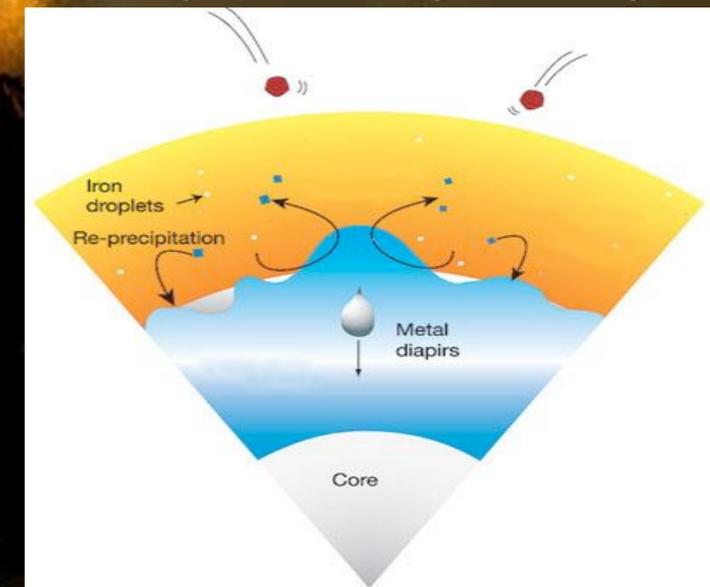


Differentiation des planetes (poesie, peinture et BD)



Yanick Ricard
CNRS/Universite de Lyon

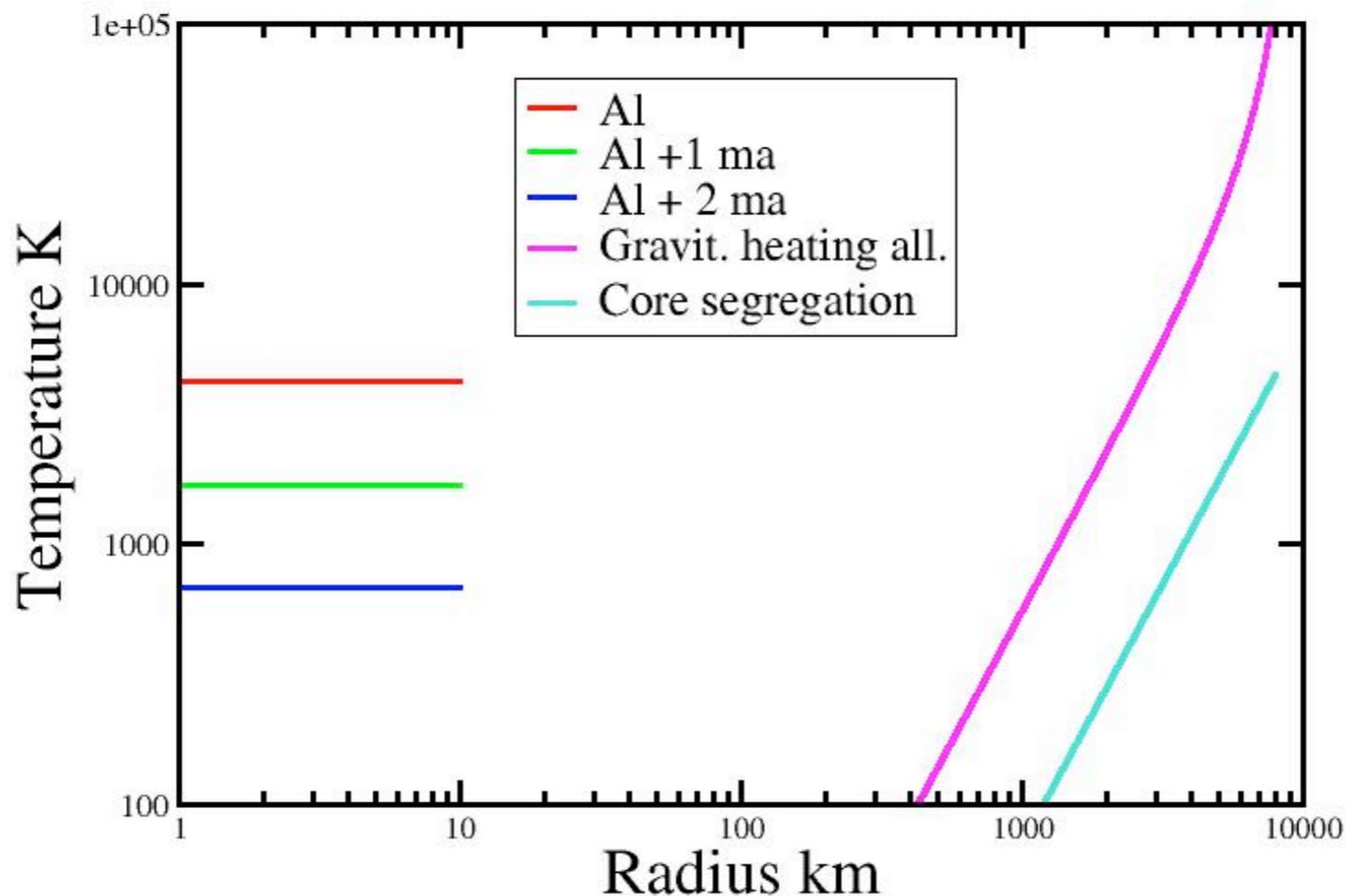


Petit resume:

Il y a beaucoup d'energie disponible

Radioactive d'abord, gravitationnelle ensuite (pour $R > 500$ km)

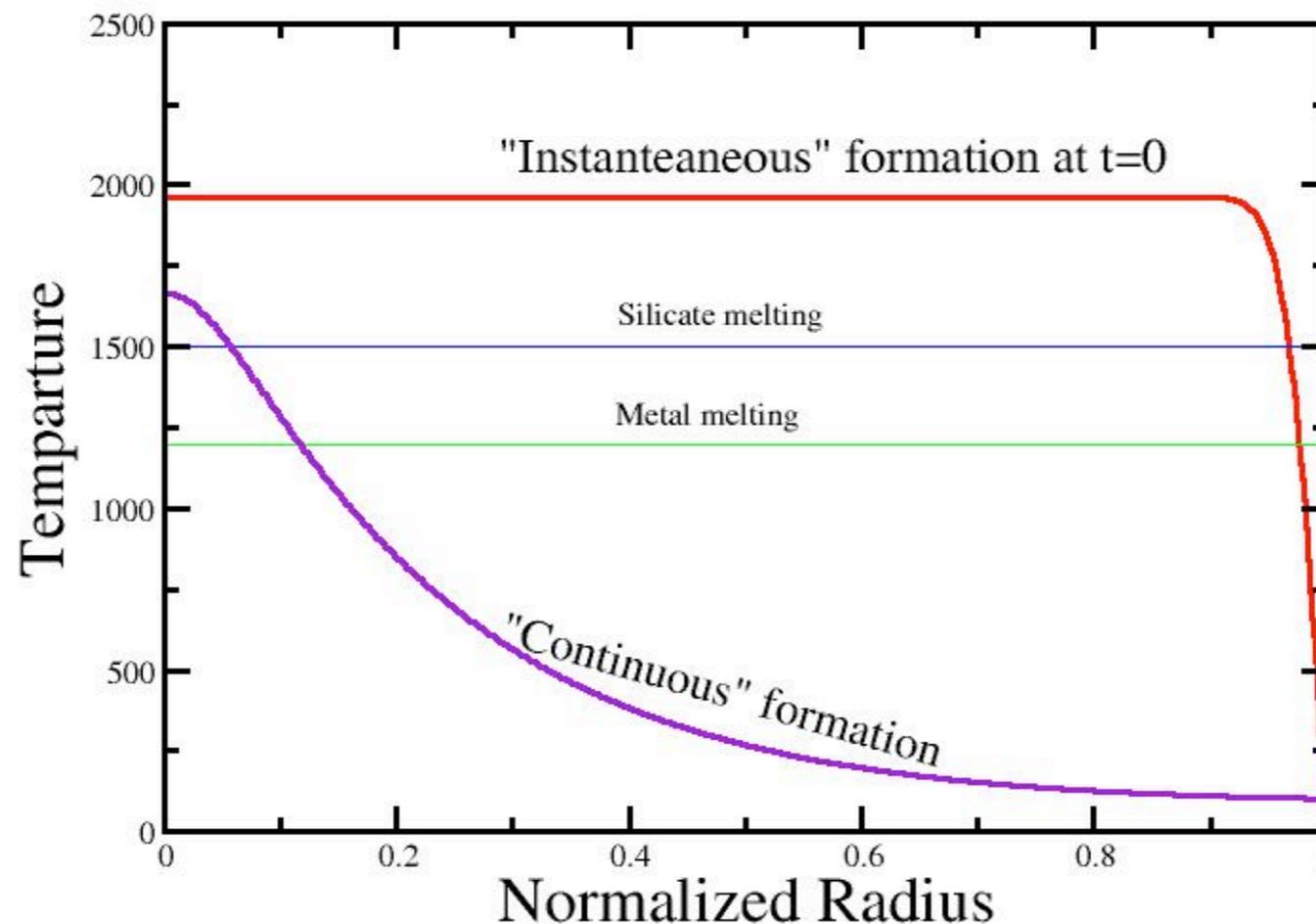
La formation du noyau re-augmente les temperatures
significativement $R > 1000$ km



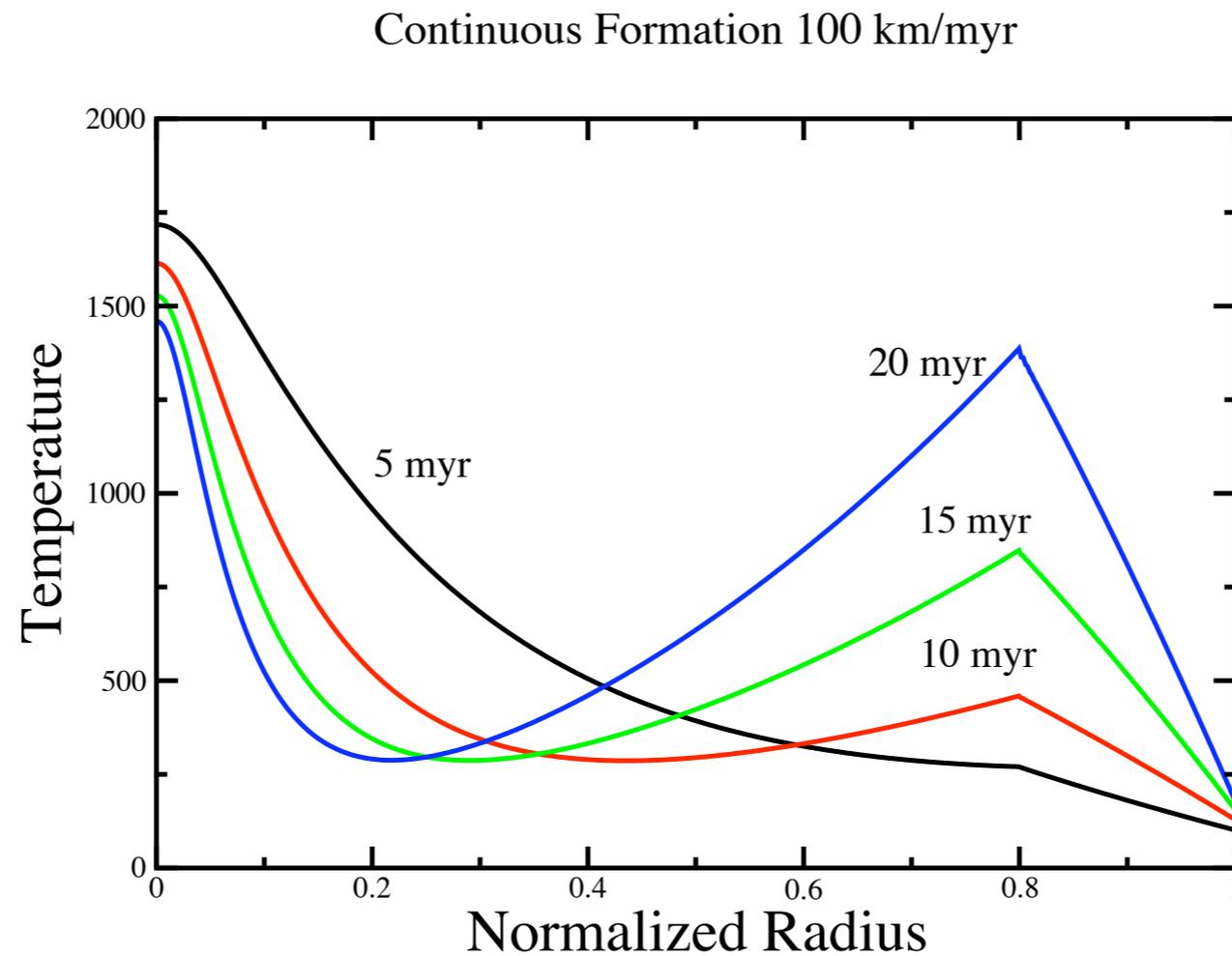
Petit resume:

Si on accrete a plus de ~ 20 km/my (il faut une epaisseur suffisamment grande pour eviter la diffusion) on a forcement fondu, metal et silicate (mais pas en grand volume, en gros seulement ce qui a accrete, au centre pendant la periode de vie de Al-26)

500 km radius, 1 myr delay, 5 myrs after start

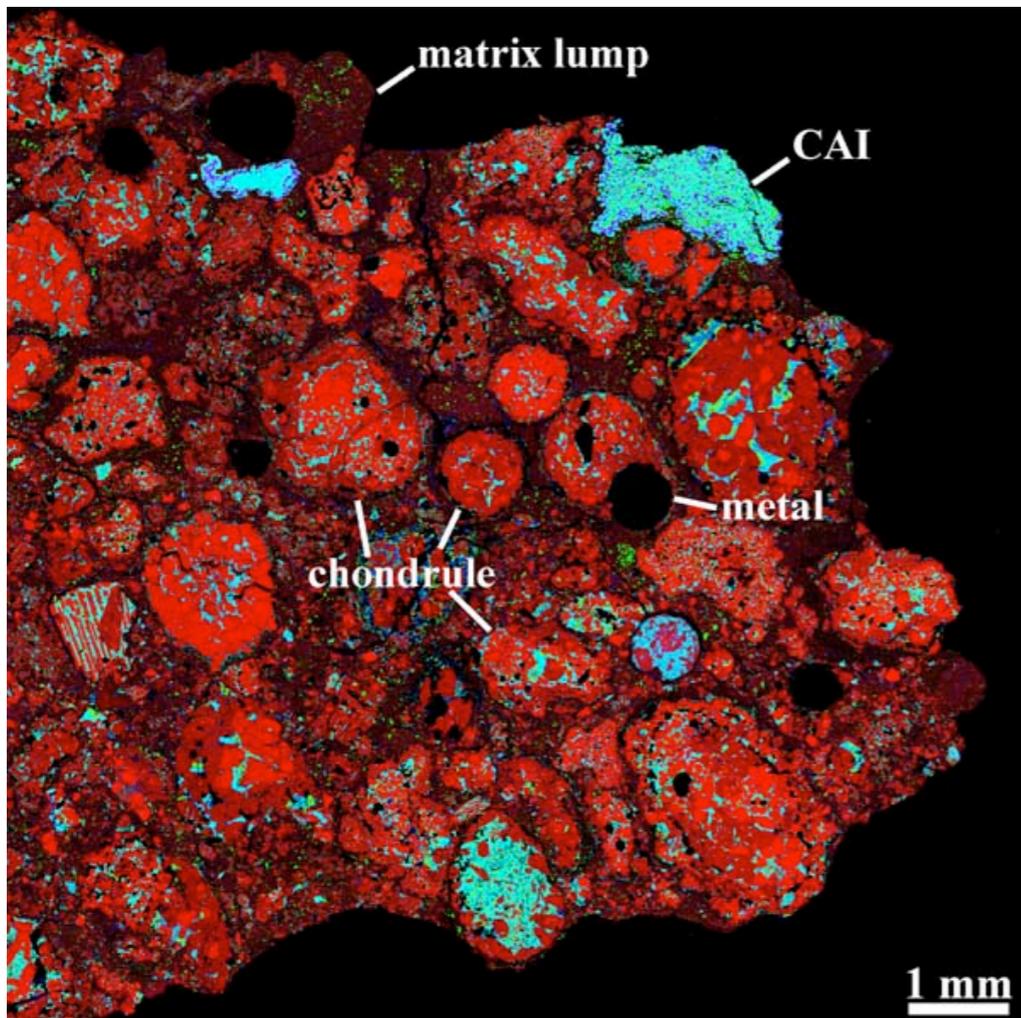


Quand on atteint ~ 800 km on commence a etre chauffe par la surface (si l'energie d'accretion est suffisamment "enterree")



Quand separe-t-on du metal des silicates?

Chondrite



Chondrites

4566 Ma

Ca-Al-
Inclusions

t_0+2-10

Iron
meteorites
chondres

t_0+30

Earth Core

t_0+50

Moon formed

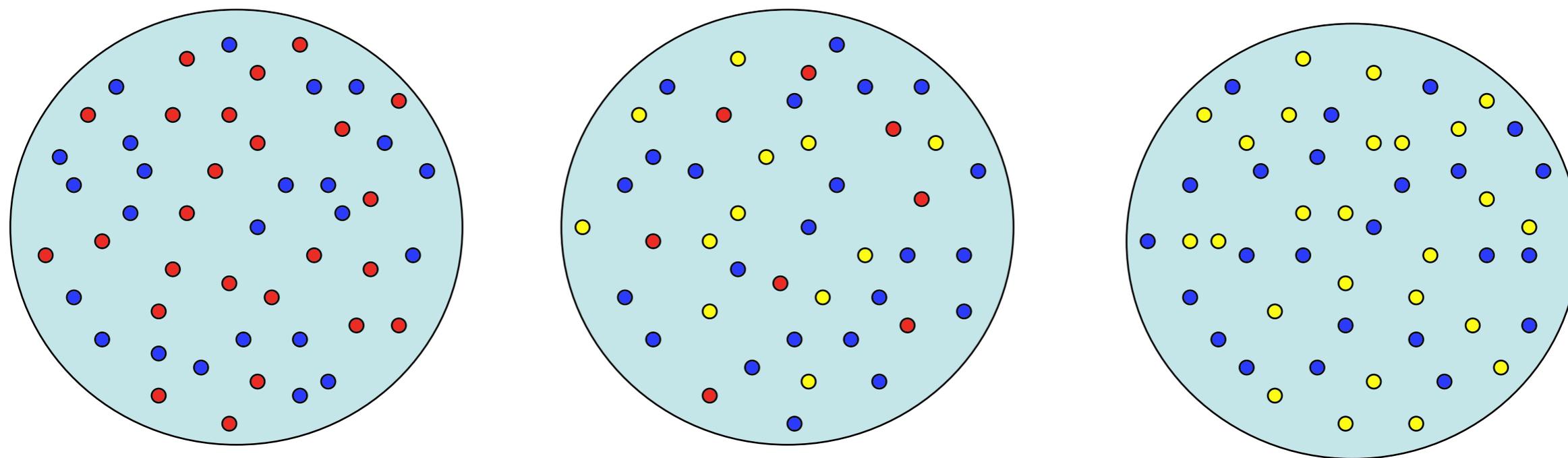
t_0+100

End of
accretion



- = stable ^{183}W
- = radioactif ^{182}Hf
- = radiogénique ^{182}W

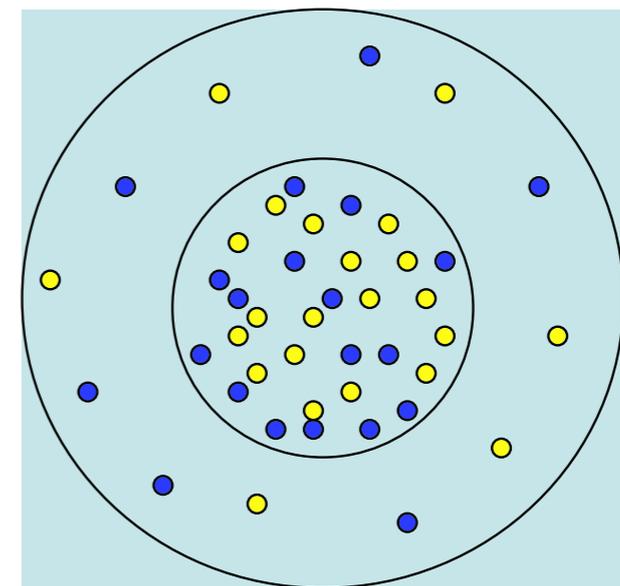
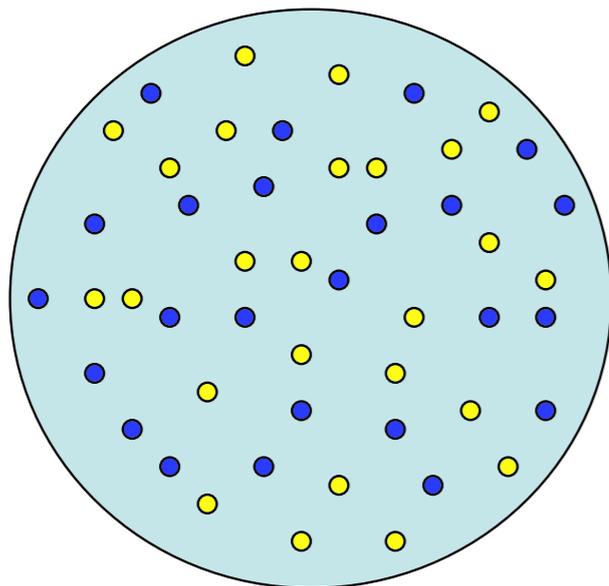
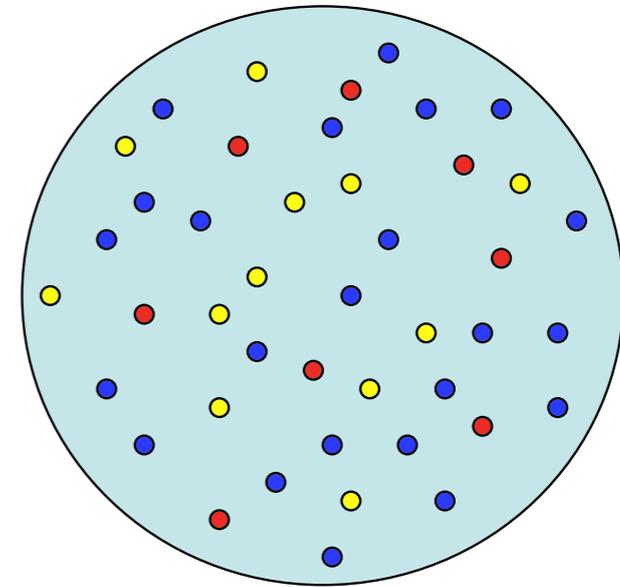
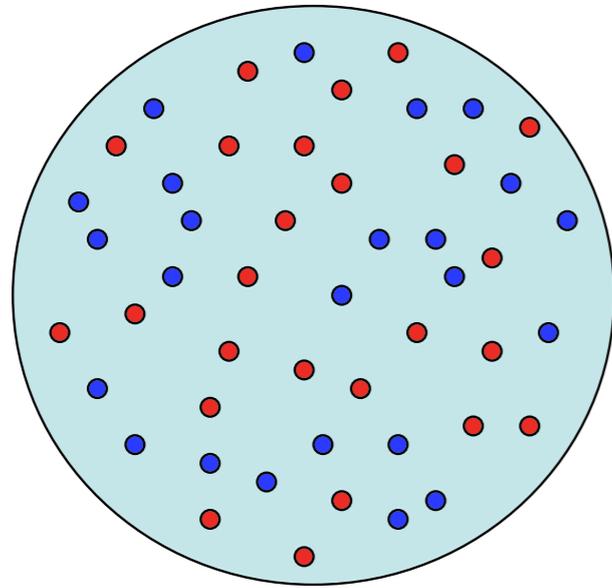
Half-life 9 ma



A given $^{182}\text{W}/^{183}\text{W}$ in undifferentiated objects...

W « siderophile » Hf « lithophile »

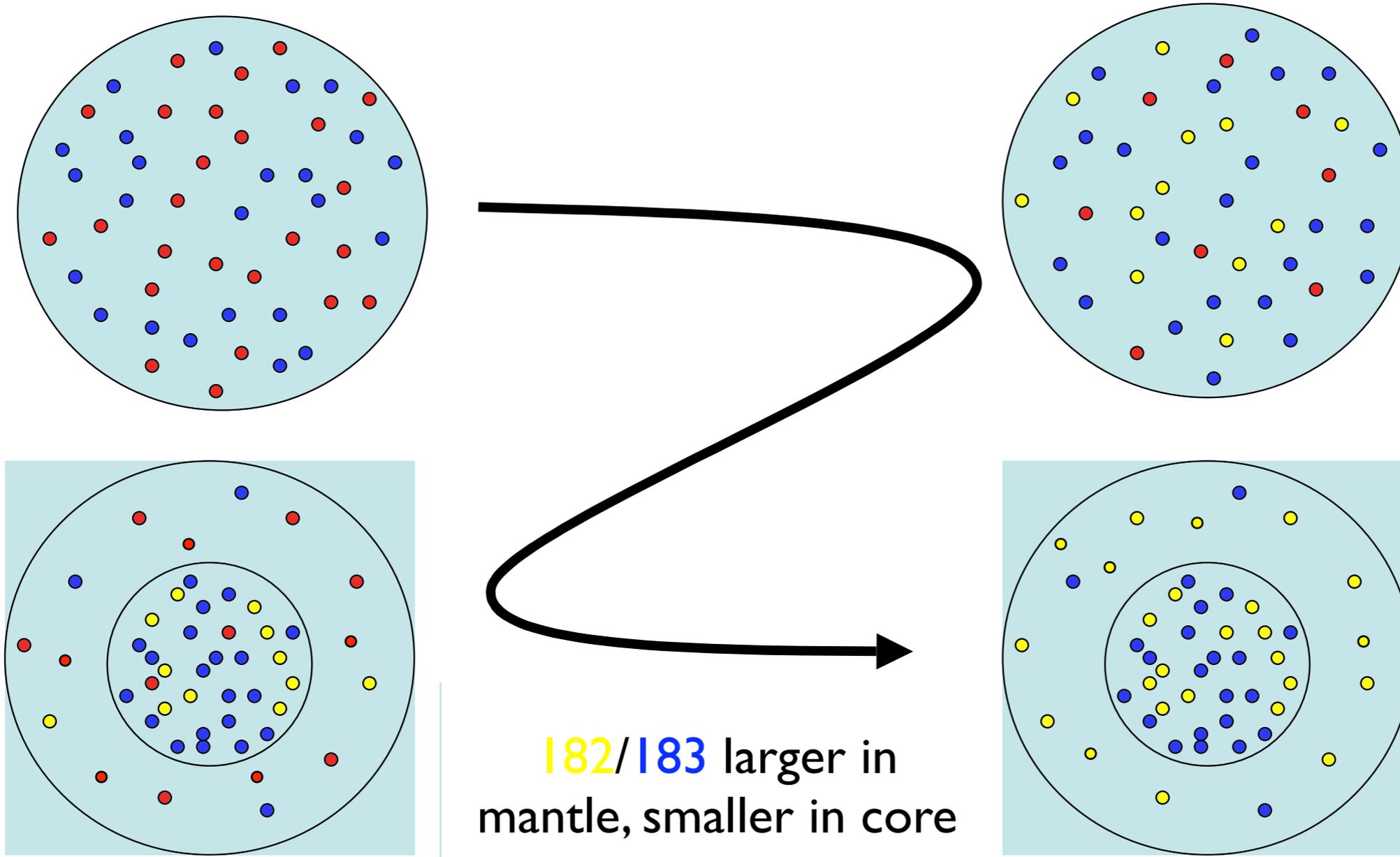
Core segregation after Hf decay

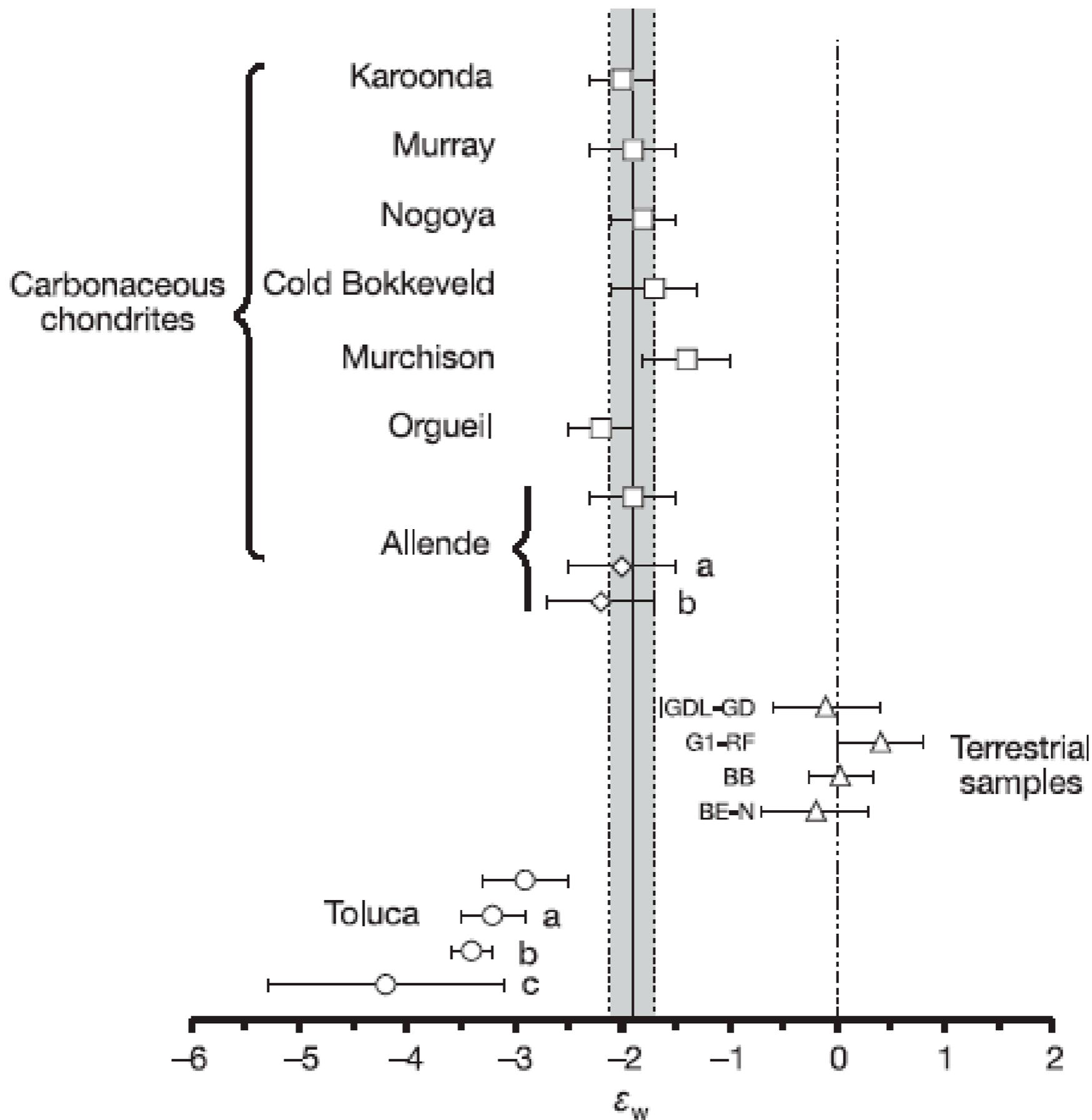


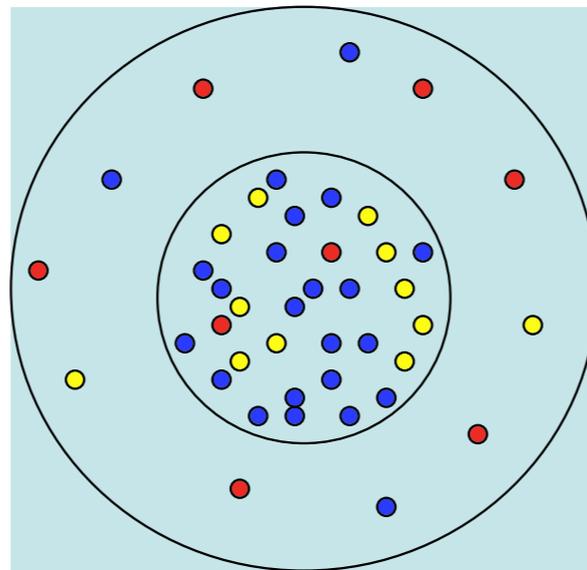
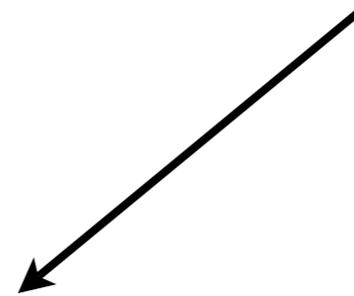
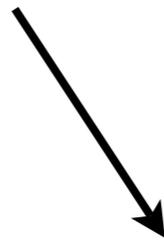
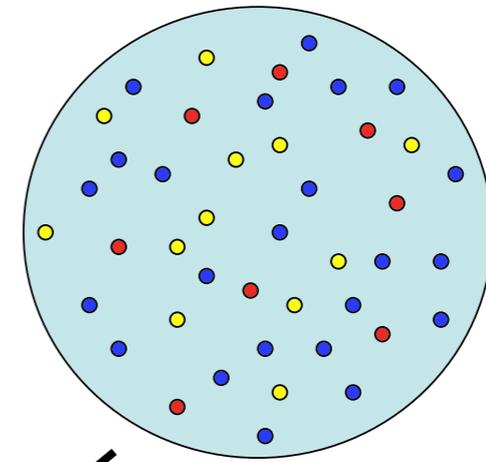
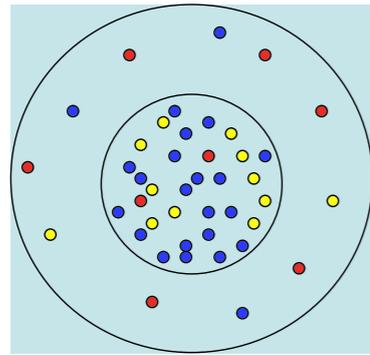
Same $^{182}\text{W}/^{183}\text{W}$ in chondrite,

W « siderophile » Hf « lithophile »

Cor formation before Hf decay







Mais si on fait plusieurs impacts et plusieurs différenciation,
c'est plus compliqué!

Fusion/percolation

$T=1250\text{K}$ [Fei 1997]

$L=240\text{ kJ kg}^{-1}$ [??]

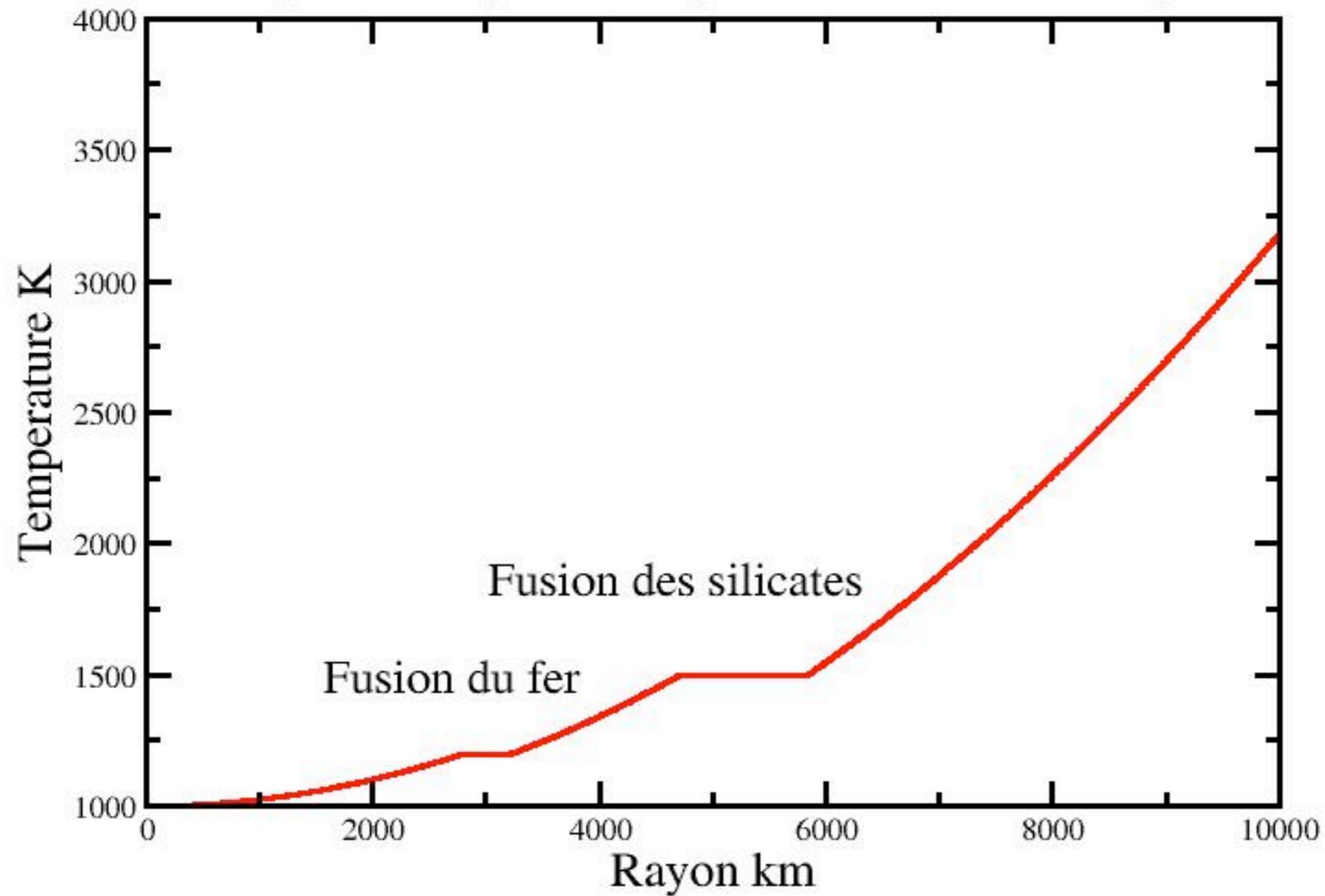
$$\rho_f L_{Fe} \phi = \bar{\rho} C \Delta T, \Delta T = 67\text{ K}$$

$T=1400\text{K}$ [Agee 1997, Hirschmann 2000]

$L=400\text{ kJ kg}^{-1}$ [Wilson 2006] or 500 kJ kg^{-1} [Kojitani 1995]

$$\rho_m L_{Si} (1 - \phi) = \bar{\rho} C \Delta T, \Delta T = 220\text{ K}$$

Temperature apres un impact dans la zone d'impact



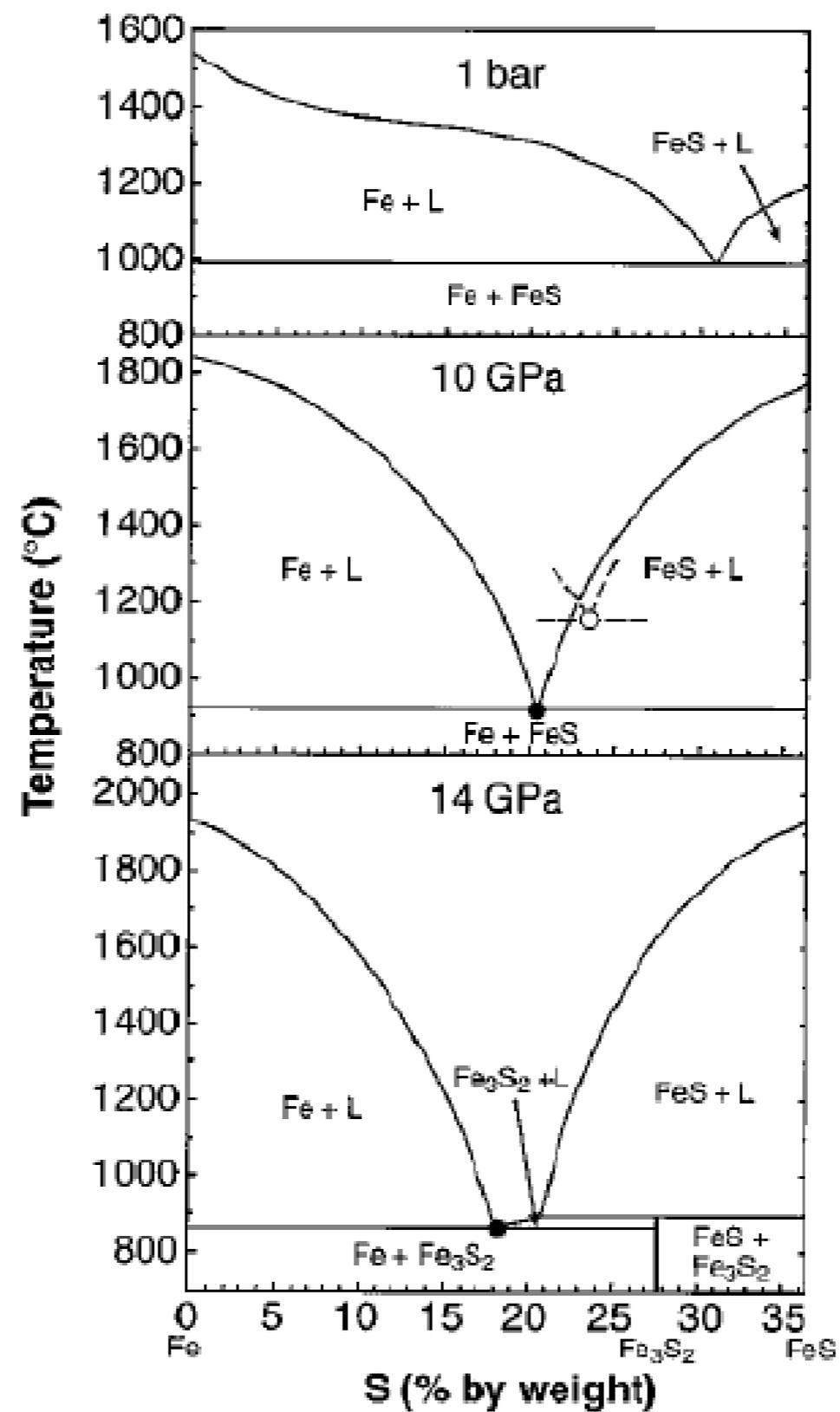
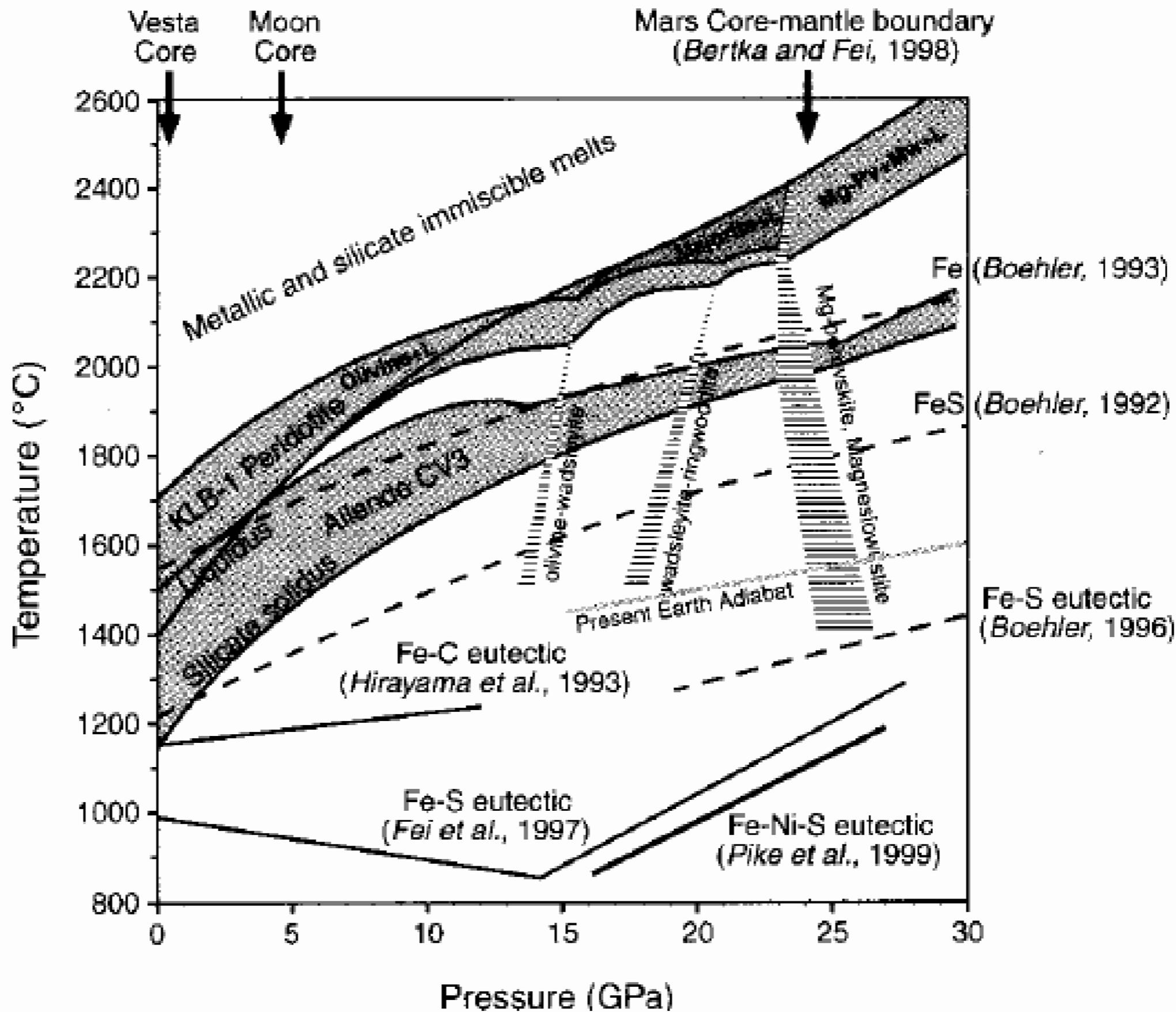
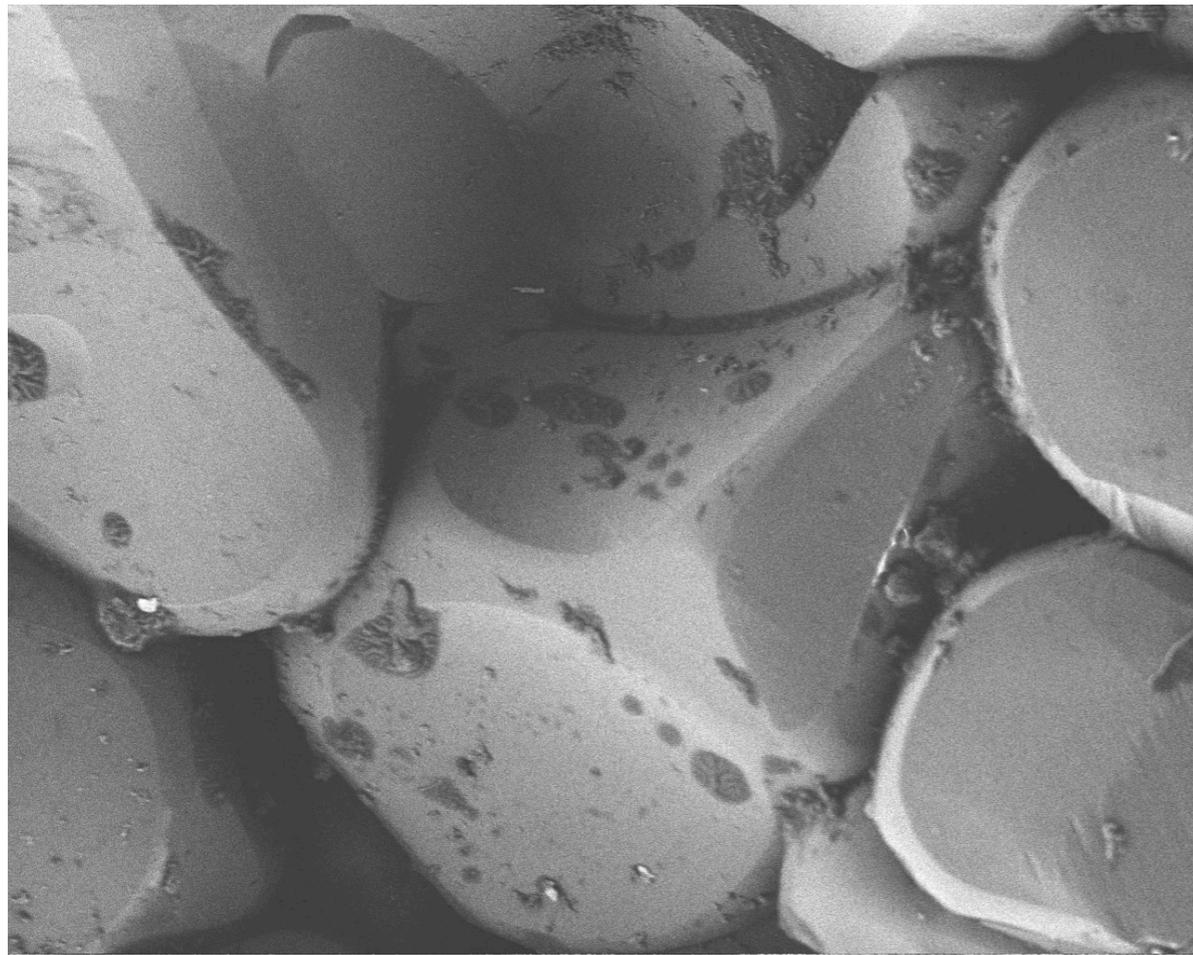


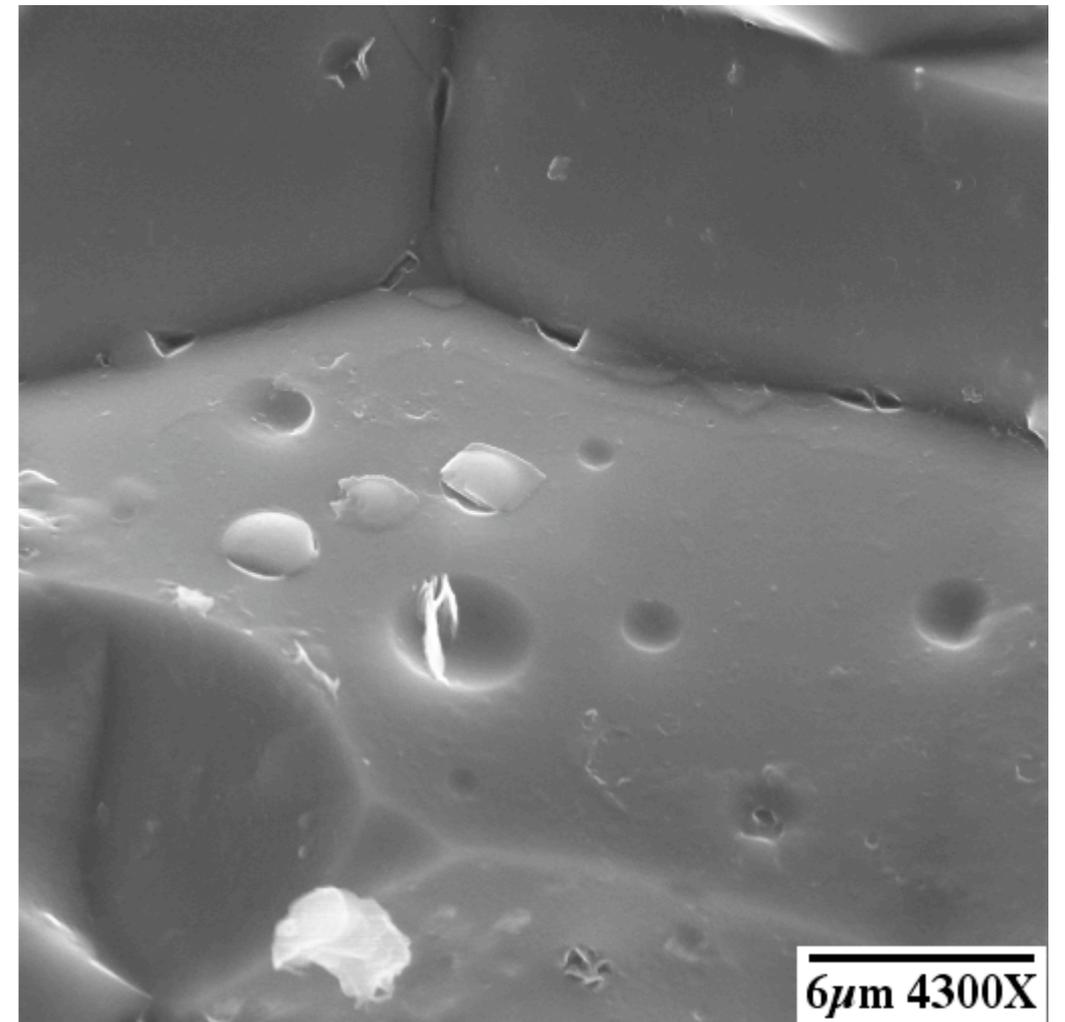
FIG. 3.1 – Diagrammes de phases du système Fe-FeS pour des pressions de 1 bar, 10 GPa et 14 GPa (d'après (Fei et al., 1997)).





Mouille

Ne mouille pas



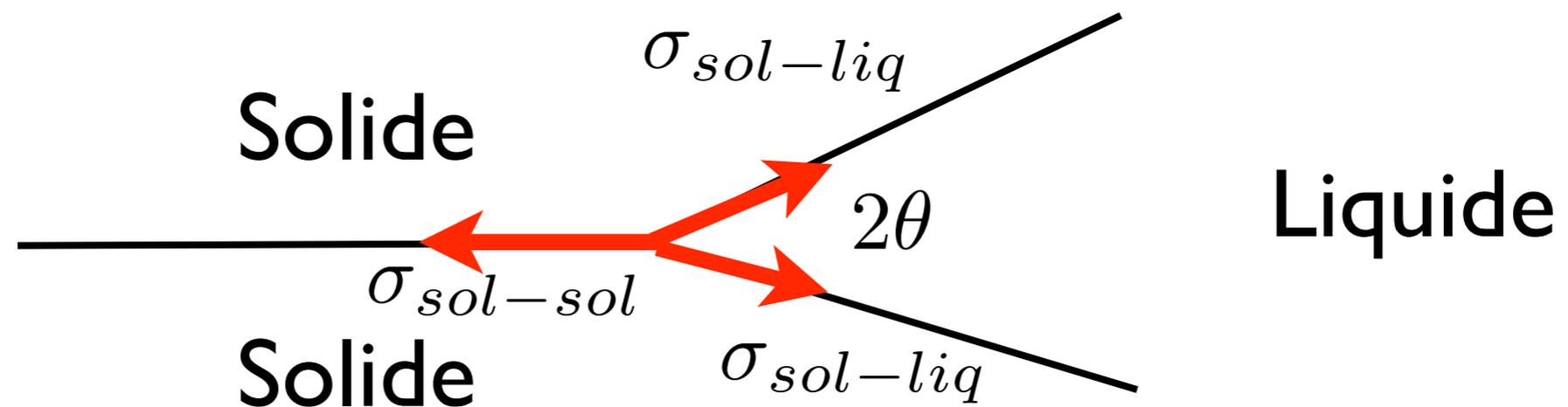
Ici fusion de l'olivine

Hier-Majumder

Angle semidiehdral $\theta = \arccos \frac{\kappa}{2}$,

Ou $\kappa = \sigma_{sol-sol} / \sigma_{sol-liq}$

Est le rapport des tensions de surface, solide-solide
versus solide-liquide



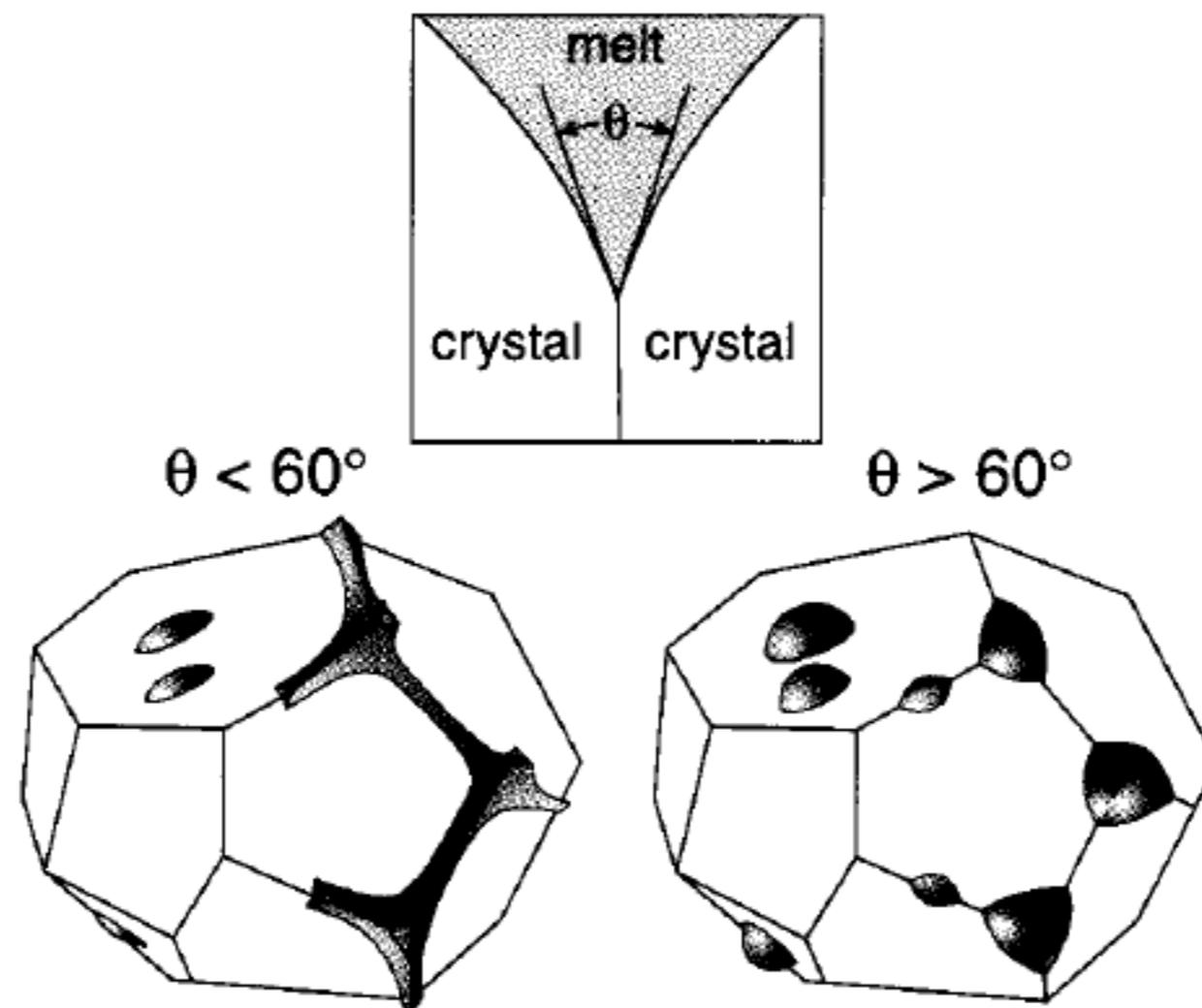
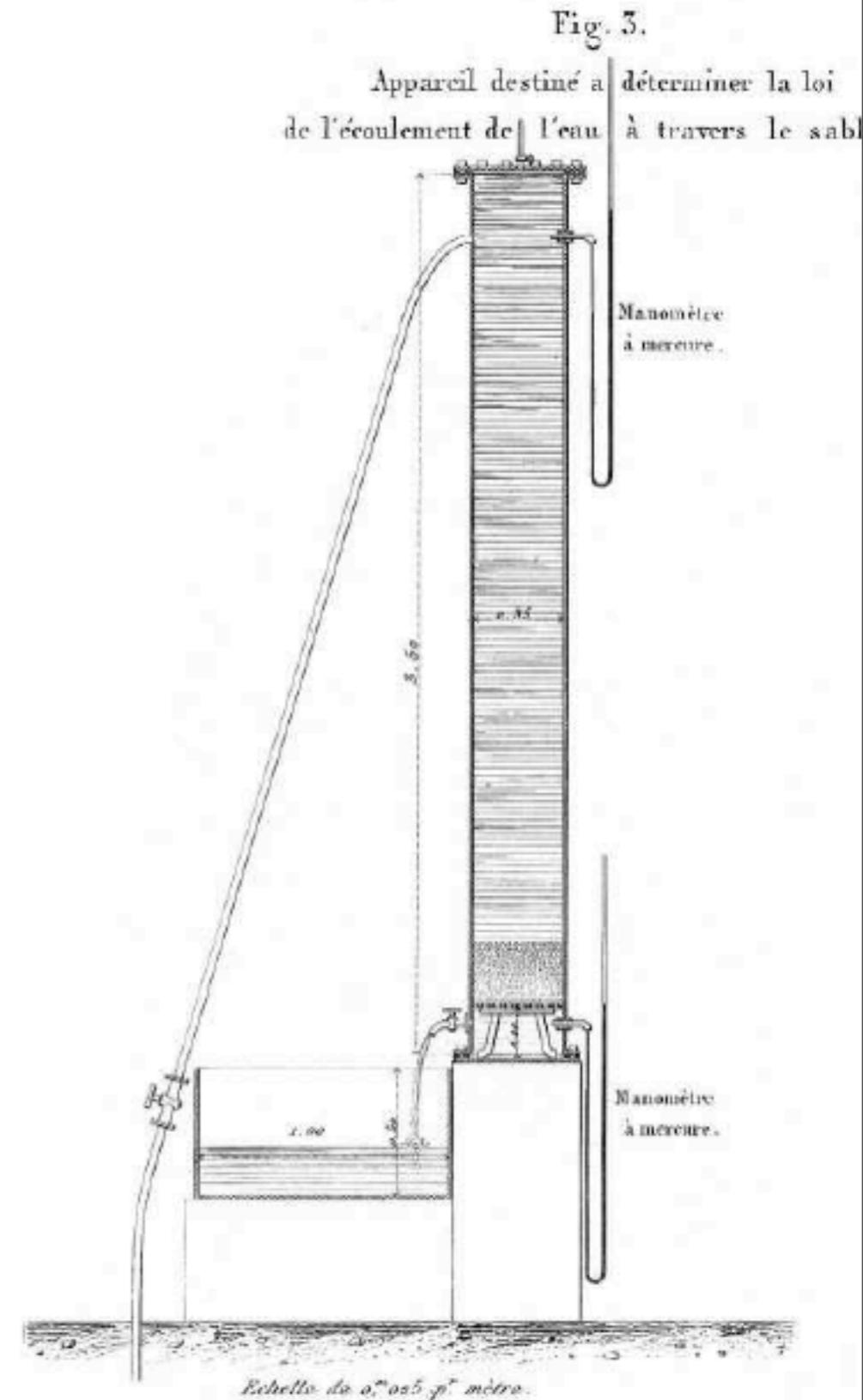


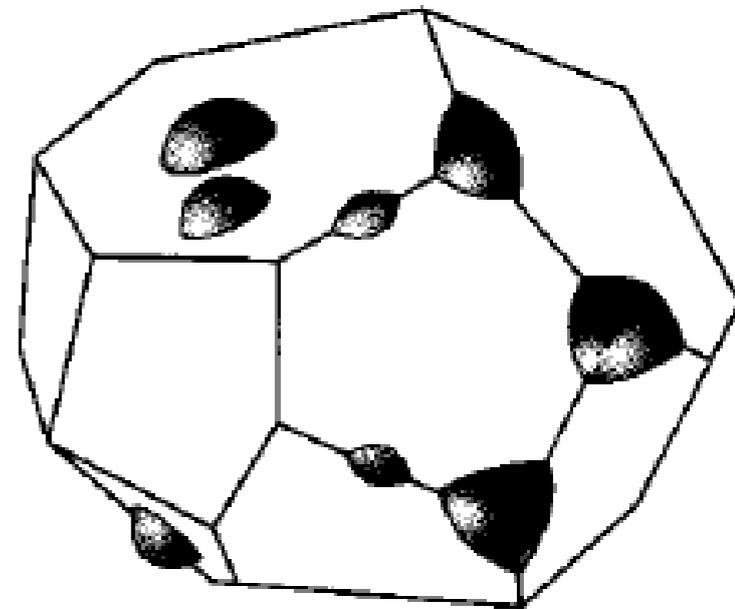
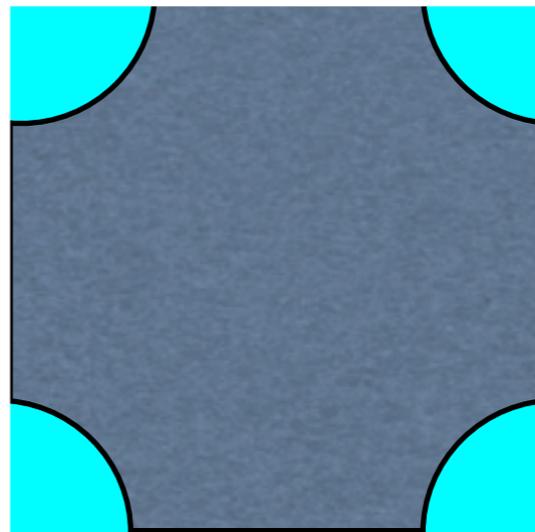
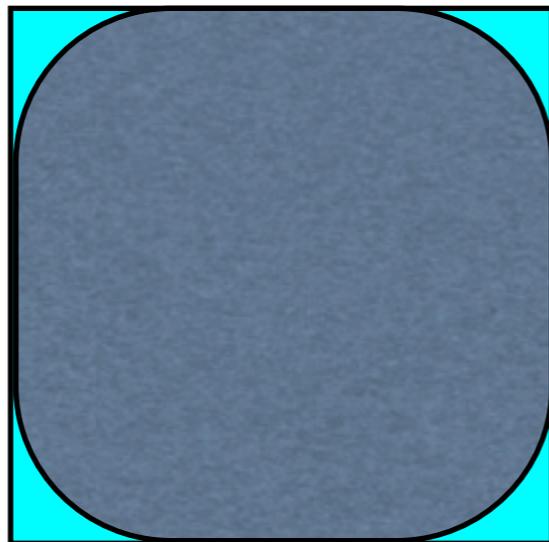
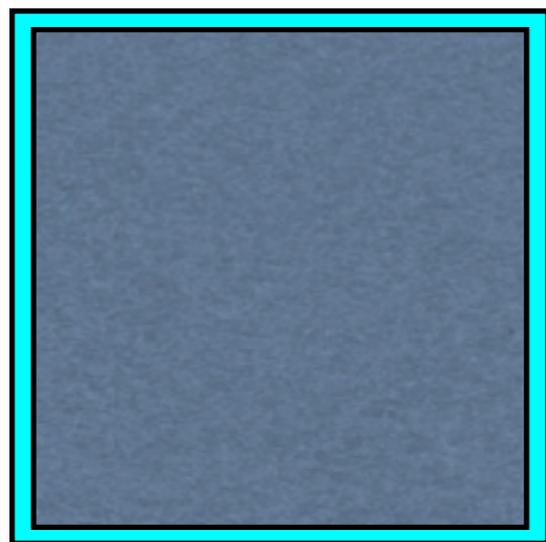
FIG. 3.6 – Illustration de l'interconnectivité du réseau liquide en fonction de l'angle de mouillage d'après (*Rushmer et al.*, 2000). Définition de l'angle de mouillage θ (haut). Si $\theta < 60^\circ$, le liquide forme un réseau interconnecté (bas gauche) et si $\theta > 60^\circ$, le liquide est isolé en poches.

S'il y a connection, on peut utiliser la loi de Darcy

$$v = (k/\eta)dP/dh$$

Ou la permeabilite k est une mesure de la section des "tubes" de liquides





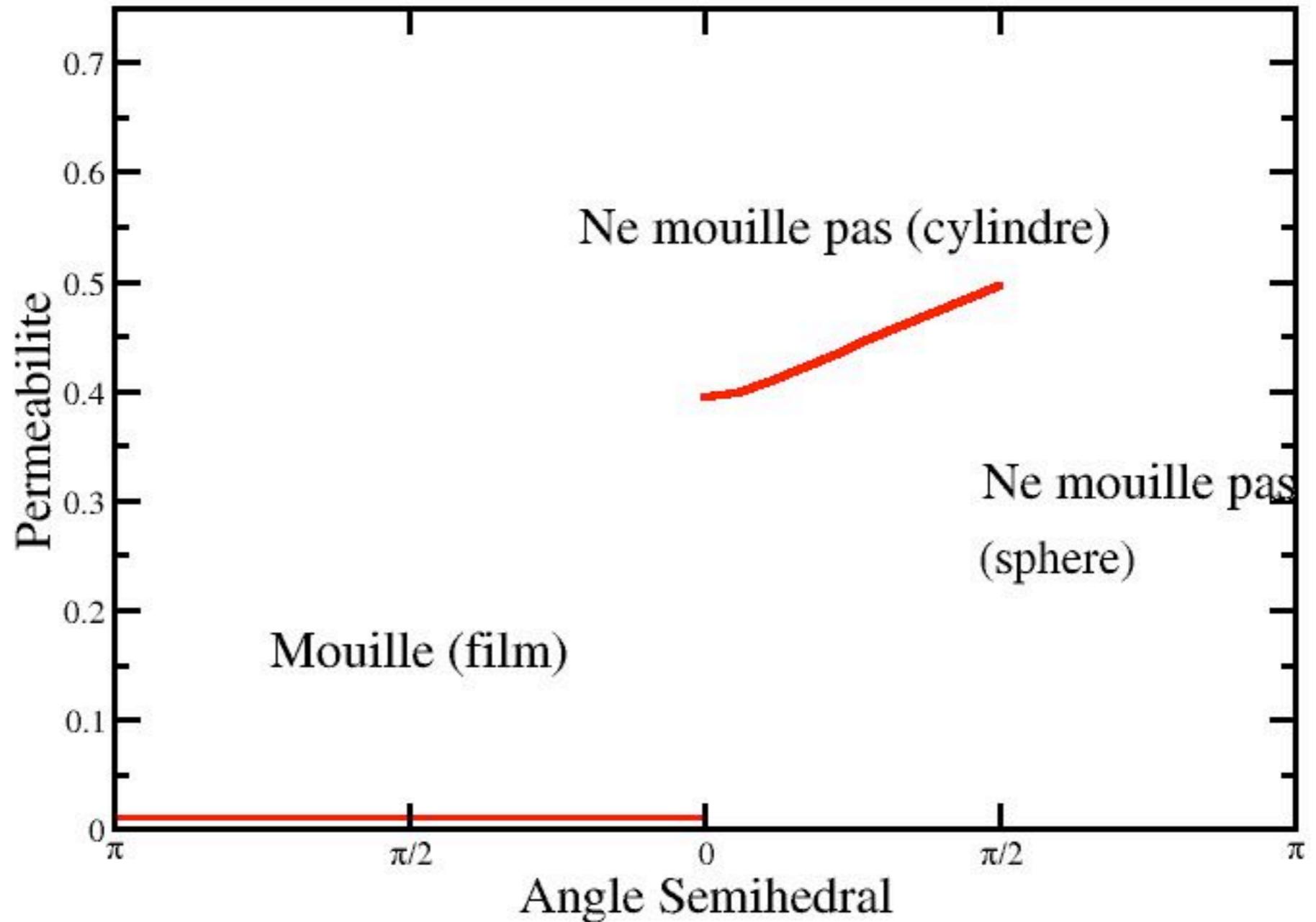
Mouille

Mouille pas



Le metal ne mouille pas a pression ambiante, devient plus mouillant a haute pression, la percolation se produit facilement en presence de deformation, 3% de metal fondu semble suffire pour une percolation

Ex avec 20%
metal



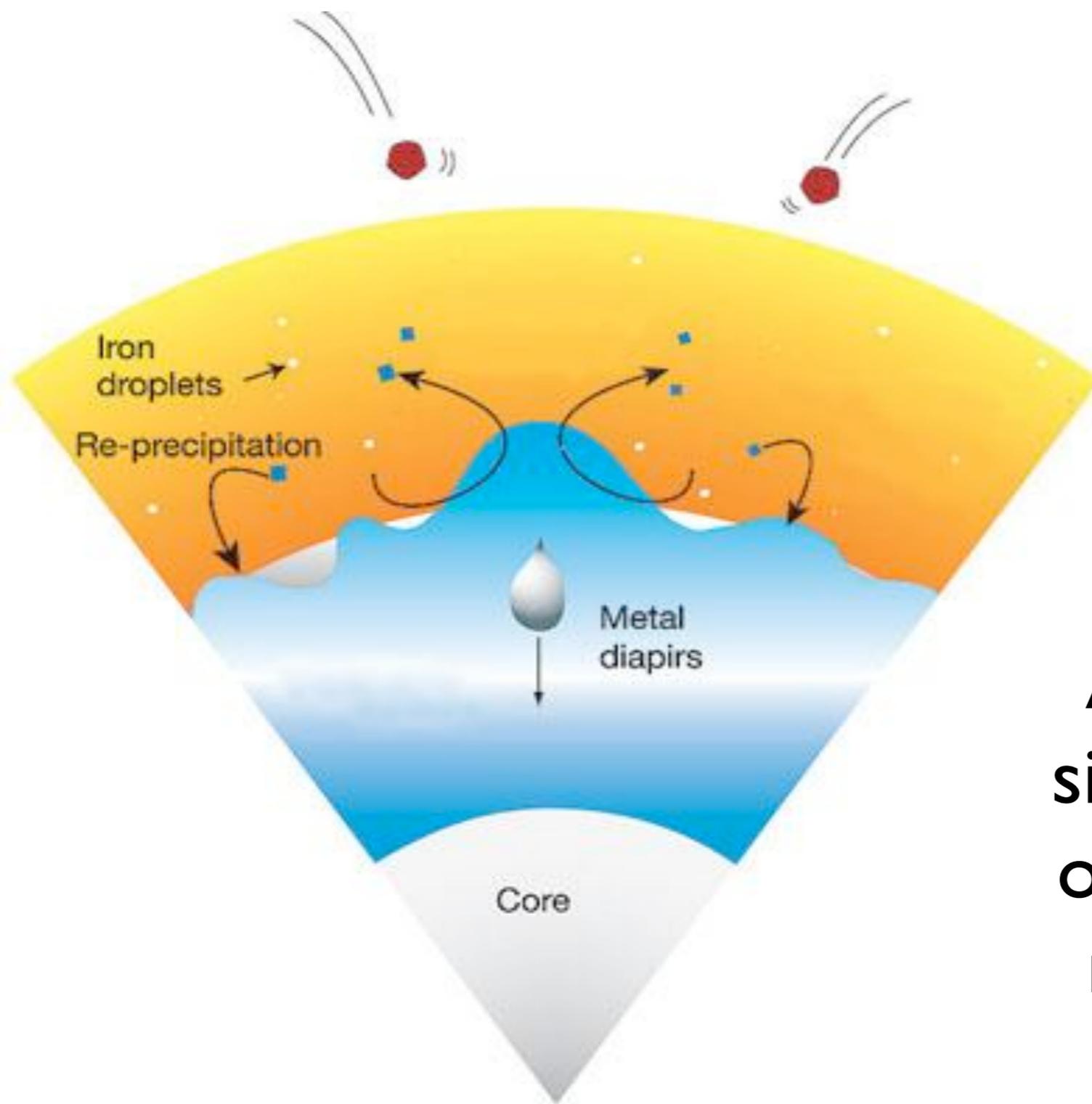
Pour un ecoulement de type Darcy, il faut que le metal mouille mal les silicates

$$\frac{R}{t} = V_{Stokes} = \frac{k_0 \Delta \rho g \phi^2 (1 - \phi)}{\mu_f} = \frac{4\pi k_0 \Delta \rho \rho G R \phi^2 (1 - \phi)}{3\mu_f}$$

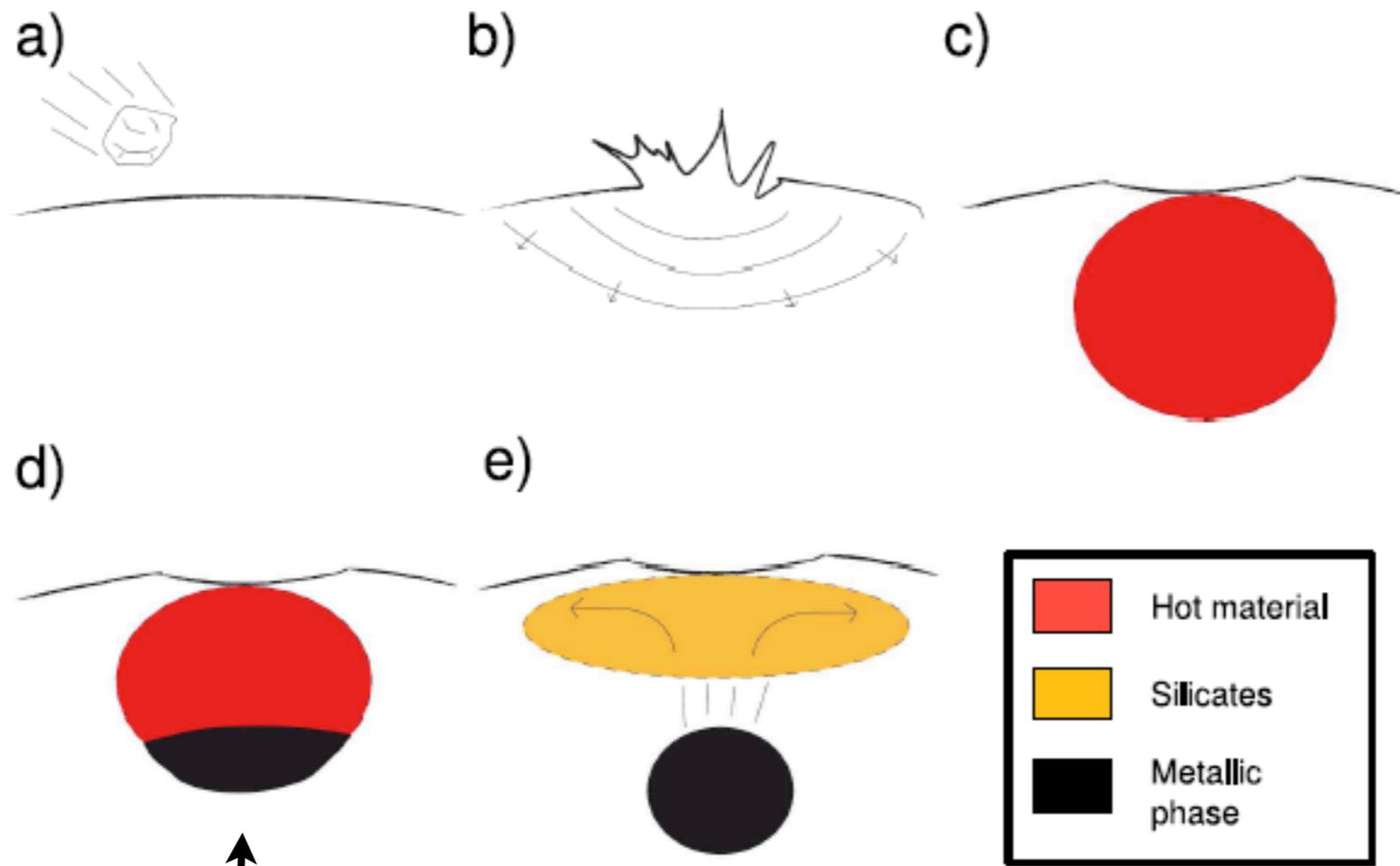
Pour traverser une planete silicatee contenant 20% de metal fondu avec une vitesse de Darcy, (quelle que soit sa taille)

il faudrait quelques milliers d'annees

Diapirs de fer



Avant de fondre les silicates ($R \sim 4000$ km) on avait déjà fondu le metal ($R \sim 2000$ km)



Hypothese
de depart

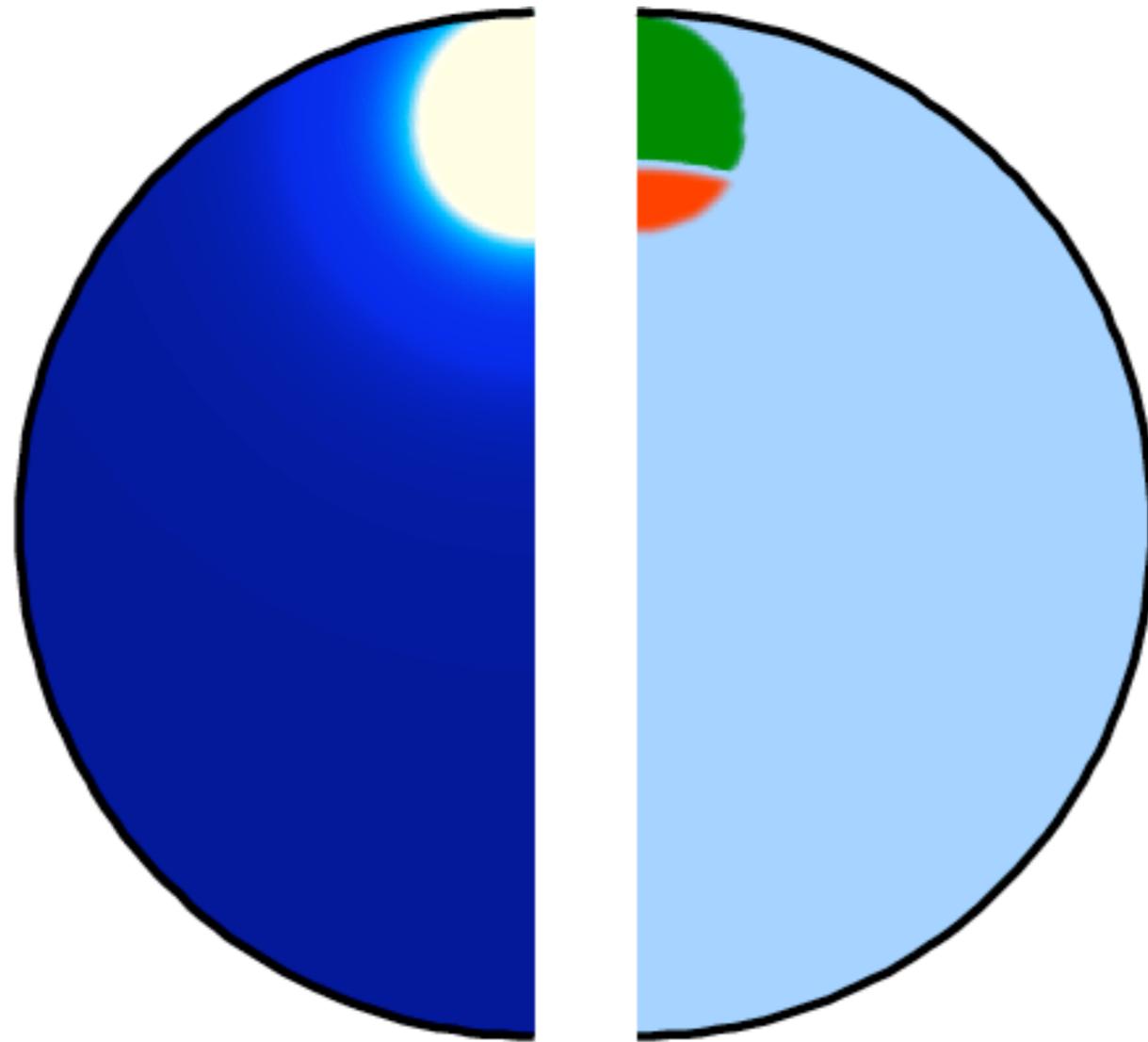
Code spherique,
axisymmetrique,
mono phase, sans changement de phase, non
Boussinesq

Temps de chute (cf loi de Stokes)

$$\tau_S = \frac{\eta_S R}{c_1 \Delta \rho_0 g_0} \frac{1}{(1 - f_0)} \frac{1}{R_{Fe}^2}$$

Viscosite ?? meme si j'utilise 10^{21} Pa s, comme dans le manteau terrestre, des diapirs ~ 10 km traversent aussi en quelques milliers d'annees (et compte tenu des enormes deviateurs de contraintes probablement bien moins)

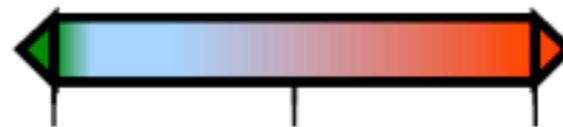
J .Monteux



0 1 2 3 4

T

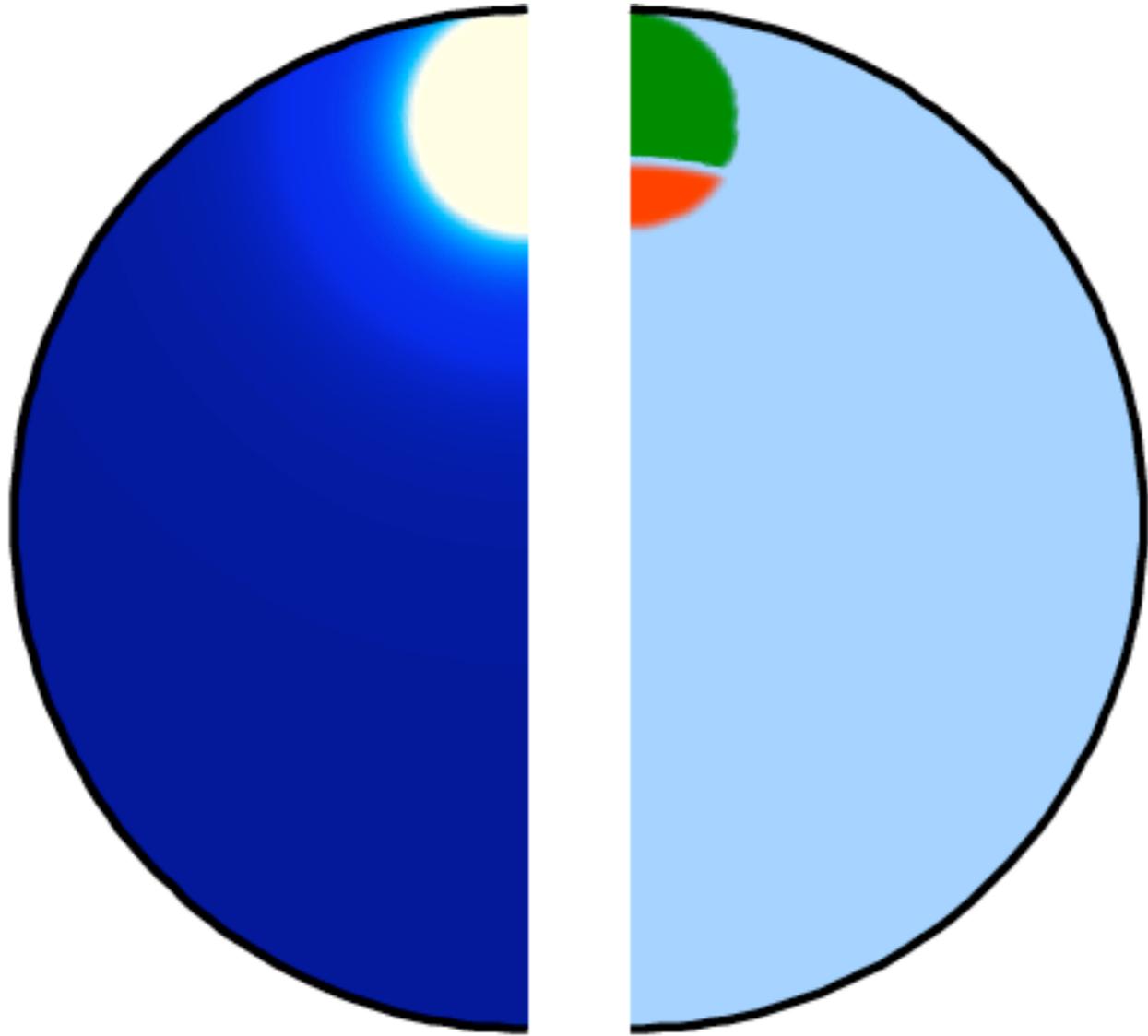
Temperature



0.0 0.5 1.0

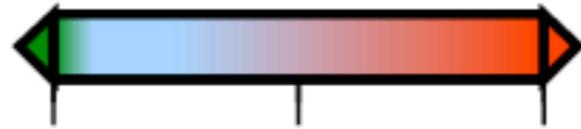
f

Chimie



0 1 2 3 4

T



0.0 0.5 1.0

f

Thermal evolution of a diapir

$$\frac{d\Delta T}{dt} = -\frac{\Delta T}{\tau_D} + 2a \frac{\Delta\Theta}{\tau_S} \exp\left(-2\frac{t}{\tau_S}\right),$$

Diffusion time

$$\tau_D \propto R_{Fe}^2$$

Stokes time

$$\tau_S \propto 1/R_{Fe}^2$$

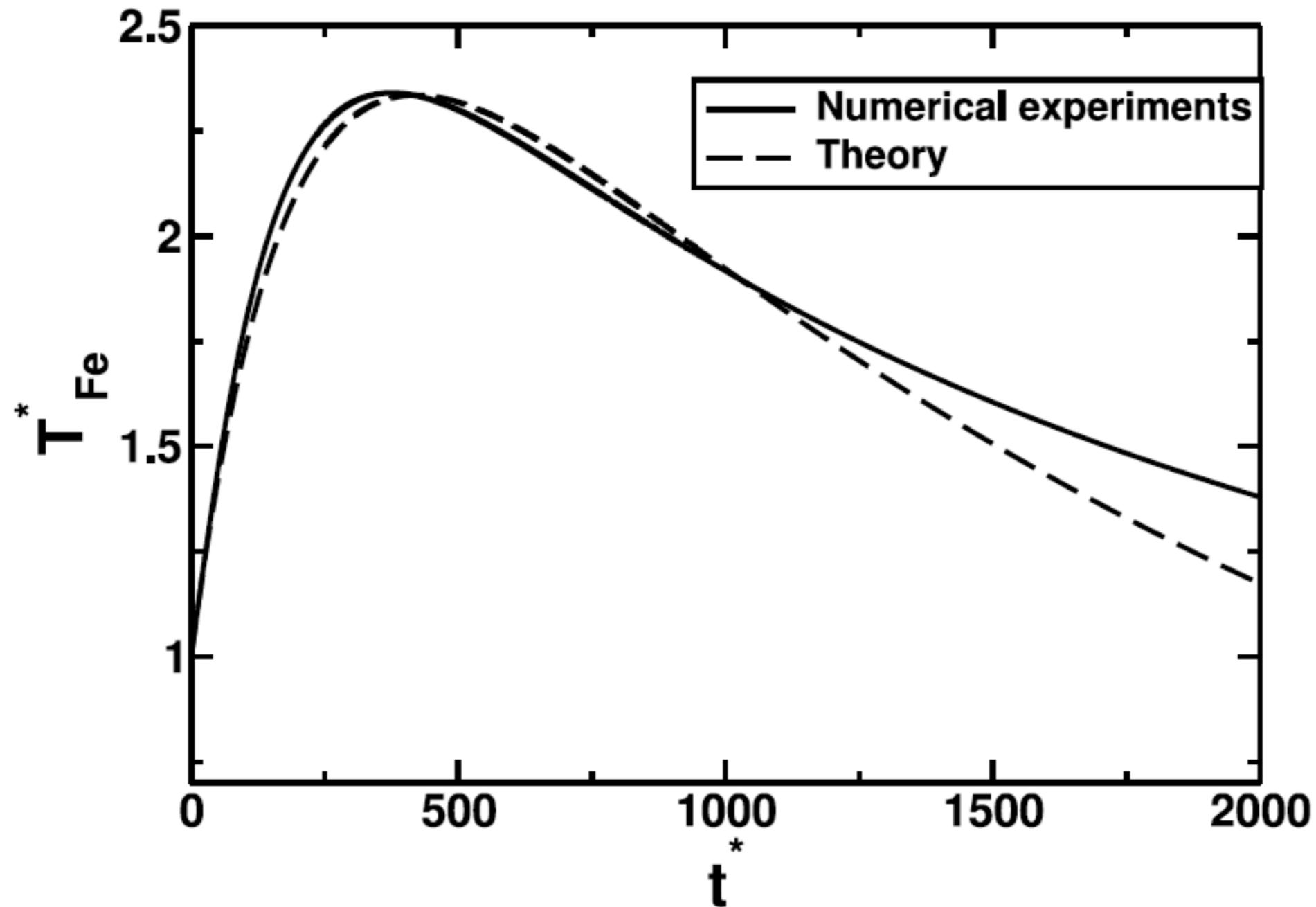
Potential energy release

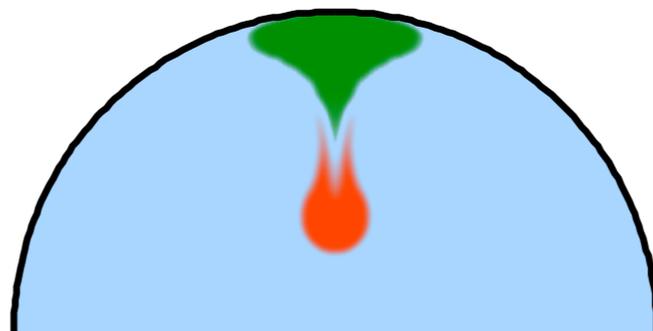
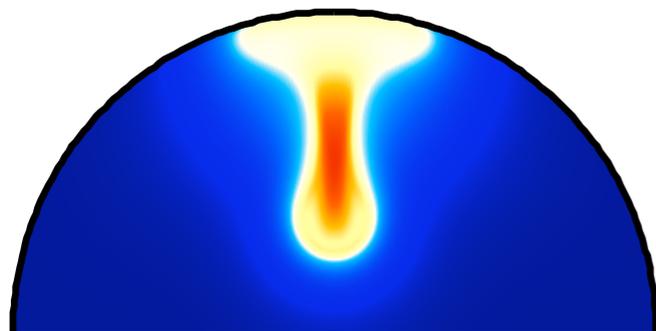
$$\Delta\Theta = \frac{1-f_0}{2} \frac{1}{\rho C_p} \Delta\rho_0 g_0 R.$$

Percentage in metal

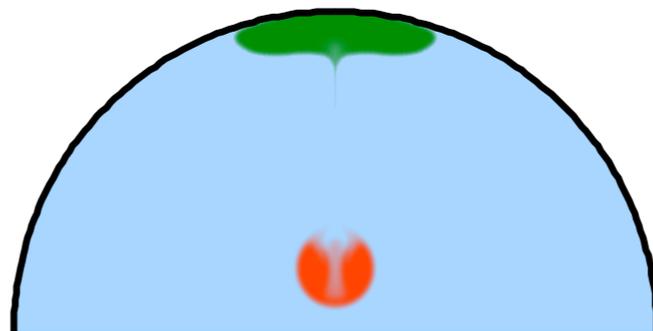
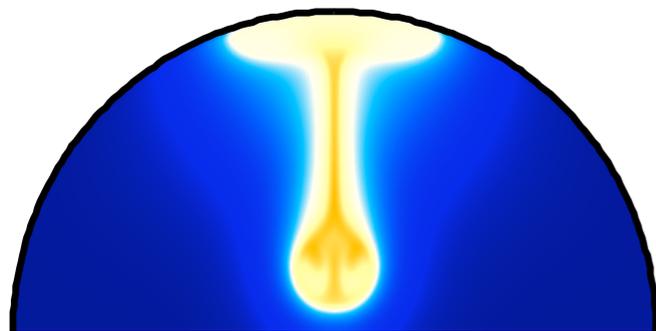
$$a \leq 0.2$$

Les petits diapirs refroidissent, les gros chauffent jusqu'à multiplier leur excès de température par ~ 3

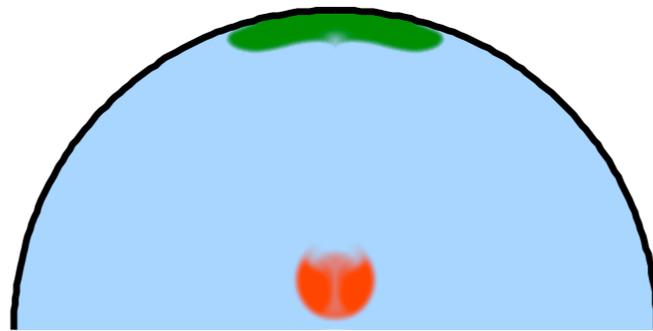
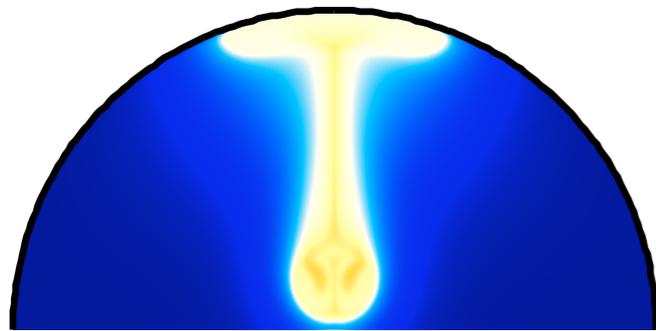




Isoviscous



Contrast ~ 25



Contrast ~ 1000



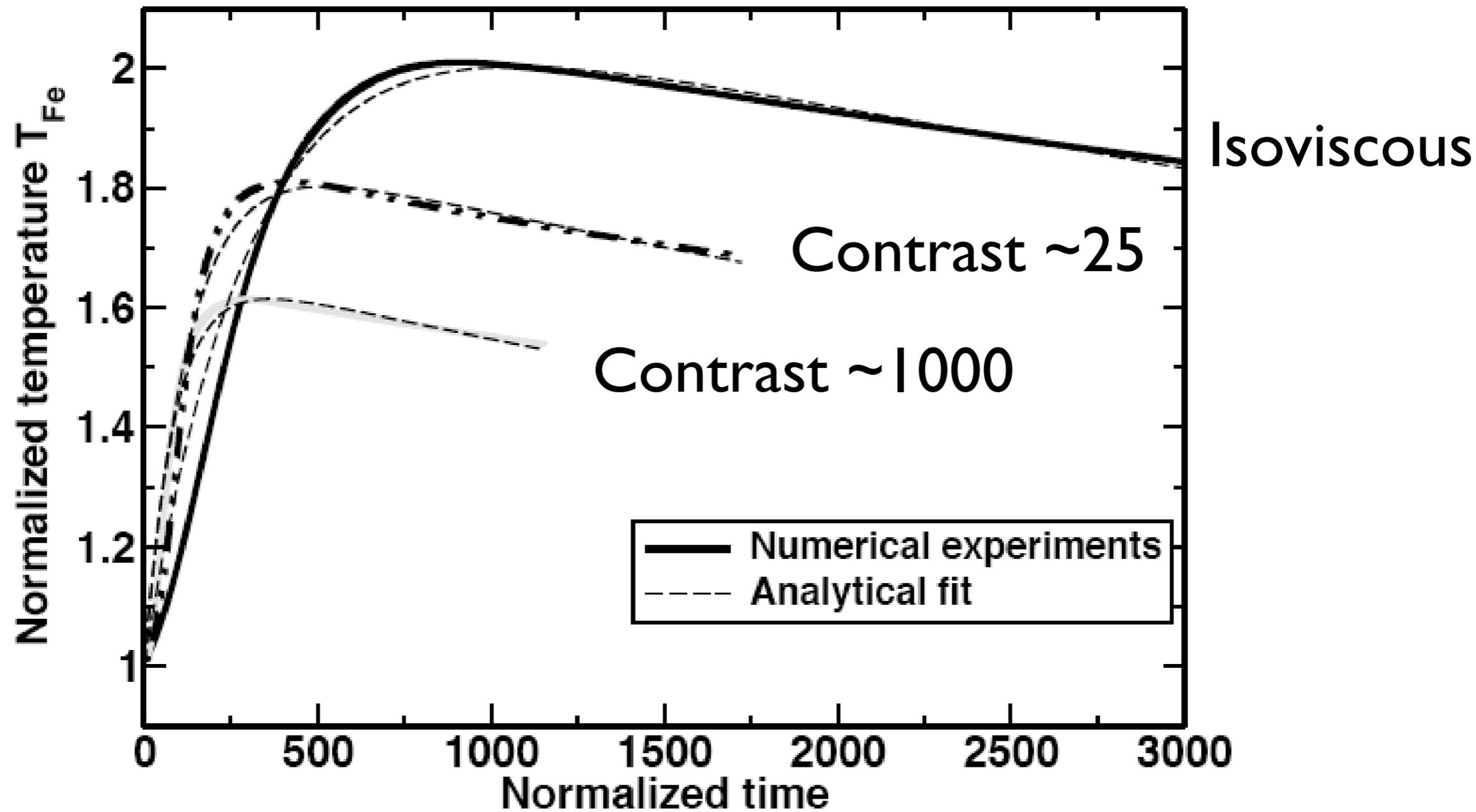
0 1 2 3 4

T



0.0 0.5 1.0

f



Modeles multiphases

Mais si il y a en general 4 phases en jeu (silicates solides, silicates solides, fer liquide, fer solide) passons aux simulations multiphasees

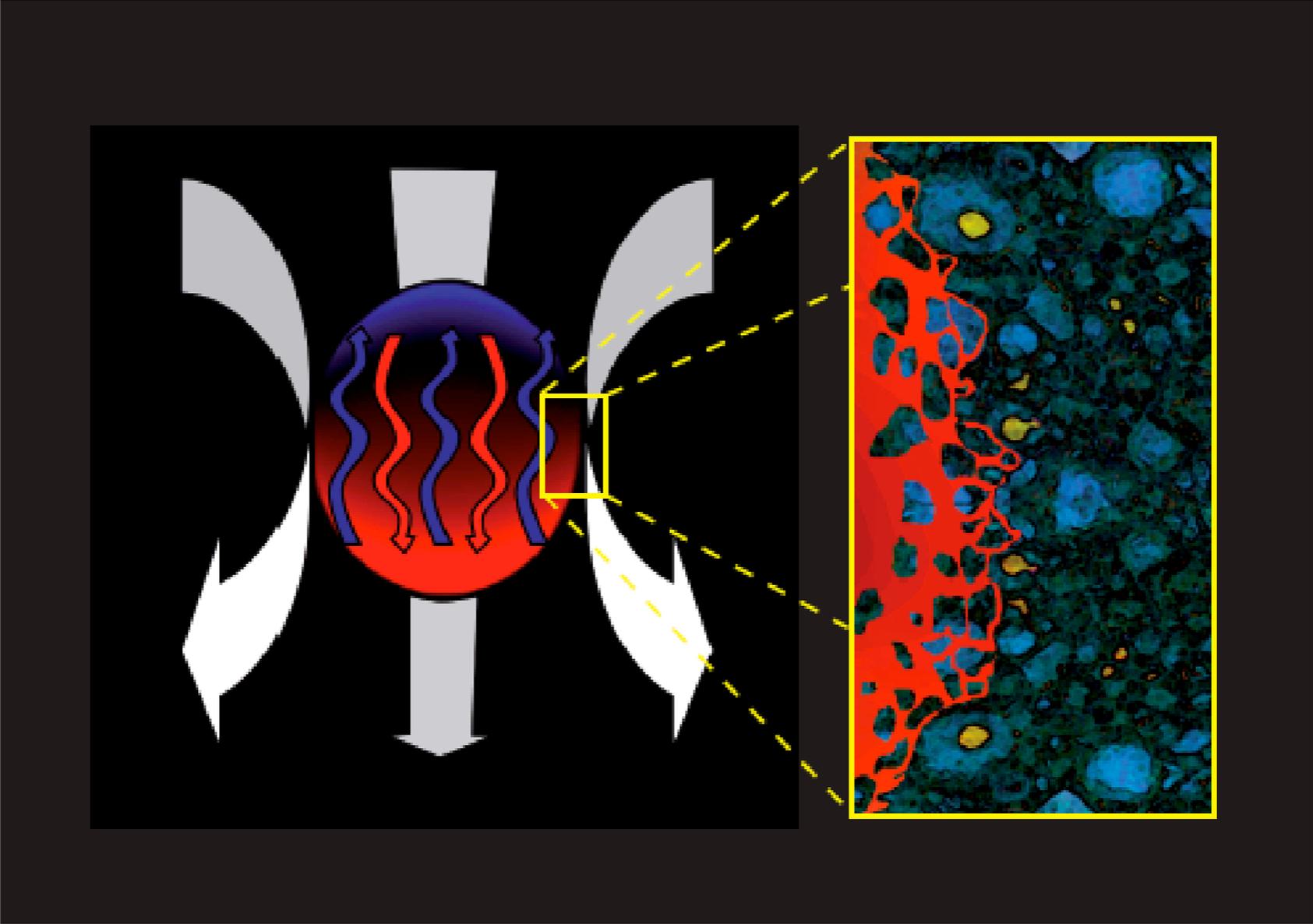
Soit a 3D, 12 conservations du moment, 1 de l'energie et 4 conservations de masse !?!?

Commencons a 2D, 3phases, negligions la chaleur latente, et on peut s'en sortir avec $(4+3+1)$ equations

Ultrabref tutorial

En chaque point les vitesses chaque composant est present en proportion ϕ pour le metal $1 - \phi$ pour les silicates. Si la temperature est au dessus de celle de fusion du fer, le fer est liquide et peut s'écouler a travers la matrice suivant une loi de Darcy, tandis que l'ensemble du melange s'écoule avec une rheologie $\eta(\phi)$

Si la temperature est au dessous de celle de fusion du fer il n'y a pas de separation de phases et le melange solide s'écoule avec une rheologie η_0



Mass conservation 

$$\nabla \cdot (\phi \mathbf{v}_f + (1 - \phi) \mathbf{v}_m) = 0,$$

Total momentum 

$$-\nabla \Pi + \nabla \cdot [\mu^* \underline{\boldsymbol{\tau}}_m] + \phi \hat{\mathbf{g}} = 0,$$

Phase separation



$$\phi \Delta \mathbf{v} = \delta^2 \phi^2 \left(\nabla \left[\Pi + \frac{1 - \phi}{\phi} \nabla \cdot (\phi \Delta \mathbf{v}) \right] - \hat{\mathbf{g}} \right),$$

Porosity change 

$$\frac{D\phi}{Dt} = \nabla \cdot [\phi(1 - \phi) \Delta \mathbf{v}].$$

Temp. change  $\frac{DT}{Dt} = \frac{1}{Ra} \nabla^2 T + \frac{1 - \phi}{\phi} (\nabla \cdot \mathbf{v}_m)^2 + \frac{(\Delta \mathbf{v})^2}{L^2} + \mu^* \nabla \mathbf{v}_m : \underline{\boldsymbol{\tau}}_m$

Where the local proportion of metal is

ϕ

Where the Rayleigh number is

$$Ra = \frac{\overline{\rho C} g R^3 \Delta \rho}{k \mu_m},$$

Where the compaction length and the viscosity are

$$\delta^2 = \frac{\mu_m}{c R^2} \quad \mu^* = 1$$

above the iron melting temperature, and

$$\delta^2 = 0 \quad \mu^* = (1 - \phi)$$

below

2D Cartesian code

Boundary conditions:

Free slip + thermal isolation

Initial conditions:

Uniform undifferentiated planet with 25% iron

Uniform temperature ~ 100 K below iron melting temperature except in a circular zone near the surface where it reaches ~ 100 K above iron melting temp.

Solid viscosity 10^{19} Pa s, $Ra=10^6$

Normalized compaction length 0.1

Stokes velocity 200 m/yr Darcy velocity 200 m/yr

Segregation of an undifferentiated planet

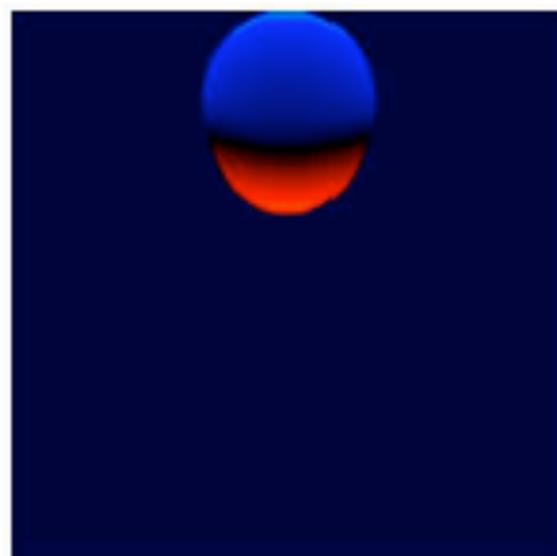
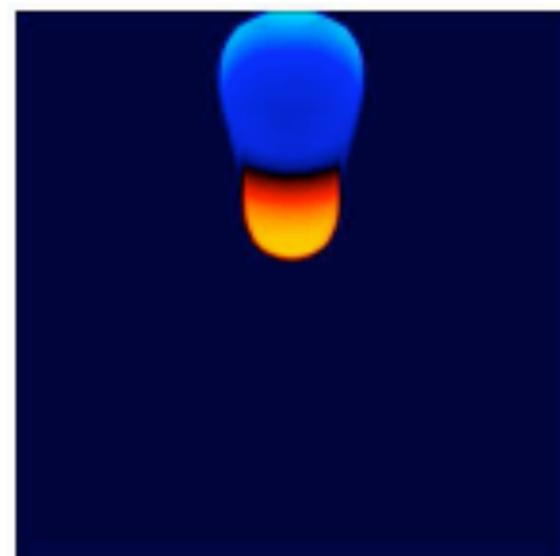
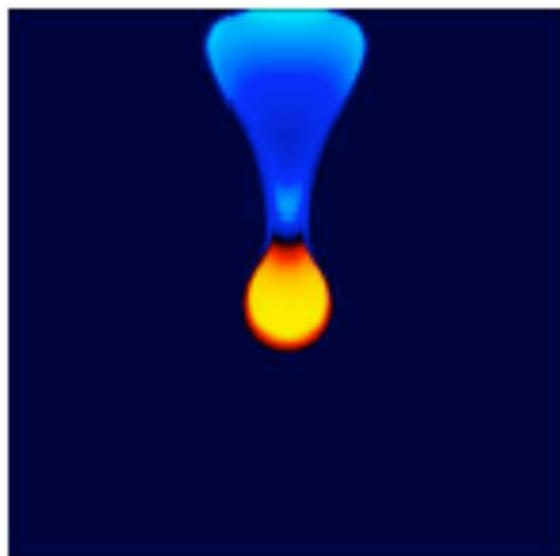
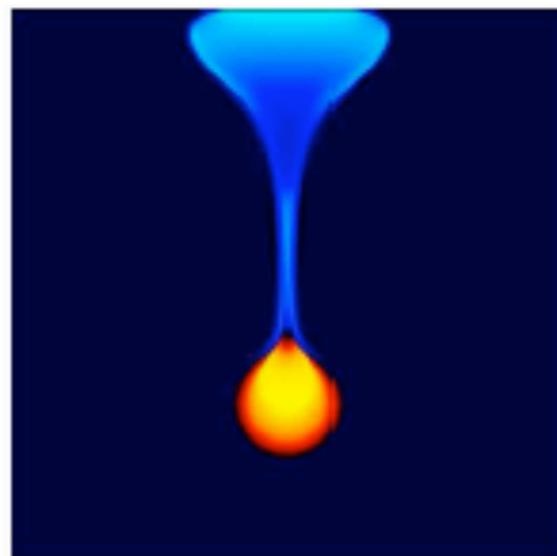
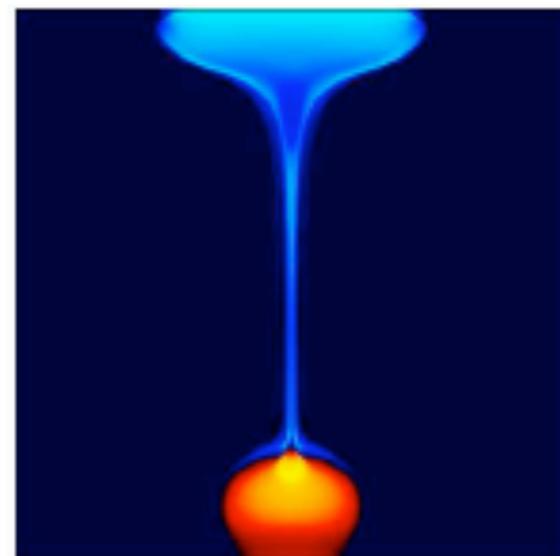
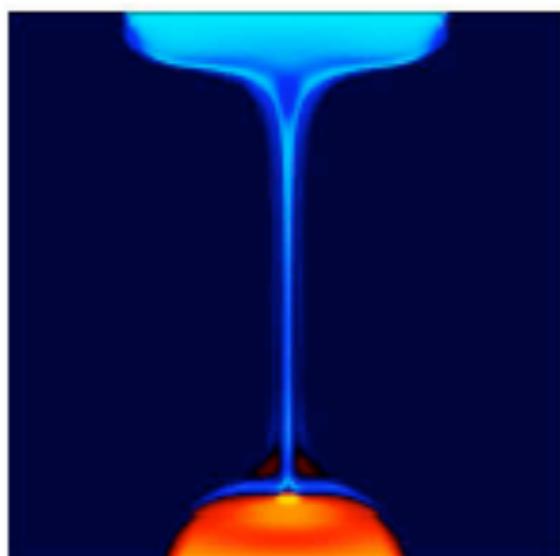
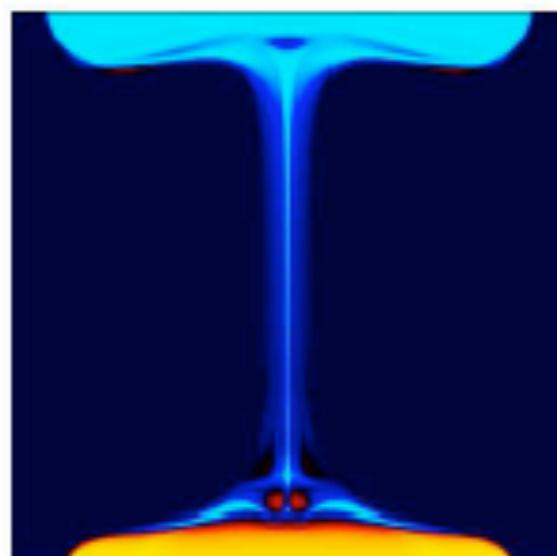
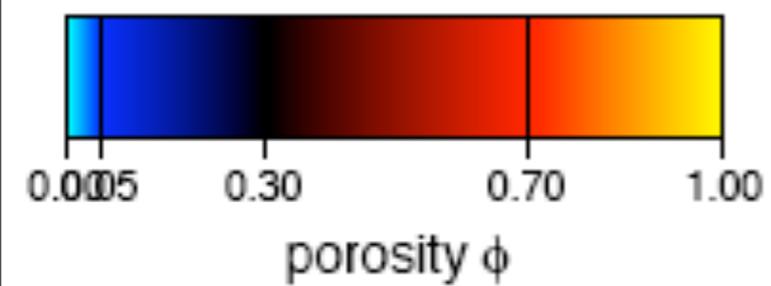
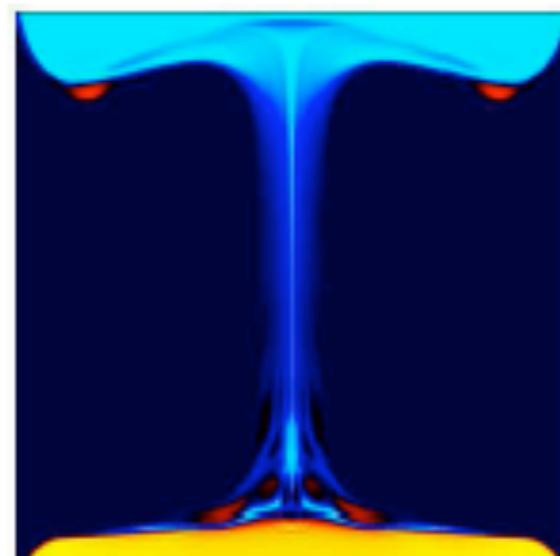
- **Black** = solid iron + solid silicates
- **Red-Yellow** = mostly liquid iron
- **Blue** = mostly liquid silicates

Segregation of an undifferentiated planet

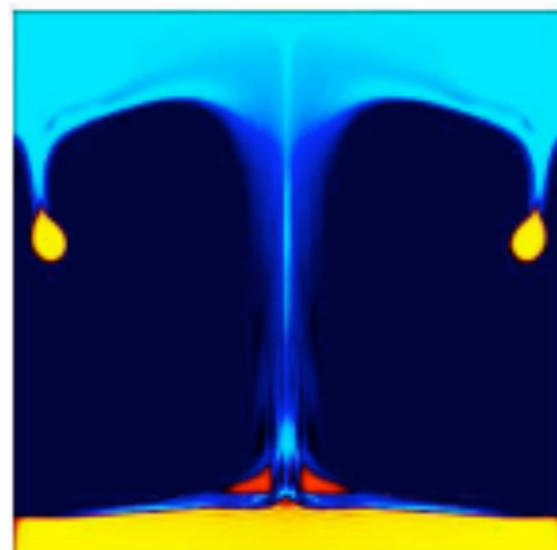
- **Black** = solid iron + solid silicates
- **Red-Yellow** = mostly liquid iron
- **Blue** = mostly liquid silicates

Que c'est il passe?

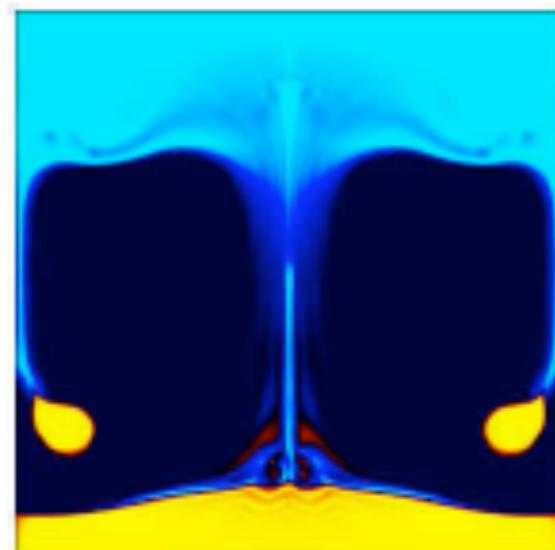
De l'énergie gravitationnelle était stockée correspondant à quelques centaines de degrés, c'est aussi l'excès de température délivré par l'impact qui a été nécessaire pour initier la fusion partielle, d'où une instabilité catastrophique

$t = 0$  $t = 75$  $t = 150$  $t = 250$  $t = 350$  $t = 475$  $t = 575$  $t = 975$  $t = 1300$ 

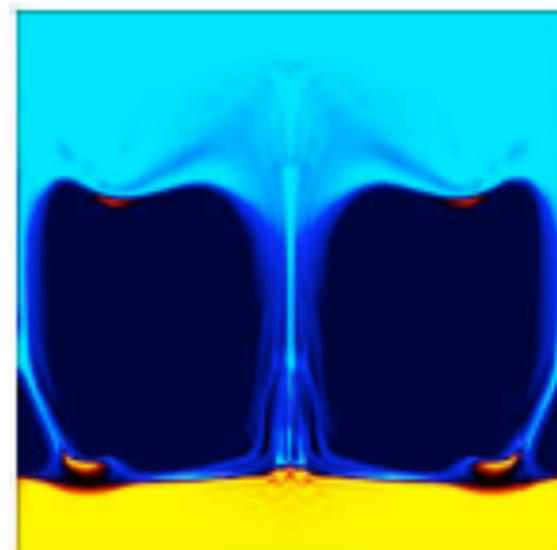
$t = 1950$



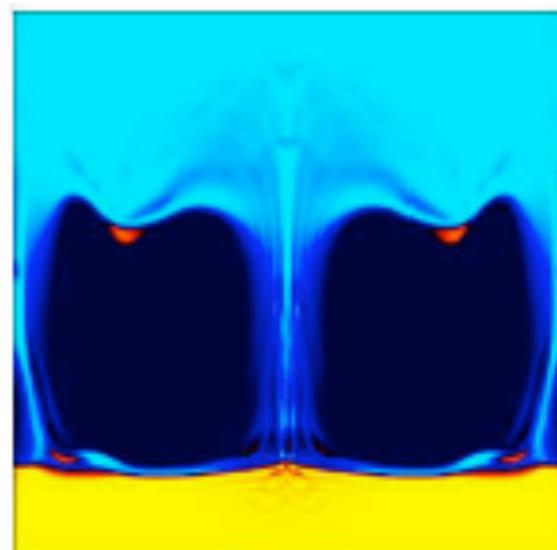
$t = 2450$



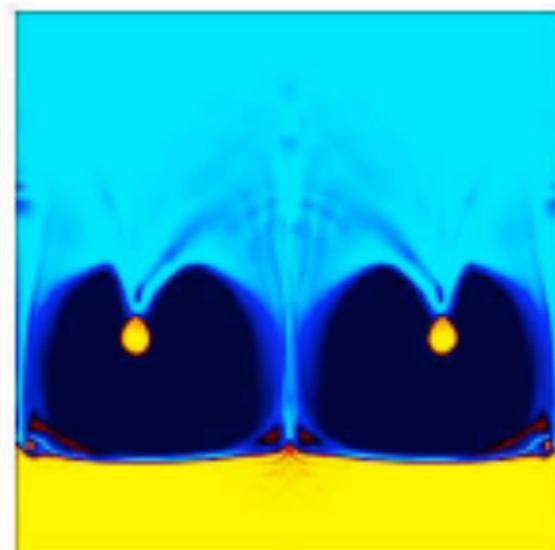
$t = 2825$



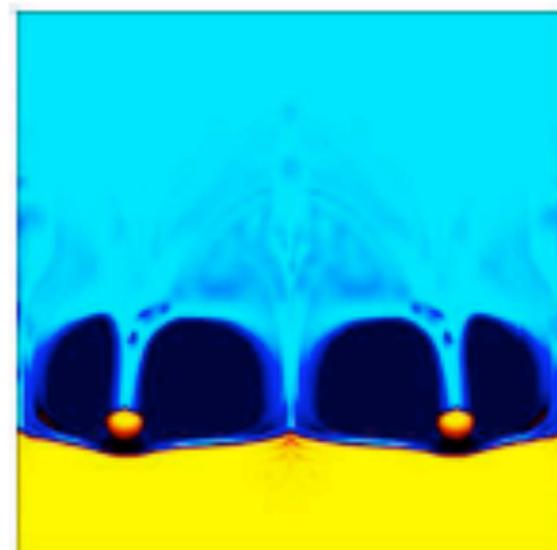
$t = 3075$



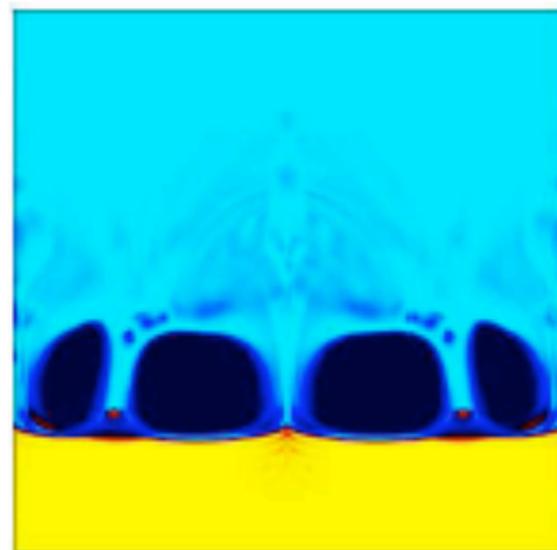
$t = 3575$



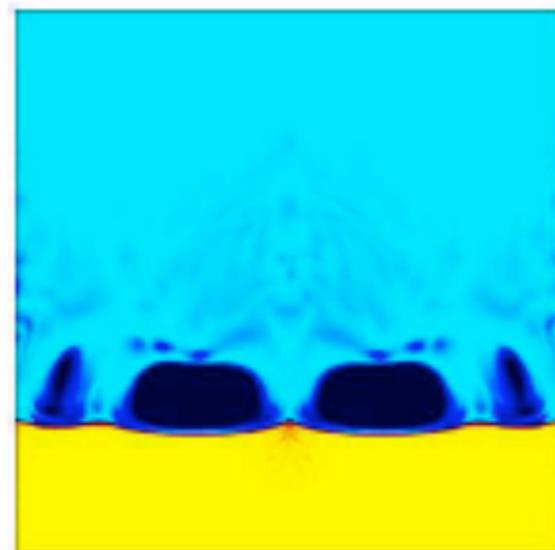
$t = 4050$



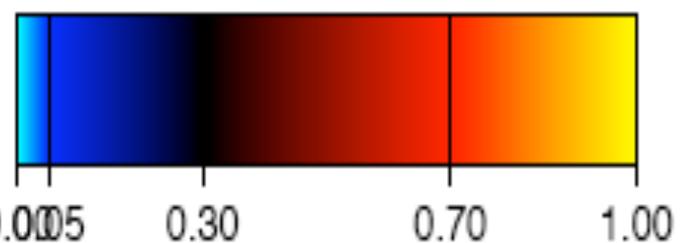
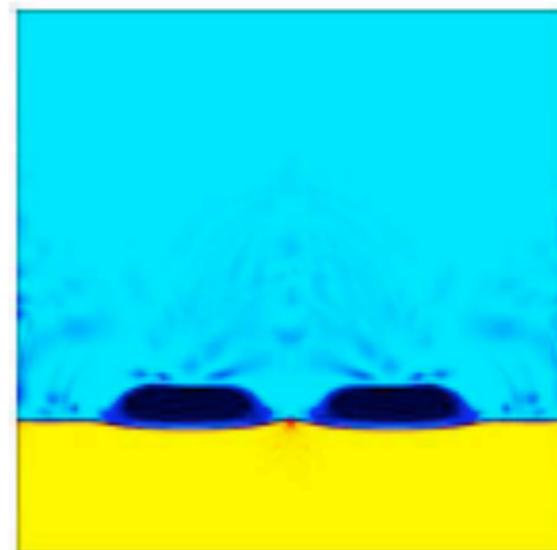
$t = 4225$



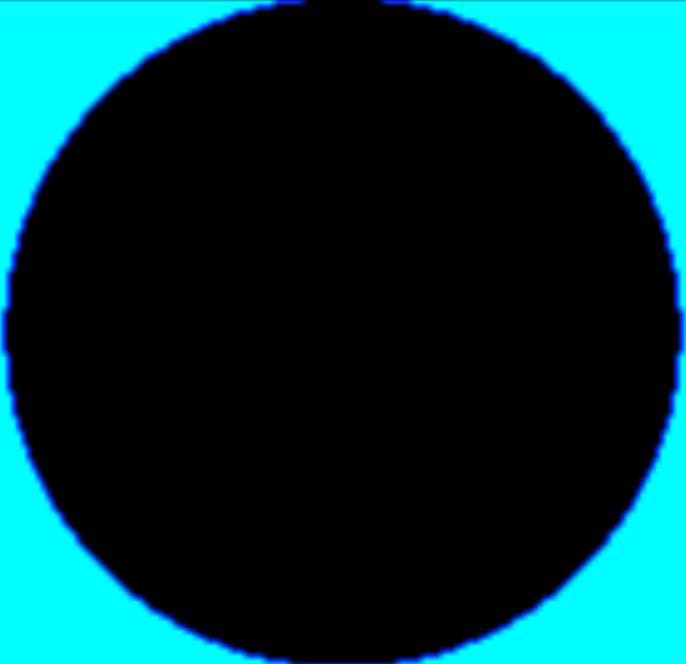
$t = 4675$

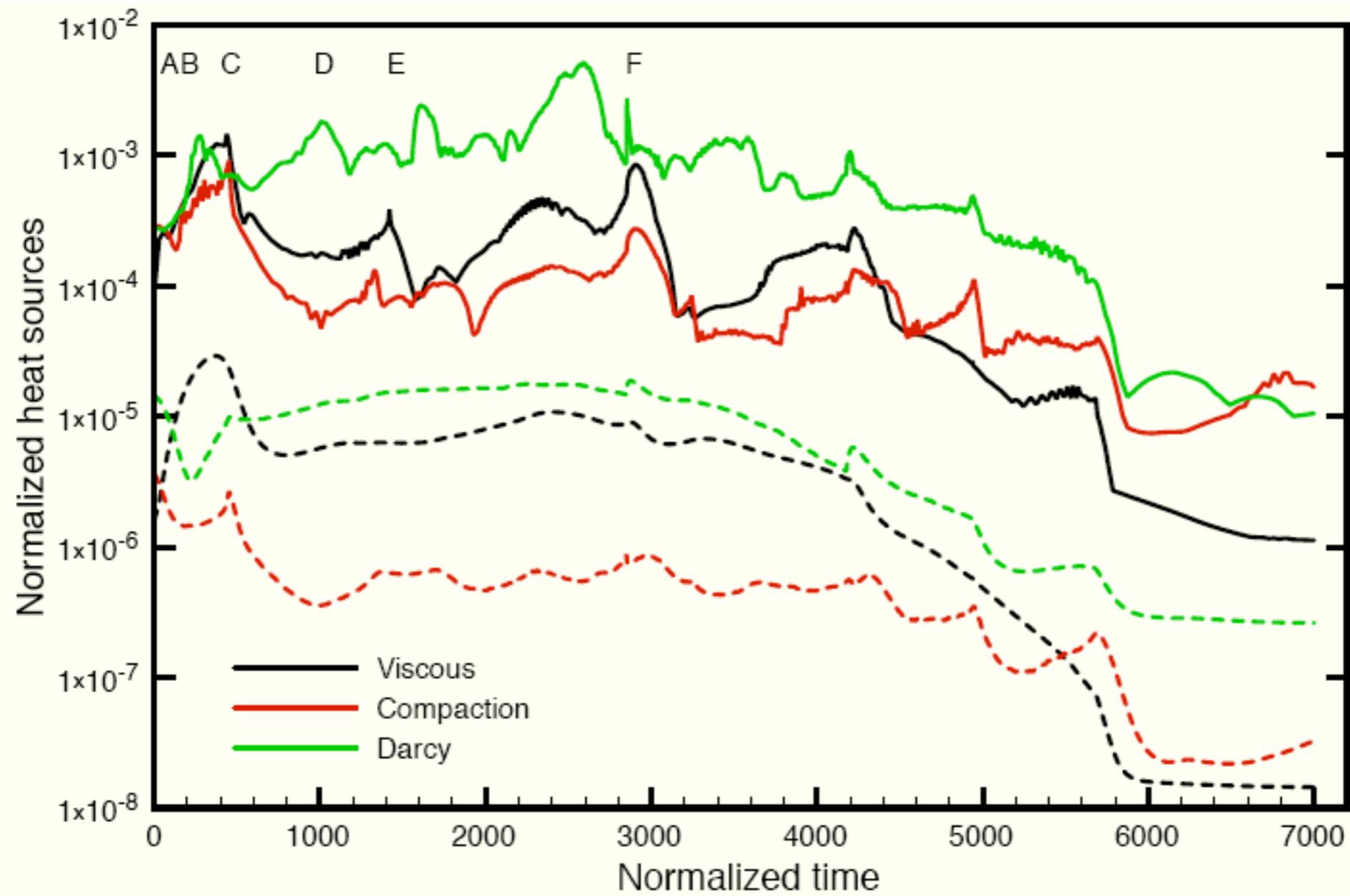


$t = 5225$



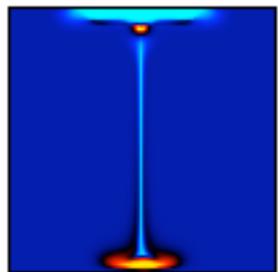
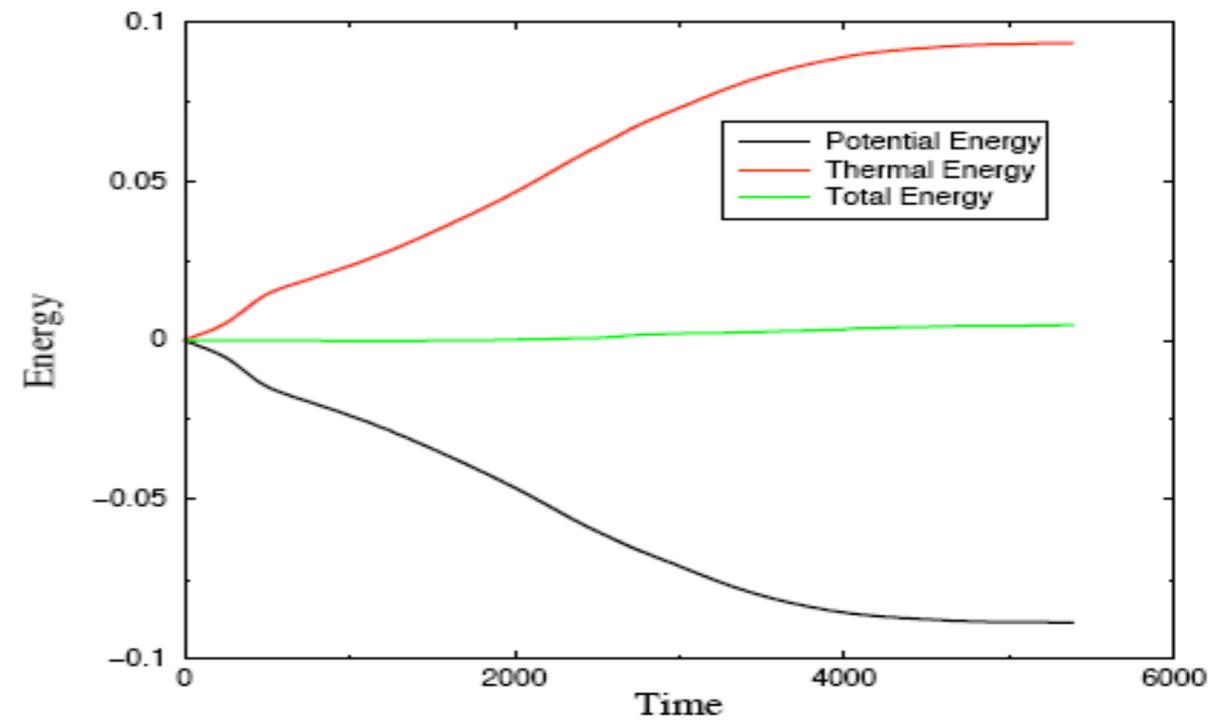
porosity ϕ



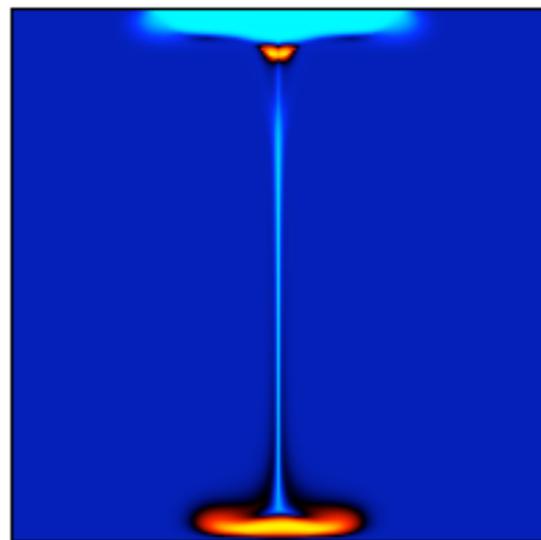


Numerical test

$$\frac{d}{dt} \int_{\tilde{V}} (T - \phi z) dV = Q$$

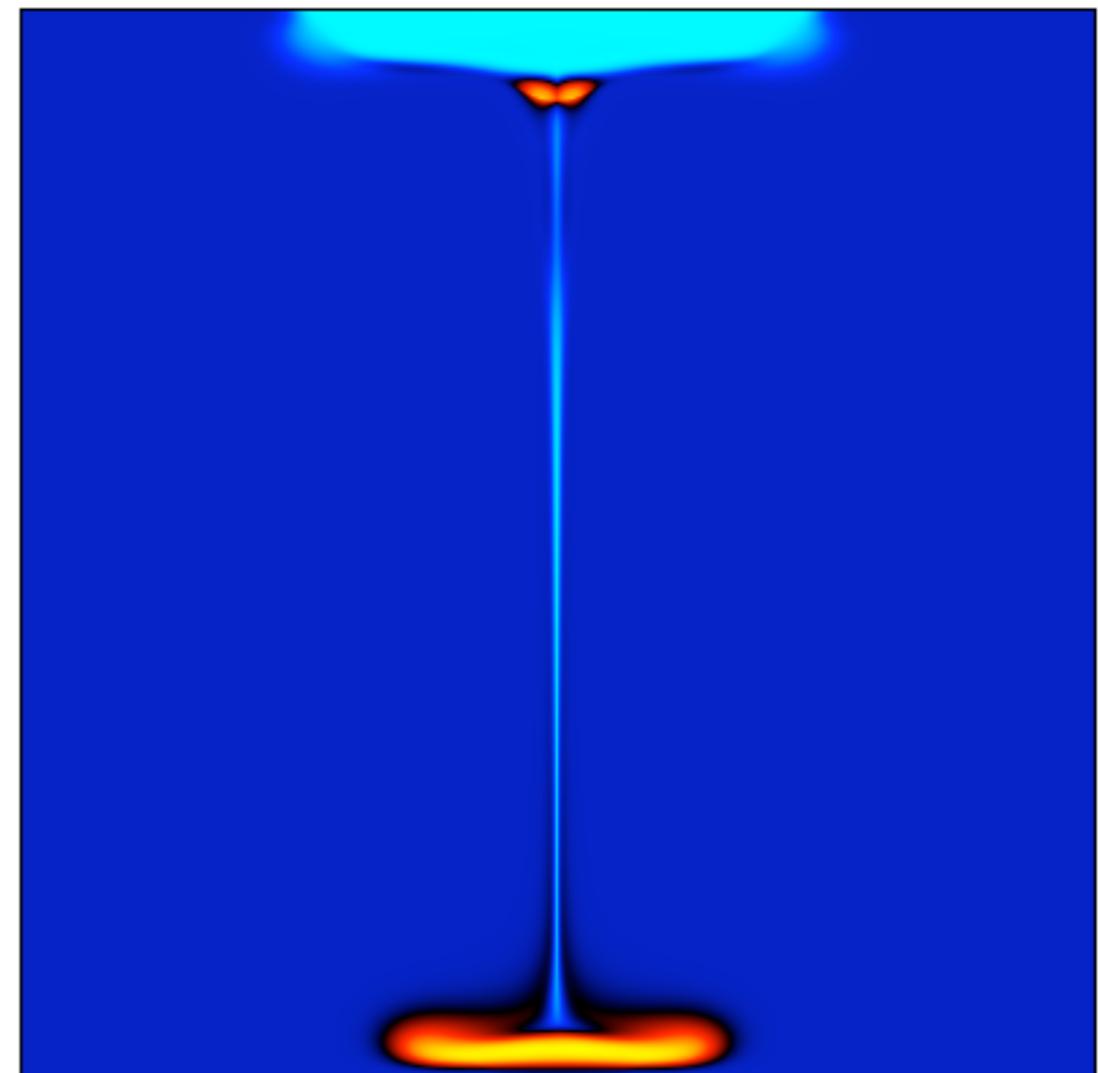


129x129

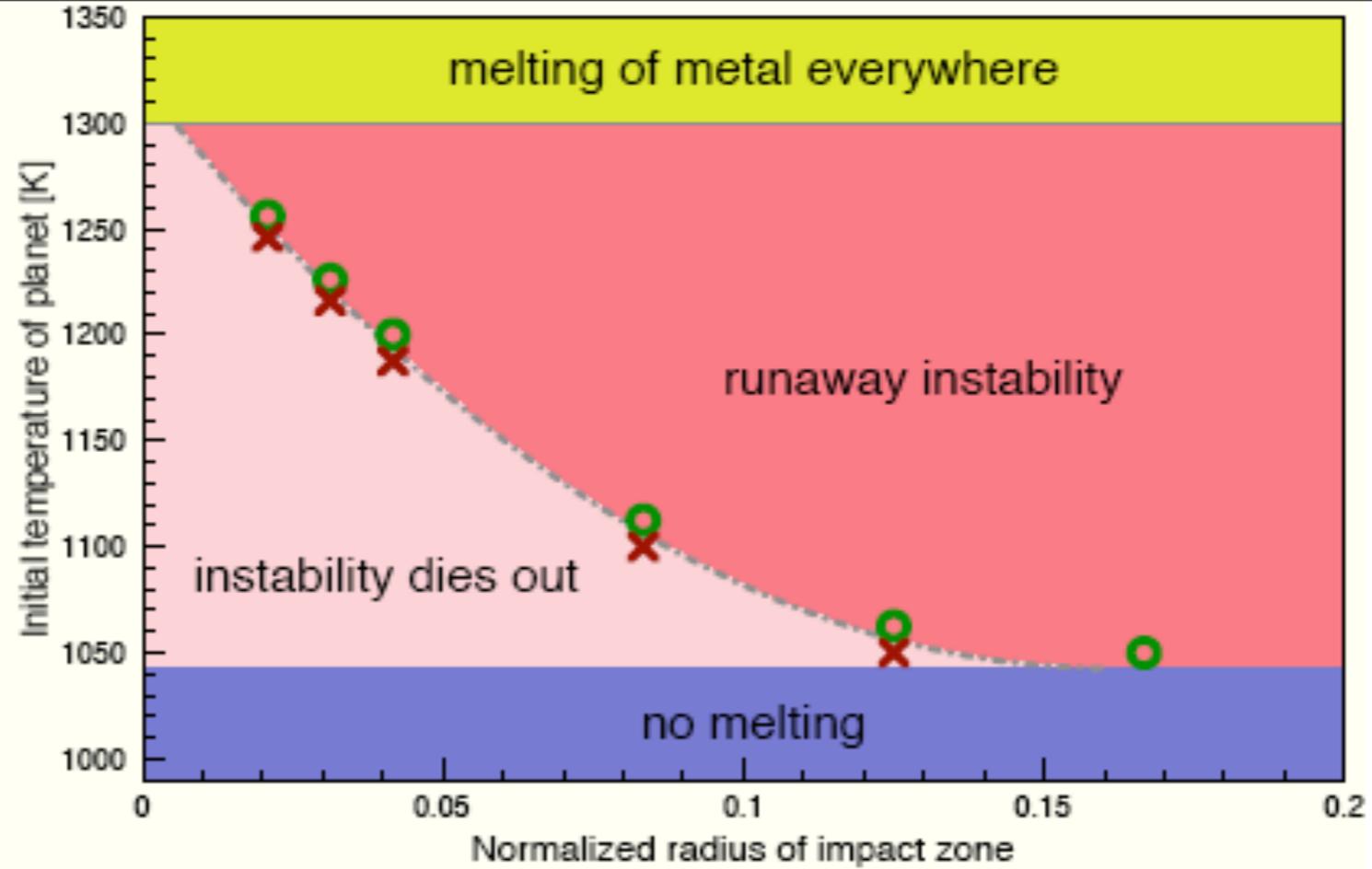


257x257

513x513

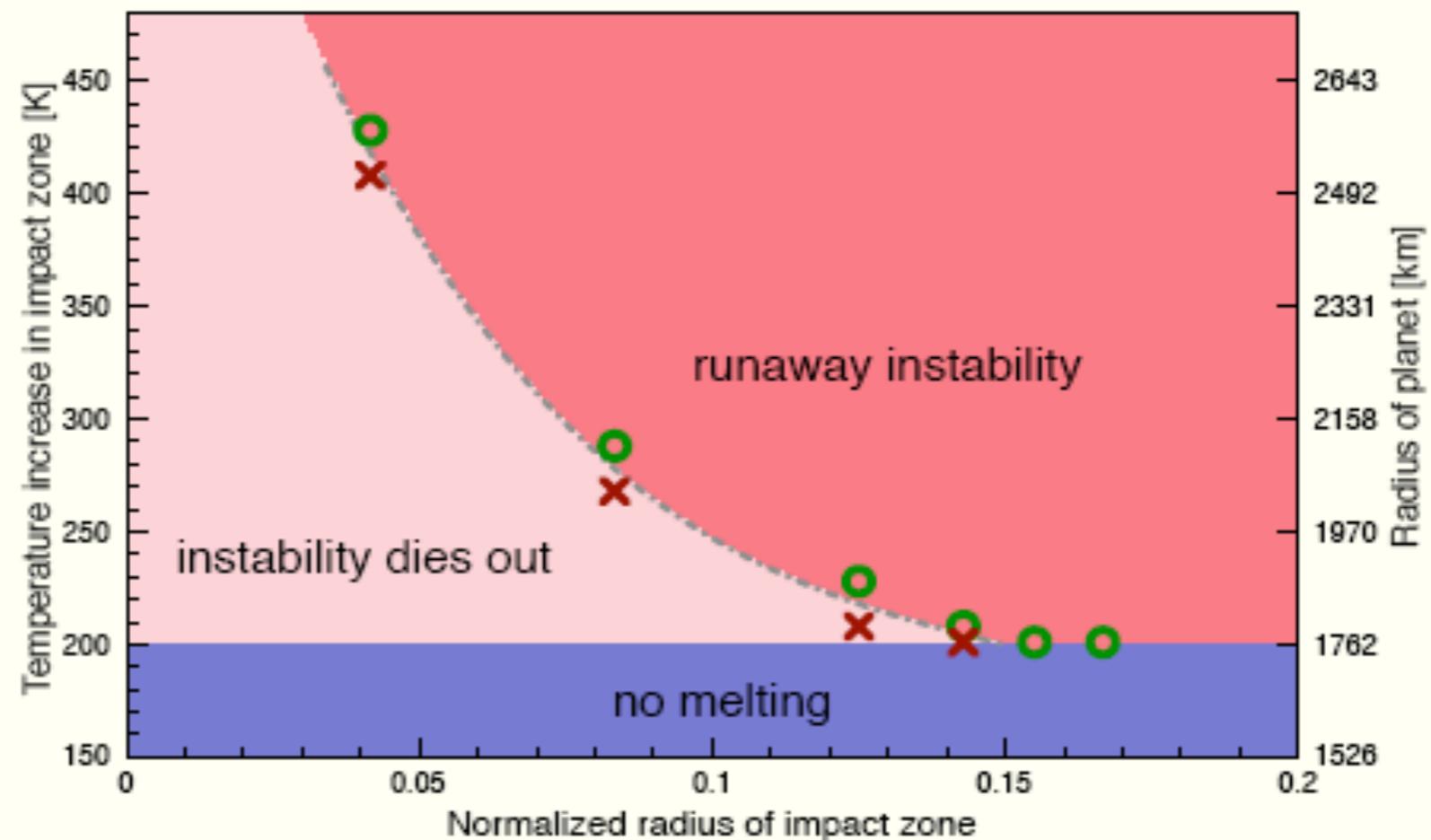


A given planetary radius...

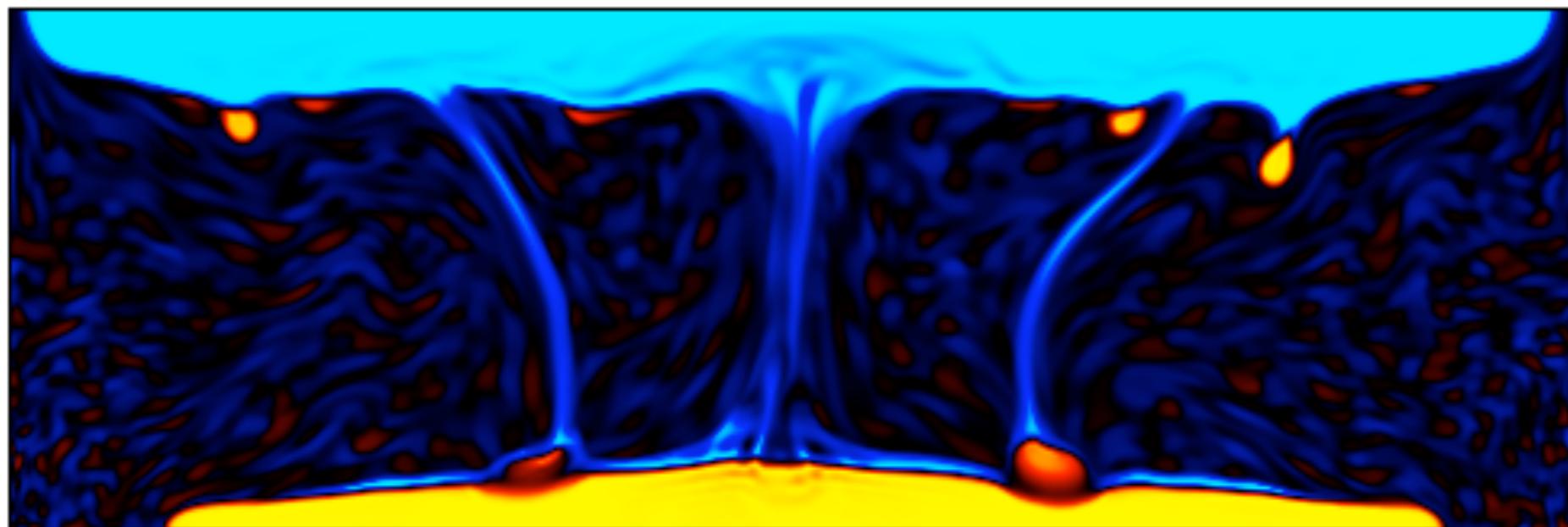
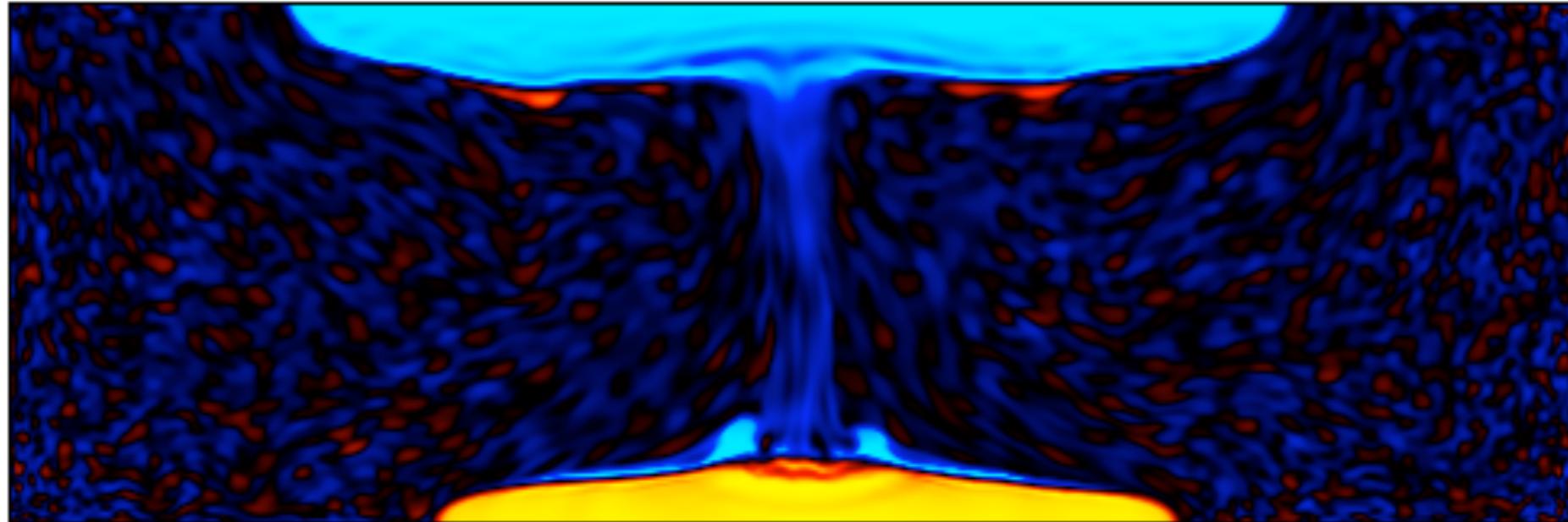


A given initial temperature

...



In rectangular box and with
random initial composition



Je ne sais pas fabriquer un embryon de planète non différenciée plus gros que $R \sim 1500$ km, car il contient trop d'énergie potentielle qui ne demande qu'à se libérer

Les noyaux de ces embryons (1-2% de celui de la Terre actuelle) sont formés à basses pressions et assez basses températures (2000 K)

Les collisions entre ces objets conduisent rapidement à des températures extrêmement élevées et des réactions en phase vapeur.

Les codes de type SPH peuvent peut-être rendre le relais mais ils sont encore loin de pouvoir simuler les résolutions qui intéressent les chimistes.

