Oscillations of a particle-laden fountain

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(Received 15 April 2022; accepted 23 June 2022; published 4 August 2022)

Different regimes are usually observed for fluid migration through an immersed granular layer. In this work, we report a puzzling behavior when injecting water at a constant flow rate through a nozzle at the bottom of an immersed granular layer in a Hele-Shaw cell. In a given range of parameters (granular layer height and fluid flow rate) the granular bed is not only fluidized, but the particle-laden jet also exhibits periodic oscillations. The frequency and amplitude of the oscillations are quantified. The Strouhal number displays a power-law behavior as a function of a nondimensional parameter, $J$, defined as the ratio between the jet velocity at the initial granular bed height and the inertial particle velocity. Fluid-particle coupling is responsible for the jet oscillations. This mechanism could be at the origin of the cyclic behavior of pockmarks and mud volcanoes in sedimentary basins.

DOI: 10.1103/PhysRevE.106.024901

I. INTRODUCTION

Particles entrained by fluid motion are not only a fascinating topic for physicists, but they are also at the core of present-day challenges in geosciences or in industrial processes. Fluid migration in sedimentary basins and their subsequent expulsion at the seafloor lead either to gentle fluid release and the formation of pockmarks or to violent subsea fluid release such as mud volcanoes [1,2]. These latter can exhibit puzzling behaviors [4]. In spite of modeling attempts [5], the underlying mechanism for this cyclicity is still unknown but could provide keys for predicting these phenomena and improve hazard mitigation. In the industrial context, fluidized beds are classically used in heterogeneous catalysis to enhance multiphase reactions [6,7]. However, the emergence of instabilities in the suspension may be at the origin of a chaotic behavior of the reactor and potentially dangerous exothermic reactions [8].

Since the work of Ergun and Orning and the first description of a fluid flow through expanded and fluidized beds [9], countless studies have focused on the complex problem of particle entrainment by fluid flows in different configurations. Examples include, in a nonexhaustive way, submarine avalanches [10], erosion processes [11,12], crater formation [13,14], or bedload transport [15,16]. The complexity of such flows, involving a strong coupling between the particles and the fluid, hydrodynamics and contact interactions, transport and sedimentation, bring forward numerous challenges for the physicists including finding a constitutive equation and the rheology of dense packings [17], quantifying the role of pore pressure and lubrication forces [18,19], friction and particle roughness [20], and compaction [21].

In practical applications, either natural or industrial, a localized fluid injection is the source of the system dynamics. In other words, the system can be described as the injection of the fluid through a single orifice at the base of an initially sedimented granular bed. Theoretical, numerical, and experimental studies have thoroughly investigated this configuration for two-phase flows [22–27]. Different invasion regimes are reported when the injected flow-rate increases: fluid percolation through a static bed, formation of a stable fluidized cavity which does not reach the granular bed surface, or fluidization over the entire granular layer height. However, to our knowledge, no previous work has described the dynamics of the particle-laden jet inside the fluidized zone.

In a recent work, Vessaire et al. investigated a liquid jet impinging on a confined saturated granular bed [28]. This configuration unraveled a puzzling phenomenon: in a given range of parameters, the jet is strong enough to form a crater containing suspended particles and exhibits oscillations. Although apparently different, this downward particle-laden jet is strongly reminiscent of the upward particle-laden jet reported for localized fluidization when injecting a fluid at the base of a granular layer.

The present work investigates the dynamics of the particle-laden jet formed after the onset of localized fluidization when injecting water at constant flow rate through a single nozzle at the base of an immersed granular layer. Experiments are performed in a Hele-Shaw cell, which allows a direct visualization (Sec. II). We report the emergence of periodic oscillations and characterize their existence (Sec. III), frequency, and amplitude (Sec. IV). Finally, we discuss the results in comparison to the impinging jet configuration and other types of hydrodynamic instabilities (Sec. V).

II. EXPERIMENTAL SETUP

Experiments are performed in a Hele-Shaw cell, consisting in two glass plates separated by a gap $e = 2 \text{ mm}$. The cell inner dimensions are $31.5 \text{ cm} \times 35 \text{ cm}$ (width). Initially, the cell is filled with a granular layer up to a height $h_g$, fully immersed in distilled water (density...
ρf = 1000 kg/m^3; viscosity ηf = 10^{-3} Pa s). The particles are spherical glass beads (USF Matrasur) of diameter d_p and of density ρ_p = 2300 kg/m^3. The density difference between the grains and surrounding fluid is Δρ = ρ_p - ρf = 1300 kg/m^3. The granular layer height varies from h_g = 1 to 8 cm. Although most experiments have been performed with d_p = 106–212 μm, additional experiments have also been analyzed with d_p = 400–600 μm, or with no grains for comparison (see Sec. III).

At time t = 0, water is injected at constant flow-rate Q from a nozzle (inner diameter d_i = 1.1 mm) located at the bottom center of the cell. A flow-rate controller (Bronkhorst mini CORI-FLOW M14-AAD-22-0-S, 2 – 100 mL/min) connected to a pump upstream (Bronkhorst Tuthill 7.11.468) made possible to explore a range of flow rates Q = [3–90] mL/min. A transparency flat viewer (Just NormLicht, Classic Line) located behind the cell ensures a homogeneous backlight. Direct visualization of the two-phase system at time t ≥ 0 is performed by means of a camera CMOS 1 in. (BASLER acA2040-90um, monochrome, full resolution 2048 × 2048 pixels) with a lens of focal distance 16 mm, located at about 26 cm from the cell. Calibration with a grid and geometrical corrections are applied (with MATLAB software) to account for small optical distortions on the sides of the image. All experiments are recorded at 50 fps, for a duration of typically 2 min. Reproducibility is checked by performing twice all experiments.

III. DIFFERENT REGIMES

Different regimes are reported for fluid migration in the granular layer and are described in the next paragraph. The most puzzling observation, which is the central part of this work, is reported for d_p = 106–212 μm in a given range of parameters (Q, h_g). In this range, the upward jet does not only fluidize the granular layer, as already reported in previous works in similar configurations [23–27,29,30], but also exhibits periodic oscillations as illustrated in Fig. 1. In this regime, after reaching the surface, the particle-laden jet oscillates regularly from one side to the other of the central crater. This latter is formed by the particles suspended by the upward flow and deposited on either side of the crater, in a way similar to crater growth in immersed granular media with gas injection [13]. The jet oscillations develop in a given region of the fluidization regime, for low h_g and above a critical flow rate (black circles, Fig. 2).

A thorough exploration of the parameter phase space (Q, h_g) makes possible to identify the regions associated with the three different regimes. (1) In the percolation regime (white squares, Fig. 2), the fluid migrates upward through the granular bed without moving significantly the grains, even if a slight decompression is observed (see movie 1 [31]). (2) In the fluidization regime (gray triangles, Fig. 2), particles are lifted and, after a transient, the central region is fluidized and a crater forms with dunes on each side. The particle-laden jet fluctuates inside this region but does not exhibit periodic oscillations (see movie 2 [31]). (3) In the oscillations regime, the central region is fluidized and, in the stationary regime (see Sec. IV B), the upward jet flaps with a given frequency (Fig. 1 and movie 3 [31]). For h_g = 4 cm, although oscillations are observed for Q ≥ 60 mL/min, a variability is reported when repeating the experiments at Q = 60 and 70 mL/min. A first set of data displays clear oscillations, whereas a second set of data shows jet fluctuations without any clear periodicity. Although in the following Qc = 60 mL/min will be considered as the critical flow rate for the emergence of oscillations,
these two data sets will not be considered in the jet oscillations analysis (Sec. IV).

For particles of larger diameter ($d_p = 400–600 \, \mu m$), although a clear transition between the percolation and fluidization regime is reported (dashed line, Fig. 2), the oscillation regime is never observed in the explored range of parameters. Experiments have also been performed without grains, injecting the fluid in the Hele-Shaw cell initially filled with the same fluid only. A stable jet is observed for the whole range $Q = 3–90 \, mL/min$ (Fig. 2, $h_g = 0$). In the following section, we focus on the oscillating regime of the particle-laden jet ($d_p = 106–212 \, \mu m$) and quantify the oscillation properties.

IV. JET OSCILLATIONS

A. Data analysis

To quantify the jet oscillations, the experimental data are analyzed as follows. First, images are cropped to keep the central zone only, including the jet, whose maximum vertical extent is, in the stationary regime, higher than the initial granular bed [see Fig. 1(d) for example]. To avoid contour detection problems when the jet has a low particle volume fraction, the computation is based on the difference between two consecutive frames. The image acquisition rate being much larger than the oscillations frequency, it improves the image analysis without introducing a significant error in the jet maximum height detection. After binarization, morphological operations are performed (imclose, imfill, imfilter with MATLAB software, MathWorks®) to identify the largest particle-laden region including both the particles remaining at rest on each side of the crater and the particle-laden jet. Its edges are detected with a Sobel method. The jet maximum point ($X_j, Z_j$), corresponding to the coordinates of its further

FIG. 2. Different regimes for fluid migration through the granular layer ($d_p = 106–212 \, \mu m$). □: percolation; △: fluidization; ○: fluidization with particle-laden jet oscillations (see text). Double symbols for $h_g = 4 \, cm$, $Q = 60–70 \, mL/min$ indicate different regimes observed when repeating the experiment. The colored regions are guides for the eye. The dashed line indicates the limit between percolation and fluidization for $d_p = 400–600 \, \mu m$ (see text). For these largest particles, no oscillations are observed. For $h_g = 0$ (no grains), a stable jet is observed for all flow rates.

extremity in the $z$ direction is then obtained as a function of time, for each experimental series.

B. Stationary regime

At the beginning of each experiment in the oscillations regime, the fluid invades the granular layer, migrates upward through local fluidization, and reaches the surface, leading to the full fluidization of the granular layer. The particle-laden jet maximum height can only be detected after this time, which lasts in average less than 5 s after the first fluid injection. Figure 3 displays the temporal evolution of the jet maximum ($X_j, Z_j$) for different flow rates. In the transient regime, the jet height $Z_j$ increases until it reaches a plateau $p^*_z$ superimposed with oscillations. Inset: zoom between $t = 110$ and 120 s showing the oscillations for the lowest flow rates.

FIG. 3. Temporal evolution of the jet maximum height (a) horizontal position $X_j$ and (b) vertical position $Z_j$ for different flow rates ($h_g = 2 \, cm$). After a transient, the jet height $Z_j$ reaches a plateau $p^*_z$ superimposed with oscillations. Inset: zoom between $t = 110$ and 120 s showing the oscillations for the lowest flow rates.

Figure 4 shows the jet maximum height in the stationary regime, $p^*_z$, as a function of the imposed fluid flow rate $Q$. The jet height is normalized by the initial granular bed thickness, $h_g$, while the flow rate is normalized by the critical flow rate $Q_c$ above which the oscillations appear for each initial granular layer height $h_g$. For $Q/Q_c < 1$, all data collapse to $p^*_z/h_g = 1$. Except small fluctuations, the jet height equals the initial granular layer height and no oscillations are reported (open symbols, Fig. 4). For $Q/Q_c > 1$, the data are scattered but display an increase of the jet height as a function of $Q/Q_c$. 

024901-3
where $Q$ function by the normalized flow rate $Q_c$, where $Q_c$ is the critical fluid flow rate above which oscillations appear, for different initial granular bed height $h_g$. Open symbols: no jet oscillations; filled symbols: jet oscillations (gray region). The error bars are smaller than the symbol size. The solid colored lines correspond to the prediction of Eq. (1) for each $h_g$. Inset: critical flow rate $Q_c$ as a function of $h_g$.

A simple fluid analogy and force balance leads to

$$p^*_i = \frac{v_i^2}{2(\rho_f/\rho_0)} \frac{1}{g},$$

where $v_i = Q/(\pi d_i^2/4)$ is the initial jet velocity at the nozzle exit and $\rho_f = \phi \rho_p + (1 - \phi) \rho_f$ the particle-laden jet density with $\phi$ its particle volume fraction. Considering $\phi \approx 15\%$ as a reasonable estimate leads to the prediction displayed by the solid lines in Fig. 4 for each $h_g$. Although based on a crude approximation, this prediction captures well the jet height order of magnitude. However, it fails in capturing the granular jet height at low flow rates (for $Q < Q_c$ in the fluidization regime, $p^*_i/h_g \approx 1$). This is due to the fact that this simple fluid analogy does not account for the initial granular layer. This latter remains present on both sides of the jet, even after total fluidization, and grains avalanching in the inner part of the crater participate to the jet dynamics, which explains why the jet maximum height does not tend to zero when the flow rate decreases to zero. Equation (1) also fails in predicting the oscillations, which appear when the jet becomes higher than $h_g$ ($p^*_i/h_g > 1$, filled symbols and gray region, Fig. 4).

Note that, for $h_g = 1 \text{ cm}$, we could not detect the jet maximum for $Q > 40 \text{ mL/min}$ due to the very low particle volume fraction inside, resulting in the failure of the image analysis presented in Sec. IV A. When increasing the initial bed height the critical flow rate increases (Fig. 4, inset). This is consistent with the fact that the particle-laden jet has to reach a maximum higher that the initial granular layer to undergo oscillations. Henceforth, we focus on the oscillation properties in the stationary regime.

C. Frequency

The jet oscillations’ frequency along the $x$ and $z$ axis, respectively $f_x$ and $f_z$, are computed from the temporal evolution of the jet maximum displayed in Figs. 3(a) and 3(b). First, the mean evolution of $Z_j$ (increase + plateau) is removed from the signal in Fig. 3(b), so that the oscillatory part with an average at zero is extracted from $Z_j$. Then spectrograms based on fast Fourier transforms are computed. In the stationary regime, the frequencies $f_x$ and $f_z$ are constant and depend on $(Q, h_g)$ only. They are extracted from the spectrograms as the average values over the stationary regime of the spectrogram maximum at each time $t$. The error on both frequencies’ estimation is estimated as the standard deviation over the same temporal window.

Figure 5(a) shows the jet oscillations frequency along $z$, $f_z$, as a function of the oscillations’ frequency along $x$, $f_x$, for different $h_g$ (different symbols) when varying the flow rate $Q$ (color filling the symbols; see color scale on the right part of the figure). When increasing the flow rate, both frequencies decrease. Interestingly, all experimental data collapse on a single curve, $f_z \approx 2f_x$, whichever the parameters $(Q, h_g)$. This indicates that the particle-laden jet experiences oscillations analog to a solid-body oscillation, reminiscent of a metronome. This result will be further analyzed in the next section IV D. As both quantities are directly proportional, in the following, we shall only use $f_x$ as the jet characteristic
Oscillations of a Particle-Laden Fountain

The oscillation frequency is directly linked to the crater geometry, $f_x$, being highest for $\theta \simeq 28^\circ$, and decreasing when the crater becomes more open.

Figure 5(b) displays the horizontal oscillations frequency $f_x$ as a function of the normalized flow rate $Q/Q_c$. The frequency strongly decreases with the normalized flow rate until reaching for high flow rate $\theta \simeq 28^\circ$, corresponding to the avalanche angle [13]. The oscillation frequency is directly linked to the crater geometry, $f_x$, being highest for $\theta \simeq 28^\circ$, and decreasing when the crater becomes more open.

D. Amplitude

To investigate further the hypothesis of a solid-body oscillation (metronome model), we can estimate theoretically the relationship between the horizontal and vertical amplitudes of oscillations in the steady-state regime, $A_x^*$ and $A_z^*$, respectively. Figure 6(a), inset, presents a sketch of the metronome model, with $L = p_x^* + A_x^*$. Simple trigonometry with $A_x^* = L \sin \delta$ and $A_z^* = L/2(1 - \cos \delta)$ leads to the theoretical expression of $A_x^*$ given by

$$A_x^* = A_z^* \sqrt{2 \left( \frac{p_x^*}{A_z^*} - 1 \right)}.$$  \hspace{1cm} (2)

From the experimental measurements of $p_x^*$ and $A_z^*$, $A_x^{\text{theo}}$ is computed and compared to its experimental value [Fig. 6(a)]. Despite some scattering, both values agree satisfactorily, showing once again the pertinence of the analogy between the jet oscillations and solid-body oscillations.

Figure 6(b) displays the amplitude variations of the jet vertical oscillations in the steady-state regime, $A_z^*$, normalized by the initial granular bed height $h_g$, as a function of the normalized flow rate $Q/Q_c$. The colored lines correspond to Eq. (3) for each data set (see text). Inset: coefficient $\xi$ [Eq. (3)].

Figure 7 displays the Strouhal number $St$ as a function of the parameter $J$. Similar to the jet impingement configuration [28], all data points collapse on a single curve and $St$ exhibits a power-law behavior, $St = J^{\gamma}$, with $\gamma \simeq -5/2$. Interestingly, although in both the jet impingement or the upward jet configuration the Strouhal number

\begin{equation}
J = \frac{Q}{(2eh_g \tan \alpha)u}.
\end{equation}

This dimensionless parameter is adapted from the erosion parameter $E$ classically used in the literature [32] for confined geometry. Note that it also corresponds to the square root of a Shields number. Here $\alpha \simeq 7^\circ$, of the order of the value measured by Vessaire et al. [28].
The emergence of jet self-sustained oscillations has been pointed out in previous works as the consequence of a feedback loop, generated either by impingement on an obstacle [33–37] or by a specific nozzle configuration or mixing chamber such as in fluidic oscillators [38,39]. One possible mechanism at the origin of the oscillations is hydrodynamic, with structures advected by a recirculation flow and interacting back with the nozzle. In this case, the subsequent oscillations frequency—and thus the Strouhal number—increases with the jet velocity [35]. Another possible origin for the feedback is the generation of acoustic waves which propagate upstream and interact back with the nozzle. At low Mach numbers, this mechanism is characterized by a constant Strouhal number [40].

Neither the acoustic nor hydrodynamic origin can explain the oscillations frequency of the particle-laden jet reported in this work, as the Strouhal number decreases with the jet speed, with all other parameters being constant for a given initial granular layer height [Eq. (4)]. Vessaire et al., who also discarded these two mechanisms, proposed that the mechanism at the origin of the decreasing frequency with the jet velocity is a coupling between the fluid and the grains—the jet oscillations in their experiment being directly coupled with the crater aperture width oscillations. This argument seems valid for the present observations. First, the oscillation frequency decreases with $J$, the exponent of the power-law behavior is drastically different ($\gamma \approx -1$ in the reverse configuration [28]).

VI. CONCLUSION

This work reports the appearance of self-sustained oscillations of a particle-laden jet when injecting water at constant flow rate at the base of a granular layer confined in a Hele-Shaw cell. These oscillations develop in a given range of jet flow rate, initial granular layer height, and grain size. In particular, while the oscillations are fully characterized for $d = 106–212 \, \mu m$, no oscillations are observed for the larger grains ($d = 400–600 \, \mu m$). Surprisingly, the particle-laden jet oscillates as a solid body, and a simple metronome model can explain the relationship between its vertical and horizontal frequency, as well as its vertical, horizontal amplitude, and jet height. Similar to previous results on the reverse configuration of an impinging jet on the surface of a granular layer in a confined geometry, the Strouhal number displays a power-law behavior but with a larger exponent $\gamma \approx -5/2$. This points out the possible origin of the self-sustained oscillations in the strong coupling between the fluid and particles. This mechanism could be a possible origin for the periodic behavior which can be exhibited by pockmarks and mud volcanoes in sedimentary basins [41].

ACKNOWLEDGMENTS

The authors acknowledge the Centre National de la Recherche Scientifique (CNRS) for MITI PhD Grant funding. We thank B. Sutherland for interesting discussions and two anonymous referees for their comments which greatly improved the manuscript.

[31] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.106.024901 for movie 1, percolation regime ($h_g = 6\, \text{cm}$, $Q = 3\, \text{mL/min}$), movie 2, fluidized regime without oscillations ($h_g = 4\, \text{cm}$, $Q = 15\, \text{mL/min}$), and movie 3, fluidized regime with oscillations ($h_g = 2\, \text{cm}$, $Q = 35\, \text{mL/min}$).