TD6: Slowing and trapping atoms

1 Zeeman slower

In this exercise we consider a common scheme to slow down a collimated atomic beam coming straight from a furnace at high temperature. This scheme is based on a spatial modulation of the detuning.

1.1

As sketched in Fig. 1, we consider a collimated atomic beam moving along the $z$ axis. As the atoms come out of their source at position $z = 0$ they have a velocity $v_0$, but they will be decelerated as they move along the $z$ axis, due to the force exerted by a laser beam with wavevector $k = -k\hat{z}$ and frequency $\omega$. The laser is detuned by $\Delta_0 = \omega_{eg} - \omega$ with respect to an internal atomic transition when the atom is at rest.

Write down the Doppler-shifted detuning $\Delta_v$ when the atom moves at velocity $v$, and draw schematically the magnitude of the force $F$ as a function of $v$. For which velocity does the force reach its maximum value $F_{\text{max}}$?

1.2

In the following we will imagine that the detuning is made position-dependent, so that $F = F_{\text{max}}$ everywhere. Treating the atomic motion with Newton’s equation, show that the velocity varies along the $z$ axis as

$$v(z) = v_0 \left( 1 - \frac{z}{L_0} \right)^{1/2}$$

where $L_0$ is a distance to be determined.

1.3

Write down the position-dependent detuning $\Delta_0 = \Delta_0(z)$ which satisfies the condition $F = F_{\text{max}}$ everywhere between $z = 0$ and $z = L_0$. 
1.4

We now imagine that the transition takes place between two hyperfine states $|g\rangle = |F, M_F\rangle$ and $|g\rangle = |F', M_F + 1\rangle$. How should the light beam be polarized to drive this transition?

In the presence of a magnetic field $B = B\hat{z}$ the hyperfine states are Zeeman-shifted by an energy $g_F\mu_B BM_F$, where $g_F > 0$ is the so-called Landé factor. Write down a convenient position-dependent magnetic field $B(z)$ which can lead to the condition $F = F_{\text{max}}$ being satisfied in $[0, L_0]$.

This magnetic field can in fact be generated via a solenoid with a variable number of turns. Could you sketch a possible design for this solenoid?

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2 Magneto-optical trap (MOT)

A widespread trapping scheme in cold atoms is the magneto-optical trap (MOT - Raab et al., Phys. Rev. Lett. 59, 2631 (1987), after an idea of J. Dalibard), which relies on both effects invoked in the previous exercise: velocity dependence in the radiation force due to the Doppler effect, and position dependence due to a spatially varying magnetic field.

A MOT is sketched in Fig. 2. It consists of two coils in a “anti-Helmholtz” configuration, namely the coils carry opposite currents. This creates a vanishing magnetic field in the center of the axis connecting the coils, and a linear magnetic field gradient when moving away from the center. The magnetic field lines are indicated in the figure.

2.1

To begin, we will only concentrate on what happens along the $z$ axis. Along this axis the magnetic field exhibits a positive linear gradient $\mathbf{B} = B(z) \hat{z}$ where $B(z) \approx \gamma z$. We consider two hyperfine levels of the atoms, a lower energy level with $F = 0$ and a higher energy level with $F = 1$, separated by the energy $\hbar\omega_0$ in zero field.

Given the Zeeman shift of hyperfine levels $g_F\mu_B BM_F$ with $g_F > 0$, draw the Zeeman-split energy levels as a function of the position along $z$. Draw the position-dependent detuning $\Delta_+(z)$ for a right-handed circularly polarized ($\sigma^+$) laser with frequency $\omega$, and the detuning $\Delta_-(z)$ for a left-handed polarized circularly polarized ($\sigma^-$) laser at the same frequency.
2.2

As indicated in the figure, we now imagine to have two counter-propagating lasers, with polarization $\sigma^+$ for the laser propagating along $+z$ and $\sigma^-$ for the laser propagating along $-z$ (beware: the polarization is here referred to the positive $z$ axis, and not to the laser propagation vector).

The previous point seems to suggest that a two-level atom description is not sufficient. In fact, if we neglect interference effects between the two lasers, we can still treat each laser as interacting with a two-level atom (the two levels in question depend on the laser). In the end this is a fairly good approximation. Indeed show that the atom cannot absorb a photon from a laser and emit it into the other laser consecutively.

If we are allowed to imagine that the absorption of a photon is followed typically by a decay via spontaneous emission, the latter is an incoherent process erasing the “memory” of the atom. Hence the force coming from the two interactions of the two atoms with the two lasers sum up incoherently. Write the expression of the total force acting on the atom along the $z$ axis due to the counter-propagating lasers.

[Note: take into account both the Doppler shift and the position dependence of the detuning, $\Delta_\sigma = \Delta_0 + f_\sigma(v, z)$ where $\Delta_0 = \hbar(\omega_0 - \omega)$].

2.3

Expand the total force along the $z$ axis for $f_\sigma(v, z) \ll \Delta_0$, and show that it can be written as

$$F_z = -\alpha vz - \kappa z$$

(2)

where $\alpha$ is the friction coefficient for optical molasses, and $\kappa = \alpha \gamma / k$.

2.4

From the divergenceless nature of the magnetic field and the cylindrical symmetry of the problem deduce that

$$B_x(x) = -\frac{\gamma}{2} x \quad B_y(y) = -\frac{\gamma}{2} y$$

(3)

Hence conclude that, applying pairs of counterpropagating lasers along the $x$ and $y$ axes as well, one can obtain a total force which reads

$$\mathbf{F} = -\alpha \mathbf{v} - \kappa \left( \frac{x}{2} \hat{x} + \frac{y}{2} \hat{y} + z \hat{z} \right)$$

(4)

Conclude that the scheme in Fig. 2 is then good for cooling and trapping the atoms.

Can you justify why in Fig. 2 the $\sigma^+$ laser propagates in the positive direction along $z$, while it moves in the negative direction along the $x$ and $y$ axes?

2.5

We consider a magnetic field gradient $\gamma = 5$ G/cm; moreover we take $\alpha/M \approx 1$ MHz ($M =$ atomic mass), and lasers with wavelength $\lambda = 500$ nm. Estimate the trapping frequency of the harmonic trap associated with the linear forces.

Appendix

Radiation pressure force

$$\mathbf{F} = \frac{s_0 \Gamma/2}{1 + s_0 + (2\Delta/\Gamma)^2} \hbar \mathbf{k}$$

$$s_0 = \frac{2|\Omega_{eg}|^2}{\Gamma^2} \quad \Gamma = \text{natural linewidth of the } e-g \text{ transition}$$

Magnetic field created by a solenoid with $n$ turns per unit length and with a current $i$ along its axis: $B = \mu_0 i n$. 