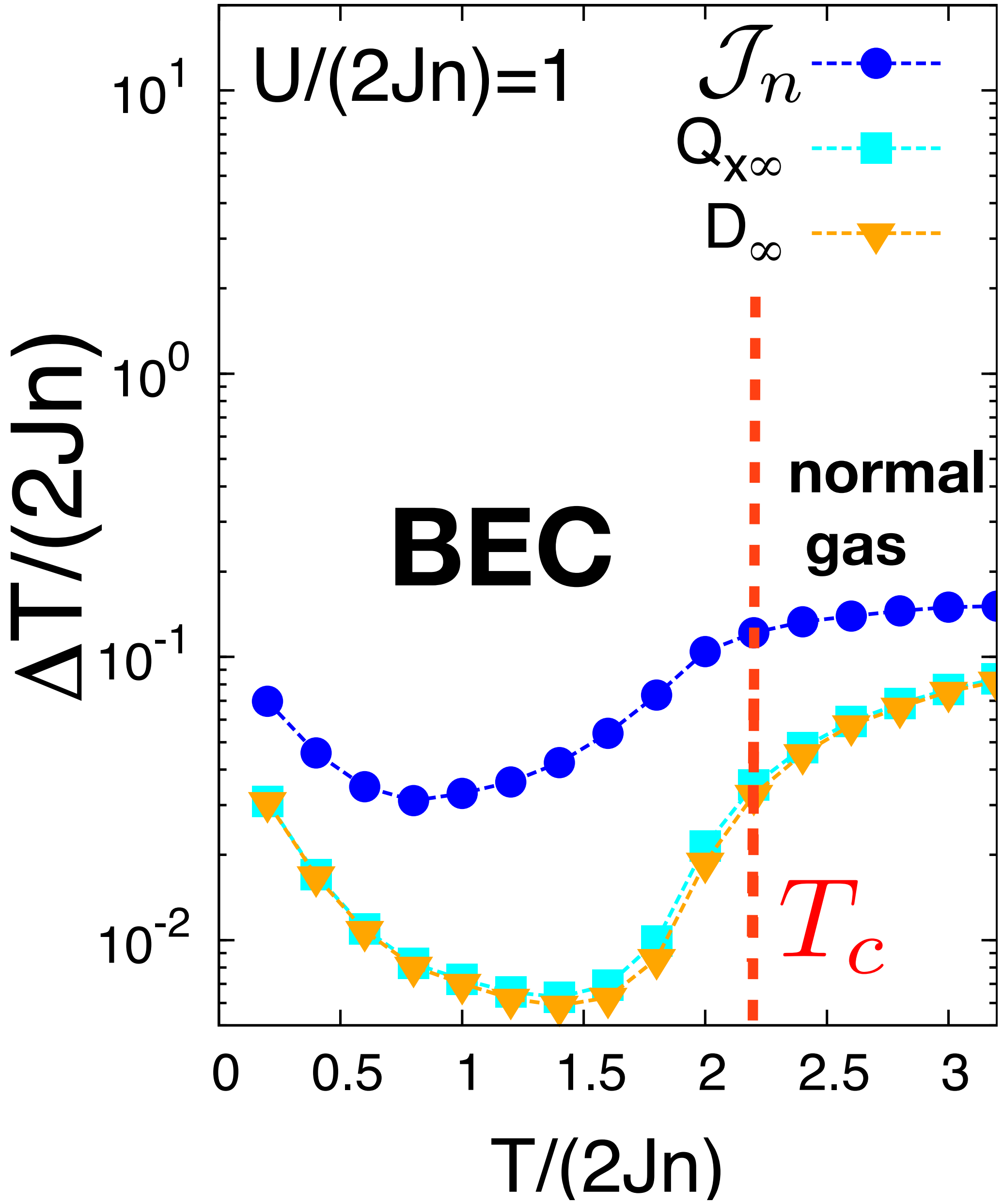


thermal fluctuations

"*quantum temperature*" from zero-point quantum fluctuations

Interacting bosons in an optical lattice





$$\Delta T = T_{\mathcal{J}_n} - T$$

quantum fluctuations are *suppressed* at the onset of Bose-Einstein condensation

TR, Phys. Rev. Lett. 2014

Conclusions

1) Asking how a quantum simulator works / might work, we can learn interesting things

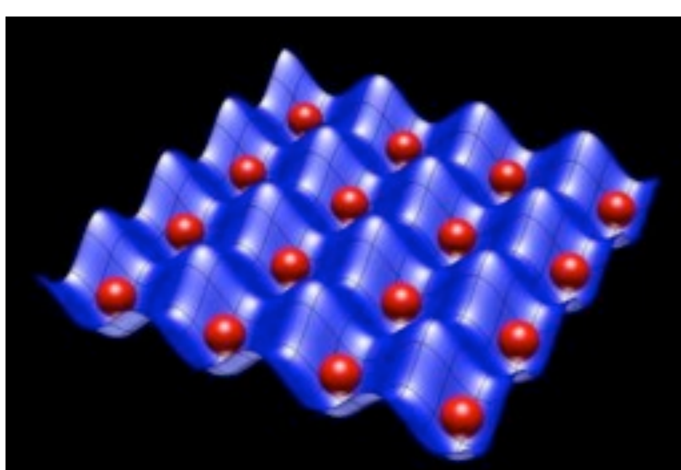
2) Hamiltonian engineering needs design engineers! (= theoreticians)

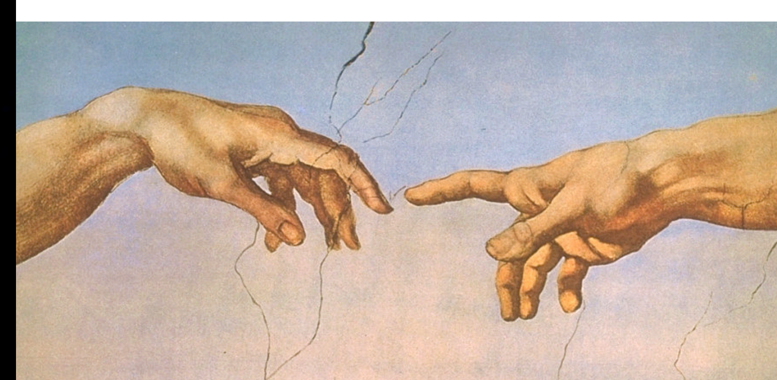
3) Novel quantum many-body phenomena are within reach, but they must be predicted/understood

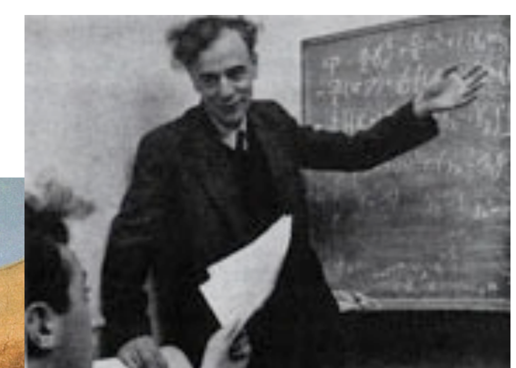
- out-of-equilibrium phenomena
- interaction between matter and (artificial) fields beyond QED/QCD/etc.
- quantum information processing
- ....

"Classical machines"

Quantum machines







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Irénée Frérot

PSMN Pôle Scientifique de Modélisation Numérique

ANR Agence Nationale de la Recherche

IUF

GdR Atomes Froids

https://sites.google.com/site/roscilde/

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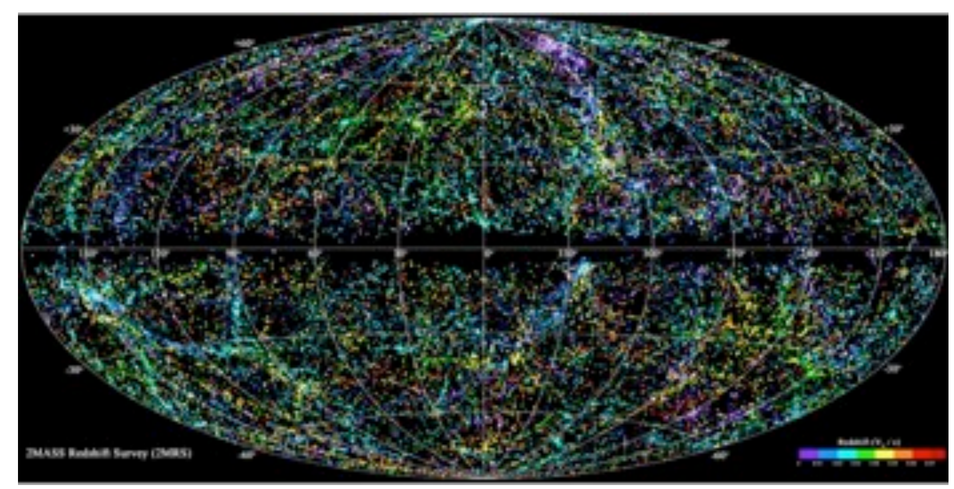
How can theory deal with an exponentially big Hilbert space?



Simulating Physics with Computers

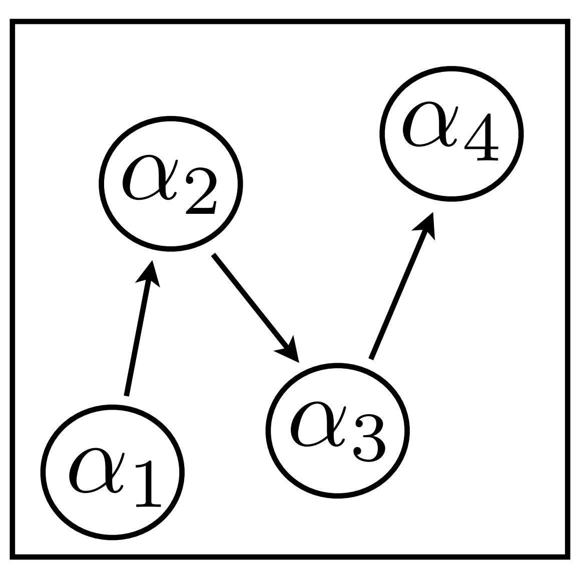
Richard P. Feynman

5. CAN QUANTUM SYSTEMS BE PROBABILISTICALLY SIMULATED BY A CLASSICAL COMPUTER?

YES: Monte Carlo methods

Stochastic sampling of statistical sums

$$\langle A \rangle_T = \frac{\sum_\alpha A_\alpha \; e^{-E_\alpha/(k_B T)}}{\sum_\alpha e^{-E_\alpha/(k_B T)}} = \sum_\alpha A_\alpha P_\alpha$$



random walk in configuration space

$$\alpha_1 \xrightarrow[P_2/P_1]{} \alpha_2$$

Quantum Monte Carlo

Quantum fields: statistical sums are traces of operators

$$\langle \hat{A} \rangle_T = \frac{1}{\mathcal{Z}} \, \mathrm{Tr} \left[\hat{A} \; e^{-\hat{\mathcal{H}}/(k_B T)} \right] \qquad \mathcal{Z} = \mathrm{Tr} \left[e^{-\hat{\mathcal{H}}/(k_B T)} \right]$$

Formally analogous to $\langle A \rangle_T = \dfrac{\sum_\alpha A_\alpha \; e^{-E_\alpha/(k_B T)}}{\sum_\alpha e^{-E_\alpha/(k_B T)}} = \sum_\alpha A_\alpha P_\alpha$

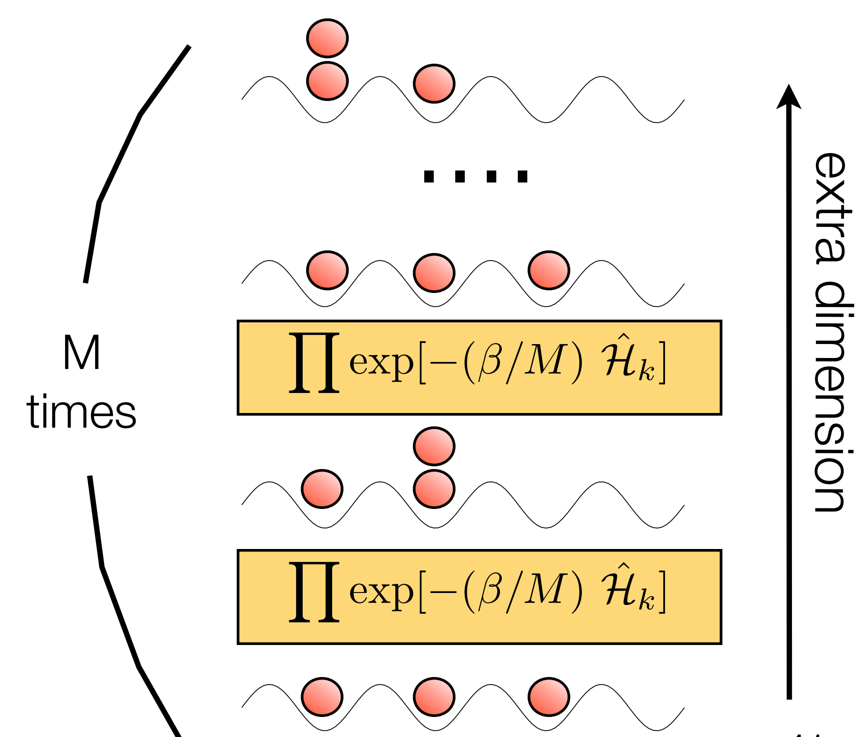
Bootstrap problem: to know P_α one needs to diagonalize $\hat{\mathcal{H}}$!!

$$\hat{\mathcal{H}} = \sum_k \hat{\mathcal{H}}_k$$

sum of local operators

$$e^{-\frac{1}{k_B T} \sum_k \hat{\mathcal{H}}_k} = \lim_{M \to \infty} \left(\prod_k e^{-\frac{1}{M k_B T} \hat{\mathcal{H}}_k} \right)^M$$

easy to diagonalize



— Douglas Adams, *The Hitchhiker's Guide to the Galaxy*

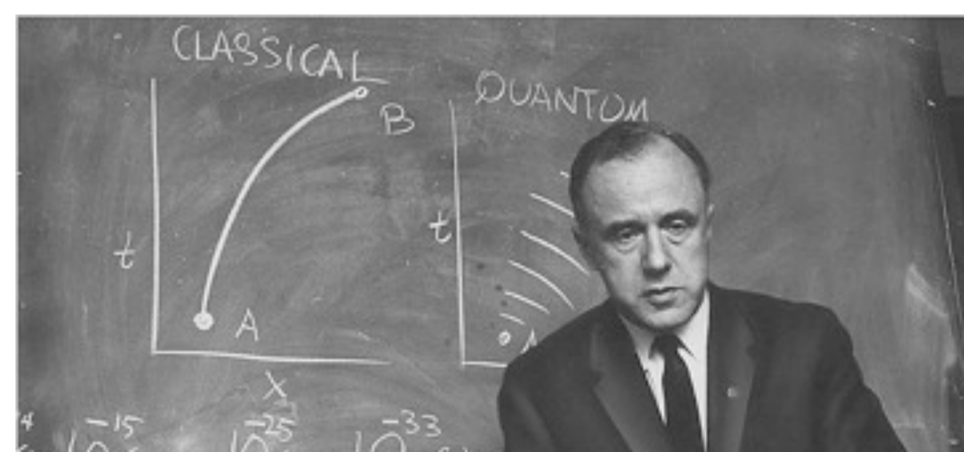
“The Answer to the Great Question... Of Life, the Universe and Everything... Is...
Forty-two,' said Deep Thought, with infinite majesty and calm.”

“It was a tough assignment,” said Deep Thought mildly... “I think that the problem, to be quite honest with you, is that you’ve never known what the question is.”...
“**Once you do know what the question actually is, you’ll know what the answer means.**”

“Can you just please tell us the question?”

“No...” “But I’ll tell you who can... A computer whose merest operational parameters I am not worthy to calculate ... a computer of such infinite an subtle complexity that organic life itself shall form part of its operational matrix... And it shall be called ... **the Earth**.”

“Never make a calculation until you know the answer”



John Wheeler