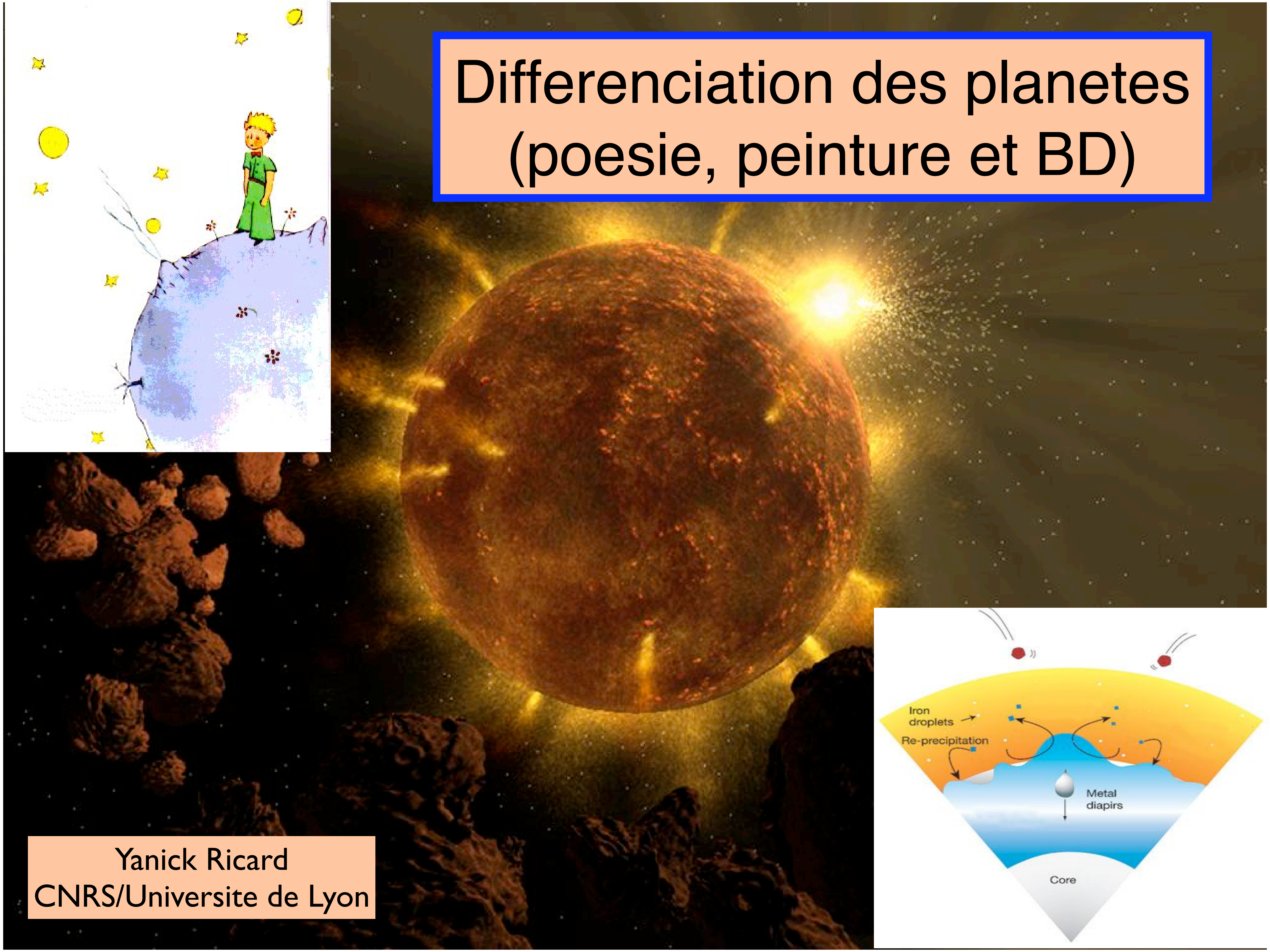
Differenciation des planetes (poesie, peinture et BD)



Yanick Ricard
CNRS/Universite de Lyon

Des energies en jeu lors de la formation d’une planete

-Radioactivites fossiles

$$\mathrm{Al\text{-}26} \xrightarrow{0.75\ \mathrm{ma}} \mathrm{Mg\text{-}26}$$

$$\mathrm{Fe\text{-}60} \xrightarrow[1.5\ \mathrm{ma}]{} \mathrm{Co\text{-}60}$$

La difference d'energie de masse - l'energie transportee par les neutrinos est dissipee

Fe chondritic abundance	18.2	wt. %	[Lodders 1998]
Al chondritic abundance	0.865	wt. %	[Lodders 1998]
^{60}Fe/Fe fractional abundance	$4.4\,10^{-6}$		[Quitte 2005]
^{26}Al/Al fractional abundance	$5\,10^{-5}$		
^{60}Fe half-life	1.5	Myr	
^{26}Al half-life	0.74	Myr	
^{60}Fe nuclear power per kg of chondrite	$5.2\,10^{-8}$	W kg^{-1}	
^{26}Al nuclear power per kg of chondrite	$1.5\,10^{-7}$	W kg^{-1}	[Schramm 1970 ??]

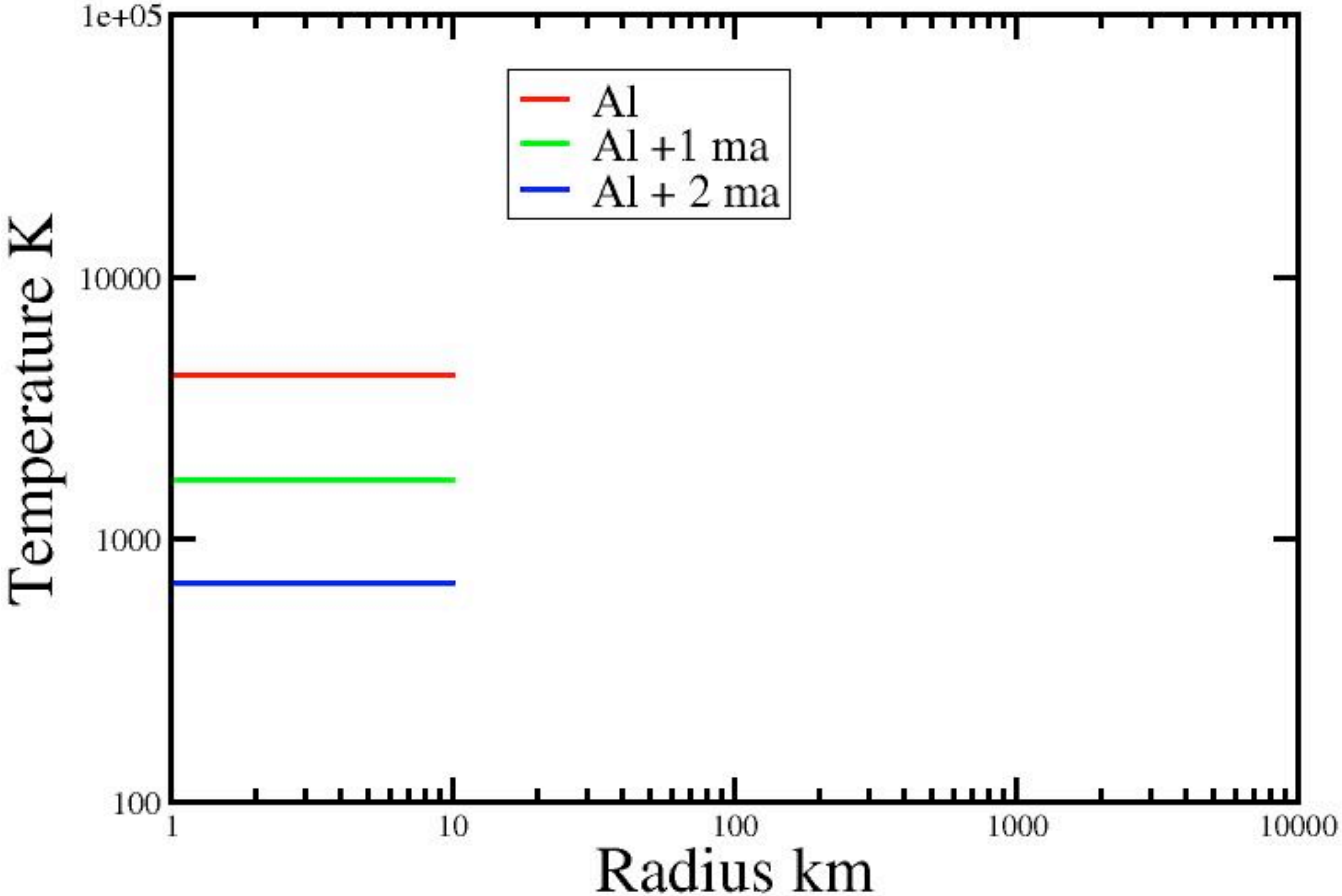
$$H = H_0 \exp(-t \log(2)/t_{1/2})$$

$$\rho C \frac{\partial T}{\partial t} = \rho H$$

$$T = \frac{H_0 t_{1/2}}{C \log 2}$$

$$H_0^{Al} = 1.5\ 10^{-7} \mathrm{W\ kg}^{-1}$$

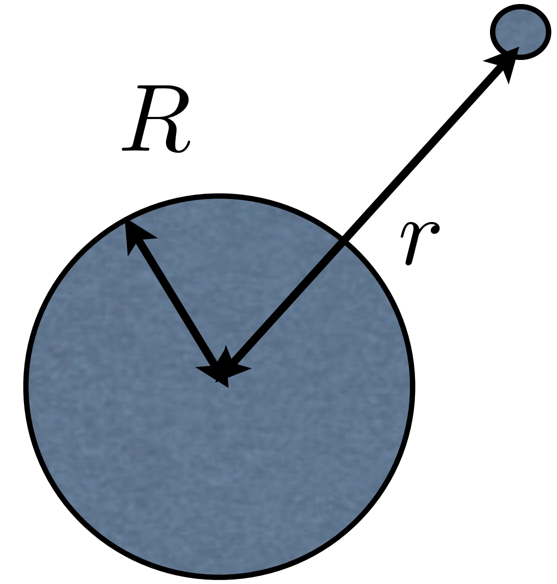
$$C = 1000\,\mathrm{J\ kg}^{-1}\mathrm{K}^{-1}$$



Il y a beaucoup d’energie disponible
pendant un temps bref

-Radioactivites fossiles
-Energie gravitationnelle

Energie pour transporter un objet de la surface de la planete a "l'infini"

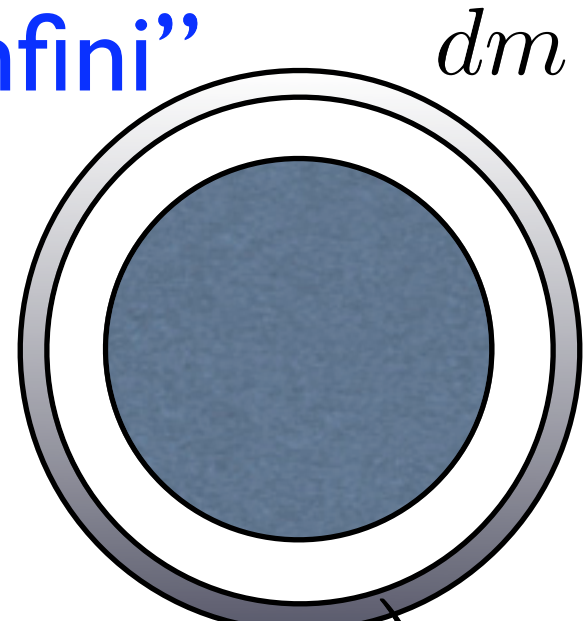


$$dW = F dr$$

$$W = mg(R) \int_R^{+\infty} \frac{R^2}{r^2} dr = mg(R) R$$

$$g(R) = \frac{4\pi G}{R^2} \int_0^R \rho(r) r^2 dr$$

Energie "gravitationnelle" pour "fabriquer" une planete avec du materiel venu de "l'infini"



$$dW = dm\, g(R) R = 4\pi \rho(R) R^3 g(R) dR$$

$$E_g = \int dW = 16\pi^2 G \int_0^a \rho(R) R \left(\int_0^R \rho(u) u^2\, du \right) dR$$

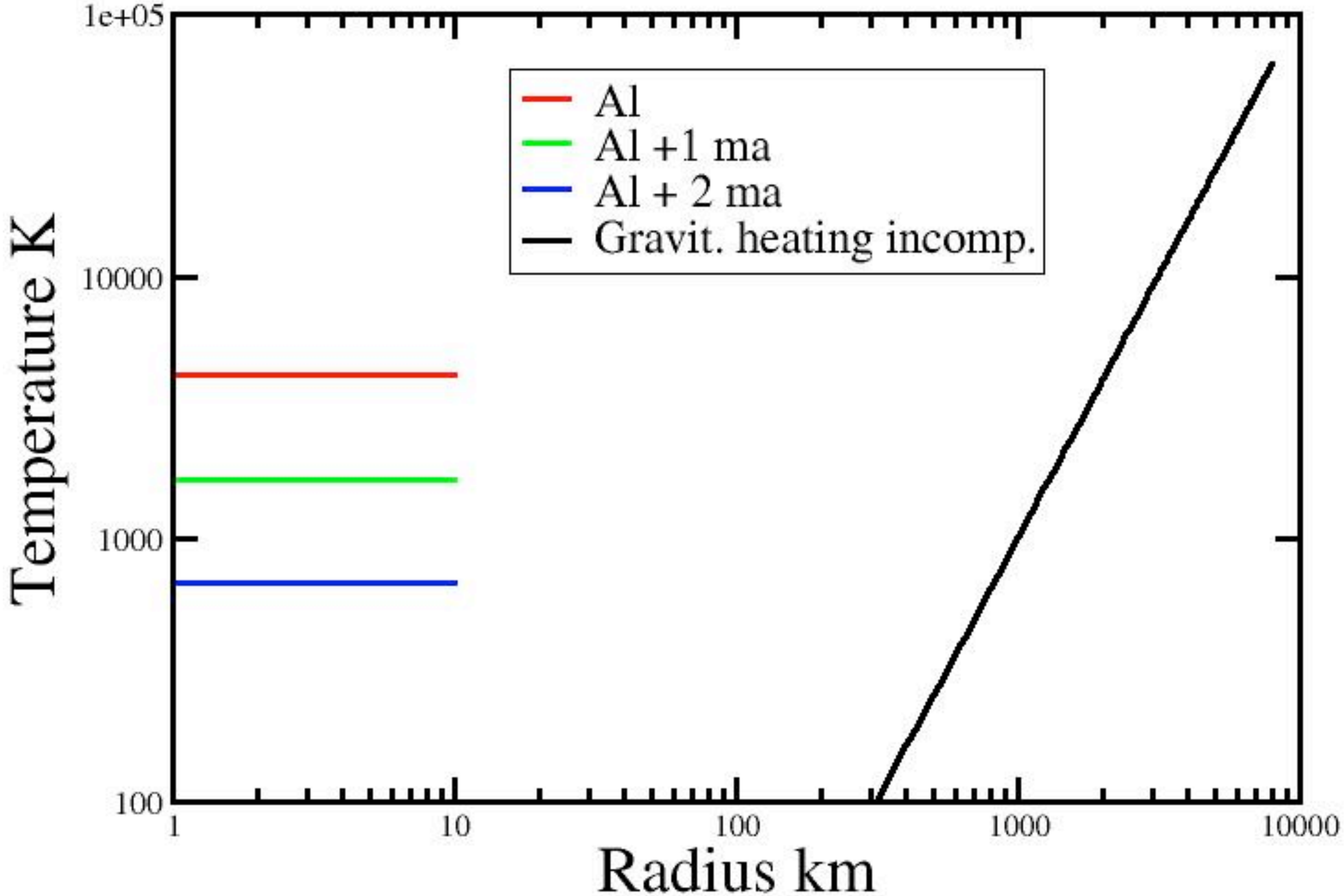
Dans le cas d’une densite uniforme

$$E_{unif} = \frac{16}{15}\pi^2 G\rho^2 R^5 = \frac{3}{5}\frac{GM^2}{R}$$

Soit une augmentation de temperature

$$T = \frac{3}{5}\frac{GM}{CR}$$

Varie en M/R~R^2 (en supposant une densite constante)



Devient preponderant pour les grosses planetes

-Radioactivites fossiles
-Energie gravitationnelle
-Energie elastique

La densite d’une planete n’est pas uniforme, de l’energie est stockee sous forme de travail

$$dW = -PdV = P/\rho^2 d\rho$$

Combien??

L'energie elastique est simplement

$$E_e = 4\pi \int_0^a e_g(r)\rho(r)r^2\, dr = 4\pi \int_0^a r^2 \rho(r) \int_{\rho_0}^{\rho(r)} \frac{P(u)}{u^2}\, dudr.$$

Pour calculer cela, il faut une equation d'etat, par exemple (Murnagham EoS)

$$P = \frac{K_0}{2}\left(\frac{\rho^2}{\rho_0^2} - 1\right)$$

La densite se deduit de l'equation de Bullen qui admet l'egalite de compressibilite "radiale"

$$K = \rho \frac{dP}{d\rho} = -\rho^2 g \frac{dr}{d\rho}$$

et de la compressibilite "thermodynamique"

$$K = \rho \frac{dP}{d\rho} = \rho \frac{d}{d\rho} \left(\frac{K_0}{2} (\frac{\rho^2}{\rho_0^2} - 1) \right) = K_0 \frac{\rho^2}{\rho_0^2}$$

Soit

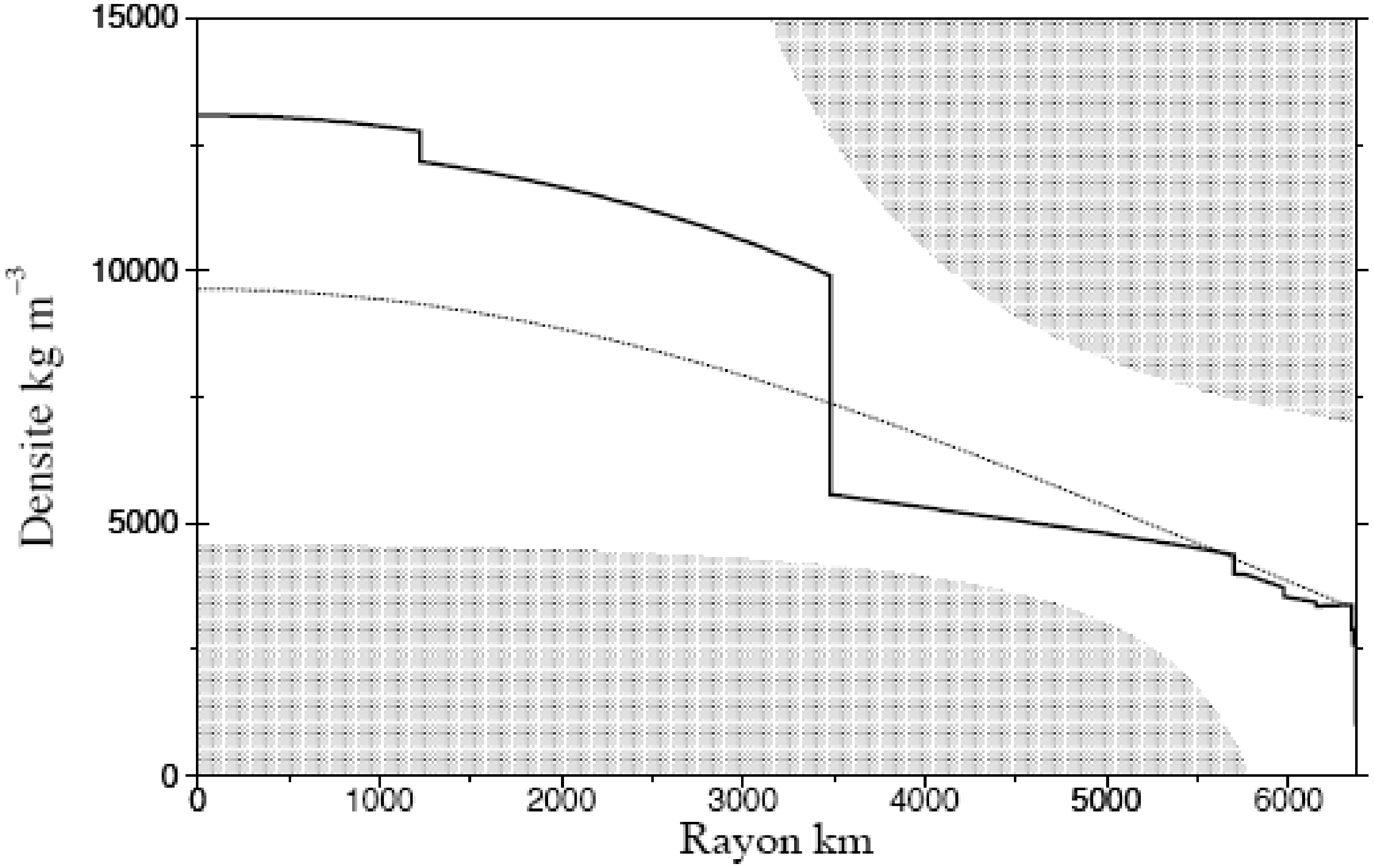
$$-\frac{1}{r^2} \int_0^r \rho(r) r^2 \, dr = K_0 \frac{1}{\rho_0^2} \frac{d\rho}{dr}$$

Cette equation (vieille histoire voir Laplace) a pour solution

$$\rho = \rho_0 \frac{R \sin(r/R_c)}{r \sin(R/Rc)}$$

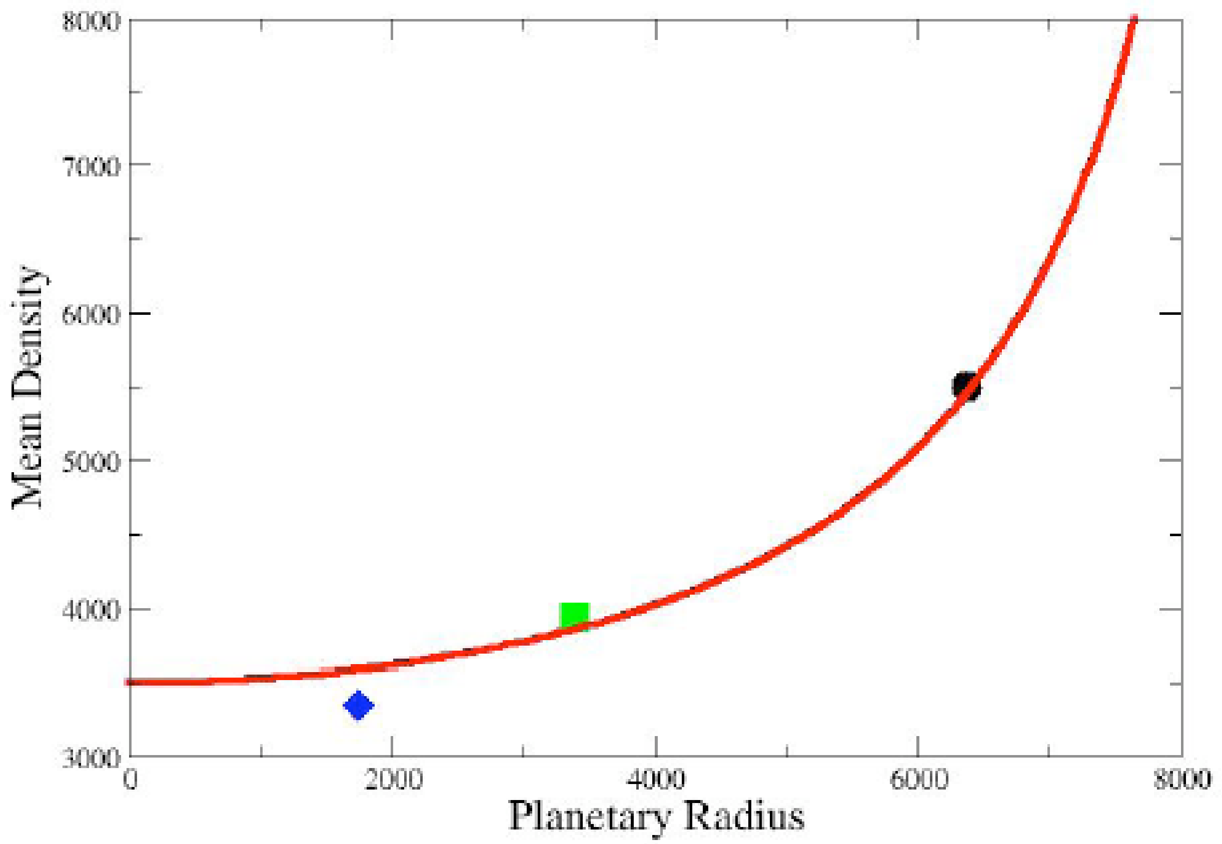
Avec $R_c = \frac{\sqrt{K_0}}{2R\sqrt{\pi G}}$

$$x = R/R_c = 2.26, \rho_0 = 3300 \mathrm{kg\ m}^{-1}$$



Densite dans la Terre

Lune, Mars et Terre



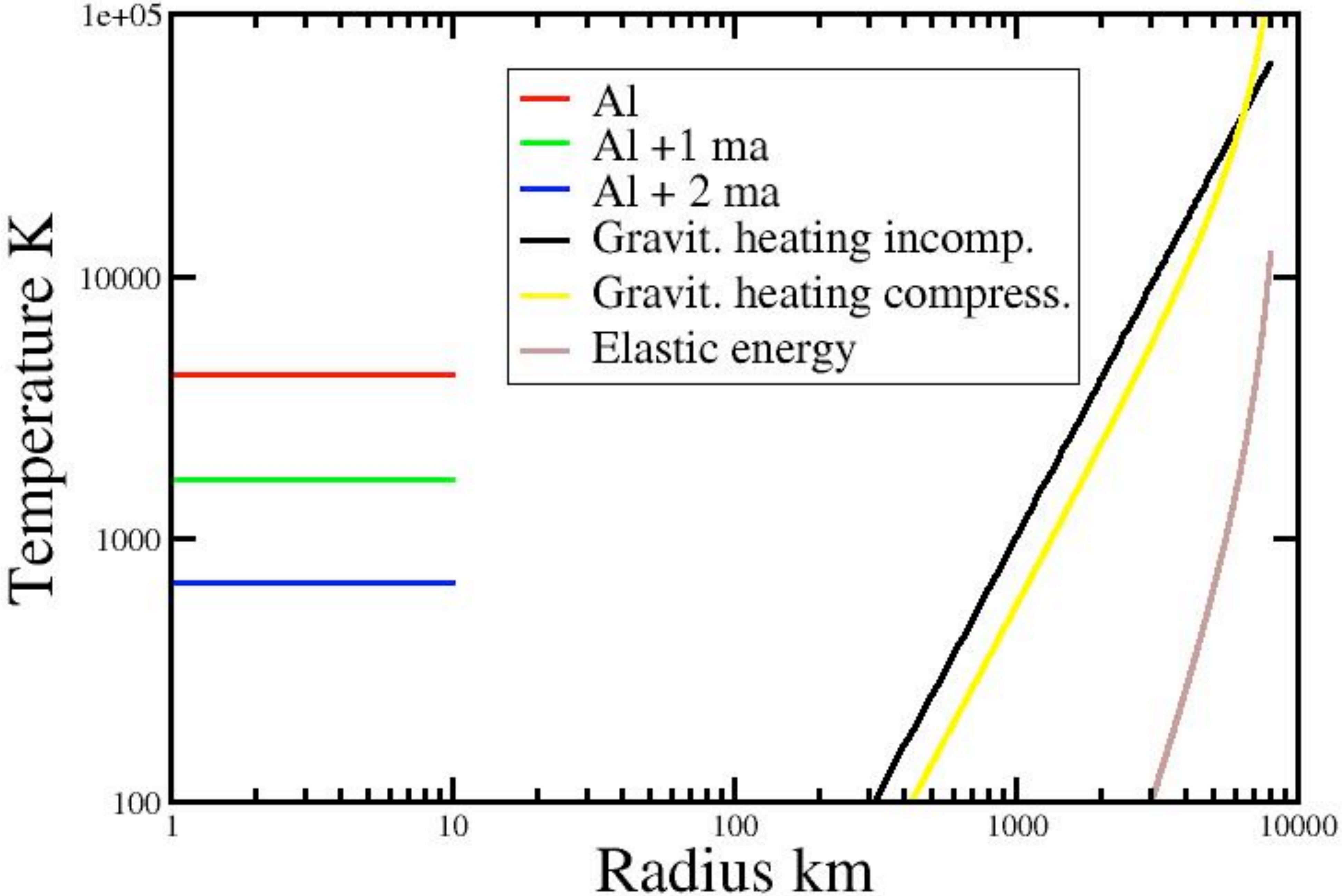
On peut maintenant recalculer l'energie gravitationnelle en tenant compte de la variation de la densite avec le rayon...

$$M = \frac{4}{3}\pi\rho_0 R^3 G(x) \quad \text{avec} \quad x = R/R_c \;\; \text{et} \;\; G(x) = 3\frac{\sin x - x\cos x}{x^2 \sin x}$$

$$E_g = \frac{16}{15}\pi^2 G\rho_0^2 R^5 F(x) \quad \text{avec} \quad F(x) = \frac{15}{4}\frac{x(2\cos^2 x + 1) - 3\cos x \sin x}{x^3 \sin^2 x}$$

et calculer l'energie elastique...

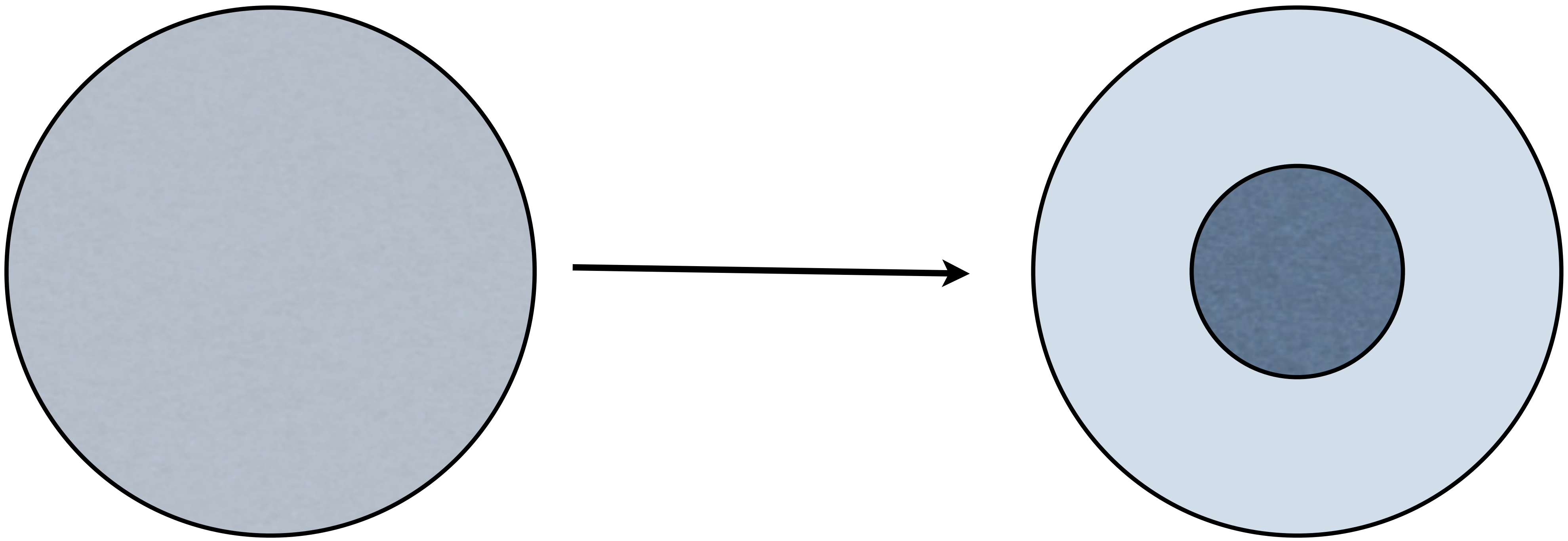
$$E_e = \frac{16}{15}\pi^2 G\rho_0^2 R^5 J(x) \quad \text{avec} \quad J(x) = \frac{5}{4}\frac{9x\cos x \sin x - x^2(2\cos^2 x - 5) - 12\sin^2 x}{x^4 \sin^2 x}$$



1e+05
10000
1000
100
1
10
100
1000
10000
Temperature K
Radius km
Al
Al +1 ma
Al + 2 ma
Gravit. heating incomp.
Gravit. heating compress.
Elastic energy

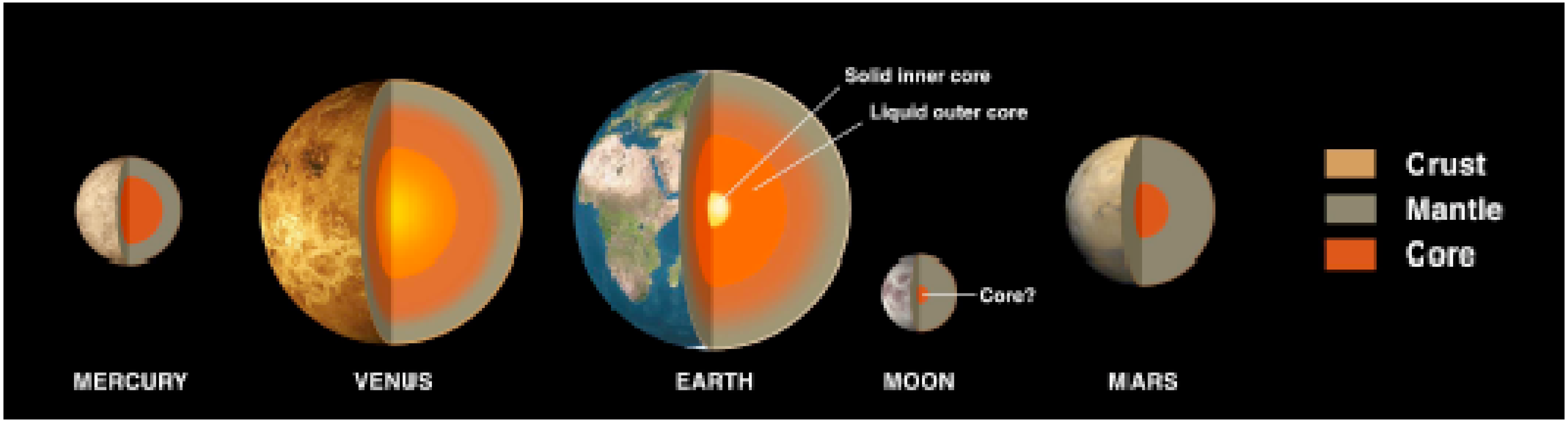
-Radioactivites fossiles
-Energie gravitationnelle
-Energie elastique
-Energie echangee durant la segregation du noyau

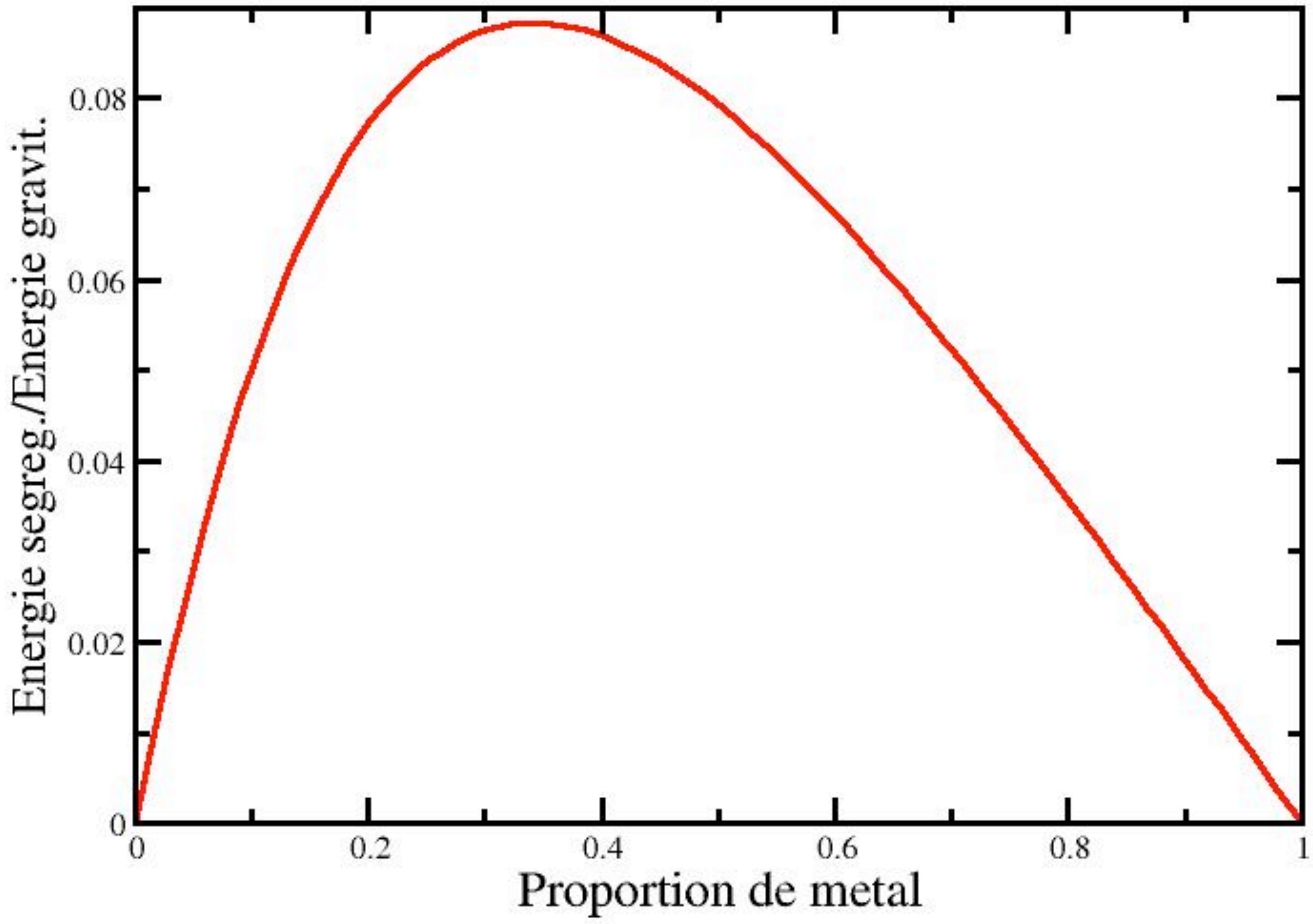
metal en proportion volumique ϕ, de densite ρ_f, et silicate de densite ρ_m

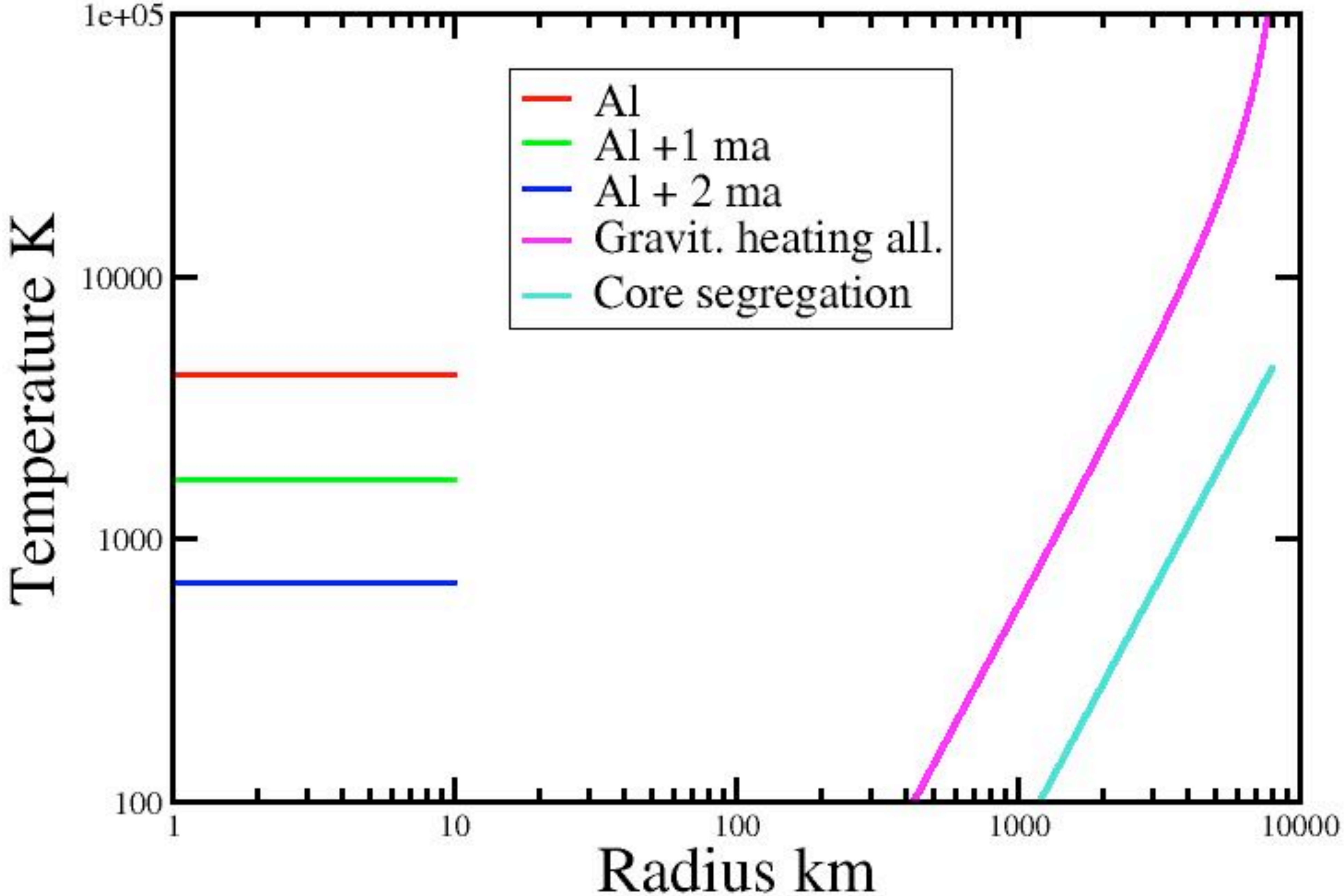


$$E_g = \int dW = 16\pi^2 G \int_0^a \rho(R) R \left(\int_0^R \rho(u) u^2 \, du \right) dR$$

$$E_N = E_g \frac{\rho_f^2 \phi^{5/3} + \rho_m^2 (1-\phi^{5/3}) + 5/2(\rho_f - \rho_m)\rho_m \phi (1-\phi^{2/3}) - \bar{\rho}^2}{\bar{\rho}^2}$$


Solid inner core
Liquid outer core
Core?
Crust
Mantle
Core
MERCURY
VENUS
EARTH
MOON
MARS


Energie segreg./Energie gravit.
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
Proportion de metal

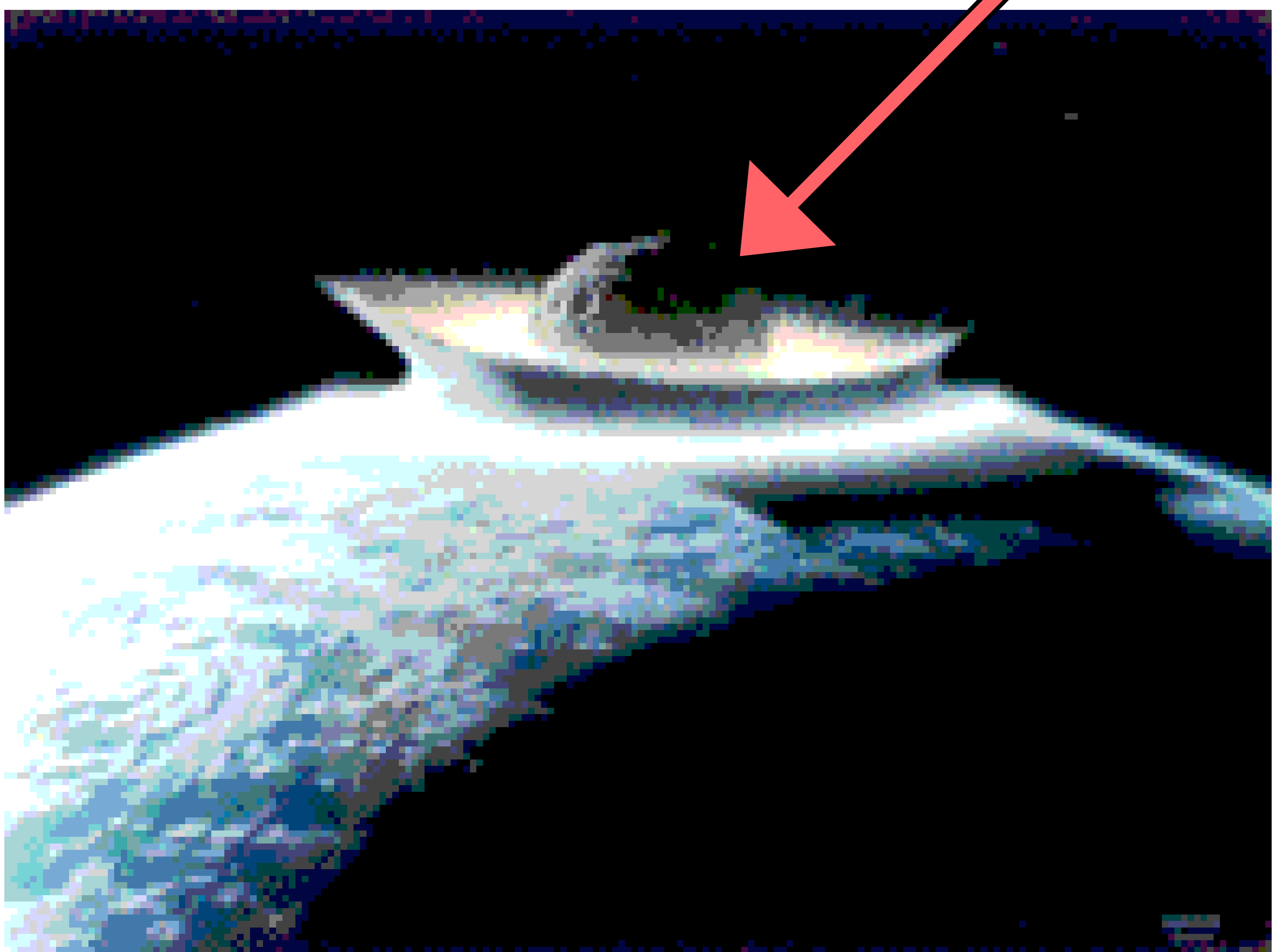


Al
Al +1 ma
Al + 2 ma
Gravit. heating all.
Core segregation
1e+05
10000
1000
100
Temperature K
1
10
100
1000
10000
Radius km

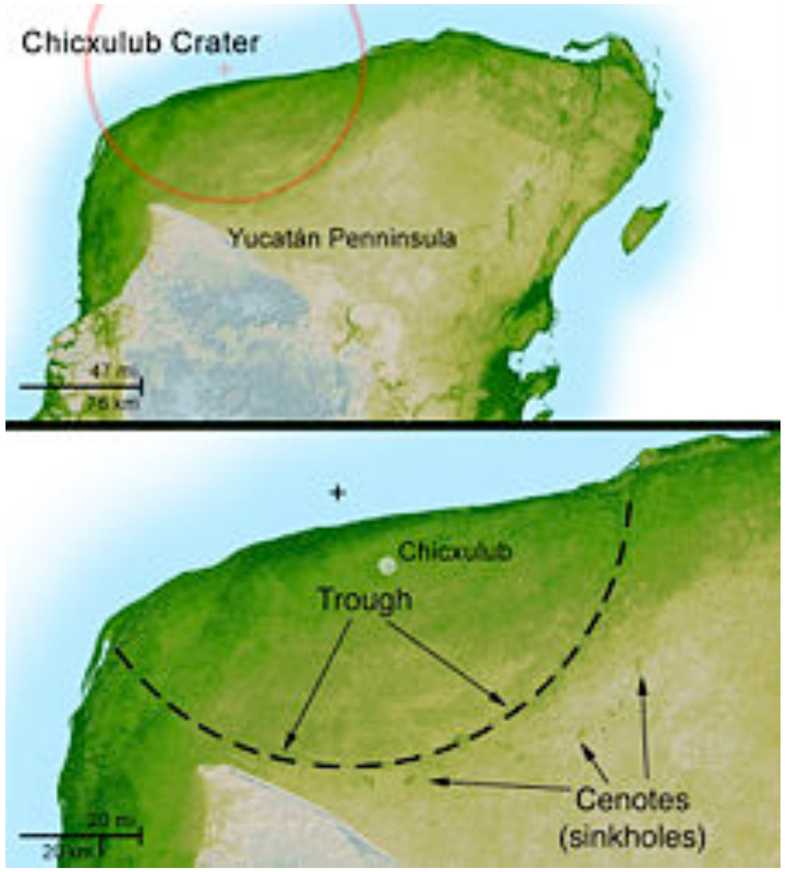
Transfert d’energie des impacteurs

$$v \sim \sqrt{2GM/R}$$

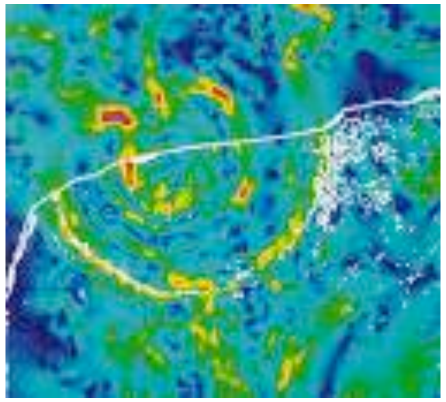
Escape velocity

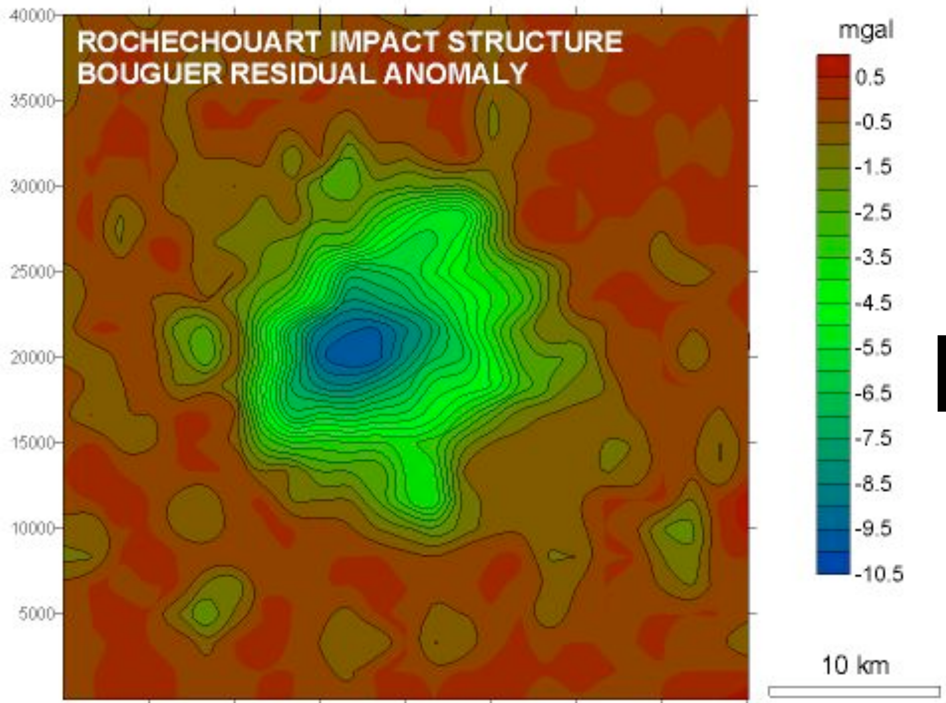


Meteor
Arizona



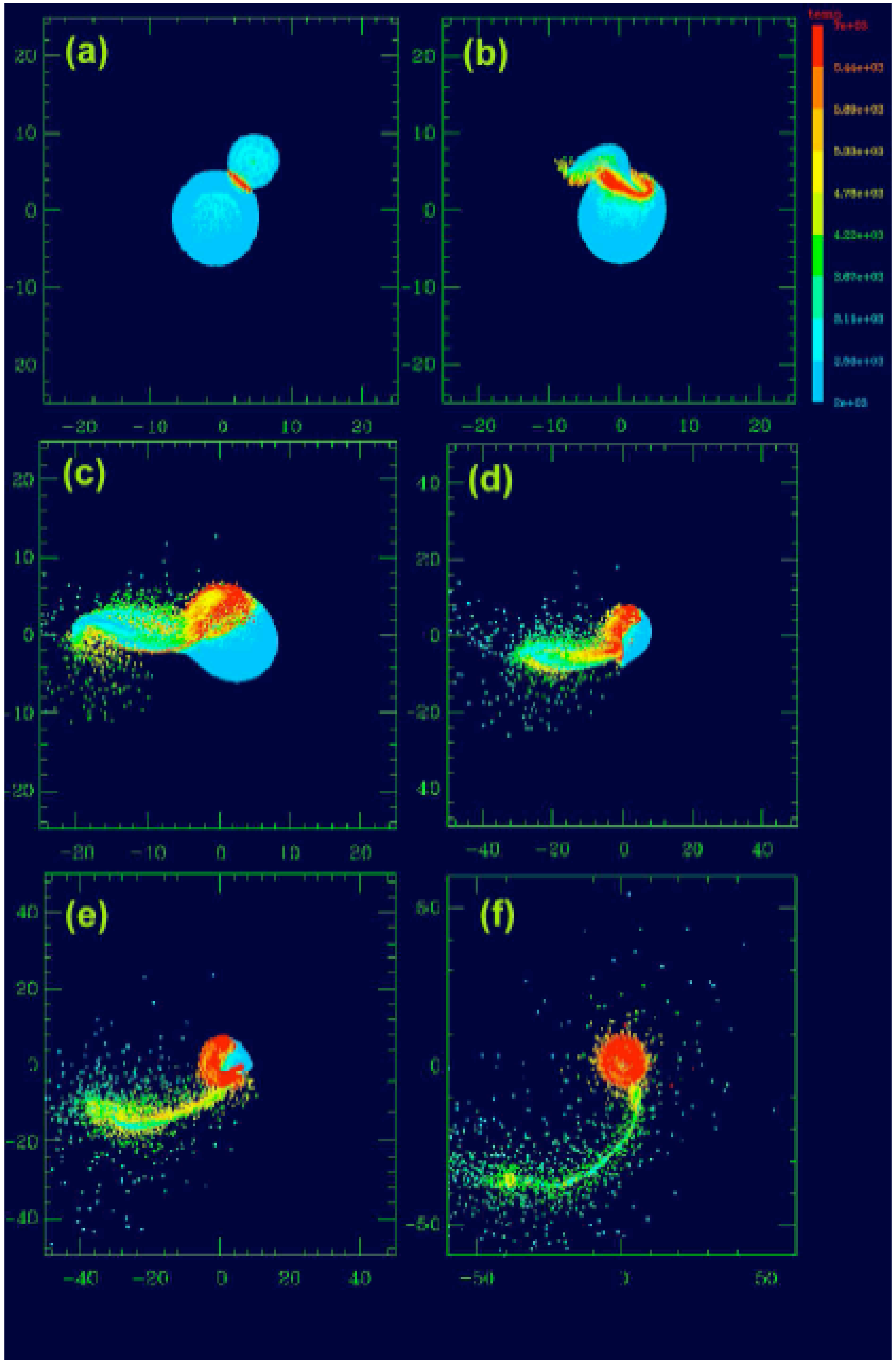
Manicouagan





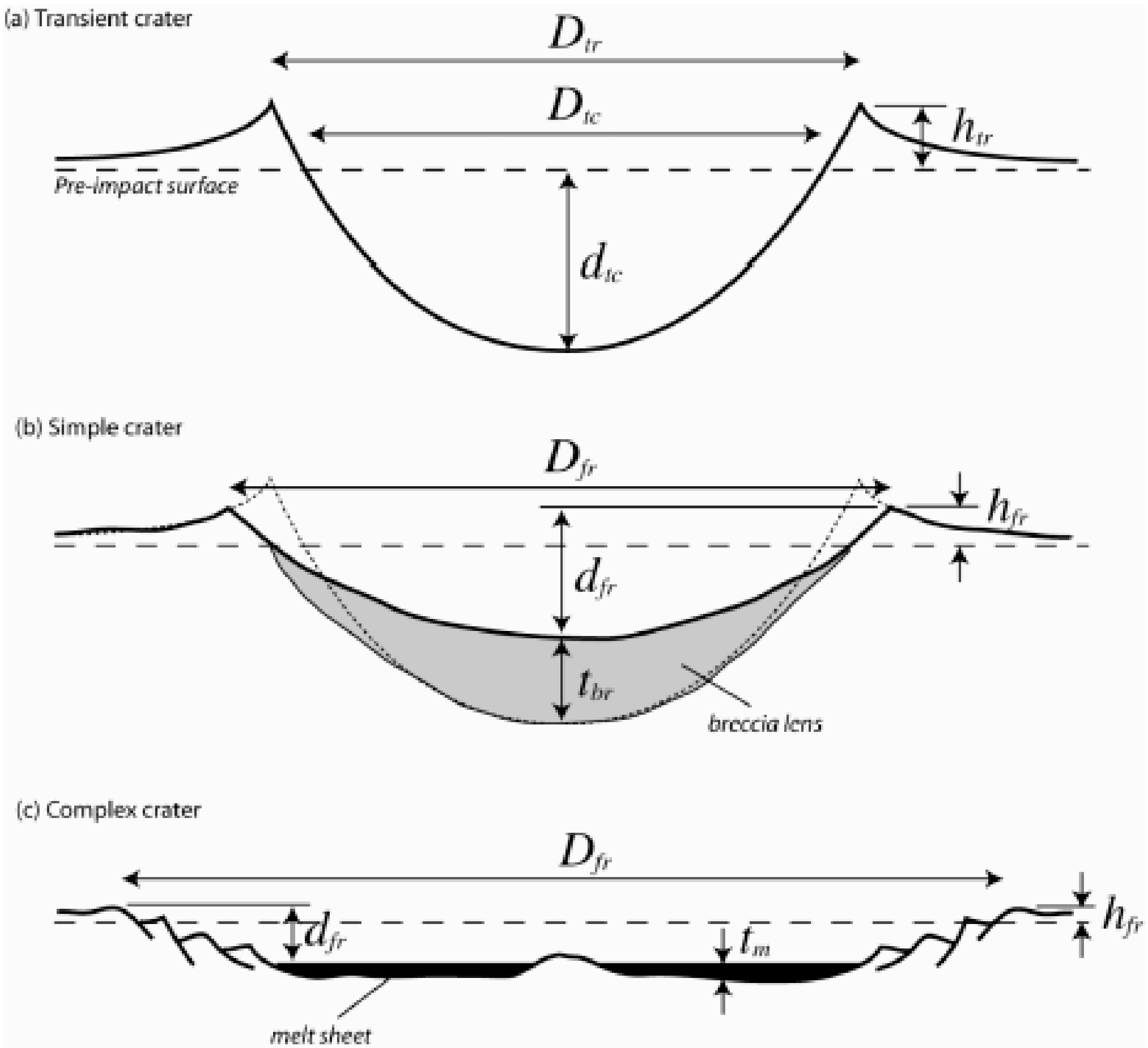
Chicxulub

Rochechouart



Formation de la Lune

Canup 2004



$$D_{tr} = 1.161 \left(\frac{\rho_i}{\rho_t}\right)^{0.33} (2R_i)^{0.78} v_i^{0.44} g_t^{-0.22} \sin(\theta)^{0.33}$$

$$D_{fr} = 1.17 D_{tr} \left(\frac{D_{tr}}{D_c}\right)^{0.13} \quad \text{lorsque} \quad D_{tr} > D_c = 3.2\text{km}$$

$$d_{fr} = 50.36 D_{fr}^{0.3}$$

(valeurs en SI)

Collins Melosh

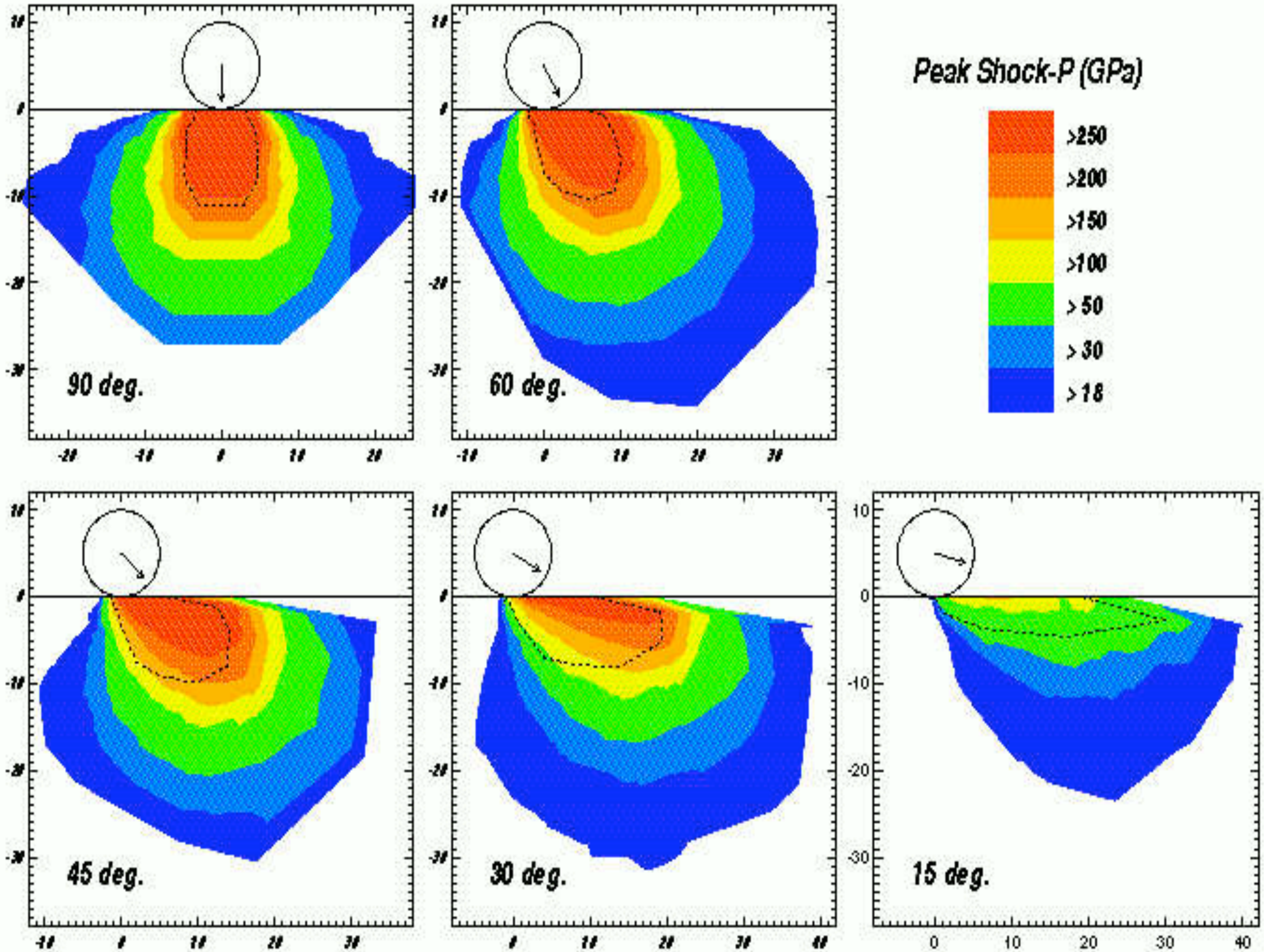
Conditions d'Hugoniot Rankine...

$$\rho(U - v_p) = \rho_0 U$$

$$\rho(P - P_0) = \rho_0 v_p U$$

$$\rho(E - E_0) = (P + P_0)(V_0 - V)/2$$

Plus decompression adiabatique...



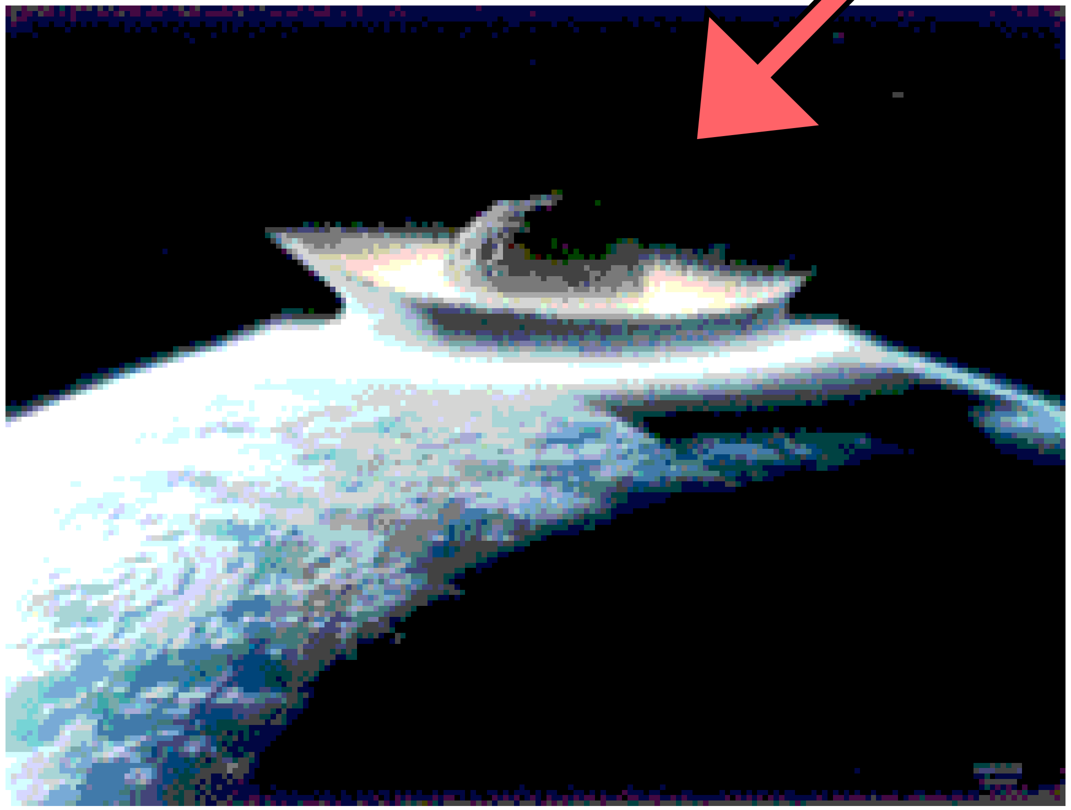
$$D_{ic} = 3^{1/3} 2R_i$$

Senshu

Typiquement,
Sur Terre un impacteur de rayon 5 km, de densite 3.2, a la vitesse d'echappement de 11 km/s, fait un cratere temporaire de 27 km de rayon, de rayon final 45 km, de profondeur 1.5 km, d'isobaric core de rayon 7.5 km.

$$v \sim \sqrt{2GM/R}$$

Escape velocity



$$\Delta T_{impact} = \frac{4\pi}{3}\frac{f_1}{f_2}\frac{G\bar{\rho}R^2}{C}.$$

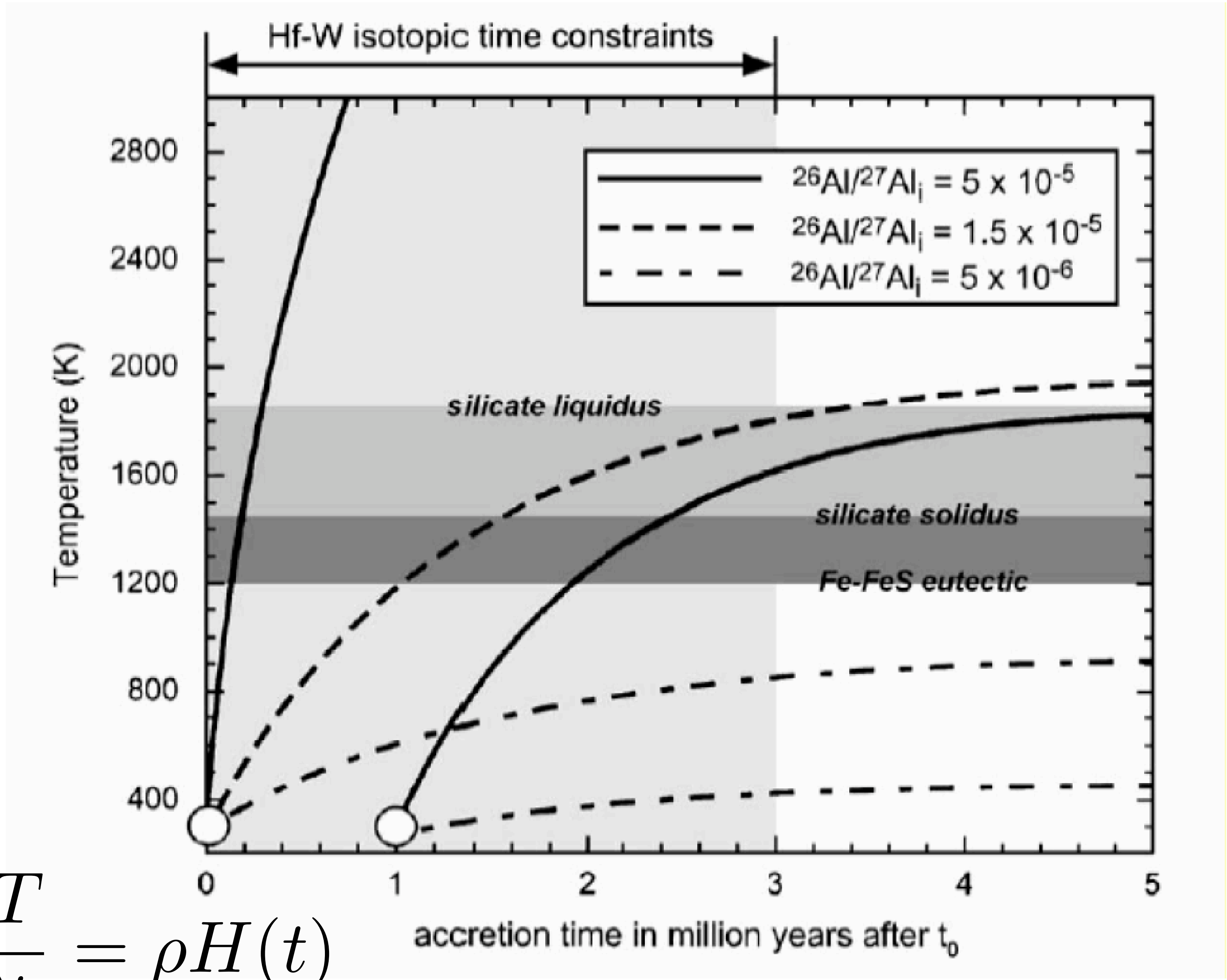
$f_1 \sim 1/3$ Proportion of energy burried

$f_2 \sim 6$ Volume heated

300 K for R=2000 km

-Radioactivites fossiles - volume
-Energie gravitationnelle moins energie elastique - surface
-Energie echangee durant la segregation du noyau - volume

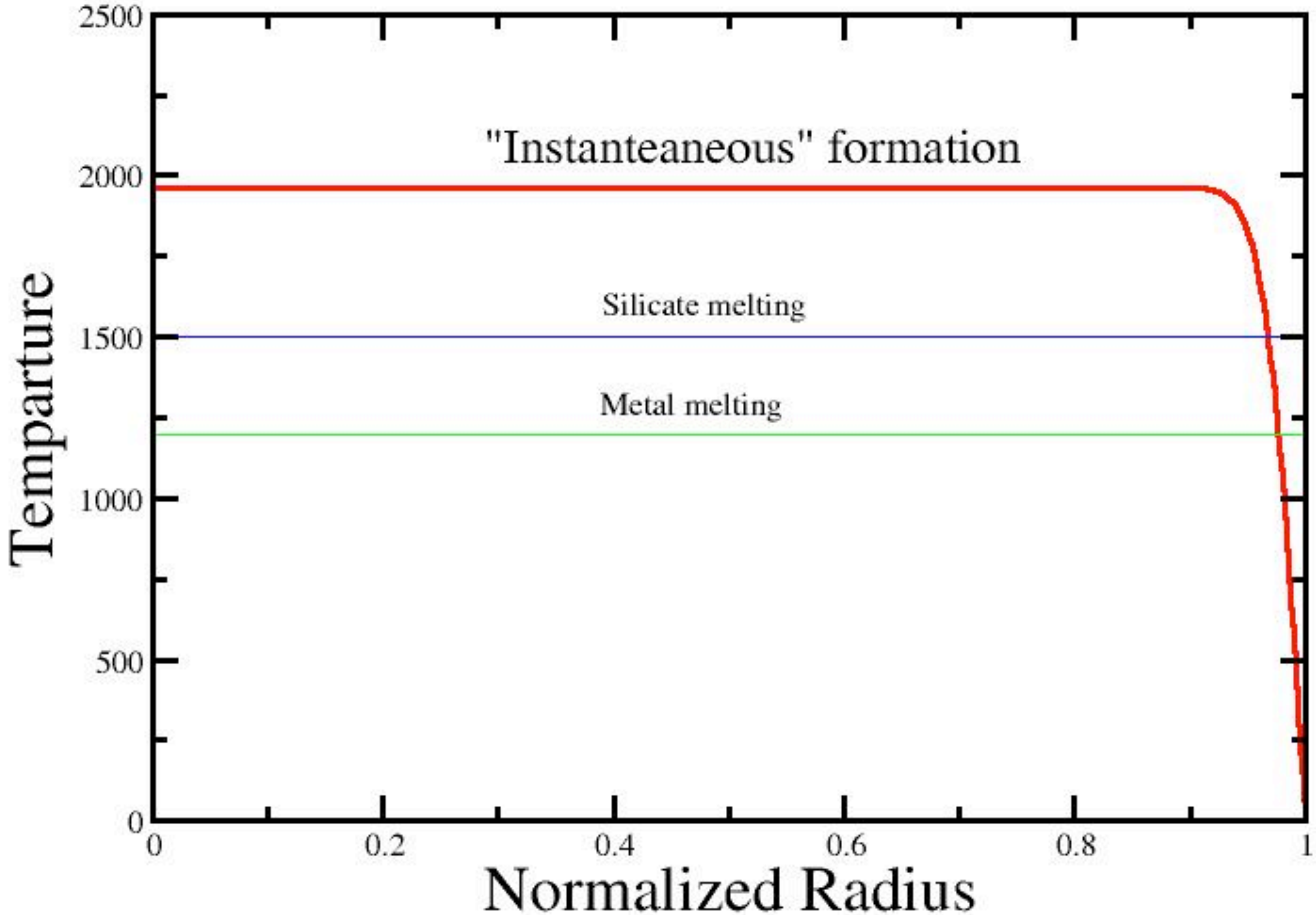
Temperature dans un planetesimal/ embryon planetaire -Le chauffage radioactif



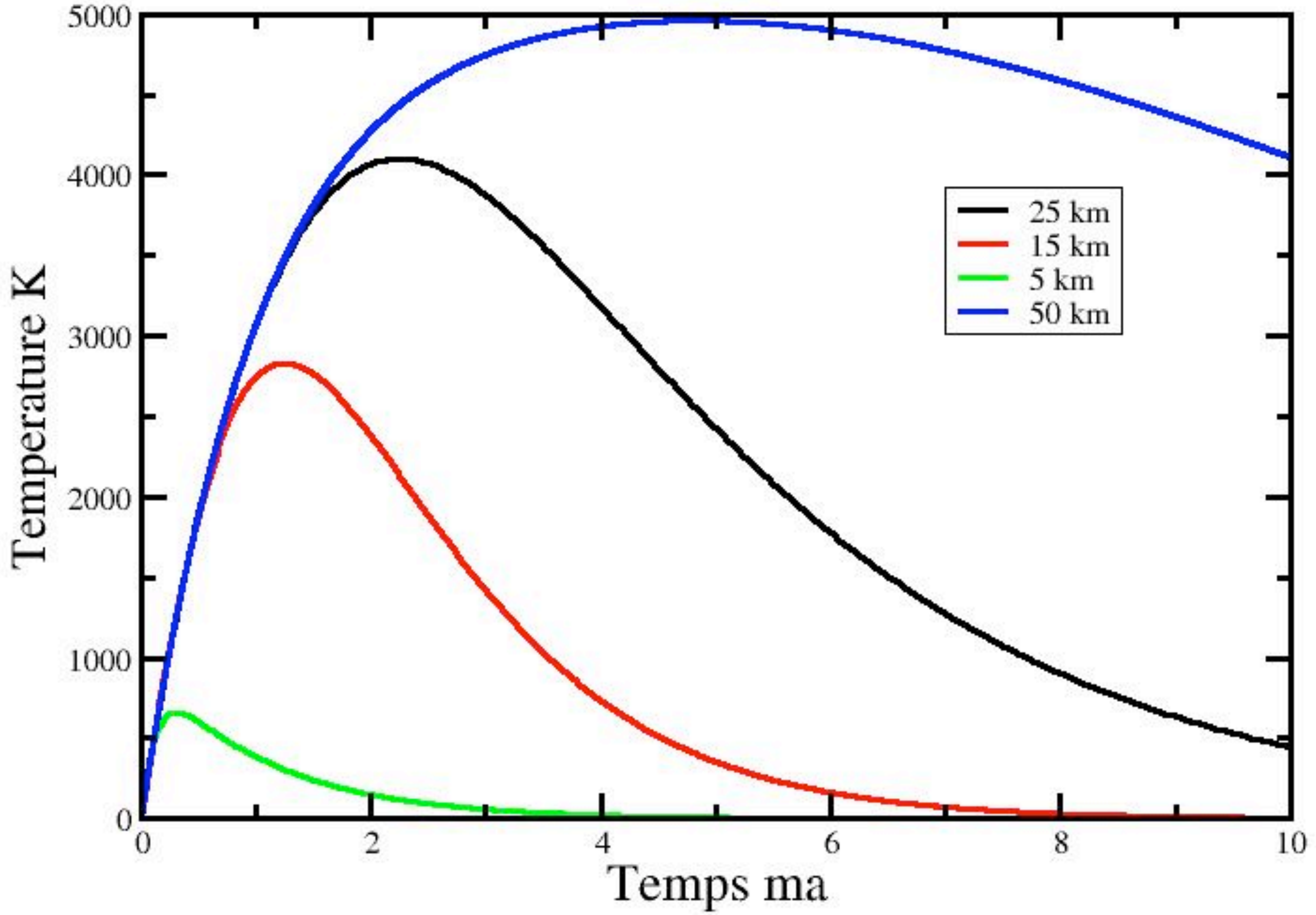
$$\rho C \frac{\partial T}{\partial t} = \rho H(t)$$

+diffusion, $L \simeq 2\sqrt{\kappa t}$, $L \simeq 20$ km en 5 ma

500 km radius, 1 myr delay, 5 myrs after start

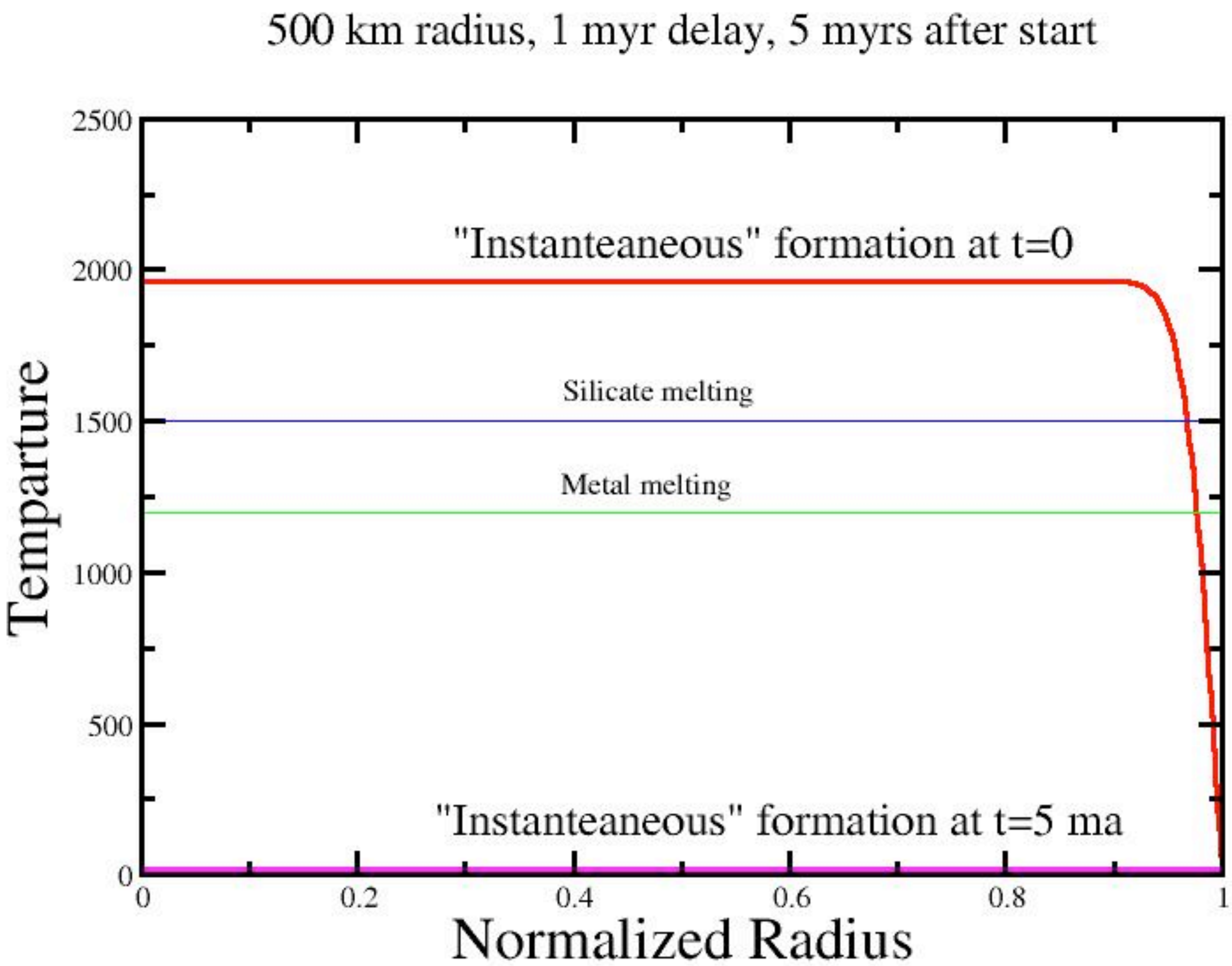


$$\frac{\partial T}{\partial t} = \frac{\kappa}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{H(t)}{C}$$



Temperature au centre (formation instantannee a t=0)

Mais le rayon varie...



Il faut faire le changement de variables...

$$(t, r) \rightarrow (\tau = t, u = \frac{r}{R(t)})$$

qui s'ecrit...

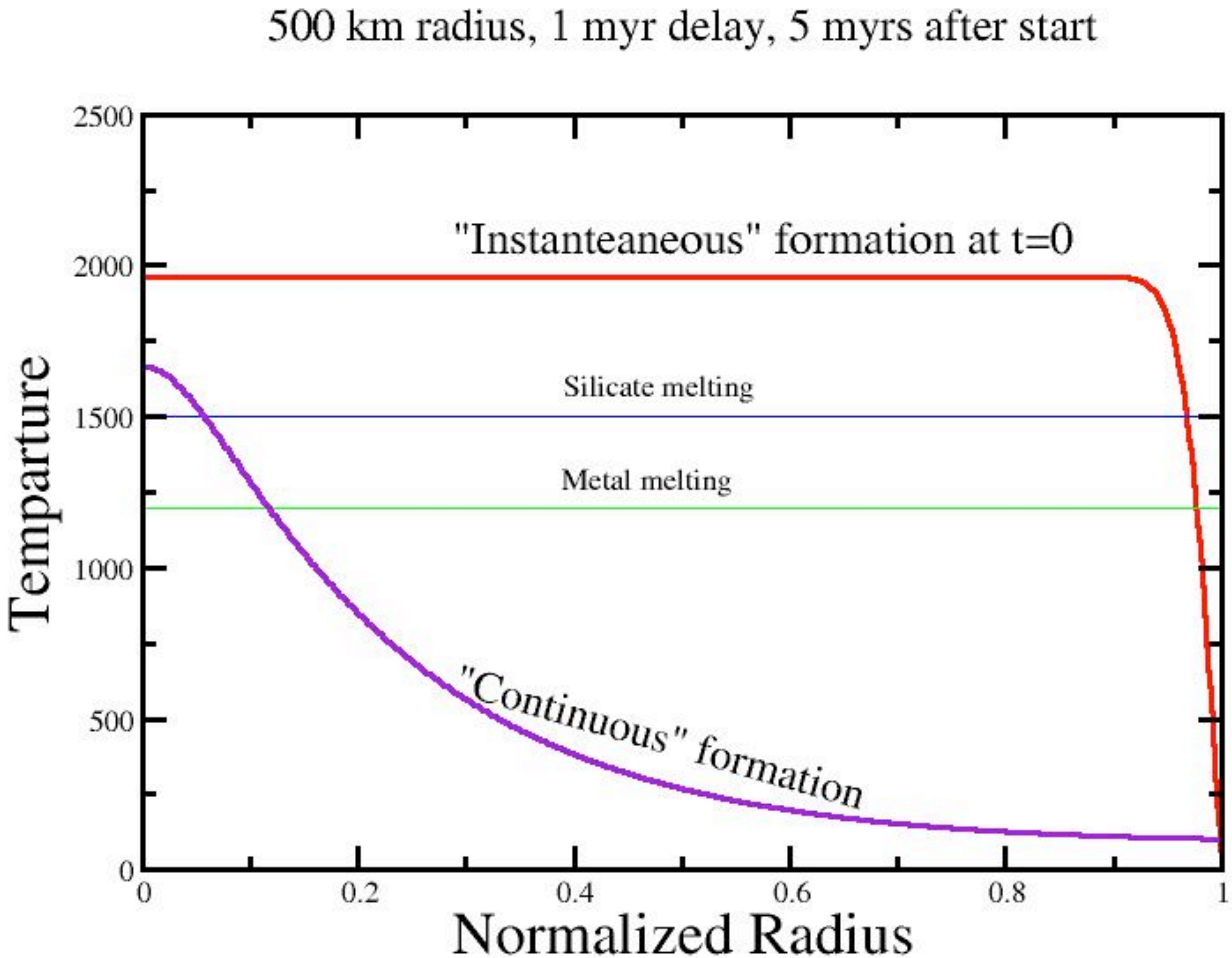
$$\frac{\partial}{\partial r} = \frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial r} + \frac{\partial}{\partial u}\frac{\partial u}{\partial r} = \frac{1}{R}\frac{\partial}{\partial u}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau}\frac{\partial \tau}{\partial t} + \frac{\partial}{\partial u}\frac{\partial u}{\partial t} = \frac{\partial}{\partial \tau} - \frac{r}{R^2}\frac{dR}{dt}\frac{\partial}{\partial u} == \frac{\partial}{\partial \tau} - u\frac{\dot{R}}{R}\frac{\partial}{\partial u}$$

d'ou l'equation de la chaleur...

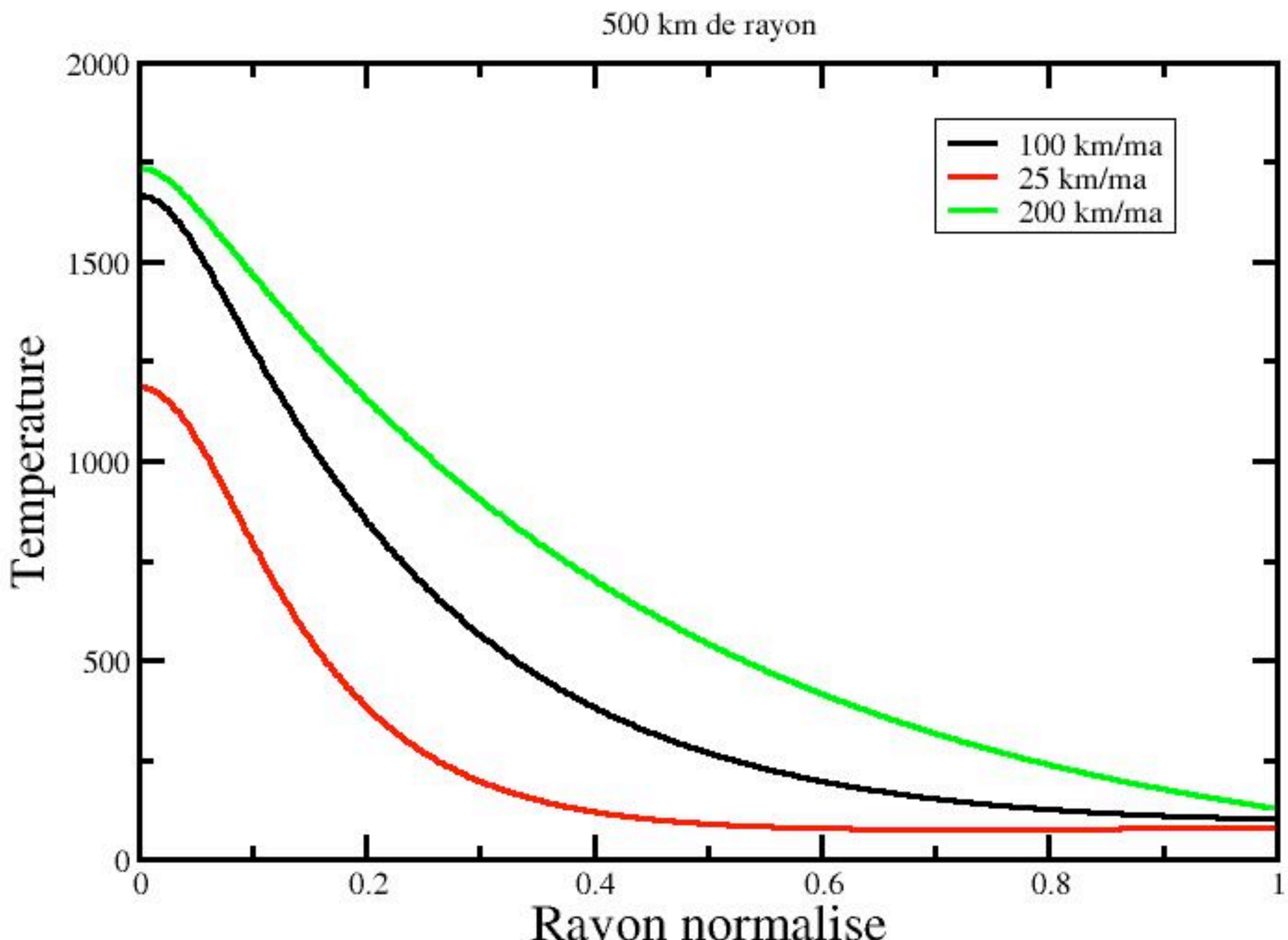
$$\frac{\partial T}{\partial t} - u\frac{\dot{R}}{R}\frac{\partial T}{\partial u} = \frac{\kappa}{R^2}\frac{1}{u^2}\frac{\partial}{\partial u}\left(u^2\frac{\partial T}{\partial u}\right) + \frac{H(t)}{C}$$

Advection "centripete" de la temperature, decroissance de la diffusion avec R



Au centre du planetesimal, la matiere possedait encore des sources radioactives, mais la diffusion etait efficace, au bord, la matiere ne possede plus de radioactivite

Il faut une vitesse minimale d'accretion pour commencer la fusion partielle



Temperature dans un planetesimal/ embryon planetaire

-Le chauffage radioactif

-Le chauffage par impact

Les impacteurs apportent une energie

$$\frac{\mathrm{d}E_g}{\mathrm{d}t} = \frac{16}{3}\pi^2 G\bar{\rho}^2 R^4 \dot{R}.$$

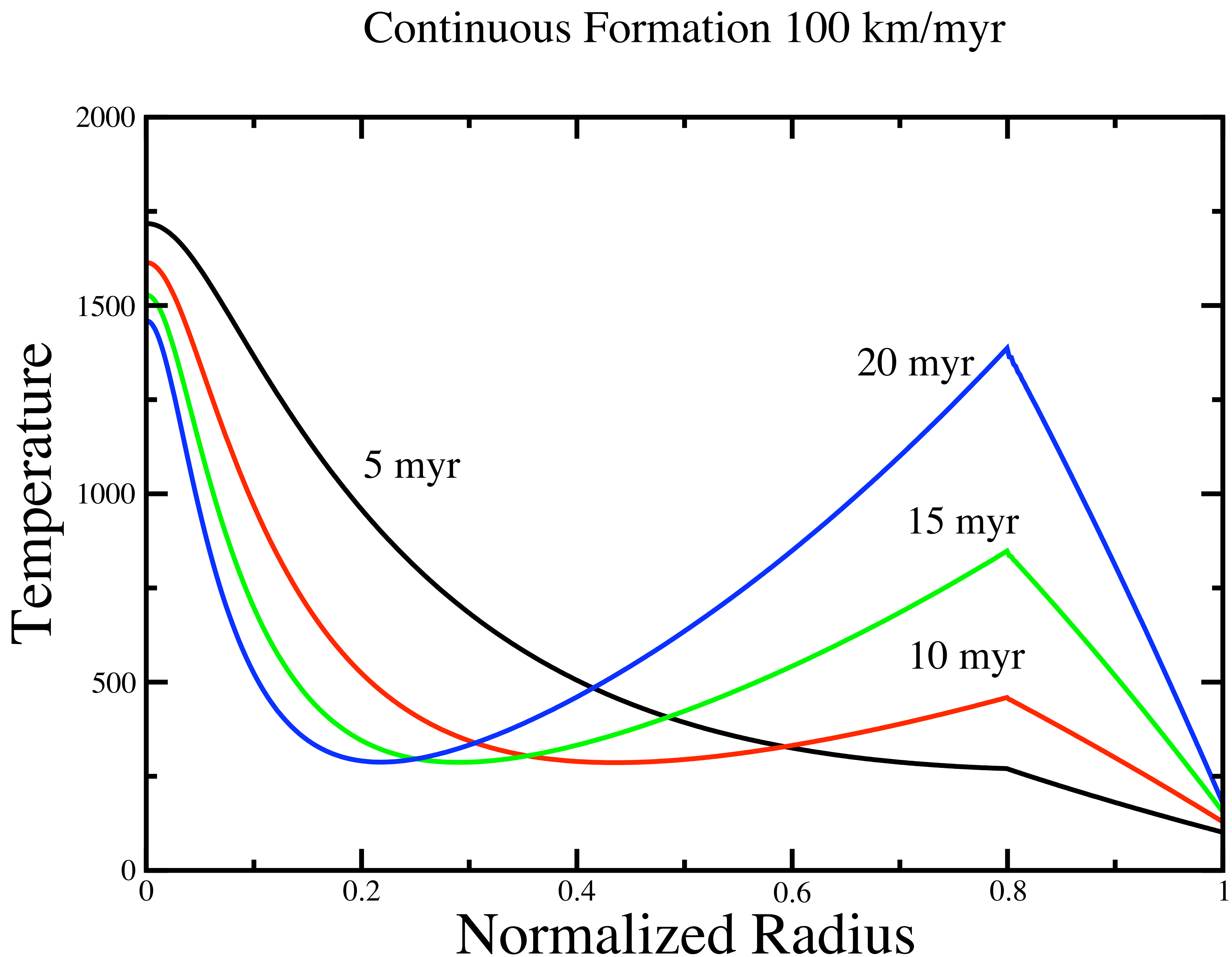
$$\frac{\mathrm{d}E_t}{\mathrm{d}t} = 4\pi C T_i \bar{\rho} R^2 \dot{R}$$

Une fraction f est “enterree” sur une epaisseur xR et est equivalente a une source de chaleur H

$$\frac{4}{3}\pi\bar{\rho}H_g R^3(1-x^3) = f\left(\frac{\mathrm{d}E_g}{\mathrm{d}t} + \frac{\mathrm{d}E_t}{\mathrm{d}t}\right)$$

Une partie est “radiee” et controle la temperature de surface (??)

$$4\pi R^2\sigma T^4 = (1-f)\left(\frac{\mathrm{d}E_g}{\mathrm{d}t} + \frac{\mathrm{d}E_t}{\mathrm{d}t}\right)$$



Corps de 500, 1000, 1500, 2000 km de rayon

Deux zones de fusions partielles...