Liquid residence time distribution of multiphase horizontal flow in packed bed milli-channel: Spherical beads versus open cell solid foams

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Highlights
- Robust access to liquid RTD in small-scale porous packings under multiphase flow conditions is presented.
- Dense micro-packed beds are compared to open cell solid foams.
- A modified liquid hold up correlation is developed and is valid for both packings.
- RTD broadening is explained by a combination of convective dispersion and mass transfer to a fraction of immobile liquid.

Abstract
A robust approach to access liquid residence time distribution (RTD) adapted to multiphase flow in porous media is presented. It is tailored to meet specific requirements of small scale systems (centimeter, millimeter or less) with Taylor segmented flow feed. The method involves direct visualization using fluorescence microscopy close to both extremities of a porous packing. Critical image treatment steps and optimization of a versatile discrete model with 4 parameters are detailed and discussed. They allow the precise and rapid determination of RTD curves and their 1st- and 2nd-order moments. The application of the method is successfully illustrated with dense micro-packed beds of sub-millimeter particles and highly porous media like open cell solid foams undergoing a preformed G-L segmented (Taylor) flow. Original results regarding the effect of fluid flowrates and different confined porous media are discussed and lead to a single two-parameter liquid hold up correlation, which is valid for both packings. As usual, RTD broadening is treated as a combination of convective dispersion and mass transfer to a fraction of immobile liquid. The predominant role of mass transfer is underlined with an analysis of characteristic times.

1. Introduction
Micro- or milli-packed beds (MPBs) of small catalyst particles (10–200 μm) became established and efficient lab-tools in the case of heterogeneously catalyzed gas–liquid (G-L) reactions (Losey et al., 2001; van Herk et al., 2005; Al-Rifai et al., 2016; Moulijn et al., 2016, Faridkhou et al., 2016). Their attractive mass and heat transfer performances and their low material inventory are the driving reasons for this development. Overall, they help speeding up the access to intrinsic chemical activity (van Herk et al., 2009) and optimizing operating conditions in potentially unconventional and risky domains where macroscale reactors are difficult to operate (Inoue et al., 2007). Additionally, these tools can be of great...
interest to mimic and understand the complex two-phase flow patterns occurring at intergranular pore scale in larger scale reactors.

For production purpose, this concept faces scalability and operability issues inherent to multiple channel geometries. Important pressure drop, clogging, uniform two-phase distribution, reproducibility issues, and handling and modeling the spreading of the reactants and products along the reactor which can lead to selectivity and conversion issues.

At millimeter scale, in a wide range of feeding conditions and in empty channels, gas-liquid flows are often self-organized as segmented flows (also well-known as Taylor flows) (Kreutzer et al., 2005a, 2005b). A global understanding of the progressive disorganization of this flow entering porous beds (spheres, foams, etc.) is of fundamental importance because it governs the phase ratio, local mixing, the gas-liquid contact surface area and eventually the mass transfer performance. Residence Time Distribution (RTD) measurements provide precious and meaningful information (mean residence time and phase hold up of each fluid phase, dispersive behavior, presence of dead-zones or by-pass, etc.) about these issues. It helps handling and modeling the spreading of the reactants and products along the reactor which can lead to selectivity and conversion issues.

### Nomenclature

- $CV_{1\text{st}}$: variation coefficient of the 1st-order moment of the RTD [%]
- $CV_{2\text{nd}}$: variation coefficient of the 2nd-order moment of the RTD [%]
- $d$: spherical particle diameter [m]
- $d_{\text{pore}}$: mean cell (OCSFs) or pore (MPB) diameter [m]
- $E(t)$: Laplace transform of the RTD [-]
- $E_p(\theta)$: RTD in reduced time coordinate [-]
- $E_{\text{mod}}(s)$: Laplace transform of the RTD in reduced time coordinate [-]
- $F_{i\text{cum}}(t)$: cumulative raw tracer intensity [a.u.]
- $F_{i\text{cum}}(t)$: cumulative tracer intensity after baseline correction [a.u.]
- $F_{\text{mod}}(s)$: modelled cumulative tracer intensity [a.u.]
- $F_{\text{mod}}(t)$: modelled cumulative tracer intensity in Laplace domain [-]
- $h$: channel height [m]
- $I_{i\text{cum}}(t)$: raw tracer intensity [a.u.]
- $J$: number of mixing cells in series [-]
- $K_{\text{im}}$: ratio of immobile to mobile liquid fractions [-]
- $L$: objective function [-]
- $Q_i$: volumetric flowrate of fluid $i$ [m$^3$/s]
- $r$: radius of the gas injection channel [m]
- $s$: Laplace variable [s$^{-1}$]
- $S_L$: liquid saturation in the porous media [-]
- $\tau_1$: first-order moment of the RTD [s]
- $\tau_i$: first-order moment of curve $i$ [s]
- $\tau_m$: residence time of the mobile liquid phase [s]
- $\tau_M$: mass transfer time between mobile and stagnant zones [s]
- $t_D$: characteristic time of hydrodynamic dispersion in the mobile zone [s]
- $t_{\text{ DT}}$: characteristic time of mass transfer between static and dynamic zones [s]
- $u_i$: superficial velocity of the fluid $i$ [m/s]
- $u_{\text{DT}}$: drift flux velocity [m/s]
- $V$: volume of liquid [m$^3$]
- $V_i$: Eigenvectors of the variance-covariance matrix ($i = 1$–4)
- $V_R$: volume of the reactor [m$^3$]

### Greek symbols:

- $\alpha$, $\beta$: parameters of the drift flux model by Molga and Westerterp (1997) [-]
- $\gamma$: surface tension [N/m]
- $\delta$: distance from channel wall [m]
- $\delta_e$: mean porosity at a distance $\delta$ from the channel walls [-]
- $\delta_{\text{bulk}}$: mean porosity far from the walls [-]
- $\sigma$: volume averaged foam or bed porosity [-]
- $e_{\text{cor}}$: reduced porosity, $e_{\text{cor}}/\delta_{\text{bulk}}$ [-]
- $e_{\text{rec}}$: liquid hold up of the reactor [-]
- $e_{\text{mod}}$: liquid hold up estimated from the drift flux model [-]
- $\theta$: reduced time coordinate, $t/\tau_i$ [-]
- $\theta_m$: fraction of mobile liquid phase [-]
- $\theta_{\text{im}}$: fraction of immobile or stagnant liquid phase [-]
- $\mu$: viscosity [-]
- $\rho$: density [kg/m$^3$]
- $\sigma_i^2$: variance of the curve $i$ [s$^2$]
- $\sigma_{\text{RTD}}^2$: reduced variance of the RTD [s$^2$]

### Subscripts:

- $0$: initial values
- $1$: inlet
- $2$: outlet
- $\text{DF}$: drift flux
- $\text{exp}$: experimental
- $\text{G}$: gas
- $\text{L}$: liquid
- $\text{min}$: minimum
- $\text{max}$: maximum
- $\text{mod}$: model
- $\text{R}$: reactor, reduced.
- $\text{S}$: solid

### Abbreviations:

- CSTR$_4$: series of continuously stirred tank reactor model with 4 parameters
- FFT: fast Fourier transform
- LHS: Latin hypercube sampling
- MPB: micro packed bed
- OCSF: open cell solid foam
- RTD: residence time distribution

Recently, milli-channels containing alternative porous structures like pillar arrays (de Loos et al., 2010; Yang et al., 2015), open-cell solid foams (Saber et al., 2012; Liu et al., 2013; Tourville et al., 2015, 2015a) or ordered porous media (Häfeli et al., 2013; Elias and von Rohr, 2016; Potdar et al. 2017) have been reported for multiphase applications. All these internals still exhibit very good mass transfer and reaction performances but with drastically reduced pressure drop which can be attractive for future lab-tools or millireactor concepts.
RTD acquisition at small scale is challenging and various artefacts can alter its proper determination (Faridkhou et al., 2013):

- The use of classical probes developed for larger volume reactors can yield biased results owing to large dead volumes that affect at least the mean residence time and the variance of the RTD. Typically, a precision of a few percent on the mean residence time requires that the probe volume is less than a few percent of the volume accessible for the fluid. For a milli-reactor of several hundreds of μL, as the one used in the present study, this implies using a probe of less than few μL.
- Whatever the probe, the specific “pulsed” nature of a G-L segmented flow inside milli-channels leads to pulsed raw experimental signals. Recovering the RTD from these signals represents a technical challenge.
- Most often, the inlet tracer signal is not recorded and is assumed to be an ideal one (Dirac pulse, instantaneous step) without experimental evidence of the validity or consequence of the assumption (van Herk et al., 2005; Marquez et al., 2008; Kulkarni and Kalyani, 2009).
- A G-L separator is often inserted between the reactor outlet and the tracer sensor (van Herk et al., 2005, Marquez et al., 2008) that again induces an extra volume and a disturbance of the liquid flow.

To overcome most of these problems, Tourvieille et al. (2015) have presented a two-measurement technique based on direct visualization using fluorescence microscopy at both ends of a porous packing. It was an adaptation of the technique developed by Trachsel et al. (2005) and Kreutzer et al. (2008) for empty micro-channels without practical details for precise signal processing. As will be discussed hereafter, the signal processing developed in the work of Tourvieille et al. (2015) gave access to the signal envelopes and can be subject to a small bias. Thus, the first original objective of this work is to present and discuss a new consolidated image processing method including its robustness and applicability limits. The second original objective is to propose a comparison of two different milli-channel packings fed with a G-L segmented flow: “conventional” dense packed beds of spherical particles and open cell solid foams. The impact of gas and liquid flow rates on the RTD is studied for different samples of each porous structure. The analysis of the first-order moments of the RTD curves makes it possible to quantify an original correlation between the liquid hold up and the feed conditions whatever the packing.

Examination of the second-order moments supports a discussion on the predominant processes of dispersion and their evolution with different operating conditions.

2. Experimental procedures

2.1. Cell design

The cell consists of a horizontal square channel of \( h = 2 \text{ mm} \) width (cross-section \( h \times h = 4 \text{ mm}^2 \)) and a total length of 24 cm, drilled in PEEK (PolyEther Ether Ketone) (Fig. 1a). The top wall is a glass plate that allows direct visualization of the tracer (see Section 2.3). To ensure the same surface properties on the channel walls, a thin (10 nm) layer of diamond-like carbon is coated by plasma deposition. Two types of porous media are studied: spherical beads (MPBs) and Open Cell Solid Foams (OCSFs) (see details in Section 2.2). They occupy the whole section of the channel along 15 cm for the MPBs and 16 cm for the OCSFs.

Pure nitrogen and ethanol are used as model fluids (fluid properties are listed in Table 1) and are delivered at ambient temperature and pressure using respectively a mass flow controller (Analyt MTC) and a syringe pump (Harvard apparatus PHD 4400) equipped with a stainless steel 50 mL syringe. Flow rates are chosen in the range 0.5–8.0 mL/min for the liquid and 2.0–35 mL/min for the gas (always in normal conditions). The liquid flow is injected directly in the main channel while the gas flow is supplied perpendicularly through a narrow circular lateral channel (radius \( r = 0.5 \text{ mm} \)), forming a T-junction and allowing the formation of a G-L segmented flow 2 cm upstream of the porous medium entrance (Fig. 1).

2.2. Porous media

Two types of porous media are investigated: (1) Milli-Packed Beds [MPB 75 (Fig. 2a) and MPB 180 (Fig. 2b)] made of poly-

Table 1

<table>
<thead>
<tr>
<th></th>
<th>Ethanol (L)</th>
<th>Nitrogen (G)</th>
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<tr>
<td>Density ( \rho ) [kg/m(^3)]</td>
<td>795</td>
<td>1.15</td>
</tr>
<tr>
<td>Viscosity ( \mu ) [mPa s]</td>
<td>1.15</td>
<td>1.76 x 10(^{-2})</td>
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Fig. 1. (a) Schematic top view of the reactor. A segmented gas–liquid flow is created upstream the porous medium in a square channel of 2 mm x 2 mm cross-section and total length of 24 cm. (b) Rhodamine-6B is injected in the liquid phase, and its fluorescence intensity is measured upstream (left) and downstream (right) of the porous medium (dashed rectangles in (a)) along a single line of pixels (dashed white lines). Further image processing allows computing the Residence Time Distribution (RTD).
dispersed spherical glass particles and (2) Open Cell Solid Foams (OCSFs) made of either vitreous carbon [80 PPI, ERG-Aerospace, (Fig. 2c)] or nickel-chrome alloy (Ni-Cr) [30 PPI, 40 PPI, 50 PPI, Recemat (Fig. 2d, e, f)]. The particles of the milli-packed beds are maintained by a downstream plug made of a 2 cm long piece of foam (NiCr, 50 PPI), whereas the upstream end is a free surface. Foams are precisely cut with electro-erosion technique to form 20 mm bars and 8 of them are inserted inside the milli-channel to build the porous medium. These six porous media have been analyzed by X-ray tomography (GE Phoenix v|tome|x s, RX tube of 160 kV with focal point of up to 1 µm) with a spatial resolution of 5 µm. Tomographic data are processed with iMorph® software (Brun et al., 2008) providing illustrative 3D images of each medium (Fig. 2), radial porosity profiles (Fig. 3) and a precise quantification of the structure (Table 2).

MPBs are mainly characterized by their (number averaged) mean particle diameter \( d \) given in the rightmost column of Table 2. OCSFs are commercially described by a coarse linear pore density expressed in Pores Per Inch (PPI). Nonetheless, (number averaged) mean window-, cell- and strut-diameters are more relevant and useful for a precise characterization. The cells correspond to the void cages enclosed by the struts whereas windows consist of common faces (not necessarily planar) between two adjacent cells (Fig. 2c). To compare the different porous media, common parameters are chosen: the bed porosity \( \varepsilon_b \) and the (number averaged) mean pore diameter \( d_{pore} \). For MPBs, because of an insufficient spatial resolution, this last parameter cannot be extracted from the X-ray tomography measurements. It is estimated as 20% of the mean particle diameter (Glover and Walker, 2009; Varas et al. 2011). In the case of foams, the mean pore diameter is taken as the mean window diameter, computed from X-ray tomography and image processing (Table 2). The identical mean pore diameters computed for the 40 PPI and 80 PPI foams illustrate the poor significance of the linear pore density as defined by the foam supplier.

In Fig. 3, the porosity \( \varepsilon_o \) is plotted as a function of the distance \( d \) from the wall of the channel, for different porous media. It is obtained by averaging the porosity between \( d \) and \( d + d \) from the channel walls, over the whole analyzed medium. Fig. 3 inset presents the same porosity profiles in reduced coordinates where the reduced porosity \( \varepsilon_r \) is the porosity \( \varepsilon_o \) normalized by \( \varepsilon_{bulk} \), the mean porosity far from the walls, close to the center of the channel.

### Table 2

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Material</th>
<th>( \varepsilon ) [-]</th>
<th>( d_{pore} ) [µm]</th>
<th>( d ) [µm]</th>
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<tr>
<td>MPB 75</td>
<td>Spheres</td>
<td>Glass</td>
<td>0.44</td>
<td>19 ± 6</td>
<td>93 ± 25</td>
</tr>
<tr>
<td>MPB 180</td>
<td>Spheres</td>
<td>Glass</td>
<td>0.40</td>
<td>41 ± 12</td>
<td>206 ± 59</td>
</tr>
<tr>
<td>80 PPI</td>
<td>Foam</td>
<td>Carbon</td>
<td>0.980 ± 0.003</td>
<td>257 ± 85</td>
<td>-</td>
</tr>
<tr>
<td>30 PPI</td>
<td>Foam</td>
<td>NiCr</td>
<td>0.899 ± 0.014</td>
<td>385 ± 120</td>
<td>-</td>
</tr>
<tr>
<td>40 PPI</td>
<td>Foam</td>
<td>NiCr</td>
<td>0.895 ± 0.007</td>
<td>257 ± 87</td>
<td>-</td>
</tr>
<tr>
<td>50 PPI</td>
<td>Foam</td>
<td>NiCr</td>
<td>0.891 ± 0.016</td>
<td>168 ± 63</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 2. X-ray tomography 3D images of the different porous media: (a) MPB 75, (b) MPB 180, (c) 80 PPI foam with the illustration of the difference between cells (cage surrounded by struts) and windows (junction between two cells); (d) 30 PPI foam, (e) 40 PPI foam, (f) 50 PPI foam.
and the distance δ is normalized by the bead or cell diameter, d. In the case of foams (here illustrated only for the 800PPI foam), it is noticeable that the porosity profile present no variations and l_{bulk} = δ because the structure of the material is quite uniform. Conversely, the two bead packings are constrained by the wall, which induces damped porosity oscillations with a wavelength close to the bead diameter d (Fig. 3 inset). This behavior was largely and formerly studied in the literature (Mueller, 1991; Papageorgiou and de Klerk, 2003) for classical tubular fixed beds.

2.3. Data acquisition & processing

The Residence Time Distribution (RTD) is obtained from a pulse injection of a Rhodamine-6B, (Sigma-Aldrich, 99%) solution at a concentration of 3.2 × 10⁻⁵ mol L⁻¹ in the liquid phase, upstream of the T-junction as in the work of Tourvieille et al. (2015). Reproducible pulse injections are obtained using a 5 μl loop mounted on a switching 6-way valve (Rheodyne 7725i). Tracer signals are recorded by direct visualization of the fluorescent dye intensity (Fig. 1b) using a fluorescence microscope (Olympus BX51M equipped with a 2.5x objective) combined with a high speed camera (SolinoCam H2D2, 113 frames per second). The movies are processed with Matlab® (Mathwork®) in order to extract the dye intensity raw signals versus time (Fig. 4a). Because tracer injections are not ideal Dirac pulses, two signals are acquired, I₁(t) at the inlet and I₂(t) at the outlet of the porous medium (later on, index 1 always refers to the inlet and 2 to the outlet). As it was not possible to acquire both signals simultaneously, successive experiments are performed with the same stationary Taylor flow conditions. Four tracer injections for each set of experimental conditions ensure a testing of the reproducibility which yields four (I₁, I₂) couples. This non-invasive visualization technique close to the packing ends avoids dead volume of in-situ sensors that are detrimental to signal quality in so small devices. As discussed in the introduction, contrary to smooth tracer signals obtained in single phase flows, the signals are spiky and pulsed due to the Taylor flow (Fig. 4a). Recovering the RTD from such signals is challenging and cannot be done through direct deconvolution of the outlet and inlet signals. As discussed in the introduction, Tourvieille et al. (2015) bypassed this problem by a smoothing treatment that defines the RTD from the envelope of the peaks (Fig. 4b). Thus, “envelope-smoothed” inlet and outlet tracer responses can be further processed by deconvolution. This processing appears physically disputable because tracer mass balance is not fulfilled, and it can lead to inaccurate RTD curves of the liquid phase. Indeed, integrals under the envelope curves overestimate the liquid phase contribution due to the flowing gas (Fig. 4b).

Therefore, a new processing method that preserves tracer mass balance is introduced as follows: The raw pulsed tracer signals I₁(t) (Fig. 5a) and I₂(t) (Fig. 5b, solid orange lines) are integrated, providing cumulative raw intensity curves I₁(0) and I₂(0) (Fig. 5a and b, dashed lines). The baseline drifts observed in these curves are the result of the random and stationary optical noise (light reflections on the gas bubbles as described in Buttler et al. (2016)). Thus, after baseline correction on the F₁(t) curves (by subtracting the linear drift due to the noise) and a normalization to unity, “cleaned” inlet and outlet cumulative experimental signals are obtained and are hereafter named F₁(t) (Fig. 5a-b, solid blue lines). The tracer mass balance in these signals is preserved because of the stationarity of the noise in the experiments and its independence of the tracer concentration. Classically, a deconvolution method – for instance via a Fast Fourier Transform (FFT method) – is used to extract the RTD from F₁-F₂ couple (Trachsel et al., 2005; Hutter et al., 2011; Saber et al., 2012; Tourvieille et al., 2015). However, deconvolution is an improper mathematical operation, very sensitive to noise in the raw signals. It may yield biased unphysical results (oscillating RTD, negative tracer concentration) or is even unstable. The problem is avoided here by developing a more robust method based on two steps:

- Smoothing of the F₁(t) curve with a suitable expression to obtain F₁^mod(t); At that point, there is no physical interpretation, but only smoothing.
- Fitting of the parameters of a versatile RTD model to reproduce as closely as possible the F₂ curve at outlet by direct convolution (proper and more robust mathematical operation) of F₁^mod(t) by the RTD model. The next section describes this new method and the RTD model in details.

3. Modeling

3.1. Model formulation

When looking at a F curve (as the solid blue lines in Fig. 5), one easily recognizes the response of a dispersive plug flow, at least from a qualitative point of view. Unless dispersion is very high, the corresponding RTD is almost symmetrical. The more or less pronounced asymmetry around the mid-point observed in Fig. 5
suggests the presence of a stagnant zone. Simple models for dispersive flow with or without stagnant zones are plethora in the literature (see Danckwerts, 1953, Wen and Fan, 1975, Sardin et al., 1991, Stegeman et al., 1996, Iliuta et al., 1999). The model used in this study is a discrete series of continuous mixing cells containing a stagnant zone and is schematically illustrated in Fig. 6 (hereafter named CSTR4). It is a simple alternative to the classical dispersive flow model based on Fick’s law. Common sense suggests that this versatile model should easily mimic the flow pattern from the injection of Rhodamine dye down to the inlet detection zone and then through the reactor zone either packed with beads or foams, although it does not necessarily represent the true physical processes responsible for tracer dispersion. This model has been used also by Tourvieille et al. (2015) in the case of multiphase flow in open cell solid foam and gave good estimates of the RTD.

Let \( V \) be the volume of liquid in the system under study. As described by Sardin et al. (1991), each mixing cell contains a volume \( V \times \theta_m \) of mobile (i.e., flowing) liquid, and a volume \( V \times \theta_{im} \) of stagnant liquid, where \( \theta_m \) and \( \theta_{im} \) are the fraction of mobile and immobile liquid respectively (\( \theta_m + \theta_{im} = 1 \)) and \( J \) is the number of mixing cells. It has been shown that the Laplace transform of the RTD, \( E(t) \), of this system is:

\[
E(s) = \left\{ 1 + \frac{s \theta_m}{J} \left[ 1 + \frac{K_{im}}{1 + s \theta_m} \right] \right\}^{-J}
\]

where \( \theta_m \) is the residence time of the mobile liquid phase and accounts for convection, \( \theta_{im} \) is the characteristic time of transfer between mobile and immobile liquid, accounting for both external and internal transfer (for more details see Villermaux (1987) and Sardin et al. (1991)), and \( K_{im} = \theta_m / \theta_{im} \) is the ratio of the immobile over mobile fraction of the liquid phase. The first-order moment (\( \bar{t} \)) and the second-order centered and reduced moment (\( \sigma^2 \)) of this \( E \) distribution function are given by Sardin et al. (1991) using the parameters of this CSTR4 model:

\[
\bar{t} = \theta_m (1 + K_{im})
\]

\[
\sigma^2 = \frac{\sigma^2}{\bar{t}^2} = \frac{1}{J + 1} \frac{K_{im}}{K_{im} \bar{t}}
\]

In order to discuss and compare the RTD curves and in particular to decouple the second-order moment from its dependence on the mean residence time, it is often helpful to use a reduced time \( \theta \) for the RTD and the corresponding reduced RTD expression (Eq. 1)). The Laplace transform of this reduced RTD model in reduced time coordinate is then given by:

\[
\bar{\theta} = \frac{\bar{t}}{\bar{t}_m (1 + K_{im})}
\]

\[
\mathcal{L}_E (s) = \left\{ 1 + \frac{s}{J(1 + K_{im})} \left[ 1 + \frac{K_{im}}{1 + s \theta_m} \right] \right\}^{-J}
\]

### 3.2. Smoothing of inlet \( F_1 \) curves

The first modelling step described in Section 2.3, consists in using the model to smooth the baseline-corrected \( F_1 \) curve with no possibility of oscillations as often encountered with purely mathematical and blind smoothing procedures. The global nonlinear parameter regression is performed using a Levenberg-Marquardt algorithm (Matlab\textsuperscript{TM}, Mathworks\textsuperscript{\textregistered}) associated with a variation of the initial conditions in order to avoid any “local” solution due to the presence of four regressed parameters (see Appendix A). Fig. 7 shows an example of the excellent agreement between the model smoothed curve \( F_1^{\text{mod}}(t) \) according to this CSTR4 model, and the baseline-corrected \( F_1(t) \) signal normalized to unity. In the present case, the four model parameters have no physical meaning, as there is no evidence that the physical phenomena described in the model are responsible for the shape on the inlet
signals. Conversely, they allow recovering the leading moments of the \( F_1 \) curve using Eqs. (2) and (3). More important for the next step, any residual baseline drift, or noise (stepwise increase due to the segmented flow nature) along the baseline-corrected curve are eliminated by this procedure.

### 3.3. Convolution method

The second step consists in extracting the RTD from \( F_{2}^{\text{mod}}(t) \) and \( F_2(t) \) curves. Although the CSTR\(_4\) model is physically debatable when representing the inlet signal, it is a good candidate to model the liquid flow in the packed section. It allows calculating the response of the system in the Laplace domain as:

\[
\tilde{F}_2^{\text{mod}}(s) = \tilde{F}_2^{\text{mod}}(s) \times \tilde{E}(s)
\]

where \( \tilde{E}(s) \) is the unknown RTD. Fitting leads to a new set of parameters \( t_{\text{exp}}, K_{\text{mis}}, J \) and \( t_{\text{exp}} \) that is considered now as significant, and will deserve interpretation. A parameter regression identical to the one already described in Section 3.2 is used to minimize by the least squares method an objective function based on the experimental baseline-corrected \( F_2(t) \) and the model \( F_{2}^{\text{mod}}(t) \), this latter being the inverse Laplace transform of \( \tilde{F}_{2}^{\text{mod}}(s) \). It is noticeable that deconvolution is not involved; only the proper and stable convolution is used. Finally, the inverse Laplace transform of \( \tilde{E}(s) \) with its adjusted parameters gives access to the RTD function \( E(t) \) using a Talbot algorithm.

Fig. 7 presents a typical example of this two-steps signal processing and the resulting RTD curve. A very good agreement is observed between baseline-corrected and modelled signals for the inlet and outlet positions. The next sections describe the results and interpretations of the parameters obtained using this procedure.

The overall impact of this novel treatment (image processing and RTD convolution) has been evaluated by comparing its responses with those obtained with the former envelope curve method. This comparison was done on three foam packings, namely 30, 40 and 50 PPI (see Table 2). Mean relative differences of nearly 0 to 8% are obtained for the first-order moment of the RTD whereas more pronounced deviations are obtained for the second-order moment, from 10 to 90%, confirming the necessity to re-investigate the RTD treatment.

### 4. Results & discussion

In this section, after a discussion on the experimental repeatability, the model ability to accurately represent the data is addressed. Finally residence time distributions and their moments are presented and analyzed for various liquid and gas flowrates and the different porous media.

#### 4.1. Experimental repeatability

As explained in the experimental part, parameter uncertainty is estimated from the two inlet and two outlet signals recorded for a given set of experimental conditions. These signals lead to four different RTD (four different couples of inlet/outlet signals are possible). A stationary flow and a synchronous triggering of the video recording with the tracer injection are assumed. This was verified in most experiments except the ones at high liquid flow rates (\( Q_L \geq 8 \text{ mL/min} \)) where the start off precision is not sufficient relatively to the short residence times obtained. In these cases, sufficiently repeatable mean residence times were not obtained and the results are therefore not considered as consistent in the following.

In the other cases, direct calculations of the first- and second-order moments of the four baseline-corrected curves (Eqs. (7)-(10)) are used to quantify the repeatability and to define what is called “experimental moments” contrary to Eqs. (2) and (3) that define “model moments”:

\[
\hat{t}_i = \int_0^\infty [1 - F_i(t)]dt; \quad i = (1, 2)
\]

\[
\sigma_i^2 = \int_0^\infty [1 - F_i(t)]tdt - \hat{t}_i^2; \quad i = (1, 2)
\]

\[
\hat{t}_{\text{exp}} = \hat{t}_2 - \hat{t}_1
\]

\[
\sigma_{\text{exp}}^2 = \sigma_2^2 - \sigma_1^2
\]

The experimental variability on the first- and second-order experimental moments of the RTD is quantified by representing their variation coefficients \( CV_{\hat{t}_{\text{exp}}} \) and \( CV_{\sigma_{\text{exp}}^2} \) respectively. They correspond to the ratio of the standard deviations with the average values for these two experimental moments. For the first-order moments (Fig. 8a), the average value of \( CV_{\hat{t}_{\text{exp}}} \) is 3% with a maximum value of 18% obtained for the shorter residence times (as discussed before). For the second-order moments (Fig. 8b), the average value of \( CV_{\sigma_{\text{exp}}^2} \) is larger and is about 33% with a maximum value of 200% (only the residence times higher than 4 s were considered). These significantly higher values were expected for \( CV_{\sigma_{\text{exp}}^2} \). Indeed, there is a systematic error induced by the uncertainty on the first-order moments \( t \) when calculating \( \sigma_{\text{exp}}^2 \). Moreover, a flow disturbance due to tracer injection was noticed and appears as a source of variability in the second-order moment even if its impact on the first-order one is weaker. Overall, the mean residence time is properly determined when it exceeds 4 s and the variance can be considered qualitatively. Some ways to reduce the variability observed in the second-order moment would be to perform a larger number of experiments or alternatively and most probably to build a heavier experimental rig with two optical systems allowing the simultaneous acquisition of the inlet and outlet signals for a single injection.

#### 4.2. Model validation

In Fig. 9, the values of the first- and second-order experimental moments of the RTD (Eqs. (9) and (10)) are compared to model moments (Eqs. (2) and (3)). For the first-order moment (Fig. 9a), a very good agreement is obtained between model and experimental values with less than 5% of deviation. The points are more scattered for the second-order moment \( \sigma_i^2 \) (Fig. 9b) with deviations up...
to 40% (only a few ones exceed this value). These deviations between the model predictions and the experimental data are of the same order of magnitude as the experimental variability. This last result appears logical, even if the model is appropriate to represent faithfully the observed experimental results, it cannot reduce mathematically the experimental variability. Fig. 9c displays the distribution of the residuals (difference between $F_2$ and $F_{mod}^2$) for a given set of experimental conditions. The distribution is almost symmetrical and centered about 0, which suggests that the deviation between the model and the experiment is close to a random additive Gaussian noise. The low values of residuals indicate that the CSTR4 model appears relevant to represent satisfactorily our data. Additional details about the uniqueness of the RTD parameter set are presented in Appendix A. As discussed earlier in Section 4.1, to improve the model validation concerning the second-order moment of the RTDs, the efforts should be experimental to suppress the tracer perturbation effects on them for future studies.

4.3. Effect of the gas and liquid flowrates

The effect of fluid flow rates is illustrated with the 80 PPI foam in Fig. 10. Fig. 10a and b present the impact of liquid flowrate $Q_L$ on the RTD curves versus time and reduced time respectively, at a constant gas flowrate of 10 mL/min. Similarly, Fig. 10c and d display the dependence of the RTD on the gas flowrate at a constant liquid flowrate of 4 mL/min. It is easily observable on the temporal curves (Fig. 10a and c) that a decrease in liquid or gas superficial velocity leads to an increase in mean residence time and a spreading of the RTD. However, the impact of the liquid flowrate appears much more pronounced than that of the gas. To appreciate the effect of fluid flow rates on the dispersion independently of the mean residence time, the RTD in reduced time coordinate $E_d(\theta)$ has to be considered (Fig. 10b and d). It helps to see that an increase in the liquid or gas flowrate results both in a decrease in the liquid dispersive behavior with a reduction of the curve tail and asymmetry (mainly due to the exchange with stagnant zones). Fig. 10e and f show that the mean residence time and the reduced variance seem to depend monotonously on $1/u_L$ at constant $u_G$, with a slope strongly dependant on $u_G$. This trend seems no longer valid for the reduced variance at high $u_G/u_L$ ratio. The qualitative behaviors described here for the impact of gas and liquid flowrates for the 80 PPI foam were observed for the others porous media (see next section) and are common in other multiphase fixed bed reactors containing foams (Saber et al., 2012 and 2012a) or beads (Stegeman et al., 1996; Specchia and Baldi, 1977) especially for the mean residence time.
4.4. Effect of the porous medium

A typical example of the impact of the porous medium on the RTD curves is presented in Fig. 11a and b for a given set of gas and liquid flowrates (Q_G = 2 mL/min and Q_L = 4 mL/min) in time (Fig. 11a) or reduced time coordinates (Fig. 11b). The large difference in bed porosity between foams and beads (≈90% vs. ≈40%) is clearly visible in the RTD curves presented in time coordinate (Fig. 11a). When moving to the RTD curves expressed in reduced time (Fig. 11b), it is clear that foams display a broader RTD curve, which in addition is asymmetric with a longer tail than the two packed beds of spherical particles. It indicates that stagnant zones and their effect on the dispersion are more noticeable for these media. The noticeably different RTD curves of the 40 PPI and 80 PPI foams, that have the same mean pore diameter (see Table 2), indicate that this mean pore diameter is not enough to discriminate between 2 foams. The nature of the material (wettability) and other geometrical characteristics (porosity, strut shapes, node connectivity, occluded faces, orientation, etc.) have also to be considered.

To be able to compare the first moments of these RTD when looking at the impact of flowrates for example, it is appropriate to introduce the liquid saturation S_L, which considers only the accessible volume instead of the total reactor volume for the liquid hold-up ε_L.

\[ S_L = \frac{\varepsilon_L}{\varepsilon_R} \]

\[ S_L = \frac{\varepsilon_L}{\varepsilon_R} \stackrel{(11)}{=} \frac{\varepsilon_L}{\varepsilon_R} \]

In Fig. 11c, the liquid saturation obtained for two foams (80 PPI and 40 PPI), a micro packed bed (MPB 75) and the empty channel are plotted as a function of the ratio of gas and liquid superficial velocities, u_G/u_L, for various couples of flowrates. Whatever the porous medium, its presence increases drastically the liquid saturation in comparison to the empty channel. What is less intuitive is that foams and packed beds, two very different bed structures, have the same level of liquid saturation and the same evolution with the ratio u_G/u_L. It is interesting to note, however, that the micro packed bed presents a slightly higher liquid saturation than foams under similar conditions. This second-order effect may probably result from differences in pore size, number and shape. A more detailed study would be necessary to quantify this effect further, with local characterization tools. Finally, when comparing all the porous media in terms of global dispersion, no clear correlation exists between the reduced variance of the liquid RTD, \( \sigma^2 \),
and the mean pore diameter, $d_{\text{pore}}$, of the porous media (Fig. 11d). This result is also counter-intuitive.

4.5. Drift flux model

In terms of a possible prediction of the mean liquid residence time for all the media (and the corresponding liquid hold up or saturation), one can try to go further in the chemical engineering approach in trying to merge all the results in a single correlation. As shown by Wallis (1962) and later by Darton and Harrison (1975), in two phase flows, each phase does not travel in the reactor with the same mean velocity, leading to the existence of a drift flux. This latter is the velocity of each dispersed units of gas slipping on the liquid. Darton and Harrison (1975) defined the drift flux in the framework of three phase fluidized beds. In this study, the solid phase is fixed, and the Darton and Harrison’s expression has been modified accordingly. The drift flux velocity, hereafter named $u_{DF}$, depends on the liquid holdup, the porosity of the solid phase ($\varepsilon$) and the velocities of liquid ($u_L$) and gas ($u_G$) phases and is given by:

$$u_{DF} = \frac{u_G \varepsilon}{\left(\frac{\varepsilon}{C_0}\right)^{\frac{1}{2}}}$$

In fixed beds, as shown by Molga and Westerterp (1997), $u_{DF}$ is mainly a function of $u_G$. Its dependence on $u_L$ is negligible and a correlation of the following form is proposed:

$$u_{DF} = \alpha(u_G)^b$$

Their expression of $u_{DF}$ was independent of the bed porosity because in dense fixed beds, the mean porosity is roughly constant and of about 40%. In the present study, the porous media have a very different porosity, and this parameter has to be considered. Therefore, a modified form with a dependency of the drift flux velocity upon gas superficial velocity and bed porosity is proposed as follows:

$$u_{DF} = \alpha(u_G)^{b \left(1 - \frac{\varepsilon}{\varepsilon_{mod}}\right)}$$

$$u_{DF} = \alpha(u_G)^{b \left(1 - \frac{\varepsilon}{\varepsilon_{mod}}\right)}$$
As shown in Fig. 12a, all the porous media collapse on a single master curve according to this single expression involving only two fitted parameters (\(\alpha', \beta'\)), as in the work of Molga and Westerterp (1997). Here the parameters take the following values: \(\alpha' = 0.64\) and \(\beta' = 0.94\). Introducing Eq. (14) into Eq. (12), provides a simple expression to predict the liquid holdup:

\[
e_L = \frac{e_{\text{uL}} + \alpha'(u_Ge)^{\beta'}}{u_L + u_G}
\]

As shown in Fig. 12b, all the experimental liquid hold ups (\(e_L\)) can be predicted by equation 15 within \(\pm 15\%\) with a mean relative deviation of 7\% (the maximum deviation observed does not exceed 35\%). Nonetheless, this correlation should be supported by a larger number of experiments, especially with a variety of fluid

Fig. 13. Evolution of the CSTR\textsubscript{4} model parameters \(t_{\text{m}}(a,b), J(c,d), K_{\text{m}}(e,f)\) and \(t_{\lambda}(g,h)\) as a function of liquid flow rate \(Q_L\) [(a, c, e, g), \(Q_L = 10\) mL/min] and gas flow rate [(b, d, f, h), \(Q_G = 4\) mL/min] for the 80 PPI foam and the micropacked bed MPB 75.
properties. Ideally, various channel sizes would also be necessary to tackle the impact of the degree of confinement.

4.6. Evolution of the model parameters and dispersion processes

In Fig. 13, the evolution of the four model parameters is represented as a function of the liquid (left column, Fig. 13a, c, e, g) and gas (right column, Fig. 13b, d, f, h) flowrates. Only two porous media (80 PPI foam and MPB75 micro-packed bed) are presented for the sake of clarity. Globally no clear and undeniable trend is noticeable. The gas flowrate seems to have a lower impact on the parameters than the liquid flowrate, except for \( t_M \), for which the effect is seems comparable. This is in agreement with the RTD curves and their first- and second-order moments obtained in the study of the effect of flowrates (see Section 4.3). \( t_m \) and \( t_M \) may tend to decrease with an increase in liquid flowrate, which can be physically relevant. The two other parameters \( J \) and \( K_{im} \) seem pretty independent of flowrate variations and might be somehow intrinsic to the media (in the experimental domain investigated). \( J \) values are often much higher than 50 indicating a weakly dispersive flow in the dynamic zone. \( K_{im} \) values correspond to a fraction of stagnant liquid in the range of 10–20% which correspond to qualitative local observations and analysis of the flow (Serres et al., 2016; 2018).

Another way to discuss the parameters responses to physical variations is to consider them by regarding the characteristic times of the two hydrodynamic contributions to the dispersion as described in Sardin et al. (1991). Rearranging Eq. (3) leads to the following expressions for the dispersion time, \( t_D \), and the mass transfer time, \( t_T \):

\[
\sigma^2 = \frac{2}{t_m} (t_D + t_T) \tag{16}
\]

\[
t_D = \frac{t_m}{2J} \tag{17}
\]

The variation of these two characteristic times with the gas and liquid flow rates are displayed in Fig. 14. At first sight it is noticeable that \( t_T \) is always one order of magnitude greater than \( t_D \). It means that the transfer with the stagnant zones is dominant with respect to the dispersion in the dynamic zone and is controlling the overall dispersion. Again, it is physically interesting to notice that \( t_D \) is more sensitive to liquid flowrate than gas flow rate variations (Fig. 14a, b). \( t_T \) decreases with an increase in liquid or gas flowrate (Fig. 14c, d). Indeed, it is not direct and intuitive that reduced characteristic times are obtained with an increase of fluid velocity inside the porous medium, especially for the diffusion process.

Fig. 15 illustrates the effect of pore diameter on fitted model parameters and characteristic times for two couples of fluid flow-rates. Concerning the model parameters (Fig. 15a-d), no clear tendency is observable. It can just be noticed that increasing \( d_{pore} \) might result in an increase of \( t_m \) and a decrease of \( t_M \). Regarding the characteristic times (Fig. 15e-f), no particular tendency is noticeable. A higher \( t_D \) is obtained with foams than with packed beds. This observation is consistent with the more pronounced tails observed in the RTD for foams. Finally the tendency observed with the fluid flowrates is confirmed here regarding the predominant effect of mass transfer. The characteristic times obtained for dynamic dispersion are still one order of magnitude smaller than the characteristic times of the mass transfer between stagnant and dynamic zones.

5. Conclusions

An original method of liquid RTD acquisition adapted to small scale reactor is presented in details in this work. Its originality lies in its ability to handle pulsed signals often encountered in

\[
t_T = \frac{K_{im}}{(1 + K_{im})^2} t_M \tag{18}
\]
multiphase flows at small scale and to be applicable as close as possible to the packing ends without inserting any dead volume. This method involving image analysis, global optimization and modelling steps, allows an accurate and robust access to the RTD curves. Fluid flowrates and bed structure effects were investigated for two different confined porous media of interest: dense MPBs and OCSFs. The method has been found highly robust to access the first-order moment of the RTD and a unique two-parameter correlation was adapted to predict successfully the similar trend of liquid saturation in the two packings. The model of Sardin et al. (1991) used in this study has been found sufficiently versatile to describe the dispersive behavior encountered and to explain it with a predominant effect of mass transfer with respect to convective dispersion. The global dispersive behavior of the liquid flow involving a stagnant liquid described here is in physical agreement with local observations of the flow inside the porous bed (Serres et al. 2018) where a liquid zone travelling slowly can coexist with a faster G-L two phase flow inside the complex porous structure.

Improvement in the quantitative treatment of the second-order moments could be performed by a simultaneous acquisition of inlet and outlet signals.

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Appendix A. Uniqueness of the global optimization solution

Before discussing the values of the model parameters ($t_m, J, K_{nm}$, and $t_M$), it is necessary to guaranty the uniqueness of the solution. To do so, we have developed a global optimization based on the “multistart” algorithm of Matlab® (Mathworks®). This method is presented first, then, the statistical results of the modelling are briefly discussed.

The multistart algorithm consists in testing a large number of initial conditions ($t_{m0}, J_0, K_{nm0},$ and $t_{M0}$) as an input to the non-linear fitting and keeping the best solution. This method can be very time-consuming depending on the parameters space coverage. Therefore we defined reasonable bounds for each parameter,
based on preliminary tests and on the literature. Then, in order to cover the maximum of initial conditions possibilities with a reduced number of optimizations, we use the Latin Hypercube Sampling (LHS Design). An example of this homogeneous distribution of 500 initials points is displayed in Fig. A1a.

For a given set of initial conditions, a nonlinear fitting of the CSTR\(_4\) model on the experimental points is performed as described in Section 3. Sometimes, the nonlinear fitting does not lead to a result (no convergence). It corresponds to the empty dots in Fig. A1a, which shows how the nonlinear fitting tool of Matlab\textsuperscript{TM} (Mathworks\textsuperscript{\textregistered}) combined to the CSTR\(_4\) model is sensitive to the initial conditions. When the nonlinear fitting gave a result (convergence, filled dots in Fig. A1a), an objective function, \(of = \sum (F_{\text{mod}}(t) - F_2(t))^2\) is calculated at the end of the fitting run to evaluate the accuracy of the CSTR\(_4\) model, \(F_{\text{mod}}(t)\), to represent the experimental points, \(F_2(t)\). Values obtained for each set of initial conditions are reported in Fig. A1b, inset. The main plot of the Fig. A1b is a zoom of the inset around the minimum objective function value, \(of_{\text{min}}\). Note that the minimum value of the objective function has a single occurrence, which proves the uniqueness of the solution.

For the sake of clarity, only the smaller values of the objective function \(of\) are represented in Fig. A2, as a function of the normalized value \(p\) of the four model parameters (normalized by their maximum value). Only the minimum values of \(of\) are represented here for the sake of clarity. The dashed line indicates the limit below which a unique set of parameters exists which minimizes the objective function.

The variance-covariance matrix of the associated fitting is reported on Table A3a. The very small covariance between parameters seems to indicate that they are not correlated. To prove so, we represent in Table A3b the eigenvectors of this matrix. They are almost parallel to the axes defined by the parameters, showing that these latter are not correlated. Hence, the system does not appear undersized, and the four parameters are relevant.

**References**


