ENS Lyon-UJM

Master in Economics

Optimization

Syllabus

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Winter 2019

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1 Basic informations

1.1 Contacts

coordinator:	Laurent Simula	Teacher: Valentine Roos
	Professor	Lecturer (*agrégée préparatrice*)
office:	-	Sud 320 (Monod, 4th floor)
office hours:	-	-
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1.2 Set-up

This is a 8-week course, typically with 3 hours of lecture and exercises per week. It will be held on Tuesdays, from 9.00 to 12.00.

1.3 Prerequisites

All the prerequisites for this course will be studied on the first semester in the course *Linear algebra and differential calculus* given by Corentin Fierobe.

2 Course Overview

2.1 Content (see also 4)

In the first part of the course we will give theoretical and practical tools to master the optimization of a function of multiple variables, with or without constraints. We will then give a solid introduction on continuous dynamic optimization. The problem consists in optimizing a quantity depending on a *state variable* which is a function of the time, and the constraint on this state variable takes the form of a differential equation mastered by a *control variable.* We will present the first emanation of this kind of problem, namely the Euler-Lagrange equation designed to find minimizing trajectories in mechanical dynamics. Then we will present the more general optimal control theory which gives a necessary condition for the optimization problem via Pontryagin's maximum principle.

The course will NOT deal with discrete time dynamic optimization, which is the counterpart of continuous dynamic optimization, with differential equations replaced by finite difference equations.

2.2 Approach

The course of static optimization will be gradual in the complexity of the considered optimization problem: first without constraints, then with equality constraints and at last with both equality and inequality constraints. We will adapt the approach regarding the complexity of the problem solved: when dealing with unconstrained optimization, we will strive to give very precise arguments and proofs for every statement. When dealing with equality constraints, we will present a geometrical point of view to give a good intuition about the Lagrange multipliers. When dealing with inequality constraints, we will focus on finding the most efficient way to concretely solve a given problem. We will also present additional results allowing to solve without effort a perturbed problem.

During the semester, I will also propose a session on (your!) computers in order to get familiar with some very useful elementary numerical methods and tools. We will work on an online collaborative computing environment named https://cocalc.com/.

For the introduction to dynamic optimization, we will strive to avoid technical difficulties and to underline the analogies with static optimization. Some ideas of proof will be given, as well as many examples of applications.

Each lecture will be followed by exercises. Many of them will consist in concretely solving optimization problems in application of the course – those exercises need to be mastered. Some others will be more aimed at a sharpest understanding of the course (elements of proof, counter-examples, theoretical questions…) for the interested students.

2.4 Credits and Examination

There will be a final exam on April 11th and a housework to do during the February holidays. Both will determine the final grade, probably with a 1/3-2/3 ratio.

3 Reading List

These books are here to help you understanding the notions seen during the course. Nothing else than what we will study is necessary for the exam.

[1] Knut Sydsæter, Peter Hammond, Atle Seierstad, and Arne Strøm. *Further Mathematics for Economics Analysis*.

[2] Knut Sydsæter and Peter Hammond. *Essential Mathematics for Economics Analysis*. Prentice Hall.

Or for a translated option :

[3] Carl P. Simon and Lawrence Blume. *Mathématiques pour économistes.* Ouvertures économiques. Série Prémisses.

4 Preliminary Program (Bound to Change)

Date	Chapter	Description
Jan 10	Static Optimization 1 : no constraints	Local and global extrema of a function, necessary and sufficient conditions.
Jan 17	Static Optimization 2 : equality constraints	Lagrange multipliers and Lagrangian formulation, geometrical point of view and practical approach.
Jan 31	Numerical approach	Workshop on CoCalc. Representation of the graph, levelsets and gradients of a function. Implementation of the gradient method.
Feb 6	Static Optimization 3 : mixed constraints	Kuhn Tucker conditions, constraint qualifications, implementation. Signification of the multipliers.
Feb 13	Static Optimization 3 : mixed constraints	Problems depending on a parameter, envelope theorem. Nonnegative constraints, linear programming.
Mar 6	Calculus of variations	Euler-Lagrange equation. Transversality conditions. Examples and applications.
Mar 13	Optimal control theory 1	Pontryagin’s maximum principle, presentation and applications. Link with the Euler-Lagrange equation.
Mar 20	Optimal control theory 2	Heuristic of proof for the Pontryagin’s maximum principle: introduction to the dynamic programming principle, Hamilton-Jacobi-Bellman equation.
Apr 11	Exam	