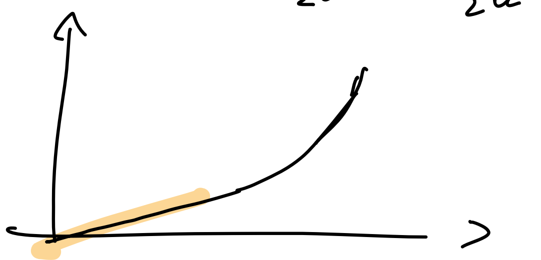
BEC is robust to interactions (weak)

(3d)

$$E_k = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2 g n_0\right)}$$

Spectrum



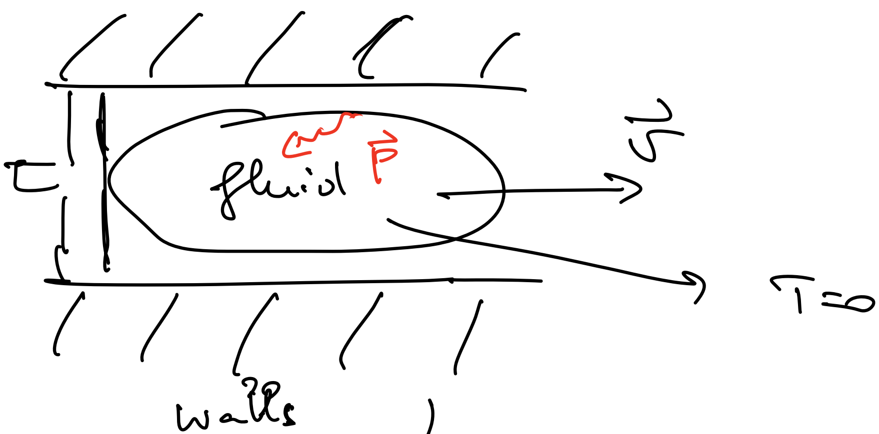
$n_0 = \frac{N_0}{V}$

$E_k \simeq \hbar c_s k$

$c_s = \sqrt{g \frac{n_0}{m}}$

Superfluidity = persistence of flow without resistance

Landau's criterion (1941 ?)



@ T=0

Galilean transform.

Rest frame $\longleftrightarrow$ Laboratory frame

excitations $\rightarrow$ $\epsilon'(\vec{p})$

$\vec{p}$ momentum

$\epsilon(\vec{p})$

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + V_{pot}$$

$$H' = \sum_{i=1}^{N} \frac{(\vec{p}_i' + m\vec{v})^2}{2m} + V_{pot}$$

$$= H + \vec{v} \cdot \underbrace{\sum_i \vec{p}_i}_{\vec{P}} + \frac{1}{2} N m v^2$$

Assumption

$$H = \sum_{\vec{p}} \epsilon(\vec{p})\, a^{+}_{\vec{p}} a_{\vec{p}} + \ldots$$

$\sum_{\vec{k}}$

$\hookrightarrow$ LoS frame

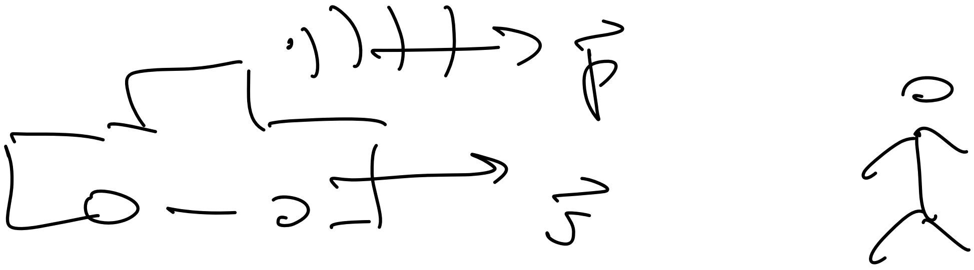
$$H' = \sum_{\vec{p}} \underbrace{\epsilon(\vec{p})}_{\geq 0} a^{+}_{\vec{p}} a_{\vec{p}} + \vec{v} \cdot \vec{P} + \ldots$$

$$\vec{P} = \sum_{\vec{p}} \vec{p}\, a^{+}_{\vec{p}} a_{\vec{p}}$$

$$= \sum_{\vec{p}} \epsilon'(\vec{p})\, a^{+}_{p} a_{p} + \ldots$$

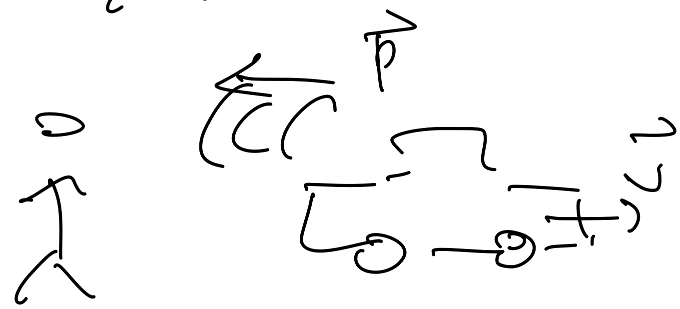
$$\epsilon'(\vec p) = \epsilon(\vec p) + \underbrace{\vec p \cdot \vec v}$$

Doppler shift



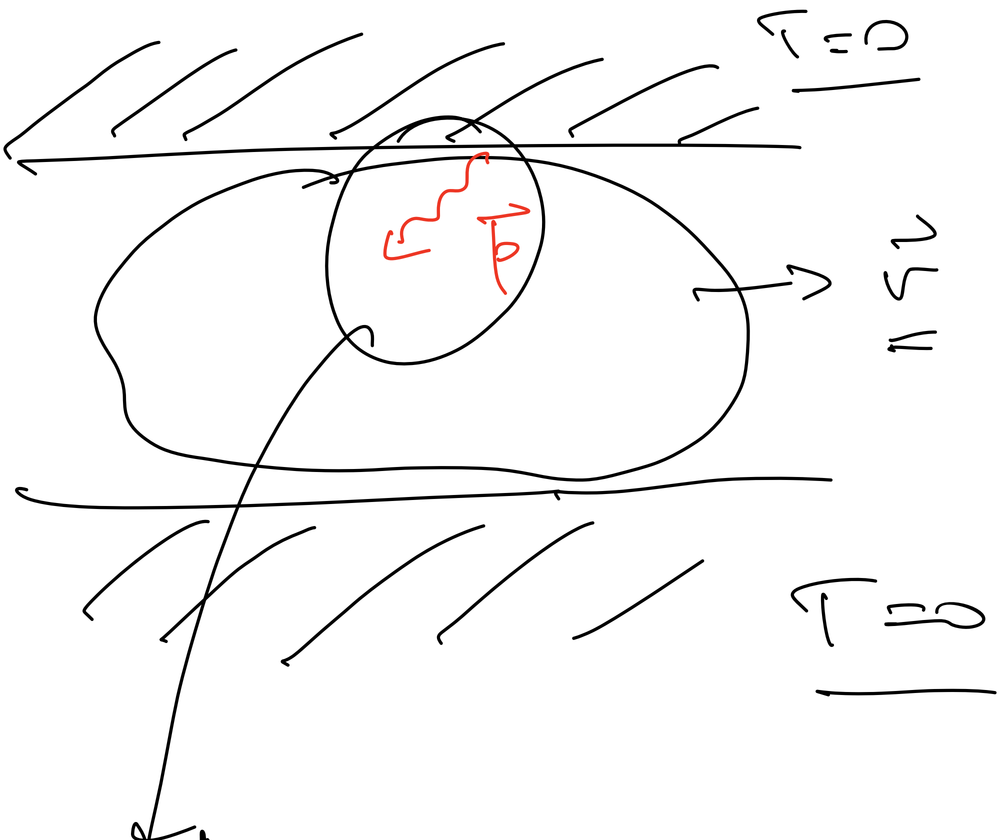
$\omega' > \omega$

$\vec p \cdot \vec v > 0$



$\omega' < \omega$

$\vec p \cdot \vec v < 0$



$\epsilon'(\vec p) \leq 0$

$$\epsilon(\vec{p}) + \vec{p}\cdot\vec{v} \leq 0$$

$$(-)\,p\,v\,\cos\vartheta \geq \epsilon(\vec{p}) \geq 0$$

$$p\,v\,|\cos\vartheta| \geq \epsilon(\vec{p})$$

$$v \geq \frac{\epsilon(\vec{p})}{p\,|\cos\vartheta|} \geq \frac{\epsilon(\vec{p})}{p} \geq \underbrace{\min_{\vec{p}} \frac{\epsilon(\vec{p})}{p} = v_c}$$

$$v_c > 0$$

Landau's criterion for superfluidity

→ Ideal Bose gas

$$\epsilon(\vec{p}) = \frac{p^2}{2m}$$

$$v_c = \min_{\vec{p}} \frac{\epsilon(\vec{p})}{p} = \min_{p} \frac{p}{2m} = 0$$

→ Bogolyubov gas

$$\epsilon(\vec{p}) \underset{p\to 0}{\simeq} c_s\,p$$

$$v_c = c_s > 0 \qquad c_s = \sqrt{\frac{g\, n_0}{m}}$$

Interactions + BEC $\Longrightarrow$ Superfluidity

?

WARNINGS

(1) Examples of systems that are superfluid but not BE condensed

ex.: interacting Bose gas in 2d.

(2) $v < v_c$ $\qquad T = 0$

entire fluid keeps moving

$\rho_s \to$ superfluid (mass) density

$\frac{\rho_s}{\rho} = \to$ superfluid fraction

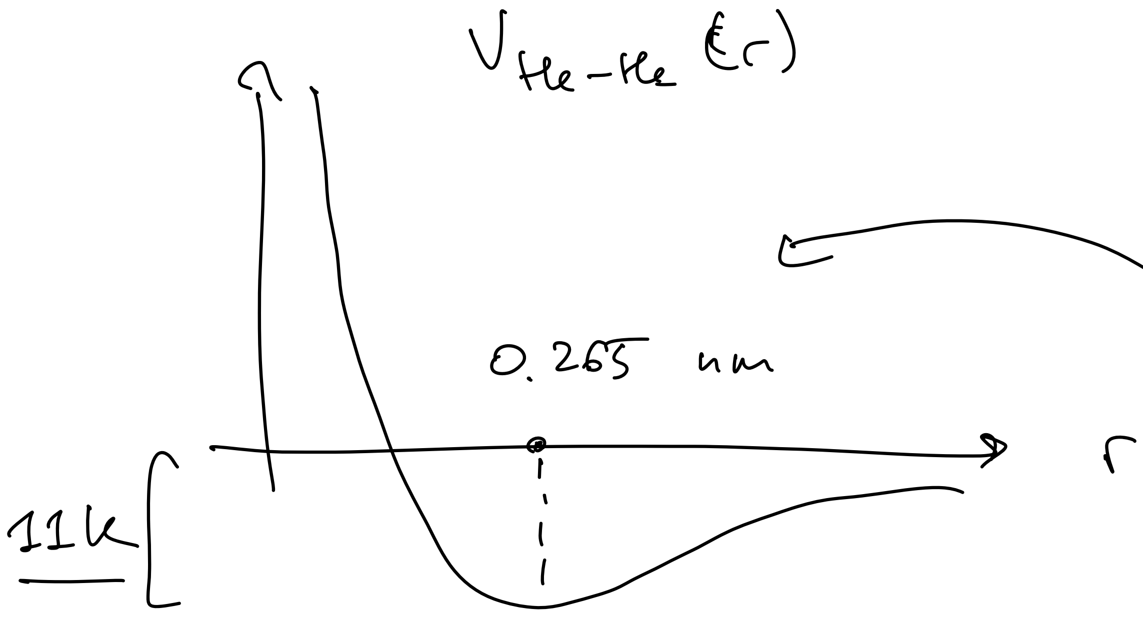
$= 1$

$\rho \to$ mass density of the fluid

Bogolyubov gas $\quad \frac{N_0}{N}$ = condensate fraction

< 1

Superfluid ^4He

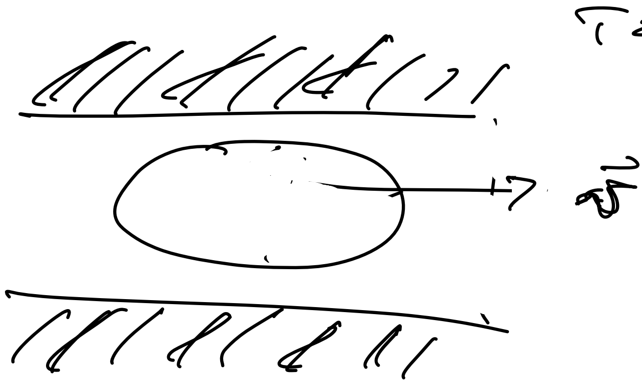


$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m_{He} k_B T}} \simeq 0.4 \text{ nm} \quad @ \ 11K$$

Torsional pendulum

superfluidity :

$T=0$

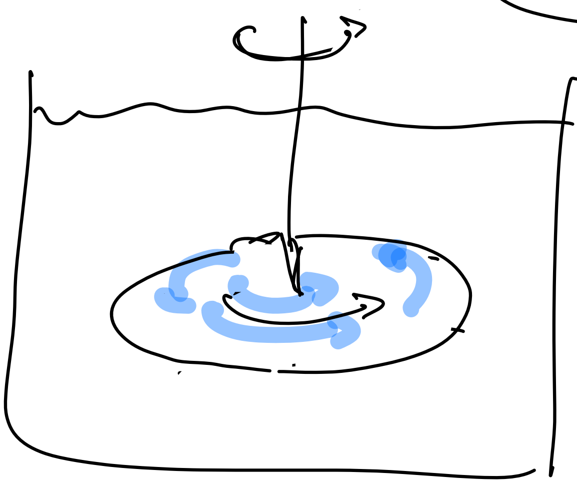


$v < v_c$

$T=0$ 100% of the fluid is superfluid

$T>0$?

$\boxed{T > T_\lambda}$



$$E = \frac{(\dot{\vartheta})^2}{2I}$$

$$L = \omega I$$

$$I = I_0 + I_{liq}$$

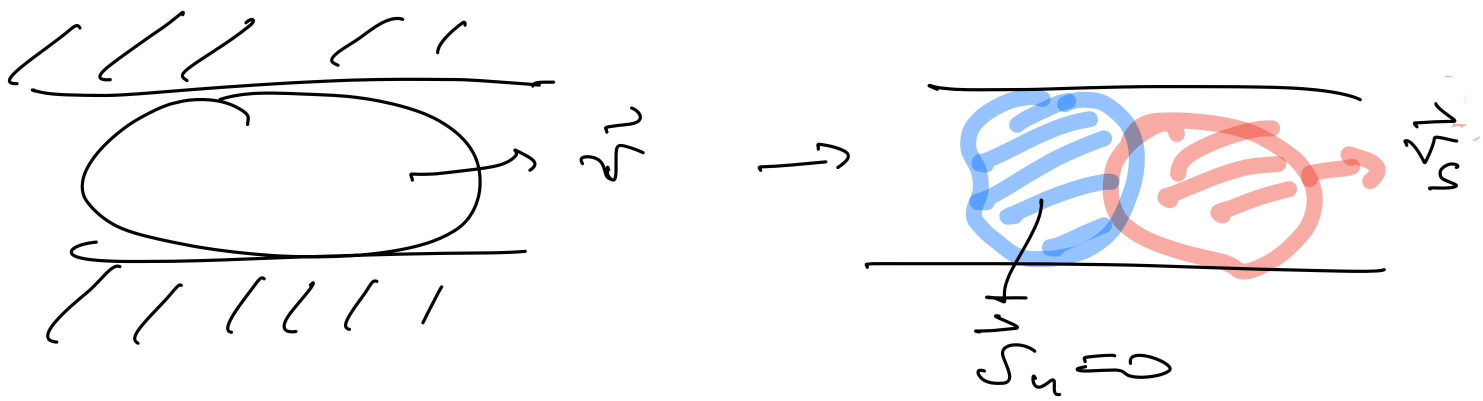
Two - fluid model

$$\rho = \rho_s^{(T)} + \rho_n^{(T)}$$ mass density

$\vec{v}_s$ $\qquad\qquad$ $\vec{J}_n$

↪ motion of the boundaries

$T>0$



Question : ρ_s related to condensate fraction ?

ρ_n related to the condensate depletion ?

NO

$$\vec{J}(\vec{r}) = m \underbrace{\boxed{\vec{J}_0}}_{\text{macroscopic}} + \underbrace{m \sum_{\alpha \neq 0} \vec{J}_\alpha}_{\text{microscopic}}$$

mass current

BEC

$\vec{J}_0 = n_0(\vec{r}) \, \vec{J}_s$

$\vec{J}_s = \frac{\hbar}{m} \vec{\nabla} \phi_0(\vec{r})$

$n_0 \sim O(N)$

$$\chi_0(\vec{r}) = |\chi_0(\vec{r})|\, e^{i\phi_0(\vec{r})}$$

In the presence of a BEC :
macroscopic irrotational flow

$$\vec{j}(\vec{r}) = \rho_s(\vec{r})\, \vec{v}_s(\vec{r}) + \rho_n(\vec{r})\, \vec{v}_n(\vec{r})$$

superfluid

normal

boundary